PyLops

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This Python library is inspired by the MATLAB Spot – A Linear-Operator Toolbox project.

Linear operators and inverse problems are at the core of many of the most used algorithms in signal processing, image processing, and remote sensing. When dealing with small-scale problems, the Python numerical scientific libraries numpy and scipy allow to perform most of the underlying matrix operations (e.g., computation of matrix-vector products and manipulation of matrices) in a simple and expressive way.

Many useful operators, however, do not lend themselves to an explicit matrix representation when used to solve large-scale problems. PyLops operators, on the other hand, still represent a matrix and can be treated in a similar way, but do not rely on the explicit creation of a dense (or sparse) matrix itself. Conversely, the forward and adjoint operators are represented by small pieces of codes that mimic the effect of the matrix on a vector or another matrix.

Luckily, many iterative methods (e.g. cg, lsqr) do not need to know the individual entries of a matrix to solve a linear system. Such solvers only require the computation of forward and adjoint matrix-vector products as done for any of the PyLops operators.

Here is a simple example showing how a dense first-order first derivative operator can be created, applied and inverted using numpy/scipy commands:

```
import numpy as np
from scipy.linalg import lstsq

nx = 7
x = np.arange(nx) - (nx-1)/2

D = np.diag(0.5*np.ones(nx-1), k=1) - \
    np.diag(0.5*np.ones(nx-1), k=-1)
D[0] = D[-1] = 0 # take away edge effects

# y = Dx
y = np.dot(D, x)
# x = D'y
xadj = np.dot(D.T, y)
# xinv = D^-1 y
xinv = lstsq(D, y)[0]
```

and similarly using PyLops commands:

```
from pylops import FirstDerivative

Dlop = FirstDerivative(nx, dtype='float64')

# y = Dx
y = Dlop*x
# x = D'y
xadj = Dlop.H*y
# xinv = D^-1 y
xinv = Dlop / y
```

Note how this second approach does not require creating a dense matrix, reducing both the memory load and the computational cost of applying a derivative to an input vector $\mathbf{x}$. Moreover, the code becomes even more compact and espressive than in the previous case letting the user focus on the formulation of equations of the forward problem to be solved by inversion.

CHAPTER 1

Terminology

A common *terminology* is used within the entire documentation of PyLops. Every linear operator and its application to a model will be referred to as **forward model (or operation)**

$$\mathbf{y} = \mathbf{A}\mathbf{x}$$

while its application to a data is referred to as **adjoint modelling (or operation)**

$$\mathbf{x} = \mathbf{A}^H\mathbf{y}$$

where $\mathbf{x}$ is called *model* and $\mathbf{y}$ is called *data*. The *operator* $\mathbf{A} : \mathbb{F}^m \to \mathbb{F}^n$ effectively maps a vector of size m in the *model space* to a vector of size n in the *data space*, conversely the * adjoint operator* $\mathbf{A}^H : \mathbb{F}^n \to \mathbb{F}^m$ maps a vector of size n in the *data space* to a vector of size m in the *model space*. As linear operators mimics the effect a matrix on a vector we can also loosely refer to m as the number of *columns* and n as the number of *rows* of the operator.

Ultimately, solving an inverse problems accounts to removing the effect of $\mathbf{A}$ from the data $\mathbf{y}$ to retrieve the model $\mathbf{x}$.

For a more detailed description of the concepts of linear operators, adjoints and inverse problems in general, you can head over to one of Jon Claerbout's books such as Basic Earth Imaging.

CHAPTER 2

Implementation

PyLops is build on top of the scipy class `scipy.sparse.linalg.LinearOperator`.

This class allows in fact for the creation of objects (or interfaces) for matrix-vector and matrix-matrix products that can ultimately be used to solve any inverse problem of the form $\mathbf{y} = \mathbf{A}\mathbf{x}$.

As explained in the scipy LinearOperator official documentation, to construct a `scipy.sparse.linalg.LinearOperator`, a user is required to pass appropriate callables to the constructor of this class, or subclass it. More specifically one of the methods `_matvec` and `_matmat` must be implemented for the *forward operator* and one of the methods `_rmatvec` or `_adjoint` may be implemented to apply the *Hermitian adjoint*. The attributes/properties `shape` (pair of integers) and `dtype` (may be None) must also be provided during `__init__` of this class.

Any linear operator developed within the PyLops library follows this philosophy. As explained more in details in *Implementing new operators* section, a linear operator is created by subclassing the `scipy.sparse.linalg.LinearOperator` class and `_matvec` and `_rmatvec` are implemented.

CHAPTER 3

History

PyLops was initially written and it is currently maintained by Equinor It is a flexible and scalable python library for large-scale optimization with linear operators that can be tailored to our needs, and as contribution to the free software community.

3.1 Installation

The PyLops project stives to create a library that is easy to install in any environment and has a very limited number of dependencies. However, since *Python2* will retire soon, we have decided to only focus on a *Python3* implementation. If you are still using *Python2*, hurry up!

For this reason you will need **Python 3.5 or greater** to get started.

3.1.1 Dependencies

Our mandatory dependencies are limited to:

- numpy
- scipy

We advise using the Anaconda Python distribution to ensure that these dependencies are installed via the `Conda` package manager. This is not just a pure stylistic choice but comes with some *hidden* advantages, such as the linking to `Intel MKL` library (i.e., a highly optimized BLAS library created by Intel).

If you simply want to use PyLops for teaching purposes or for small-scale examples, this should not really affect you. However, if you are interested in getting better code performance, read carefully the *Advanced installation* page.

3.1.2 Optional dependencies

PyLops's optional dependencies refer to those dependencies that we do not include in our `requirements.txt` and `environment.yml` files and thus are not strictly needed nor installed directly as part of a standar installation (see

below for details)

However, we sometimes implement additional back-ends (referred to as `engine` in the code) for some of our operators in order to improve their performance. To do so, we rely on third-party libraries. Those libraries are generally added to the list of our optional dependencies. If you are not after code performance, you may simply stick to the mandatory dependencies and pylops will ensure to always fallback to one of those for any linear operator.

If you are instead after code performance, take a look at the *Optional dependencies* section in the *Advanced installation* page.

3.1.3 Step-by-step installation for users

Python environment

Activate your Python environment, and simply type the following command in your terminal to install the PyPi distribution:

```
>> pip install pylops
```

If using Conda, you can also install our conda-forge distribution via:

```
>> conda install -c conda-forge pylops
```

Note that using the `conda-forge` distribution is reccomended as all the dependencies (both mandatory and optional) will be correctly installed for you, while only mandatory dependencies are installed using the `pip` distribution.

Alternatively, to access the latest source from github:

```
>> pip install https://github.com/equinor/pylops/archive/master.zip
```

or just clone the repository

```
>> git clone https://github.com/equinor/pylops.git
```

or download the zip file from the repository (green button in the top right corner of the main github repo page) and install PyLops from terminal using the command:

```
>> make install
```

Docker

If you want to try PyLops but do not have Python in your local machine, you can use our Docker image instead.

After installing Docker in your computer, type the following command in your terminal (note that this will take some time the first time you type it as you will download and install the docker image):

```
>> docker run -it -v /path/to/local/folder:/home/jupyter/notebook -p 8888:8888␣
↪mrava87/pylops:notebook
```

This will give you an address that you can put in your browser and will open a jupyter-notebook enviroment with PyLops and other basic Python libraries installed. Here */path/to/local/folder* is the absolute path of a local folder on your computer where you will create a notebook (or containing notebooks that you want to continue working on). Note that anything you do to the notebook(s) will be saved in your local folder.

A larger image with Conda distribution is also available. Simply use *conda_notebook* instead of *notebook* in the previous command.

3.1.4 Step-by-step installation for developers

Fork and clone the repository by executing the following in your terminal:

```
>> git clone https://github.com/your_name_here/pylops.git
```

The first time you clone the repository run the following command:

```
>> make dev-install
```

If you prefer to build a new Conda enviroment just for PyLops, run the following command:

```
>> make dev-install_conda
```

To ensure that everything has been setup correctly, run tests:

```
>> make tests
```

Make sure no tests fail, this guarantees that the installation has been successfull.

If using Conda environment, always remember to activate the conda environment every time you open a new *bash* shell by typing:

```
>> source activate pylops
```

3.2 Advanced installation

In this section we discuss some *important details* regarding code performance when using PyLops.

To get the most out of PyLops operators in terms of speed you will need to follow these guidelines as much as possible or ensure that the Python libraries used by PyLops are efficiently installed (e.g., allow multithreading) in your systemt.

3.2.1 Dependencies

PyLops relies on the numpy and scipy libraries and being able to link these to the most performant BLAS will ensure optimal performance of PyLops when using only *required dependencies*.

As already mentioned in the *Installation* page, we strongly encourage using the Anaconda Python distribution as *numpy* and *scipy* will be automatically linked to the `Intel MKL` library, which is per today the most performant library for basic linear algebra operations (if you don't believe it, take a read at this blog post).

The best way to understand which `BLAS` library is currently linked to your *numpy* and *scipy* libraries is to run the following commands in ipython:

```
import numpy as np
import scipy as sp
print(np.__config__.show())
print(scipy.__config__.show())
```

You should be able to understand if your *numpy* and *scipy* are linked to `Intel MKL` or something else.

Note: Unfortunately, PyLops is so far only shipped with PyPI, meaning that if you have not already installed *numpy* and *scipy* in your environment they will be installed as part of the installation process of the *pylops* library, all of those using `pip`. This comes with the disadvantage that *numpy* and *scipy* are linked to `OpenBlas` instead of `Intel MKL`, leading to a loss of performance. To prevent this, we suggest the following strategy:

- create conda environment, e.g. `conda create -n envname python=3.6.4 numpy scipy`
- install pylops using `pip install pylops`

Finally, it is always important to make sure that your environment variable `OMP_NUM_THREADS` is correctly set to the maximum number of threads you would like to use in your code. If that is not the case *numpy* and *scipy* will underutilize your hardware even if linked to a performant `BLAS` library.

For example, first set `OMP_NUM_THREADS=1` (single-threaded) in your terminal:

```
>> export OMP_NUM_THREADS=1
```

and run the following code in python:

```
import os
import numpy as np
from timeit import timeit

size = 4096
A = np.random.random((size, size)),
B = np.random.random((size, size))
print('Time with %s threads: %f s' \
      %(os.environ.get('OMP_NUM_THREADS'),
        timeit(lambda: np.dot(A, B), number=4)))
```

Subsequently set `OMP_NUM_THREADS=2`, or any higher number of threads available in your hardware (multi-threaded):

```
>> export OMP_NUM_THREADS=2
```

and run the same python code. By both looking at your processes (e.g. using `top`) and at the python print statement you should see a speed-up in the second case.

Alternatively, you could set the `OMP_NUM_THREADS` variable directly inside your script using `os.environ['OMP_NUM_THREADS']=str(2)`. Moreover, note that when using `Intel MKL` you can alternatively set the `MKL_NUM_THREADS` instead of `OMP_NUM_THREADS`: this could be useful if your code runs other parallel processes which you can control indipendently from the `Intel MKL` ones using `OMP_NUM_THREADS`.

Note: Always remember to set `OMP_NUM_THREADS` (or `MKL_NUM_THREADS`) in your enviroment when using PyLops

3.2.2 Optional dependencies

To avoid increasing the number of *required* dependencies, which may lead to conflicts with other libraries that you have in your system, we have decided to build some of the additional features of PyLops in such a way that if an *optional* dependency is not present in your python environment, a safe fallback to one of the required dependencies will be enforced.

When available in your system, we reccomend using the Conda package manager and install all the mandatory and optional dependencies of PyLops at once using the command:

```
>> conda install -c conda-forge pylops
```

in this case all dependencies will be installed from their conda distributions.

Alternatively, from version `1.4.0` optional dependencies can also be installed as part of the pip installation via:

```
>> pip install pylops[advanced]
```

Dependencies are however installed from their PyPI wheels.

numba

Although we always stive to write code for forward and adjoint operators that takes advantage of the perks of numpy and scipy (e.g., broadcasting, ufunc), in some case we may end up using for loops that may lead to poor performance. In those cases we may decide to implement alternative (optional) back-ends in numba.

In this case a user can simply switch from the native, always available implementation to the numba implementation by simply providing the following additional input parameter to the operator `engine='numba'`. This is for example the case in the *pylops.signalprocessing.Radon2D*.

If interested to use `numba` backend from conda, you will need to manually install it:

```
>> conda install numba
```

Finally, it is also advised to install the additional package icc_rt.

```
>> conda install -c numba icc_rt
```

or pip equivalent. Similarly to `Intel MKL`, you need to set the environment variable `NUMBA_NUM_THREADS` to tell numba how many threads to use. If this variable is not present in your environment, numba code will be compiled with `parallel=False`.

fft routines

Two different *engines* are provided by the *pylops.signalprocessing.FFT* operator for `fft` and `ifft` routines in the forward and adjoint modes: `engine='numpy'` (default) and `engine='fftw'`.

The first engine comes as default as numpy is part of the dependencies of PyLops and automatically installed when PyLops is installed if not already available in your Python distribution.

The second engine implements the well-known FFTW via the python wrapper `pyfftw.FFTW`. This optimized fft tends to outperform the one from numpy in many cases, however it has not been inserted in the mandatory requirements of PyLops, meaning that when installing PyLops with `pip`, `pyfftw.FFTW` will *not* be installed automatically.

Again, if interested to use `FFTW` backend from conda, you will need to manually install it:

```
>> conda install -c conda-forge pyfftw
```

or pip equivalent.

skfmm

This library is used to compute traveltime tables with the fast-marching method in the initialization of the *pylops.waveeqprocessing.Demigration* operator when choosing `mode == 'eikonal'`.

As this may not be of interest for many users, this library has not been inserted in the mandatory requirements of PyLops. If interested to use `skfmm`, you will need to manually install it:

```
>> conda install -c conda-forge scikit-fmm
```

or pip equivalent.

spgl1

This library is used to solve sparsity-promoting BP, BPDN, and LASSO problems in *pylops.optimization.sparsity.SPGL1* solver.

If interested to use `spgl1`, you can manually install it:

```
>> pip install spgl1
```

Note: If you are a developer, all the optional dependencies can also be installed automatically by cloning the repository and installing pylops via `make dev-install` or `make dev-install_conda`.

3.3 Extensions

PyLops brings to users the power of linear operators in a simple and easy to use programming interface.

While very powerful on its own, this library is further extended to take advantage of more advanced computational resources, either in terms of **multiple-node clusters** or **GPUs**.

Two spin-off projects aim at extending the capabilities of PyLops to such computational environments:

- PyLops-GPU
- PyLops-Distributed

3.4 Tutorials

Note: Click *here* to download the full example code

3.4.1 01. The LinearOpeator

This first tutorials is aimed at easing the use of the PyLops library for both new users and developers.

Since PyLops heavily relies on the use of the `scipy.sparse.linalg.LinearOperator` class of scipy, we will start by looking at how to initialize a linear operator as well as different ways to apply the forward and adjoint operations. Finally we will investigate various *special methods*, also called *magic methods* (i.e., methods with the double underscores at the beginning and the end) that have been implemented for such a class and will allow summing, subtractring, chaining, etc. multiple operators in very easy and expressive way.

Let's start by defining a simple operator that applies element-wise multiplication of the model with a vector `d` in forward mode and element-wise multiplication of the data with the same vector `d` in adjoint mode. This operator is present in PyLops under the name of *pylops.Diagonal* and its implementation is discussed in more details in the *Implementing new operators* page.

```
import timeit
import matplotlib.pyplot as plt
import numpy as np
import pylops
```

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```
n = 10
d = np.arange(n) + 1.
x = np.ones(n)
Dop = pylops.Diagonal(d)
```

First of all we apply the operator in the forward mode. This can be done in four different ways:

- `_matvec`: directly applies the method implemented for forward mode
- `matvec`: performs some checks before and after applying `_matvec`
- `*`: operator used to map the special method `__matmul__` which checks whether the input `x` is a vector or matrix and applies `_matvec` or `_matmul` accordingly.
- `@`: operator used to map the special method `__mul__` which performs like the `*` opetator

We will time these 4 different executions and see how using `_matvec` (or `matvec`) will result in the faster computation. It is thus advised to use `*` (or `@`) in examples when expressivity has priority but prefer `_matvec` (or `matvec`) for efficient implementations.

```
# setup command
cmd_setup ="""\
import numpy as np
import pylops
n = 10
d = np.arange(n) + 1.
x = np.ones(n)
Dop = pylops.Diagonal(d)
DopH = Dop.H
"""

# _matvec
cmd1 = 'Dop._matvec(x)'

# matvec
cmd2 = 'Dop.matvec(x)'

# @
cmd3 = 'Dop@x'

# *
cmd4 = 'Dop*x'

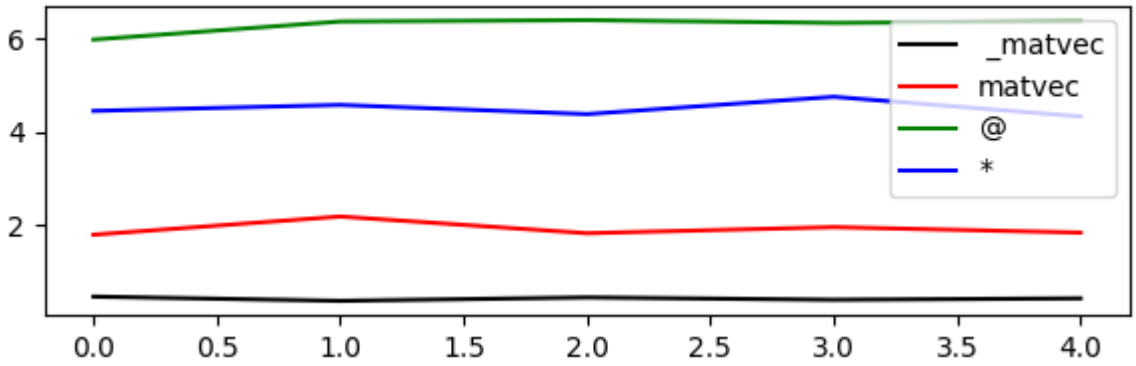
# timing
t1 = 1.e3 * np.array(timeit.repeat(cmd1, setup=cmd_setup,
                                   number=500, repeat=5))
t2 = 1.e3 * np.array(timeit.repeat(cmd2, setup=cmd_setup,
                                   number=500, repeat=5))
t3 = 1.e3 * np.array(timeit.repeat(cmd3, setup=cmd_setup,
                                   number=500, repeat=5))
t4 = 1.e3 * np.array(timeit.repeat(cmd4, setup=cmd_setup,
                                   number=500, repeat=5))

plt.figure(figsize=(7, 2))
plt.plot(t1, 'k', label=' _matvec')
plt.plot(t2, 'r', label='matvec')
plt.plot(t3, 'g', label='@')
plt.plot(t4, 'b', label='*')
```

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```
plt.legend()
plt.axis('tight')
```



Similarly we now consider the adjoint mode. This can be done in three different ways:

- `_rmatvec`: directly applies the method implemented for adjoint mode
- `rmatvec`: performs some checks before and after applying `_rmatvec`
- `.H*`: first applies the adjoint `.H` which creates a new *scipy.sparse.linalg._CustomLinearOperator*‘ where `_matvec` and `_rmatvec` are swapped and then applies the new `_matvec`.

Once again, after timing these 3 different executions we can see see how using `_rmatvec` (or `rmatvec`) will result in the faster computation while `.H*` is very unefficient and slow. Note that if the adjoint has to be applied multiple times it is at least advised to create the adjoint operator by applying `.H` only once upfront. Not surprisingly, the linear solvers in scipy as well as in PyLops actually use `matvec` and `rmatvec` when dealing with linear operators.

```
# _rmatvec
cmd1 = 'Dop._rmatvec(x)'

# rmatvec
cmd2 = 'Dop.rmatvec(x)'

# .H* (pre-computed H)
cmd3 = 'DopH*x'

# .H*
cmd4 = 'Dop.H*x'

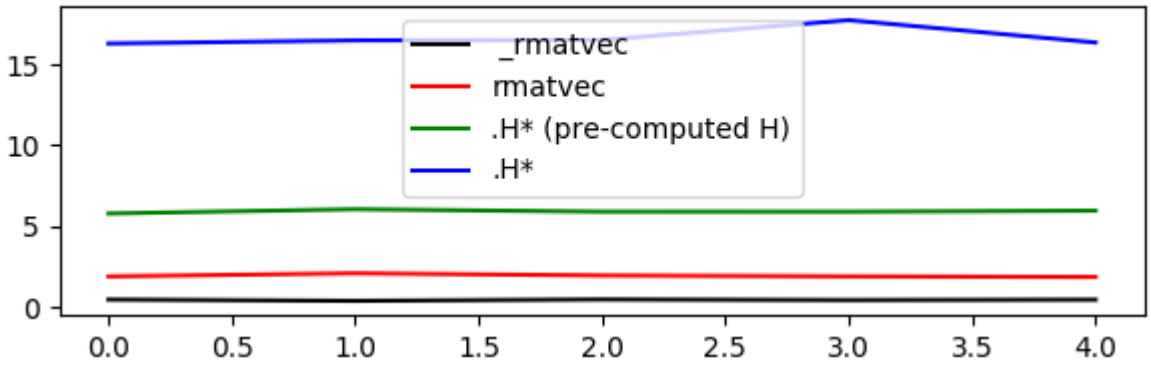
# timing
t1 = 1.e3 * np.array(timeit.repeat(cmd1, setup=cmd_setup,
                                   number=500, repeat=5))
t2 = 1.e3 * np.array(timeit.repeat(cmd2, setup=cmd_setup,
                                   number=500, repeat=5))
t3 = 1.e3 * np.array(timeit.repeat(cmd3, setup=cmd_setup,
                                   number=500, repeat=5))
t4 = 1.e3 * np.array(timeit.repeat(cmd4, setup=cmd_setup,
                                   number=500, repeat=5))

plt.figure(figsize=(7, 2))
plt.plot(t1, 'k', label=' _rmatvec')
plt.plot(t2, 'r', label='rmatvec')
plt.plot(t3, 'g', label='.H* (pre-computed H)')
```

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```
plt.plot(t4, 'b', label='.H*')
plt.legend()
plt.axis('tight')
```



Just to reiterate once again, it is advised to call `matvec` and `rmatvec` unless PyLops linear operators are used for teaching purposes.

Finally we go through some other *methods* and *special methods* that are implemented in `scipy.sparse.linalg.LinearOperator` (and *`pylops.LinearOperator`*):

- `Op1+Op2`: maps the special method `__add__` and performs summation between two operators
- `-Op`: maps the special method `__neg__` and performs negation of an operators
- `Op1-Op2`: maps the special method `__sub__` and performs summation between two operators
- `Op1**N`: maps the special method `__pow__` and performs exponentiation of an operator
- `Op/y` (and `Op.div(y)`): maps the special method `__truediv__` and performs inversion of an operator
- `Op.eigs()`: estimates the eigenvalues of the operator
- `Op.cond()`: estimates the condition number of the operator
- `Op.conj()`: create complex conjugate operator

```
# +
print(Dop+Dop)

# -
print(-Dop)
print(Dop-0.5*Dop)

# **
print(Dop**3)

#* and /
y = Dop*x
print(Dop/y)

# eigs
print(Dop.eigs(neigs=3))

# cond
print(Dop.cond())
```

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```
# conj
print(Dop.conj())
```

Out:

```
<10x10 _SumLinearOperator with dtype=float64>
<10x10 _ScaledLinearOperator with dtype=float64>
<10x10 _SumLinearOperator with dtype=float64>
<10x10 _PowerLinearOperator with dtype=float64>
[1. 1. 1. 1. 1. 1. 1. 1. 1. 1.]
[10.+0.j  9.+0.j  8.+0.j]
(9.999999999999986+0j)
<10x10 _ConjLinearOperator with dtype=float64>
```

To understand the effect of `conj` we need to look into a problem with an operator in the complex domain. Let's create again our *pylops.Diagonal* operator but this time we populate it with complex numbers. We will see that the action of the operator and its complex conjugate is different even if the model is real.

```
n = 5
d = 1j*(np.arange(n) + 1.)
x = np.ones(n)
Dop = pylops.Diagonal(d)

print('y = Dx', Dop*x)
print('y = conj(D)x', Dop.conj()*x)
```

Out:

```
y = Dx [0.+1.j 0.+2.j 0.+3.j 0.+4.j 0.+5.j]
y = conj(D)x [0.-1.j 0.-2.j 0.-3.j 0.-4.j 0.-5.j]
```

Finally it is worth remembering a useful trick. If two linear operators are combined by means of the algebraical operations shown above, the resulting operator will lose some of the convenience methods (e.g. `\`) as it will be turned into a `scipy.sparse.linalg.LinearOperator` instead of a *pylops.LinearOperator*. To transform it back to a PyLops linear operator simply do the following

```
Dop1 = Dop + Dop.conj()
Dop1 = pylops.LinearOperator(Dop, explicit=False)

y = Dop1 * x
print('x = (Dop + conj(Dop))/y', Dop1 / y)
```

Out:

```
x = (Dop + conj(Dop))/y [1.+0.j 1.+0.j 1.+0.j 1.+0.j 1.+0.j]
```

This first tutorial is completed. You have seen the basic operations that can be performed using `scipy.sparse.linalg.LinearOperator` and our overload of such a class *pylops.LinearOperator* and you should be able to get started combining various PyLops operators and solving your own inverse problems.

Total running time of the script: (0 minutes 0.540 seconds)

Note: Click *here* to download the full example code

3.4.2 02. The Dot-Test

One of the most important aspect of writing a *Linear operator* is to be able to verify that the code implemented in *forward mode* and the code implemented in *adjoint mode* are effectively adjoint to each other. If this is the case, your Linear operator will successfully pass the so-called **dot-test**. Refer to the *Notes* section of *pylops.utils.dottest*) for a more detailed description.

In this example, I will show you how to use the dot-test for a variety of operator when model and data are either real or complex numbers.

```
# pylint: disable=C0103
import numpy as np
import matplotlib.pyplot as plt
import matplotlib.gridspec as pltgs

import pylops
from pylops.utils import dottest

plt.close('all')
```

Let's start with something very simple. We will make a *pylops.MatrixMult* operator and verify that its implementation passes the dot-test. For this time, we will do this step-by-step, replicating what happens in the *pylops.utils.dottest* routine.

```
N, M = 5, 3
Mat = np.arange(N*M).reshape(N, M)
Op = pylops.MatrixMult(Mat)

v = np.random.randn(N)
u = np.random.randn(M)

# Op * u
y = Op.matvec(u)
# Op'* v
x = Op.rmatvec(v)

yy = np.dot(y, v)  # (Op  * u)' * v
xx = np.dot(u, x)  # u' * (Op' * v)

print('Dot-test %e' % np.abs((yy - xx) / ((yy + xx + 1e-15) / 2)))
```

Out:

```
Dot-test 1.366060e-16
```

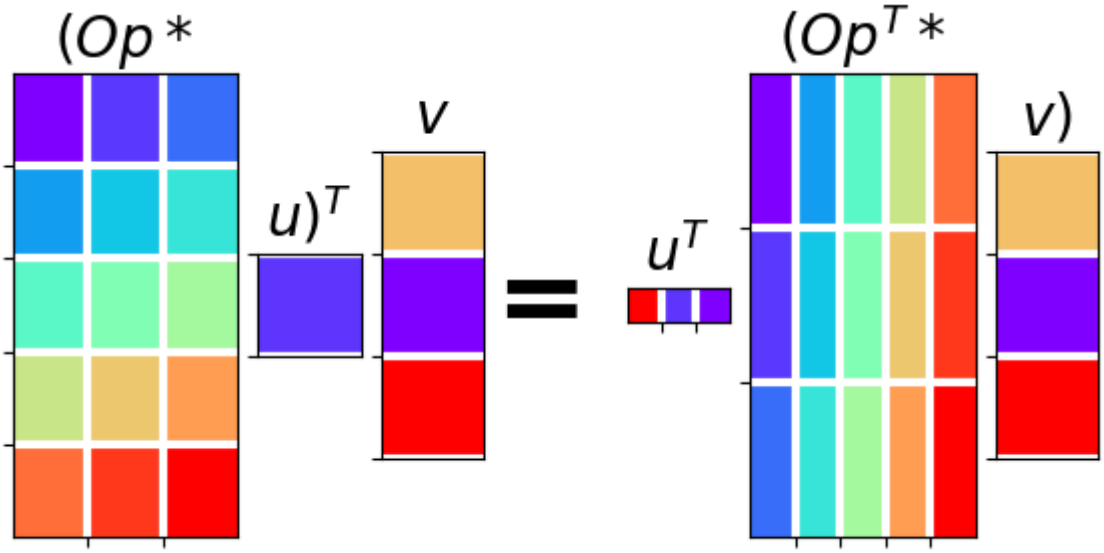
And here is a visual intepretation of what a dot-test is

```
gs = pltgs.GridSpec(1, 9)
fig = plt.figure(figsize=(7, 3))
ax = plt.subplot(gs[0, 0:2])
ax.imshow(Op.A, cmap='rainbow')
ax.set_title(r'$(Op*$', size=20, fontweight='bold')
ax.set_xticks(np.arange(M-1)+0.5)
ax.set_yticks(np.arange(N-1)+0.5)
ax.grid(linewidth=3, color='white')
ax.xaxis.set_ticklabels([])
ax.yaxis.set_ticklabels([])
```

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```
ax.axis('tight')
ax = plt.subplot(gs[0, 2])
ax.imshow(u[:, np.newaxis], cmap='rainbow')
ax.set_title(r'$u)^T$', size=20, fontweight='bold')
ax.set_xticks([])
ax.set_yticks(np.arange(M-1)+0.5)
ax.grid(linewidth=3, color='white')
ax.xaxis.set_ticklabels([])
ax.yaxis.set_ticklabels([])
ax = plt.subplot(gs[0, 3])
ax.imshow(v[:, np.newaxis], cmap='rainbow')
ax.set_title(r'$v$', size=20, fontweight='bold')
ax.set_xticks([])
ax.set_yticks(np.arange(N-1)+0.5)
ax.grid(linewidth=3, color='white')
ax.xaxis.set_ticklabels([])
ax.yaxis.set_ticklabels([])
ax = plt.subplot(gs[0, 4])
ax.text(0.35, 0.5, '=', horizontalalignment='center',
        verticalalignment='center', size=40, fontweight='bold')
ax.axis('off')
ax = plt.subplot(gs[0, 5])
ax.imshow(u[:, np.newaxis].T, cmap='rainbow')
ax.set_title(r'$u^T$', size=20, fontweight='bold')
ax.set_xticks(np.arange(M-1)+0.5)
ax.set_yticks([])
ax.grid(linewidth=3, color='white')
ax.xaxis.set_ticklabels([])
ax.yaxis.set_ticklabels([])
ax = plt.subplot(gs[0, 6:8])
ax.imshow(Op.A.T, cmap='rainbow')
ax.set_title(r'$(Op^T*$', size=20, fontweight='bold')
ax.set_xticks(np.arange(N-1)+0.5)
ax.set_yticks(np.arange(M-1)+0.5)
ax.grid(linewidth=3, color='white')
ax.xaxis.set_ticklabels([])
ax.yaxis.set_ticklabels([])
ax.axis('tight')
ax = plt.subplot(gs[0, 8])
ax.imshow(v[:, np.newaxis], cmap='rainbow')
ax.set_title(r'$v)$', size=20, fontweight='bold')
ax.set_xticks([])
ax.set_yticks(np.arange(N-1)+0.5)
ax.grid(linewidth=3, color='white')
ax.xaxis.set_ticklabels([])
ax.yaxis.set_ticklabels([])
```



From now on, we can simply use the *pylops.utils.dottest* implementation of the dot-test and pass the operator we would like to validate, its size in the model and data spaces and optionally the tolerance we will be accepting for the dot-test to be considered succesfull. Finally we need to specify if our data or/and model vectors contain complex numbers using the `complexflag` parameter. While the dot-test will return `True` when succesfull and `False` otherwise, we can also ask to print its outcome putting the `verb` parameters to `True`.

```
N = 10
d = np.arange(N)
Dop = pylops.Diagonal(d)

dottest(Dop, N, N, tol=1e-6, complexflag=0, verb=True)
```

Out:

```
Dot test passed, v^T(Opu)=-6.919654 - u^T(Op^Tv)=-6.919654
```

We move now to a more complicated operator, the *pylops.signalprocessing.FFT* operator. We use once again the *pylops.utils.dottest* to verify its implementation and since we are dealing with a transform that can be applied to both real and complex array, we try different combinations using the `complexflag` input.

```
dt = 0.005
nt = 100
nfft = 2**10

FFTop = pylops.signalprocessing.FFT(dims=(nt,), nfft=nfft, sampling=dt)
dottest(FFTop, nfft, nt, complexflag=2, verb=True)
dottest(FFTop, nfft, nt, complexflag=3, verb=True)
```

Out:

```
Dot test passed, v^T(Opu)=-17.139104 - u^T(Op^Tv)=-17.139104
Dot test passed, v^T(Opu)=11.420800 - u^T(Op^Tv)=11.420800
```

Total running time of the script: (0 minutes 0.316 seconds)

Note: Click *here* to download the full example code

3.4.3 03. Solvers

This tutorial will guide you through the `pylops.optimization` module and show how to use various solvers that are included in the PyLops library.

The main idea here is to provide the user of PyLops with very high-level functionalities to quickly and easily set up and solve complex systems of linear equations as well as include regularization and/or preconditioning terms (all of those constructed by means of PyLops linear operators).

To make this tutorial more interesting, we will present a real life problem and show how the choice of the solver and regularization/preconditioning terms is vital in many circumstances to successfully retrieve an estimate of the model. The problem that we are going to consider is generally referred to as the *data reconstruction* problem and aims at reconstructing a regularly sampled signal of size M from N randomly selected samples:

$$\mathbf{y} = \mathbf{R}\mathbf{x}$$

where the restriction operator $\mathbf{R}$ that selects the M elements from $\mathbf{x}$ at random locations is implemented using *pylops.Restriction*, and

$$\mathbf{y} = [y_1, y_2, ..., y_N]^T, \qquad \mathbf{x} = [x_1, x_2, ..., x_M]^T,$$

with $M >> N$.

```
# pylint: disable=C0103
import numpy as np
import matplotlib.pyplot as plt

import pylops

plt.close('all')
np.random.seed(10)
```

Let's first create the data in the frequency domain. The data is composed by the superposition of 3 sinusoids with different frequencies.

```
# Signal creation in frequency domain
ifreqs = [41, 25, 66]
amps = [1., 1., 1.]
N = 200
nfft = 2**11
dt = 0.004
t = np.arange(N)*dt
f = np.fft.rfftfreq(nfft, dt)

FFTop = 10*pylops.signalprocessing.FFT(N, nfft=nfft, real=True)

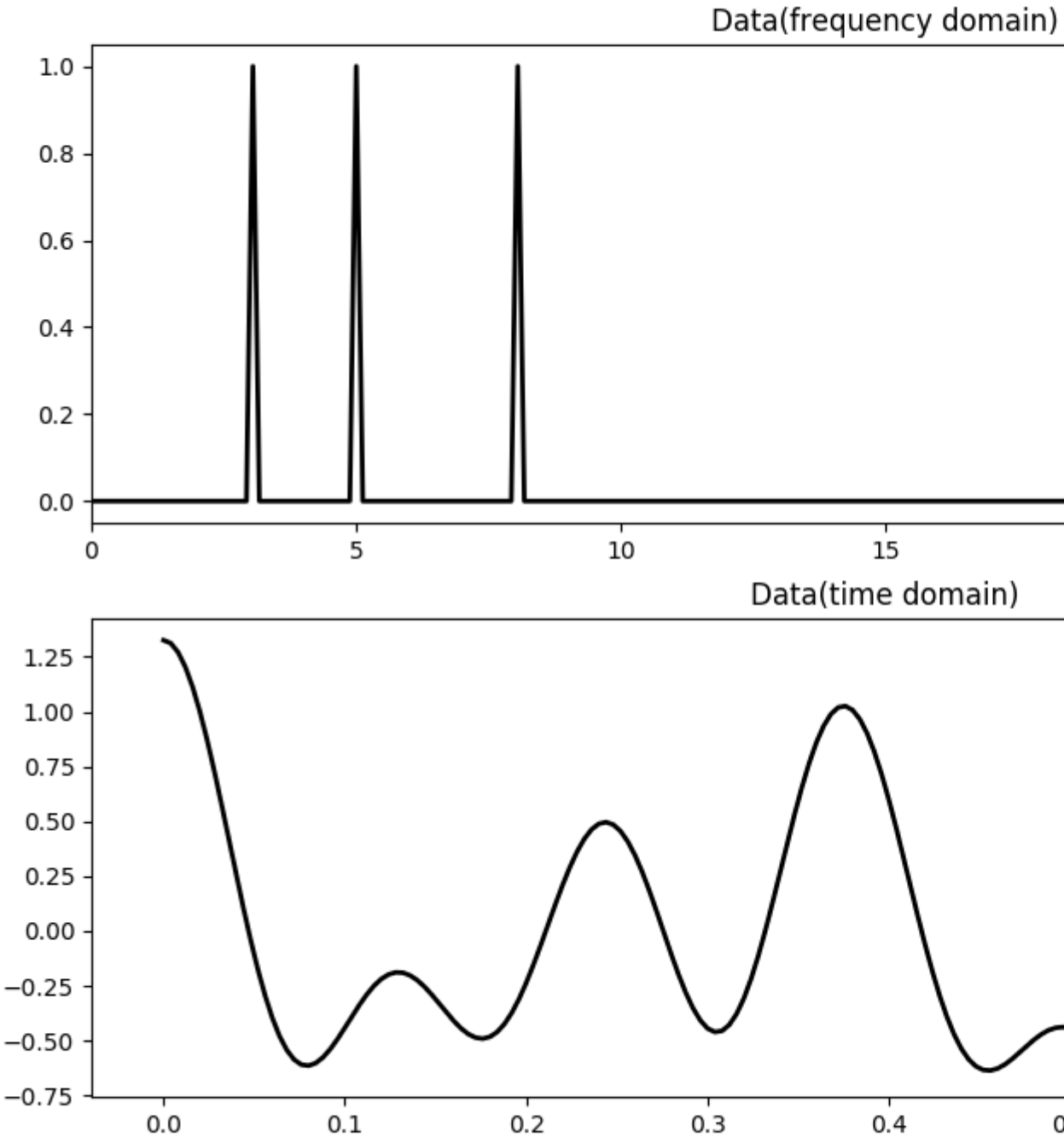
X = np.zeros(nfft//2+1, dtype='complex128')
X[ifreqs] = amps
x = FFTop.H*X

fig, axs = plt.subplots(2, 1, figsize=(12, 8))
axs[0].plot(f, np.abs(X), 'k', LineWidth=2)
```

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```
axs[0].set_xlim(0, 30)
axs[0].set_title('Data(frequency domain)')
axs[1].plot(t, x, 'k', LineWidth=2)
axs[1].set_title('Data(time domain)')
axs[1].axis('tight')
```



We now define the locations at which the signal will be sampled.

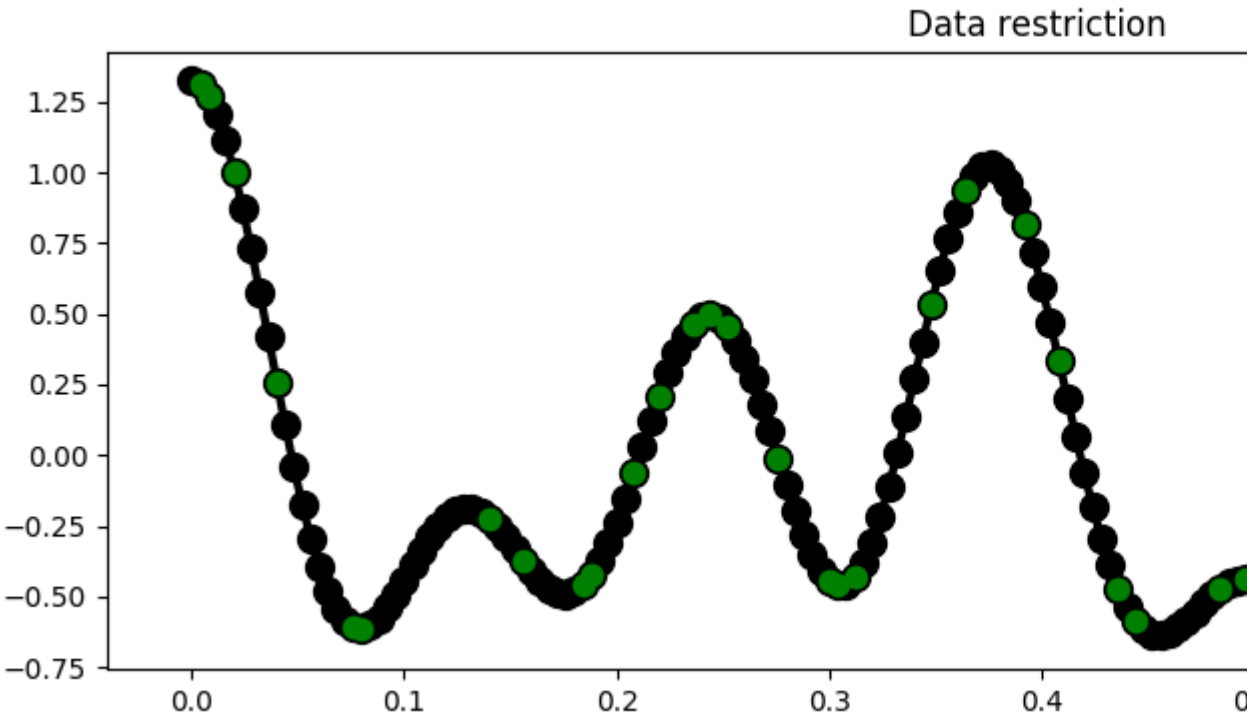
```
# subsampling locations
perc_subsampling = 0.2
Nsub = int(np.round(N*perc_subsampling))

iava = np.sort(np.random.permutation(np.arange(N))[:Nsub])

# Create restriction operator
Rop = pylops.Restriction(N, iava, dtype='float64')

y = Rop*x
ymask = Rop.mask(x)

# Visualize data
fig = plt.figure(figsize=(12, 4))
plt.plot(t, x, 'k', lw=3)
plt.plot(t, x, '.k', ms=20, label='all samples')
plt.plot(t, ymask, '.g', ms=15, label='available samples')
plt.legend()
plt.title('Data restriction')
```



To start let's consider the simplest *'solver'*, i.e., *least-square inversion without regularization*. We aim here to minimize the following cost function:

$$J = ||\mathbf{y} - \mathbf{Rx}||_2$$

Depending on the choice of the operator $\mathbf{R}$, such problem can be solved using explicit matrix solvers as well as iterative solvers. In this case we will be using the latter approach (more specifically the scipy implementation of the *LSQR* solver - i.e., `scipy.sparse.linalg.lsqr`) as we do not want to explicitly create and invert a matrix. In

most cases this will be the only viable approach as most of the large-scale optimization problems that we are interested to solve using PyLops do not lend naturally to the creation and inversion of explicit matrices.

This first solver can be very easily implemented using the / for PyLops operators, which will automatically call the `scipy.sparse.linalg.lsqr` with some default parameters.

```
xinv = Rop / y
```

We can also use *pylops.optimization.leastsquares.RegularizedInversion* (without regularization term for now) and customize our solvers using `kwargs`.

```
xinv = \
    pylops.optimization.leastsquares.RegularizedInversion(Rop, [], y,
                                                          **dict(damp=0,
                                                                 iter_lim=10,
                                                                 show=1))
```

Out:

```
LSQR            Least-squares solution of  Ax = b
The matrix A has       40 rows  and      200 cols
damp = 0.00000000000000e+00   calc_var =        0
atol = 1.00e-08                 conlim = 1.00e+08
btol = 1.00e-08               iter_lim =       10

   Itn      x[0]       r1norm     r2norm   Compatible    LS      Norm A   Cond A
     0  0.00000e+00   3.759e+00  3.759e+00    1.0e+00  2.7e-01
     1  0.00000e+00   0.000e+00  0.000e+00    0.0e+00  0.0e+00   0.0e+00  0.0e+00

LSQR finished
Ax - b is small enough, given atol, btol

istop =       1   r1norm = 0.0e+00   anorm = 0.0e+00   arnorm = 0.0e+00
itn   =       1   r2norm = 0.0e+00   acond = 0.0e+00   xnorm  = 3.8e+00
```

Finally we can select a different starting guess from the null vector

```
xinv_fromx0 = \
    pylops.optimization.leastsquares.RegularizedInversion(Rop, [], y,
                                                          x0=np.ones(N),
                                                          **dict(damp=0,
                                                                 iter_lim=10,
                                                                 show=0))
```

The cost function above can be also expanded in terms of its *normal equations*

$$\mathbf{x}_{ne} = (\mathbf{R}^T\mathbf{R})^{-1}\mathbf{R}^T\mathbf{y}$$

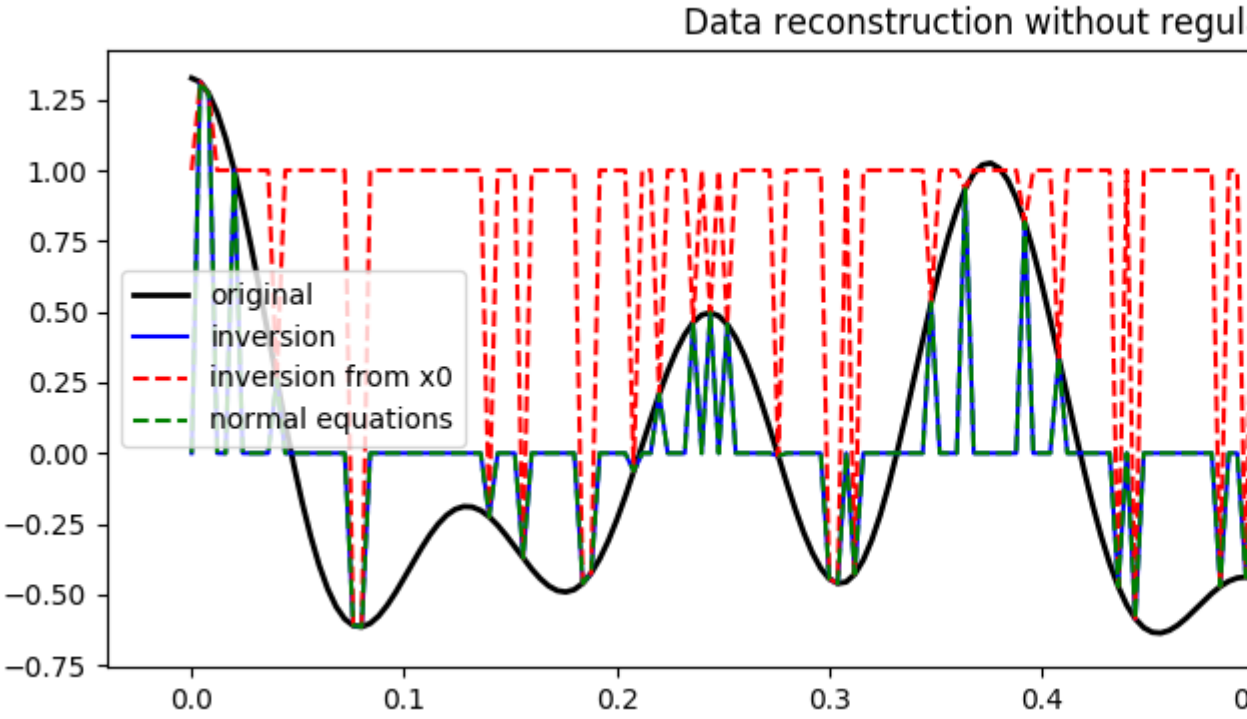
The method *pylops.optimization.leastsquares.NormalEquationsInversion* implements such system of equations explicitly and solves them using an iterative scheme suitable for square matrices (i.e., $M = N$).

While this approach may seem not very useful, we will soon see how regularization terms could be easily added to the normal equations using this method.

```
xne = pylops.optimization.leastsquares.NormalEquationsInversion(Rop, [], y)
```

Let's now visualize the different inversion results

```
fig = plt.figure(figsize=(12, 4))
plt.plot(t, x, 'k', lw=2, label='original')
plt.plot(t, xinv, 'b', ms=10, label='inversion')
plt.plot(t, xinv_fromx0, '--r', ms=10, label='inversion from x0')
plt.plot(t, xne, '--g', ms=10, label='normal equations')
plt.legend()
plt.title('Data reconstruction without regularization')
```



Regularization

You may have noticed that none of the inversion has been successfull in recovering the original signal. This is a clear indication that the problem we are trying to solve is highly ill-posed and requires some prior knowledge from the user.

We will now see how to add prior information to the inverse process in the form of regularization (or preconditioning). This can be done in two different ways

- regularization via *pylops.optimization.leastsquares.NormalEquationsInversion* or *pylops.optimization.leastsquares.RegularizedInversion*)
- preconditioning via *pylops.optimization.leastsquares.PreconditionedInversion*

Let's start by regularizing the normal equations using a second derivative operator

$$\mathbf{x} = (\mathbf{R^T R} + \epsilon_\nabla^2 \nabla^T \nabla)^{-1} \mathbf{R^T y}$$

```
# Create regularization operator
D2op = pylops.SecondDerivative(N, dims=None, dtype='float64')

# Regularized inversion
epsR = np.sqrt(0.1)
epsI = np.sqrt(1e-4)

xne = \
    pylops.optimization.leastsquares.NormalEquationsInversion(Rop, [D2op], y,
                                                              epsI=epsI,
                                                              epsRs=[epsR],
                                                              returninfo=False,
                                                              **dict(maxiter=50))
```

We can do the same while using *pylops.optimization.leastsquares.RegularizedInversion* which solves the following augmented problem

$$\begin{bmatrix} \mathbf{R} \\ \epsilon_\nabla \nabla \end{bmatrix} \mathbf{x} = \begin{bmatrix} \mathbf{y} \\ 0 \end{bmatrix}$$

```
xreg = \
    pylops.optimization.leastsquares.RegularizedInversion(Rop, [D2op], y,
                                                          epsRs=[np.sqrt(0.1)],
                                                          returninfo=False,
                                                          **dict(damp=np.sqrt(1e-4),
                                                                 iter_lim=50,
                                                                 show=0))
```

We can also write a preconditioned problem, whose cost function is

$$J = ||\mathbf{y} - \mathbf{RPp}||_2$$

where $\mathbf{P}$ is the precondioned operator, $\mathbf{p}$ is the projected model in the preconditioned space, and $\mathbf{x} = \mathbf{Pp}$ is the model in the original model space we want to solve for. Note that a preconditioned problem converges much faster to its solution than its corresponding regularized problem. This can be done using the routine *pylops.optimization.leastsquares.PreconditionedInversion*.

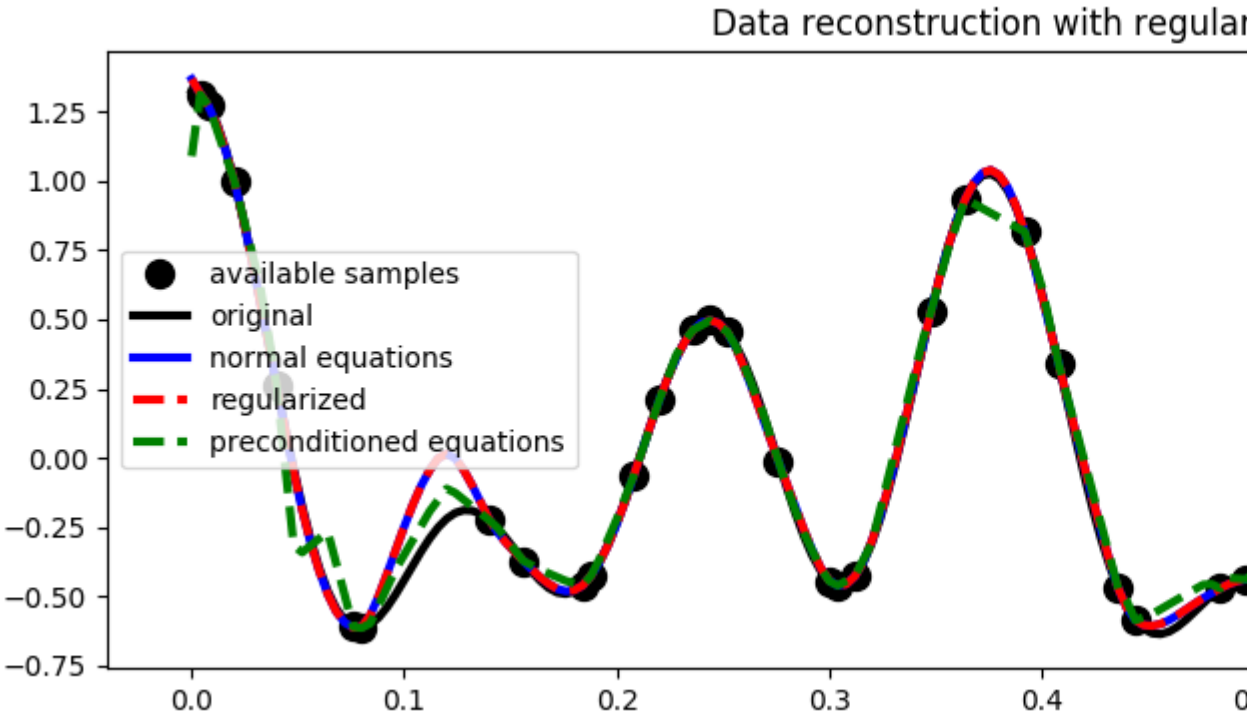
```
# Create regularization operator
Sop = pylops.Smoothing1D(nsmooth=11, dims=[N], dtype='float64')

# Invert for interpolated signal
xprec = \
    pylops.optimization.leastsquares.PreconditionedInversion(Rop, Sop, y,
                                                             returninfo=False,
                                                             **dict(damp=np.sqrt(1e-
↪9),
                                                                    iter_lim=20,
                                                                    show=0))
```

Let's finally visualize these solutions

```
# sphinx_gallery_thumbnail_number=4
fig = plt.figure(figsize=(12, 4))
plt.plot(t[iava], y, '.k', ms=20, label='available samples')
plt.plot(t, x, 'k', lw=3, label='original')
plt.plot(t, xne, 'b', lw=3, label='normal equations')
plt.plot(t, xreg, '--r', lw=3, label='regularized')
plt.plot(t, xprec, '--g', lw=3, label='preconditioned equations')
plt.legend()
plt.title('Data reconstruction with regularization')

subax = fig.add_axes([0.7, 0.2, 0.15, 0.6])
subax.plot(t[iava], y, '.k', ms=20)
subax.plot(t, x, 'k', lw=3)
subax.plot(t, xne, 'b', lw=3)
subax.plot(t, xreg, '--r', lw=3)
subax.plot(t, xprec, '--g', lw=3)
subax.set_xlim(0.05, 0.3)
```



Much better estimates! We have seen here how regularization and/or preconditioning can be vital to succesfully solve some ill-posed inverse problems.

We have however so far only considered solvers that can include additional norm-2 regularization terms. A very active area of research is that of *sparsity-promoting* solvers (also sometimes referred to as *compressive sensing*): the regularization term added to the cost function to minimize has norm-p ($p \leq 1$) and the problem is generally recasted by considering the model to be sparse in some domain. We can follow this philosophy as our signal to invert was actually created as superposition of 3 sinusoids (i.e., three spikes in the Fourier domain). Our new cost function is:

$$J_1 = ||\mathbf{y} - \mathbf{RFp}||_2 + \epsilon||\mathbf{p}||_1$$

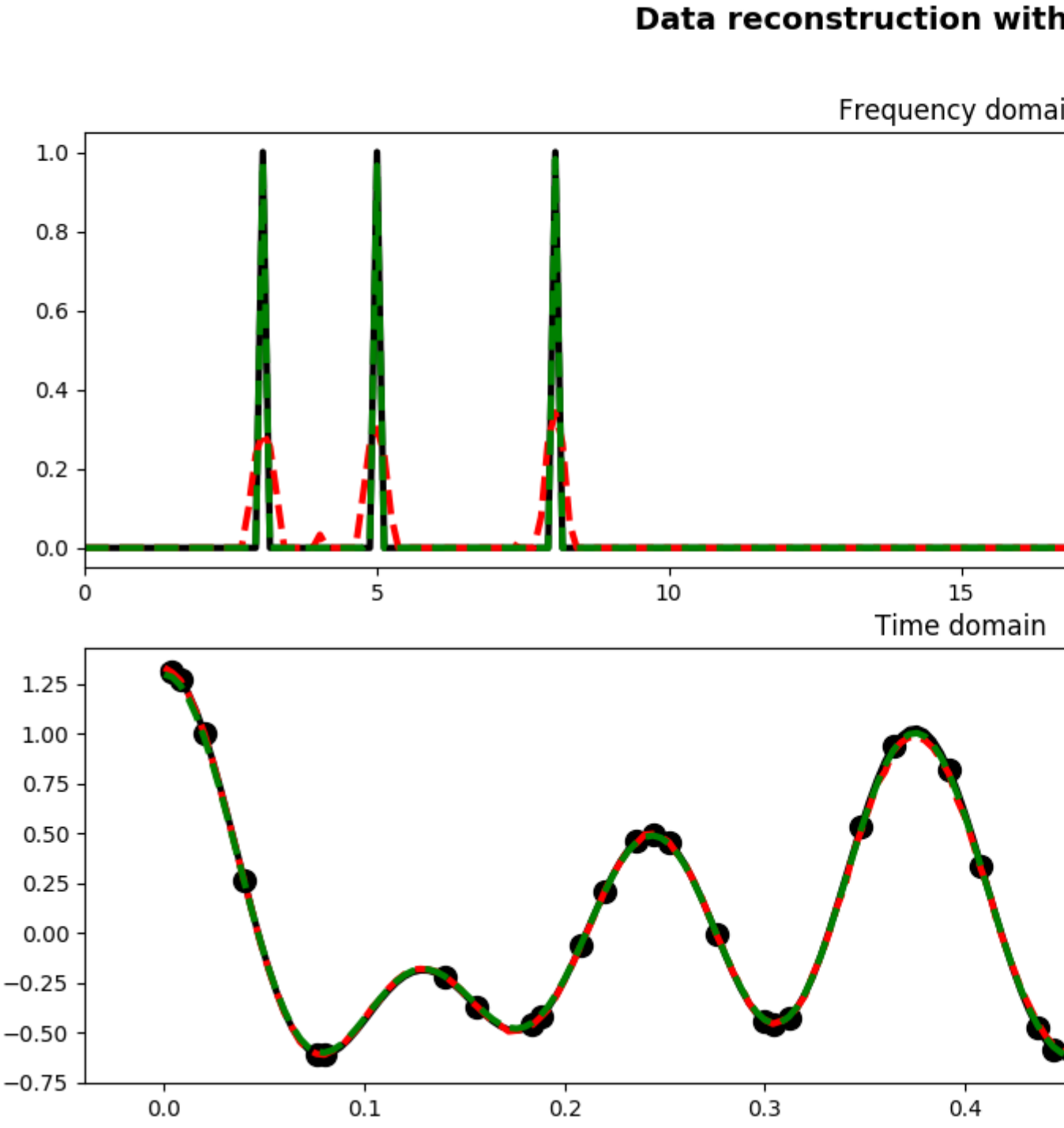
where $\mathbf{F}$ is the FFT operator. We will thus use the *pylops.optimization.sparsity.ISTA* and *pylops.optimization.sparsity.FISTA* solvers to estimate our input signal.

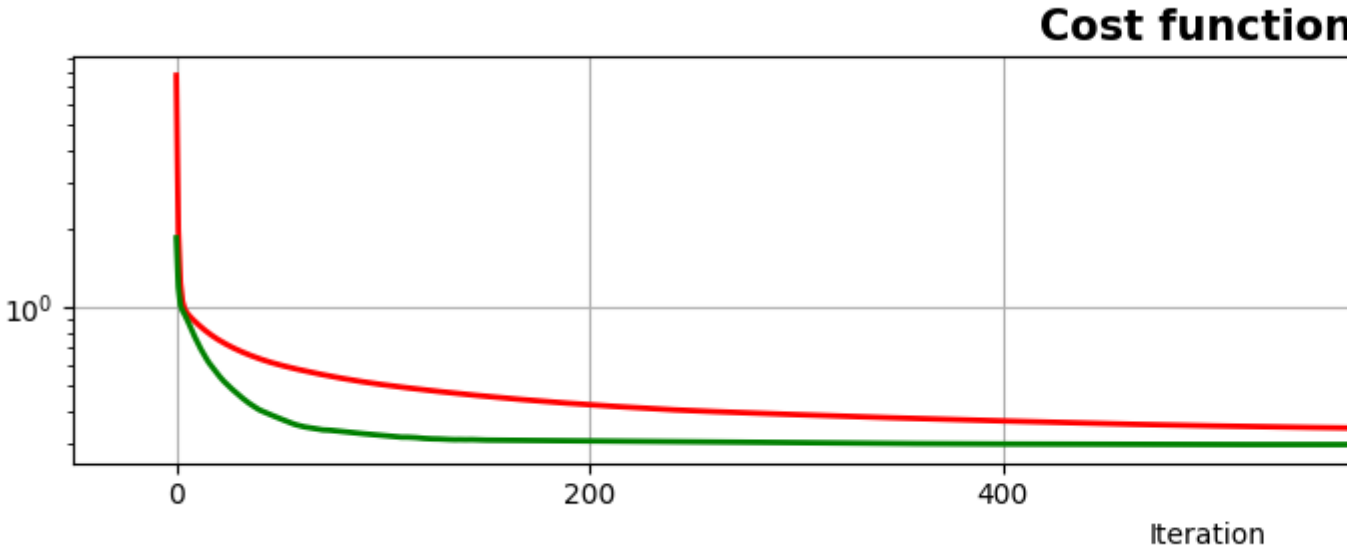
```
pista, niteri, costi = \
    pylops.optimization.sparsity.ISTA(Rop*FFTop.H, y, niter=1000,
                                      eps=0.1, tol=1e-7, returninfo=True)
xista = FFTop.H*pista

pfista, niterf, costf = \
    pylops.optimization.sparsity.FISTA(Rop*FFTop.H, y, niter=1000,
                                       eps=0.1, tol=1e-7, returninfo=True)
xfista = FFTop.H*pfista

fig, axs = plt.subplots(2, 1, figsize=(12, 8))
fig.suptitle('Data reconstruction with sparsity', fontsize=14,
             fontweight='bold', y=0.9)
axs[0].plot(f, np.abs(X), 'k', lw=3)
axs[0].plot(f, np.abs(pista), '--r', lw=3)
axs[0].plot(f, np.abs(pfista), '--g', lw=3)
axs[0].set_xlim(0, 30)
axs[0].set_title('Frequency domain')
axs[1].plot(t[iava], y, '.k', ms=20, label='available samples')
axs[1].plot(t, x, 'k', lw=3, label='original')
axs[1].plot(t, xista, '--r', lw=3, label='ISTA')
axs[1].plot(t, xfista, '--g', lw=3, label='FISTA')
axs[1].set_title('Time domain')
axs[1].axis('tight')
axs[1].legend()
plt.tight_layout()
plt.subplots_adjust(top=0.8)

fig, ax = plt.subplots(1, 1, figsize=(12, 3))
ax.semilogy(costi, 'r', lw=2, label='ISTA')
ax.semilogy(costf, 'g', lw=2, label='FISTA')
ax.set_title('Cost functions', size=15, fontweight='bold')
ax.set_xlabel('Iteration')
ax.legend()
ax.grid(True)
plt.tight_layout()
```


Data reconstruction with
Frequency domai
1.0
0.8
0.6
0.4
0.2
0.0
0
5
10
15
Time domain
1.25
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
0.0
0.1
0.2
0.3
0.4



-

As you can see, changing parametrization of the model and imposing sparsity in the Fourier domain has given an extra improvement to our ability of recovering the underlying densely sampled input signal. Moreover, FISTA converges much faster than ISTA as expected and should be preferred when using sparse solvers.

Finally we consider a slightly different cost function (note that in this case we try to solve a constrained problem):

$$J_1 = ||\mathbf{p}||_1 \quad subj.to \quad ||\mathbf{y} - \mathbf{RFp}||$$

A very popular solver to solve such kind of cost function is called *spgl1* and can be accessed via *pylops.optimization.sparsity.SPGL1*.

```
xspgl1, pspgl1, info = \
    pylops.optimization.sparsity.SPGL1(Rop, y, FFTop, tau=3, iter_lim=200)

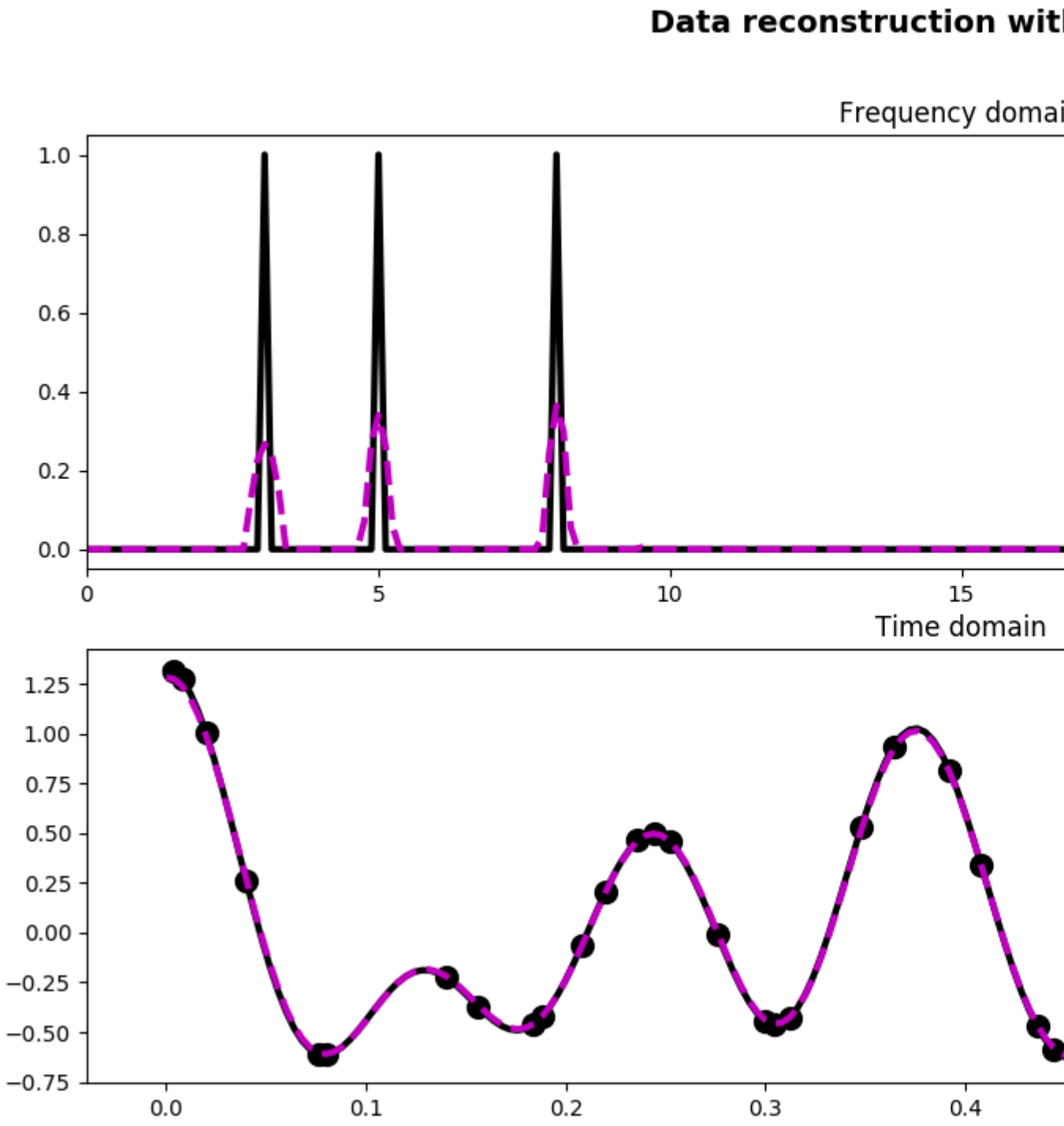
fig, axs = plt.subplots(2, 1, figsize=(12, 8))
fig.suptitle('Data reconstruction with SPGL1', fontsize=14,
             fontweight='bold', y=0.9)
axs[0].plot(f, np.abs(X), 'k', lw=3)
axs[0].plot(f, np.abs(pspgl1), '--m', lw=3)
axs[0].set_xlim(0, 30)
axs[0].set_title('Frequency domain')
axs[1].plot(t[iava], y, '.k', ms=20, label='available samples')
axs[1].plot(t, x, 'k', lw=3, label='original')
axs[1].plot(t, xspgl1, '--m', lw=3, label='SPGL1')
axs[1].set_title('Time domain')
axs[1].axis('tight')
axs[1].legend()
plt.tight_layout()
plt.subplots_adjust(top=0.8)

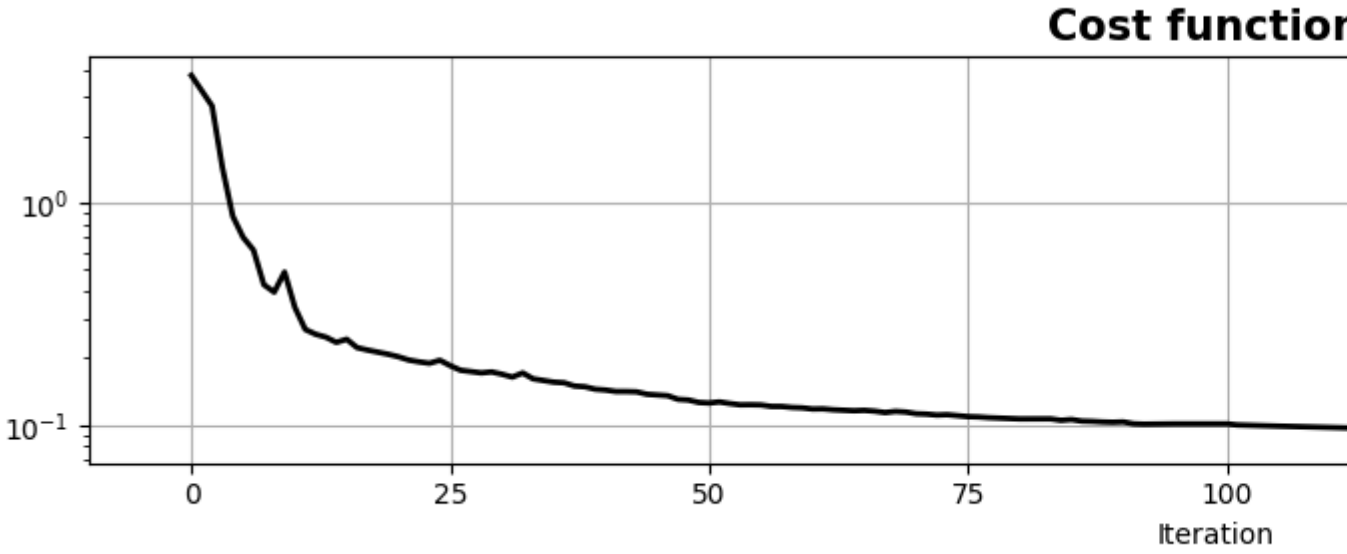
fig, ax = plt.subplots(1, 1, figsize=(12, 3))
ax.semilogy(info['rnorm2'], 'k', lw=2, label='ISTA')
ax.set_title('Cost functions', size=15, fontweight='bold')
ax.set_xlabel('Iteration')
ax.legend()
```

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```
ax.grid(True)
plt.tight_layout()
```





-

Total running time of the script: (0 minutes 3.773 seconds)

Note: Click *here* to download the full example code

3.4.4 04. Image deblurring

Deblurring is the process of removing blurring effects from images, caused for example by defocus aberration or motion blur.

In forward mode, such blurring effect is typically modelled as a 2-dimensional convolution between the so-called *point spread function* and a target sharp input image, where the sharp input image (which has to be recovered) is unknown and the point-spread function can be either known or unknown.

In this tutorial, an example of 2d blurring and deblurring will be shown using the *pylops.signalprocessing.Convolve2D* operator assuming knowledge of the point-spread function.

```
import numpy as np
import matplotlib.pyplot as plt
import pylops
```

Let's start by importing a 2d image and defining the blurring operator

```
im = np.load('../testdata/python.npy')[::5, ::5, 0]

Nz, Nx = im.shape

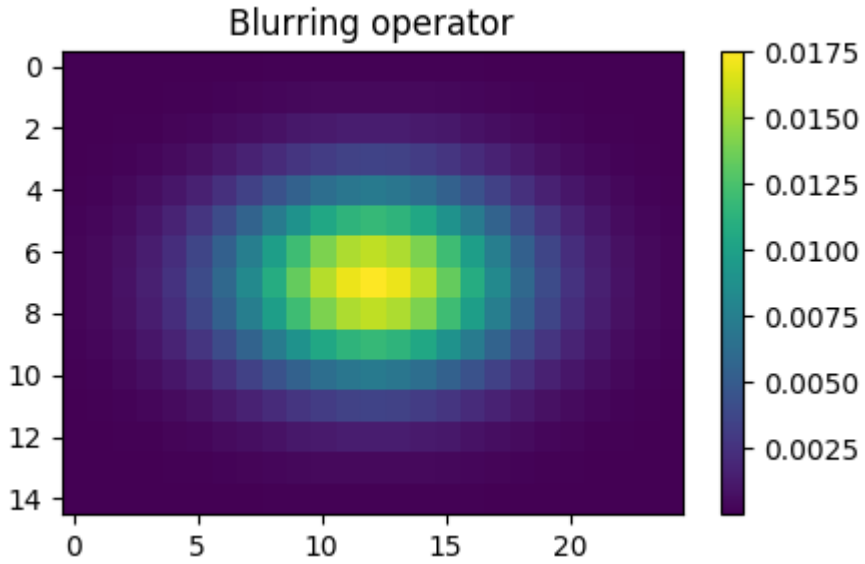
# Blurring guassian operator
nh = [15, 25]
hz = np.exp(-0.1*np.linspace(-(nh[0]//2), nh[0]//2, nh[0])**2)
hx = np.exp(-0.03*np.linspace(-(nh[1]//2), nh[1]//2, nh[1])**2)
hz /= np.trapz(hz) # normalize the integral to 1
hx /= np.trapz(hx) # normalize the integral to 1
h = hz[:, np.newaxis] * hx[np.newaxis, :]
```

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```
fig, ax = plt.subplots(1, 1, figsize=(5, 3))
him = ax.imshow(h)
ax.set_title('Blurring operator')
fig.colorbar(him, ax=ax)
ax.axis('tight')

Cop = pylops.signalprocessing.Convolve2D(Nz * Nx, h=h,
                                         offset=(nh[0] // 2,
                                                 nh[1] // 2),
                                         dims=(Nz, Nx), dtype='float32')
```



We will now apply the blurring operator to the sharp image. Finally we try to recover the sharp input image by inverting the convolution operator from the blurred image. Note that when we perform inversion without any regularization, the deblurred image will show some ringing due to the instabilities of the inverse process. Adding TV regularization allows to recover sharp contrasts.

```
imblur = Cop * im.flatten()

imdeblur = \
    pylops.optimization.leastsquares.NormalEquationsInversion(Cop, None,
                                                              imblur,
                                                              maxiter=50)

Dop = [pylops.FirstDerivative(Nz * Nx, dims=(Nz, Nx), dir=0, edge=False),
       pylops.FirstDerivative(Nz * Nx, dims=(Nz, Nx), dir=1, edge=False)]
imdeblurtv = \
    pylops.optimization.sparsity.SplitBregman(Cop, Dop, imblur.flatten(),
                                              niter_outer=10, niter_inner=5,
                                              mu=1.5, epsRL1s=[1e0, 1e0],
                                              tol=1e-4, tau=1., show=False,
                                              ** dict(iter_lim=5, damp=1e-4))[0]

# Reshape images
imblur = imblur.reshape((Nz, Nx))
imdeblur = imdeblur.reshape((Nz, Nx))
```

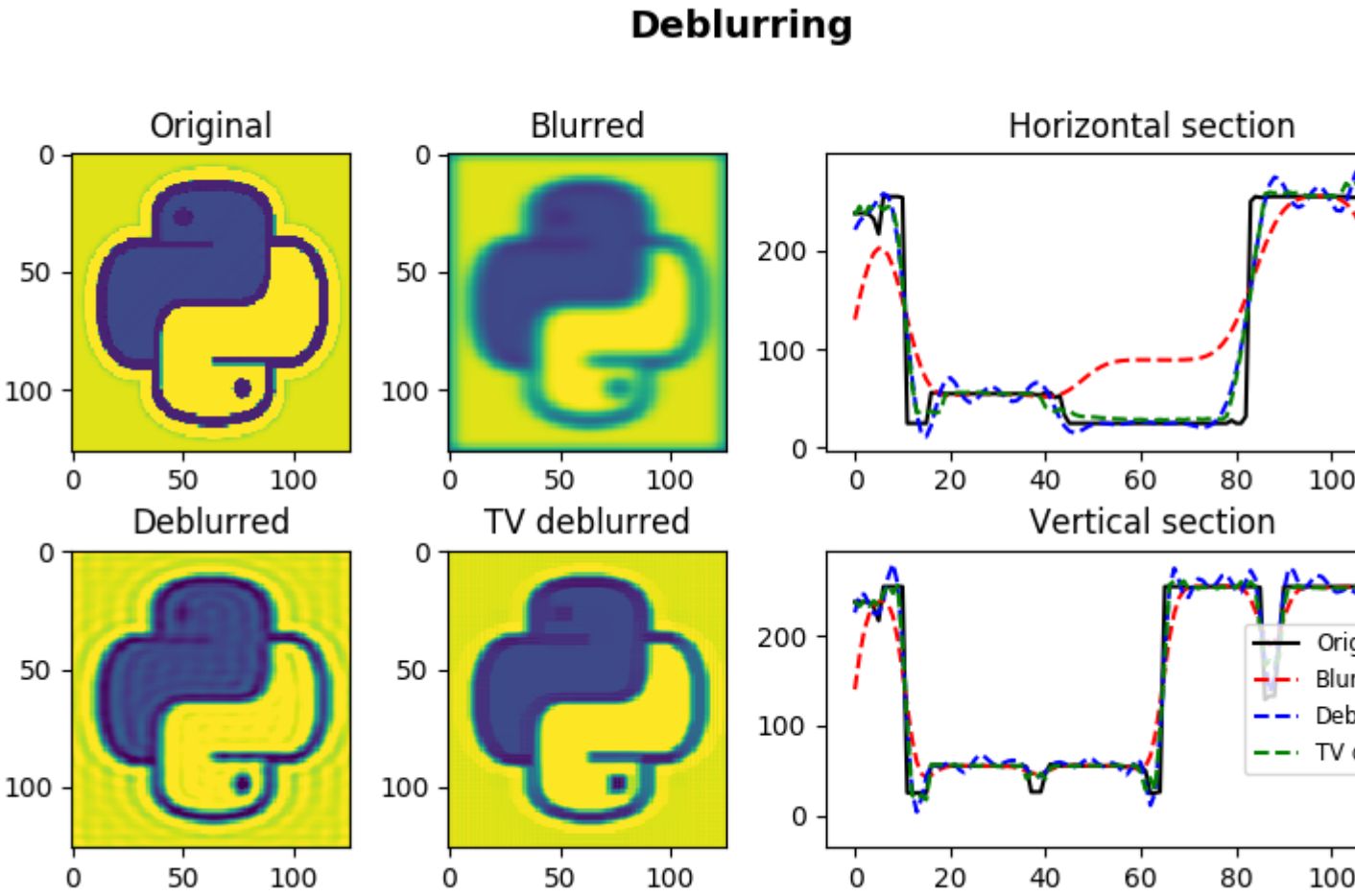
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```
imdeblurtv = imdeblurtv.reshape((Nz, Nx))
```

Finally we visualize the original, blurred, and recovered images.

```
# sphinx_gallery_thumbnail_number = 2
fig = plt.figure(figsize=(8, 5))
fig.suptitle('Deblurring', fontsize=14, fontweight='bold', y=0.95)
ax1 = plt.subplot2grid((2, 4), (0, 0))
ax2 = plt.subplot2grid((2, 4), (0, 1))
ax3 = plt.subplot2grid((2, 4), (1, 0))
ax4 = plt.subplot2grid((2, 4), (1, 1))
ax5 = plt.subplot2grid((2, 4), (0, 2), colspan=2)
ax6 = plt.subplot2grid((2, 4), (1, 2), colspan=2)
ax1.imshow(im, cmap='viridis', vmin=0, vmax=250)
ax1.axis('tight')
ax1.set_title('Original')
ax2.imshow(imblur, cmap='viridis', vmin=0, vmax=250)
ax2.axis('tight')
ax2.set_title('Blurred')
ax3.imshow(imdeblur, cmap='viridis', vmin=0, vmax=250)
ax3.axis('tight')
ax3.set_title('Deblurred')
ax4.imshow(imdeblurtv, cmap='viridis', vmin=0, vmax=250)
ax4.axis('tight')
ax4.set_title('TV deblurred')
ax5.plot(im[Nz//2], 'k')
ax5.plot(imblur[Nz//2], '--r')
ax5.plot(imdeblur[Nz//2], '--b')
ax5.plot(imdeblurtv[Nz//2], '--g')
ax5.axis('tight')
ax5.set_title('Horizontal section')
ax6.plot(im[:, Nx//2], 'k', label='Original')
ax6.plot(imblur[:, Nx//2], '--r', label='Blurred')
ax6.plot(imdeblur[:, Nx//2], '--b', label='Deblurred')
ax6.plot(imdeblurtv[:, Nx//2], '--g', label='TV deblurred')
ax6.axis('tight')
ax6.set_title('Vertical section')
ax6.legend(loc=5, fontsize='small')
plt.tight_layout()
plt.subplots_adjust(top=0.8)
```



Total running time of the script: (0 minutes 3.464 seconds)

Note: Click *here* to download the full example code

3.4.5 05. 2D Interpolation

In the mathematical field of numerical analysis, interpolation is the problem of constructing new data points within the range of a discrete set of known data points. In signal and image processing, the data may be recorded at irregular locations and it is often required to *regularize* the data into a regular grid.

In this tutorial, an example of 2d interpolation of an image is carried out using a combination of PyLops operators (`pylops.Restriction` and `pylops.Laplacian`) and the `pylops.optimization` module.

Mathematically speaking, if we want to interpolate a signal using the theory of inverse problems, we can define the following forward problem:

$$\mathbf{y} = \mathbf{R}\mathbf{x}$$

where the restriction operator $\mathbf{R}$ selects M elements from the regularly sampled signal $\mathbf{x}$ at random locations. The input and output signals are:

$$\mathbf{y} = [y_1, y_2, ..., y_N]^T, \qquad \mathbf{x} = [x_1, x_2, ..., x_M]^T,$$

with $M >> N$.

```
import numpy as np
import matplotlib.pyplot as plt
import pylops

plt.close('all')
np.random.seed(0)
```

To start we import a 2d image and define our restriction operator to irregularly and randomly sample the image for 30% of the entire grid

```
im = np.load('../testdata/python.npy')[:, :, 0]

Nz, Nx = im.shape
N = Nz * Nx

# Subsample signal
perc_subsampling = 0.2

Nsub2d = int(np.round(N*perc_subsampling))
iava = np.sort(np.random.permutation(np.arange(N))[:Nsub2d])

# Create operators and data
Rop = pylops.Restriction(N, iava, dtype='float64')
D2op = pylops.Laplacian((Nz, Nx), weights=(1, 1), dtype='float64')

x = im.flatten()
y = Rop * x
y1 = Rop.mask(x)
```

We will now use two different routines from our optimization toolbox to estimate our original image in the regular grid.

```
xcg_reg_lop = \
    pylops.optimization.leastsquares.NormalEquationsInversion(Rop, [D2op], y,
                                                              epsRs=[np.sqrt(0.1)],
                                                              returninfo=False,
                                                              **dict(maxiter=200))

# Invert for interpolated signal, lsqrt
xlsqr_reg_lop, istop, itn, r1norm, r2norm = \
    pylops.optimization.leastsquares.RegularizedInversion(Rop, [D2op], y,
                                                          epsRs=[np.sqrt(0.1)],
                                                          returninfo=True,
                                                          **dict(damp=0,
                                                                 iter_lim=200,
                                                                 show=0))

# Reshape estimated images
im_sampled = y1.reshape((Nz, Nx))
im_rec_lap_cg = xcg_reg_lop.reshape((Nz, Nx))
im_rec_lap_lsqr = xlsqr_reg_lop.reshape((Nz, Nx))
```

Finally we visualize the original image, the reconstructed images and their error

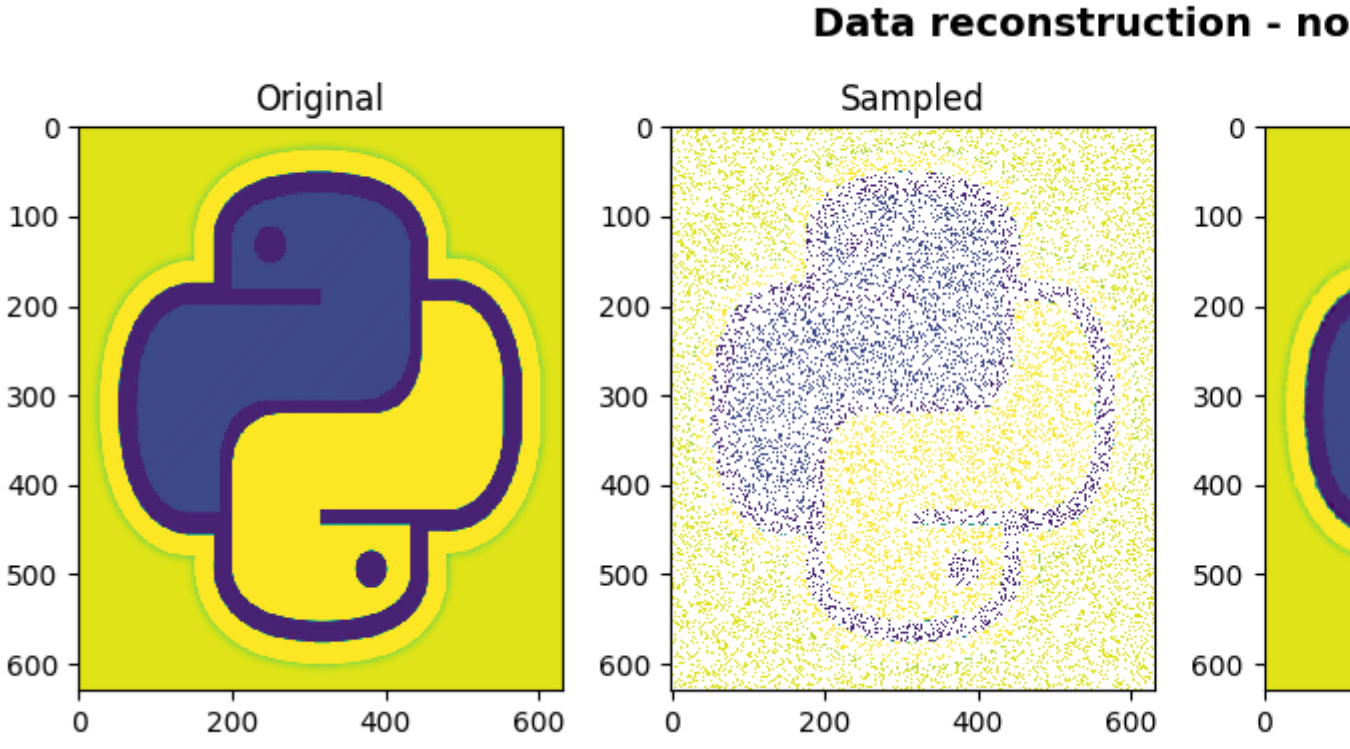
```
fig, axs = plt.subplots(1, 4, figsize=(12, 4))
fig.suptitle('Data reconstruction - normal eqs', fontsize=14,
```

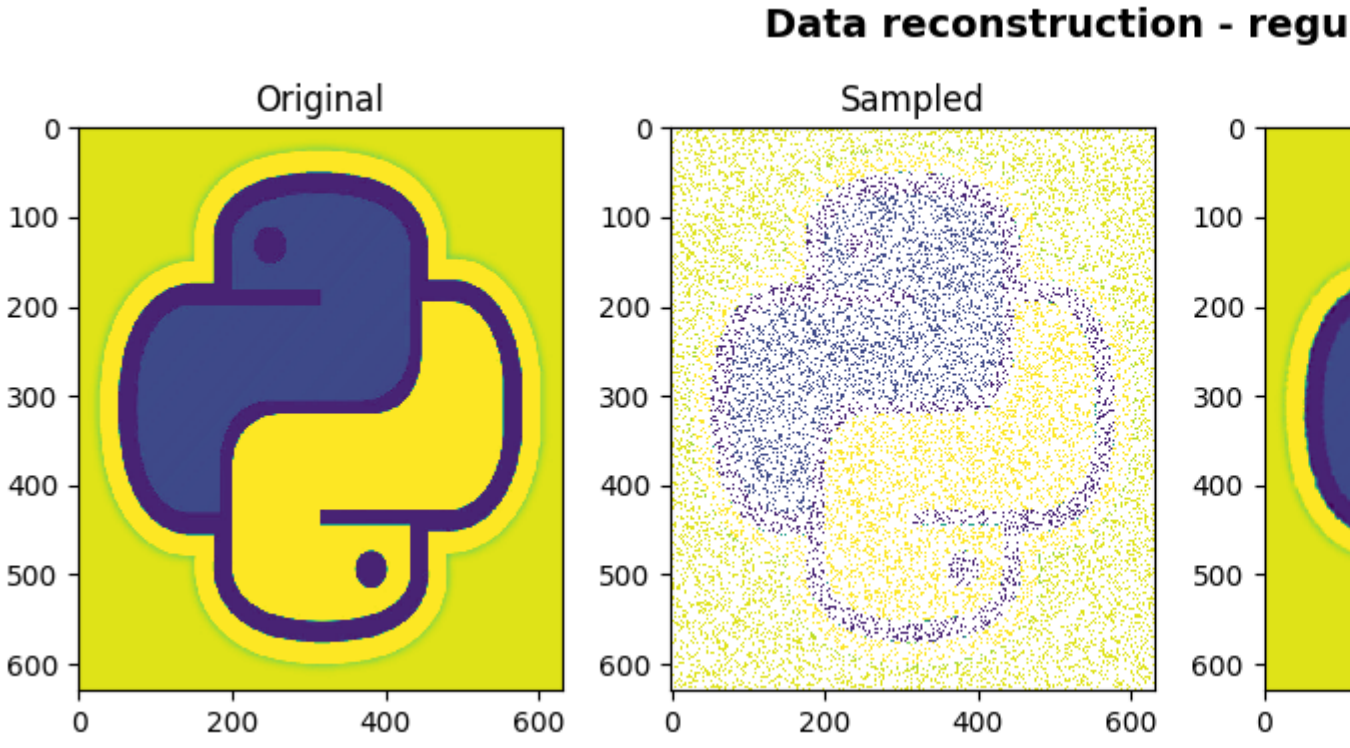
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```
             fontweight='bold', y=0.95)
axs[0].imshow(im, cmap='viridis', vmin=0, vmax=250)
axs[0].axis('tight')
axs[0].set_title('Original')
axs[1].imshow(im_sampled, cmap='viridis', vmin=0, vmax=250)
axs[1].axis('tight')
axs[1].set_title('Sampled')
axs[2].imshow(im_rec_lap_cg, cmap='viridis', vmin=0, vmax=250)
axs[2].axis('tight')
axs[2].set_title('2D Regularization')
axs[3].imshow(im - im_rec_lap_cg, cmap='gray', vmin=-80, vmax=80)
axs[3].axis('tight')
axs[3].set_title('2D Regularization Error')
plt.tight_layout()
plt.subplots_adjust(top=0.8)

fig, axs = plt.subplots(1, 4, figsize=(12, 4))
fig.suptitle('Data reconstruction - regularized eqs', fontsize=14,
             fontweight='bold', y=0.95)
axs[0].imshow(im, cmap='viridis', vmin=0, vmax=250)
axs[0].axis('tight')
axs[0].set_title('Original')
axs[1].imshow(im_sampled, cmap='viridis', vmin=0, vmax=250)
axs[1].axis('tight')
axs[1].set_title('Sampled')
axs[2].imshow(im_rec_lap_lsqr, cmap='viridis', vmin=0, vmax=250)
axs[2].axis('tight')
axs[2].set_title('2D Regularization')
axs[3].imshow(im - im_rec_lap_lsqr, cmap='gray', vmin=-80, vmax=80)
axs[3].axis('tight')
axs[3].set_title('2D Regularization Error')
plt.tight_layout()
plt.subplots_adjust(top=0.8)
```





Total running time of the script: (0 minutes 17.910 seconds)

Note: Click *here* to download the full example code

3.4.6 06. Post-stack inversion

Estimating subsurface properties from band-limited seismic data represents an important task for geophysical subsurface characterization.

In this tutorial, the *pylops.avo.poststack.PoststackLinearModelling* operator is used for modelling of both 1d and 2d synthetic post-stack seismic data from a profile or 2d model of the subsurface acoustic impedence.

$$d(t, \theta = 0) = w(t) * \frac{d \ln(AI(t))}{dt}$$

where $AI(t)$ is the acoustic impedance profile and $w(t)$ is the time domain seismic wavelet. In compact form:

$$\mathbf{d} = \mathbf{WDai}$$

where $\mathbf{W}$ is a convolution operator, $\mathbf{D}$ is a first derivative operator, and $\mathbf{ai}$ is the input model. Subsequently the acoustic impedance model is estimated via the *pylops.avo.poststack.PoststackInversion* module. A two-steps inversion strategy is finally presented to deal with the case of noisy data.

```
# sphinx_gallery_thumbnail_number = 4
import numpy as np
import matplotlib.pyplot as plt
from scipy.signal import filtfilt

import pylops
from pylops.utils.wavelets import ricker

plt.close('all')
np.random.seed(10)
```

Let's start with a 1d example. A synthetic profile of acoustic impedance is created and data is modelled using both the dense and linear operator version of *pylops.avo.poststack.PoststackLinearModelling* operator.

```
# model
nt0 = 301
dt0 = 0.004
t0 = np.arange(nt0)*dt0
vp = 1200 + np.arange(nt0) + \
     filtfilt(np.ones(5)/5., 1, np.random.normal(0, 80, nt0))
rho = 1000 + vp + \
      filtfilt(np.ones(5)/5., 1, np.random.normal(0, 30, nt0))
vp[131:] += 500
rho[131:] += 100
m = np.log(vp*rho)

# smooth model
nsmooth = 100
mback = filtfilt(np.ones(nsmooth)/float(nsmooth), 1, m)

# wavelet
ntwav = 41
wav, twav, wavc = ricker(t0[:ntwav//2+1], 20)
```

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```
# dense operator
PPop_dense = \
    pylops.avo.poststack.PoststackLinearModelling(wav, nt0=nt0, explicit=True)

# lop operator
PPop = pylops.avo.poststack.PoststackLinearModelling(wav, nt0=nt0)

# data
d_dense = PPop_dense*m.flatten()
d = PPop*m

# add noise
dn_dense = d_dense + np.random.normal(0, 2e-2, d_dense.shape)
```

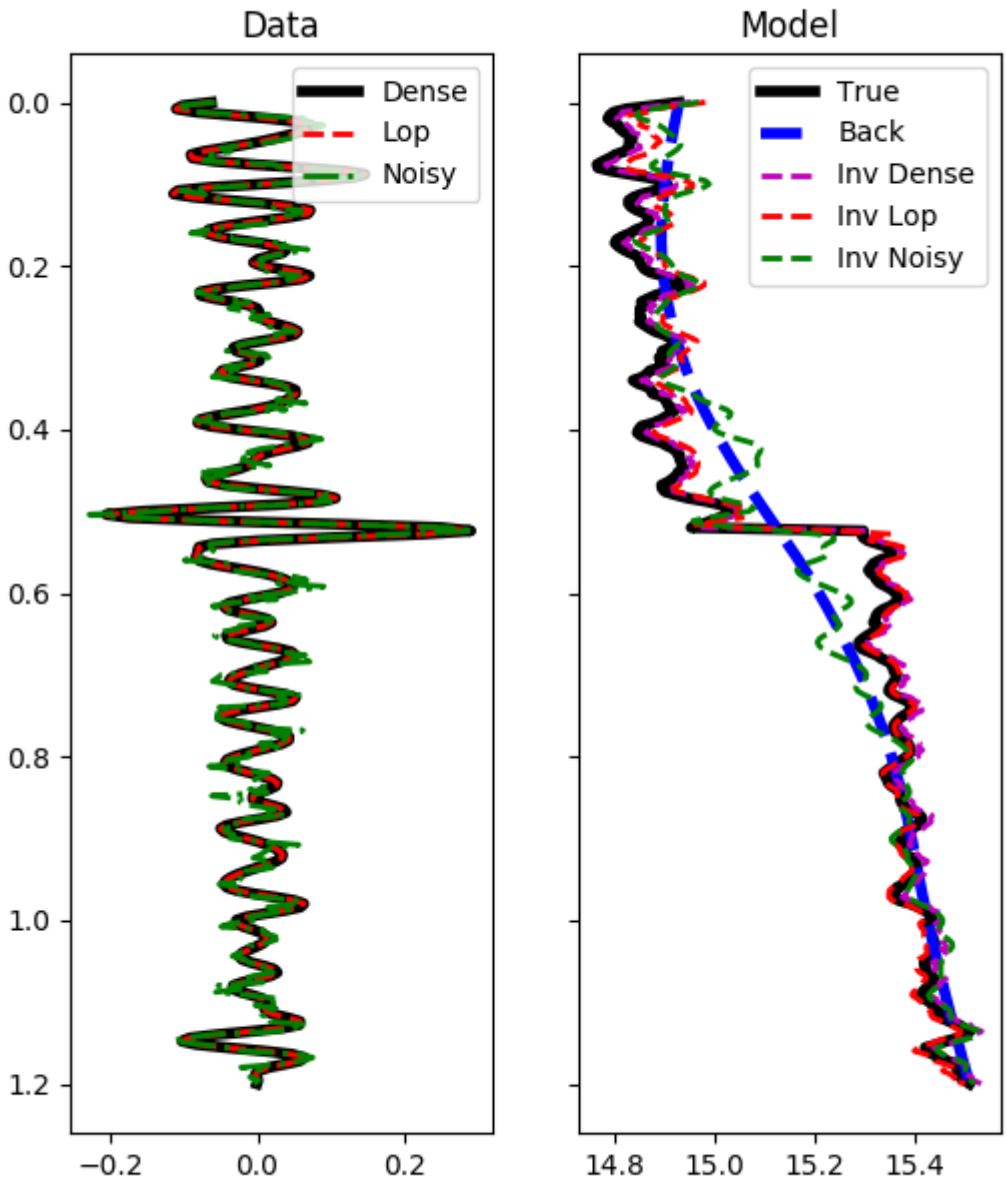
We can now estimate the acoustic profile from band-limited data using either the dense operator or linear operator.

```
# solve dense
minv_dense = \
    pylops.avo.poststack.PoststackInversion(d, wav, m0=mback, explicit=True,
                                            simultaneous=False)[0]

# solve lop
minv = \
    pylops.avo.poststack.PoststackInversion(d_dense, wav, m0=mback,
                                            explicit=False,
                                            simultaneous=False,
                                            **dict(iter_lim=2000))[0]

# solve noisy
mn = \
    pylops.avo.poststack.PoststackInversion(dn_dense, wav, m0=mback,
                                            explicit=True,
                                            epsR=1e0, **dict(damp=1e-1))[0]

fig, axs = plt.subplots(1, 2, figsize=(6, 7), sharey=True)
axs[0].plot(d_dense, t0, 'k', lw=4, label='Dense')
axs[0].plot(d, t0, '--r', lw=2, label='Lop')
axs[0].plot(dn_dense, t0, '-.g', lw=2, label='Noisy')
axs[0].set_title('Data')
axs[0].invert_yaxis()
axs[0].axis('tight')
axs[0].legend(loc=1)
axs[1].plot(m, t0, 'k', lw=4, label='True')
axs[1].plot(mback, t0, '--b', lw=4, label='Back')
axs[1].plot(minv_dense, t0, '--m', lw=2, label='Inv Dense')
axs[1].plot(minv, t0, '--r', lw=2, label='Inv Lop')
axs[1].plot(mn, t0, '--g', lw=2, label='Inv Noisy')
axs[1].set_title('Model')
axs[1].axis('tight')
axs[1].legend(loc=1)
```



We see how inverting a dense matrix is in this case faster than solving for the linear operator (a good estimate of the model is in fact obtained only after 2000 iterations of lsqr). Nevertheless, having a linear operator is useful when we deal with larger dimensions (2d or 3d) and we want to couple our modelling operator with different types of spatial regularizations or preconditioning.

Before we move onto a 2d example, let's consider the case of non-stationary wavelet and see how we can easily use the same routines in this case

```
# wavelet
```

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```
ntwav = 41
f0s = np.flip(np.arange(nt0) * 0.05 + 3)
wavs = np.array([ricker(t0[:ntwav], f0)[0] for f0 in f0s])
wavc = np.argmax(wavs[0])

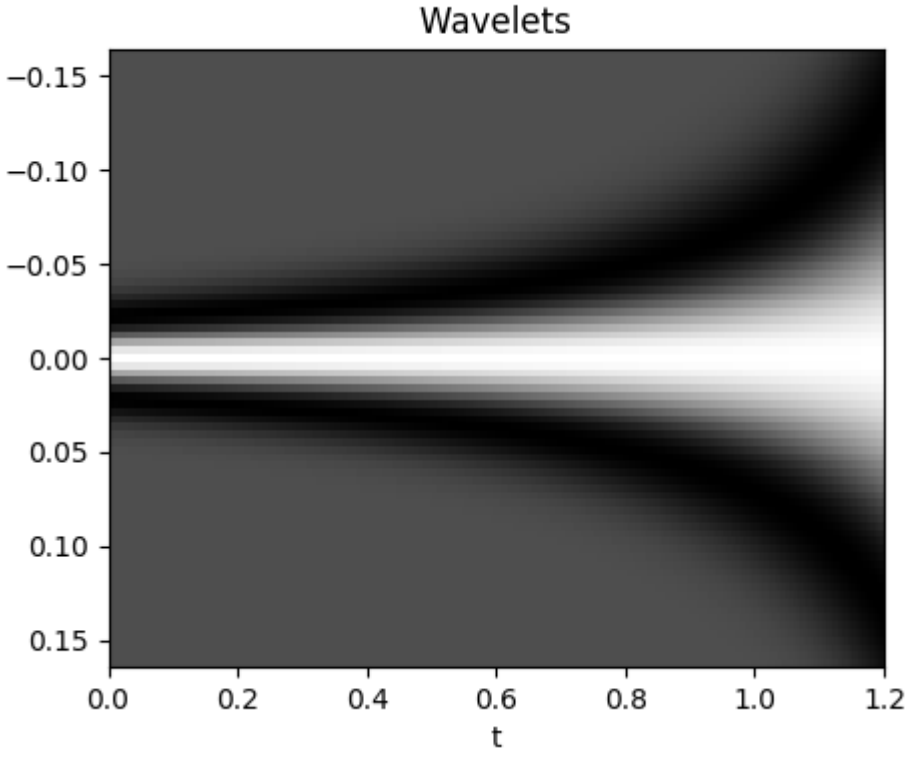
plt.figure(figsize=(5, 4))
plt.imshow(wavs.T, cmap='gray', extent=(t0[0], t0[-1], t0[ntwav], -t0[ntwav]))
plt.xlabel('t')
plt.title('Wavelets')
plt.axis('tight')

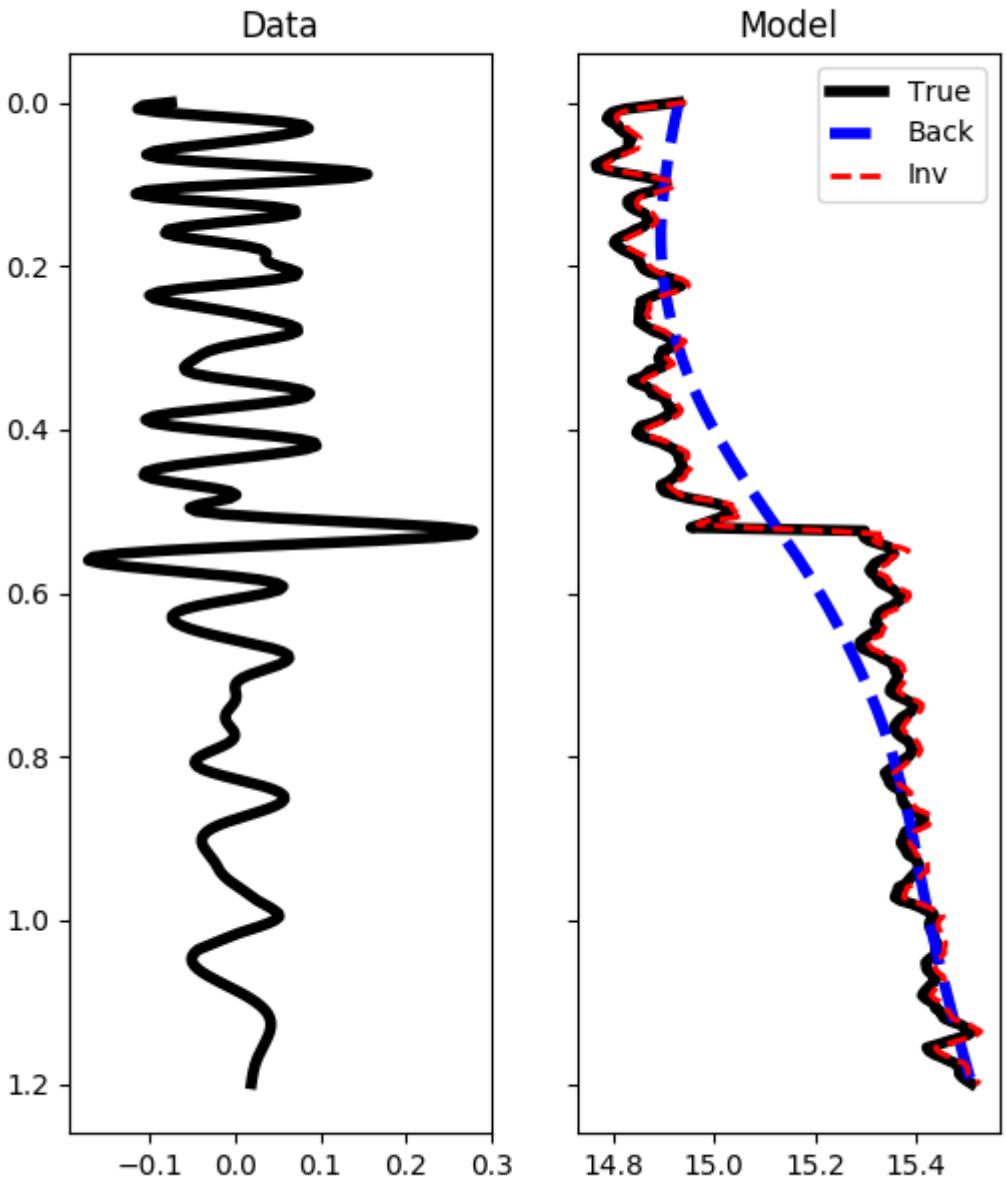
# operator
PPop = \
    pylops.avo.poststack.PoststackLinearModelling(wavs, nt0=nt0, explicit=True)

# data
d = PPop * m

# solve
minv = \
    pylops.avo.poststack.PoststackInversion(d, wavs, m0=mback, explicit=True,
                                            **dict(cond=1e-10))[0]

fig, axs = plt.subplots(1, 2, figsize=(6, 7), sharey=True)
axs[0].plot(d, t0, 'k', lw=4)
axs[0].set_title('Data')
axs[0].invert_yaxis()
axs[0].axis('tight')
axs[1].plot(m, t0, 'k', lw=4, label='True')
axs[1].plot(mback, t0, '--b', lw=4, label='Back')
axs[1].plot(minv, t0, '--r', lw=2, label='Inv')
axs[1].set_title('Model')
axs[1].axis('tight')
axs[1].legend(loc=1)
```


Wavelets
−0.15
−0.10
−0.05
0.00
0.05
0.10
0.15
0.0
0.2
0.4
0.6
0.8
1.0
1.2
t



•

We move now to a 2d example. First of all the model is loaded and data generated.

```
# model
inputfile = '../testdata/avo/poststack_model.npz'

model = np.load(inputfile)
m = np.log(model['model'][:, ::3])
x, z = model['x'][::3]/1000., model['z']/1000.
nx, nz = len(x), len(z)
```

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```
# smooth model
nsmoothz, nsmoothx = 60, 50
mback = filtfilt(np.ones(nsmoothz)/float(nsmoothz), 1, m, axis=0)
mback = filtfilt(np.ones(nsmoothx)/float(nsmoothx), 1, mback, axis=1)

# dense operator
PPop_dense = \
    pylops.avo.poststack.PoststackLinearModelling(wav, nt0=nz,
                                                  spatdims=nx, explicit=True)

# lop operator
PPop = pylops.avo.poststack.PoststackLinearModelling(wav, nt0=nz,
                                                     spatdims=nx)

# data
d = (PPop_dense * m.flatten()).reshape(nz, nx)
n = np.random.normal(0, 1e-1, d.shape)
dn = d + n
```

Finally we perform 4 different inversions:

- trace-by-trace inversion with explicit solver and dense operator with noise-free data
- trace-by-trace inversion with explicit solver and dense operator with noisy data
- multi-trace regularized inversion with iterative solver and linear operator using the result of trace-by-trace inversion as starting guess

$$J = ||\Delta\mathbf{d} - \mathbf{W}\Delta\mathbf{ai}||_2 + \epsilon_\nabla^2||\nabla\mathbf{ai}||_2$$

  where $\Delta\mathbf{d} = \mathbf{d} - \mathbf{WAI_0}$ is the residual data

- multi-trace blocky inversion with iterative solver and linear operator

```
# dense inversion with noise-free data
minv_dense = \
    pylops.avo.poststack.PoststackInversion(d, wav, m0=mback,
                                            explicit=True,
                                            simultaneous=False)[0]

# dense inversion with noisy data
minv_dense_noisy = \
    pylops.avo.poststack.PoststackInversion(dn, wav, m0=mback,
                                            explicit=True, epsI=4e-2,
                                            simultaneous=False)[0]

# spatially regularized lop inversion with noisy data
minv_lop_reg = \
    pylops.avo.poststack.PoststackInversion(dn, wav, m0=minv_dense_noisy,
                                            explicit=False, epsR=5e1,
                                            **dict(damp=np.sqrt(1e-4),
                                                   iter_lim=80))[0]

# blockiness promoting inversion with noisy data
minv_lop_blocky = \
    pylops.avo.poststack.PoststackInversion(dn, wav, m0=mback,
                                            explicit=False,
```

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```
                                            epsR=[0.4], epsRL1=[0.1],
                                            **dict(mu=0.1,
                                                   niter_outer=5,
                                                   niter_inner=10,
                                                   iter_lim=5, damp=1e-3))[0]

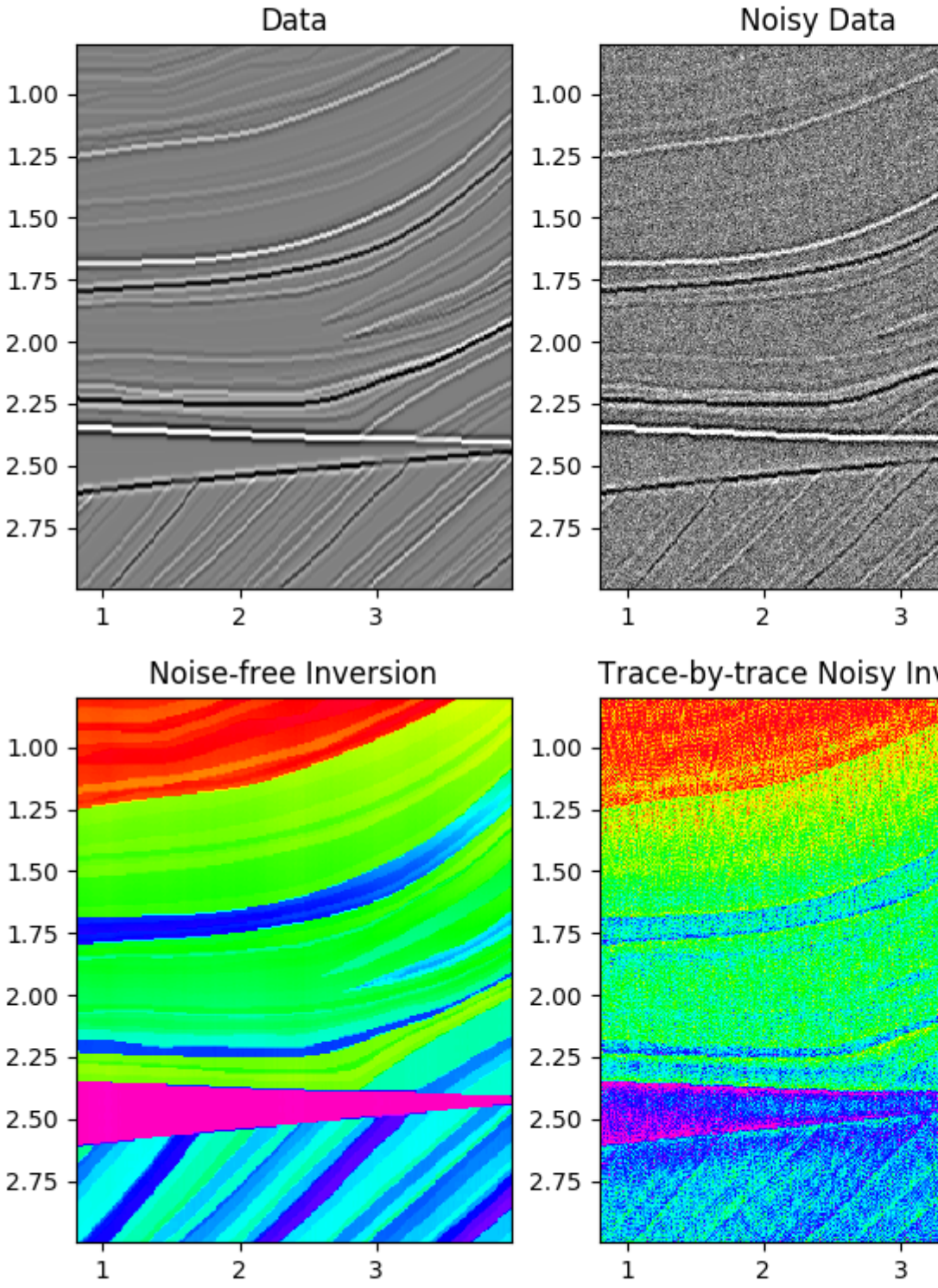
fig, axs = plt.subplots(2, 4, figsize=(15, 9))
axs[0][0].imshow(d, cmap='gray',
                 extent=(x[0], x[-1], z[-1], z[0]),
                 vmin=-0.4, vmax=0.4)
axs[0][0].set_title('Data')
axs[0][0].axis('tight')
axs[0][1].imshow(dn, cmap='gray',
                 extent=(x[0], x[-1], z[-1], z[0]),
                 vmin=-0.4, vmax=0.4)
axs[0][1].set_title('Noisy Data')
axs[0][1].axis('tight')
axs[0][2].imshow(m, cmap='gist_rainbow',
                 extent=(x[0], x[-1], z[-1], z[0]),
                 vmin=m.min(), vmax=m.max())
axs[0][2].set_title('Model')
axs[0][2].axis('tight')
axs[0][3].imshow(mback, cmap='gist_rainbow',
                 extent=(x[0], x[-1], z[-1], z[0]),
                 vmin=m.min(), vmax=m.max())
axs[0][3].set_title('Smooth Model')
axs[0][3].axis('tight')
axs[1][0].imshow(minv_dense, cmap='gist_rainbow',
                 extent=(x[0], x[-1], z[-1], z[0]),
                 vmin=m.min(), vmax=m.max())
axs[1][0].set_title('Noise-free Inversion')
axs[1][0].axis('tight')
axs[1][1].imshow(minv_dense_noisy, cmap='gist_rainbow',
                 extent=(x[0], x[-1], z[-1], z[0]),
                 vmin=m.min(), vmax=m.max())
axs[1][1].set_title('Trace-by-trace Noisy Inversion')
axs[1][1].axis('tight')
axs[1][2].imshow(minv_lop_reg, cmap='gist_rainbow',
                 extent=(x[0], x[-1], z[-1], z[0]),
                 vmin=m.min(), vmax=m.max())
axs[1][2].set_title('Regularized Noisy Inversion - lop ')
axs[1][2].axis('tight')
axs[1][3].imshow(minv_lop_blocky, cmap='gist_rainbow',
                 extent=(x[0], x[-1], z[-1], z[0]),
                 vmin=m.min(), vmax=m.max())
axs[1][3].set_title('Blocky Noisy Inversion - lop ')
axs[1][3].axis('tight')

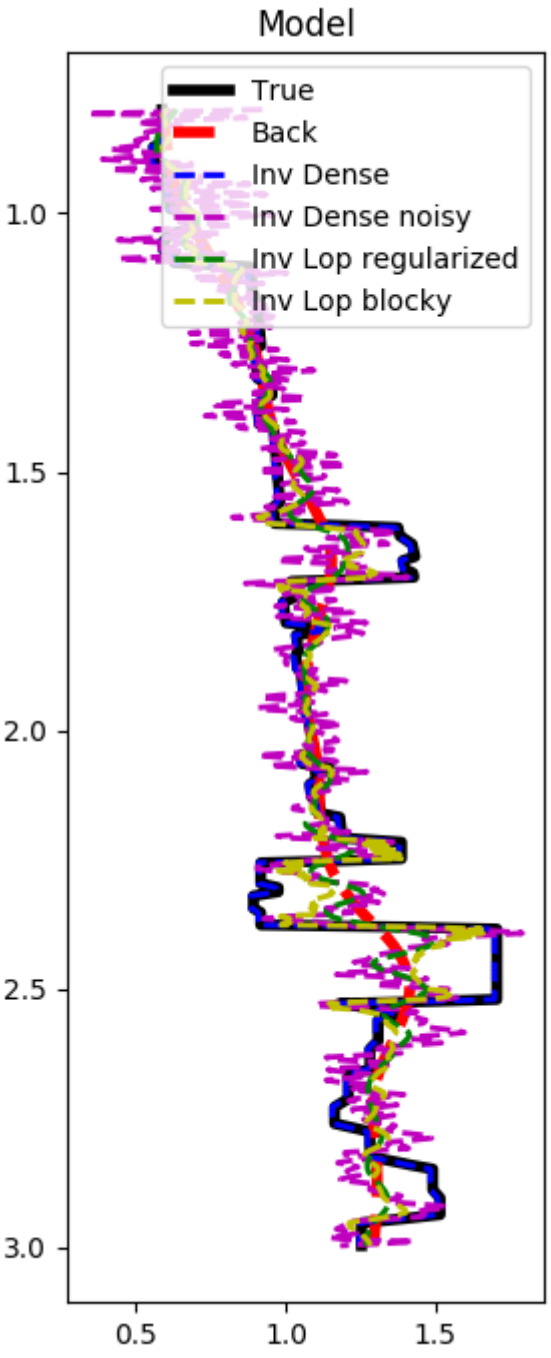
fig, ax = plt.subplots(1, 1, figsize=(3, 7))
ax.plot(m[:, nx//2], z, 'k', lw=4, label='True')
ax.plot(mback[:, nx//2], z, '--r', lw=4, label='Back')
ax.plot(minv_dense[:, nx//2], z, '--b', lw=2, label='Inv Dense')
ax.plot(minv_dense_noisy[:, nx//2], z, '--m', lw=2, label='Inv Dense noisy')
ax.plot(minv_lop_reg[:, nx//2], z, '--g', lw=2, label='Inv Lop regularized')
ax.plot(minv_lop_blocky[:, nx//2], z, '--y', lw=2, label='Inv Lop blocky')
ax.set_title('Model')
ax.invert_yaxis()
```

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```
ax.axis('tight')
ax.legend()
plt.tight_layout()
```


Data
Noisy Data
Noise-free Inversion
Trace-by-trace Noisy Inv
1.00
1.25
1.50
1.75
2.00
2.25
2.50
2.75
1
2
3



That's almost it. If you wonder how this can be applied to real data, head over to the following notebook where the open-source segyio library is used alongside pylops to create an end-to-end open-source seismic inversion workflow with SEG-Y input data.

Total running time of the script: (0 minutes 19.125 seconds)

Note: Click *here* to download the full example code

3.4.7 07. Pre-stack (AVO) inversion

Pre-stack inversion represents one step beyond post-stack inversion in that not only the profile of acoustic impedance can be inferred from seismic data, rather a set of elastic parameters is estimated from pre-stack data (i.e., angle gathers) using the information contained in the so-called AVO (amplitude versus offset) response. Such elastic parameters represent vital information for more sophisticated geophysical subsurface characterization than it would be possible to achieve working with post-stack seismic data.

In this tutorial, the *pylops.avo.prestack.PrestackLinearModelling* operator is used for modelling of both 1d and 2d synthetic pre-stack seismic data using 1d profiles or 2d models of different subsurface elastic parameters (P-wave velocity, S-wave velocity, and density) as input.

$$d(t, \theta) = w(t) * \sum_{i=1}^{N} G_i(t, \theta) \frac{d \ln(m_i(t))}{dt}$$

where $\mathbf{m}(t) = [V_P(t), V_S(t), \rho(t)]$ is a vector containing three elastic parameters at time t, $G_i(t, \theta)$ are the coefficients of the AVO parametrization used to model pre-stack data and $w(t)$ is the time domain seismic wavelet. In compact form:

$$\mathbf{d} = \mathbf{WGDm}$$

where $\mathbf{W}$ is a convolution operator, $\mathbf{G}$ is the AVO modelling operator, $\mathbf{D}$ is a block-diagonal derivative operator, and $\mathbf{m}$ is the input model. Subsequently the elastic parameters are estimated via the *pylops.avo.prestack.PrestackInversion* module. Once again, a two-steps inversion strategy can also be used to deal with the case of noisy data.

```
import numpy as np
import matplotlib.pyplot as plt
from scipy.signal import filtfilt

import pylops
from pylops.utils.wavelets import ricker

plt.close('all')
np.random.seed(0)
```

Let's start with a 1d example. A synthetic profile of acoustic impedance is created and data is modelled using both the dense and linear operator version of *pylops.avo.prestack.PrestackLinearModelling* operator

```
# sphinx_gallery_thumbnail_number = 5

# model
nt0 = 301
dt0 = 0.004

t0 = np.arange(nt0)*dt0
vp = 1200 + np.arange(nt0) + \
     filtfilt(np.ones(5)/5., 1, np.random.normal(0, 80, nt0))
vs = 600  + vp/2 + \
     filtfilt(np.ones(5)/5., 1, np.random.normal(0, 20, nt0))
rho = 1000 + vp + \
      filtfilt(np.ones(5)/5., 1, np.random.normal(0, 30, nt0))
vp[131:] += 500
vs[131:] += 200
rho[131:] += 100
vsvp = 0.5
```

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```
m = np.stack((np.log(vp), np.log(vs), np.log(rho)), axis=1)

# background model
nsmooth = 50
mback = filtfilt(np.ones(nsmooth)/float(nsmooth), 1, m, axis=0)

# angles
ntheta = 21
thetamin, thetamax = 0, 40
theta = np.linspace(thetamin, thetamax, ntheta)

# wavelet
ntwav = 41
wav = ricker(t0[:ntwav//2+1], 15)[0]

# lop
PPop = \
    pylops.avo.prestack.PrestackLinearModelling(wav, theta,
                                                vsvp=vsvp, nt0=nt0,
                                                linearization='akirich')

# dense
PPop_dense = \
    pylops.avo.prestack.PrestackLinearModelling(wav, theta,
                                                vsvp=vsvp, nt0=nt0,
                                                linearization='akirich',
                                                explicit=True)

# data lop
dPP = PPop*m.flatten()
dPP = dPP.reshape(nt0, ntheta)

# data dense
dPP_dense = PPop_dense*m.T.flatten()
dPP_dense = dPP_dense.reshape(ntheta, nt0).T

# noisy data
dPPn_dense = dPP_dense + np.random.normal(0, 1e-2, dPP_dense.shape)
```

We can now invert our data and retreive elastic profiles for both noise-free and noisy data using *pylops.avo.prestack.PrestackInversion*.

```
# dense
minv_dense, dPP_dense_res = \
    pylops.avo.prestack.PrestackInversion(dPP_dense, theta, wav, m0=mback,
                                          linearization='akirich',
                                          explicit=True, returnres=True,
                                          **dict(cond=1e-10))

# lop
minv, dPP_res = \
    pylops.avo.prestack.PrestackInversion(dPP, theta, wav, m0=mback,
                                          linearization='akirich',
                                          explicit=False, returnres=True,
                                          **dict(damp=1e-10, iter_lim=2000))
```

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```
# dense noisy
minv_dense_noise, dPPn_dense_res = \
    pylops.avo.prestack.PrestackInversion(dPPn_dense, theta, wav, m0=mback,
                                          linearization='akirich',
                                          explicit=True,
                                          returnres=True, **dict(cond=1e-1))

# lop noisy (with vertical smoothing)
minv_noise, dPPn_res = \
    pylops.avo.prestack.PrestackInversion(dPPn_dense, theta, wav, m0=mback,
                                          linearization='akirich',
                                          explicit=False,
                                          epsR=5e-1, returnres=True,
                                          **dict(damp=1e-1, iter_lim=100))
```

The data, inverted models and residuals are now displayed.

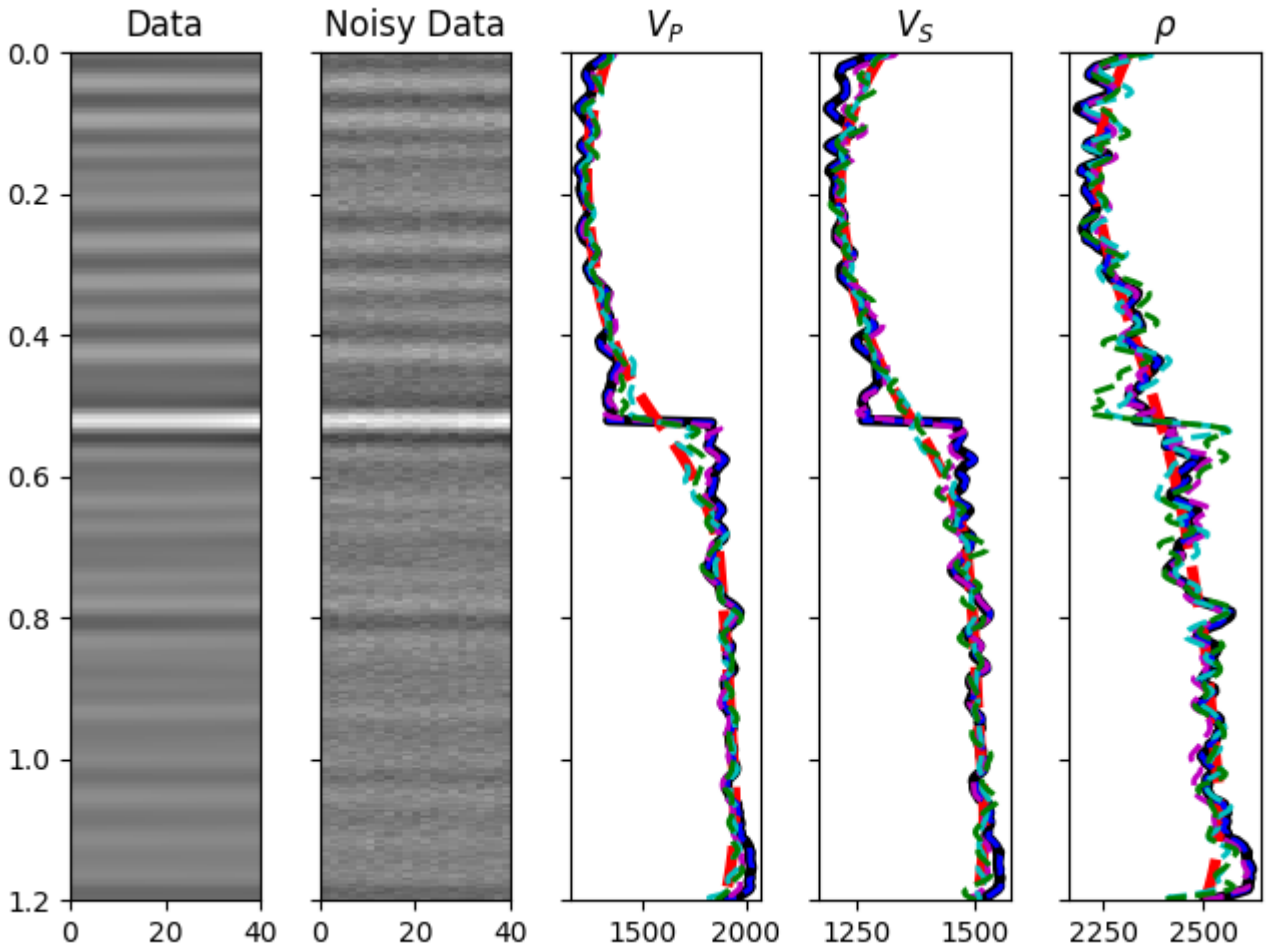
```
# Data and model
fig, (axd, axdn, axvp, axvs, axrho) = \
    plt.subplots(1, 5, figsize=(8, 5), sharey=True)
axd.imshow(dPP_dense, cmap='gray',
           extent=(theta[0], theta[-1], t0[-1], t0[0]),
           vmin=-np.abs(dPP_dense).max(), vmax=np.abs(dPP_dense).max())
axd.set_title('Data')
axd.axis('tight')
axdn.imshow(dPPn_dense, cmap='gray',
            extent=(theta[0], theta[-1], t0[-1], t0[0]),
            vmin=-np.abs(dPP_dense).max(), vmax=np.abs(dPP_dense).max())
axdn.set_title('Noisy Data')
axdn.axis('tight')
axvp.plot(vp, t0, 'k', lw=4, label='True')
axvp.plot(np.exp(mback[:, 0]), t0, '--r', lw=4, label='Back')
axvp.plot(np.exp(minv_dense[:, 0]), t0, '--b', lw=2, label='Inv Dense')
axvp.plot(np.exp(minv[:, 0]), t0, '--m', lw=2, label='Inv Lop')
axvp.plot(np.exp(minv_dense_noise[:, 0]), t0, '--c', lw=2, label='Noisy Dense')
axvp.plot(np.exp(minv_noise[:, 0]), t0, '--g', lw=2, label='Noisy Lop')
axvp.set_title(r'$V_P$')
axvs.plot(vs, t0, 'k', lw=4, label='True')
axvs.plot(np.exp(mback[:, 1]), t0, '--r', lw=4, label='Back')
axvs.plot(np.exp(minv_dense[:, 1]), t0, '--b', lw=2, label='Inv Dense')
axvs.plot(np.exp(minv[:, 1]), t0, '--m', lw=2, label='Inv Lop')
axvs.plot(np.exp(minv_dense_noise[:, 1]), t0, '--c', lw=2, label='Noisy Dense')
axvs.plot(np.exp(minv_noise[:, 1]), t0, '--g', lw=2, label='Noisy Lop')
axvs.set_title(r'$V_S$')
axrho.plot(rho, t0, 'k', lw=4, label='True')
axrho.plot(np.exp(mback[:, 2]), t0, '--r', lw=4, label='Back')
axrho.plot(np.exp(minv_dense[:, 2]), t0, '--b', lw=2, label='Inv Dense')
axrho.plot(np.exp(minv[:, 2]), t0, '--m', lw=2, label='Inv Lop')
axrho.plot(np.exp(minv_dense_noise[:, 2]),
           t0, '--c', lw=2, label='Noisy Dense')
axrho.plot(np.exp(minv_noise[:, 2]), t0, '--g', lw=2, label='Noisy Lop')
axrho.set_title(r'$\rho$')
axrho.legend(loc='center left', bbox_to_anchor=(1, 0.5))
axd.axis('tight')
plt.tight_layout()

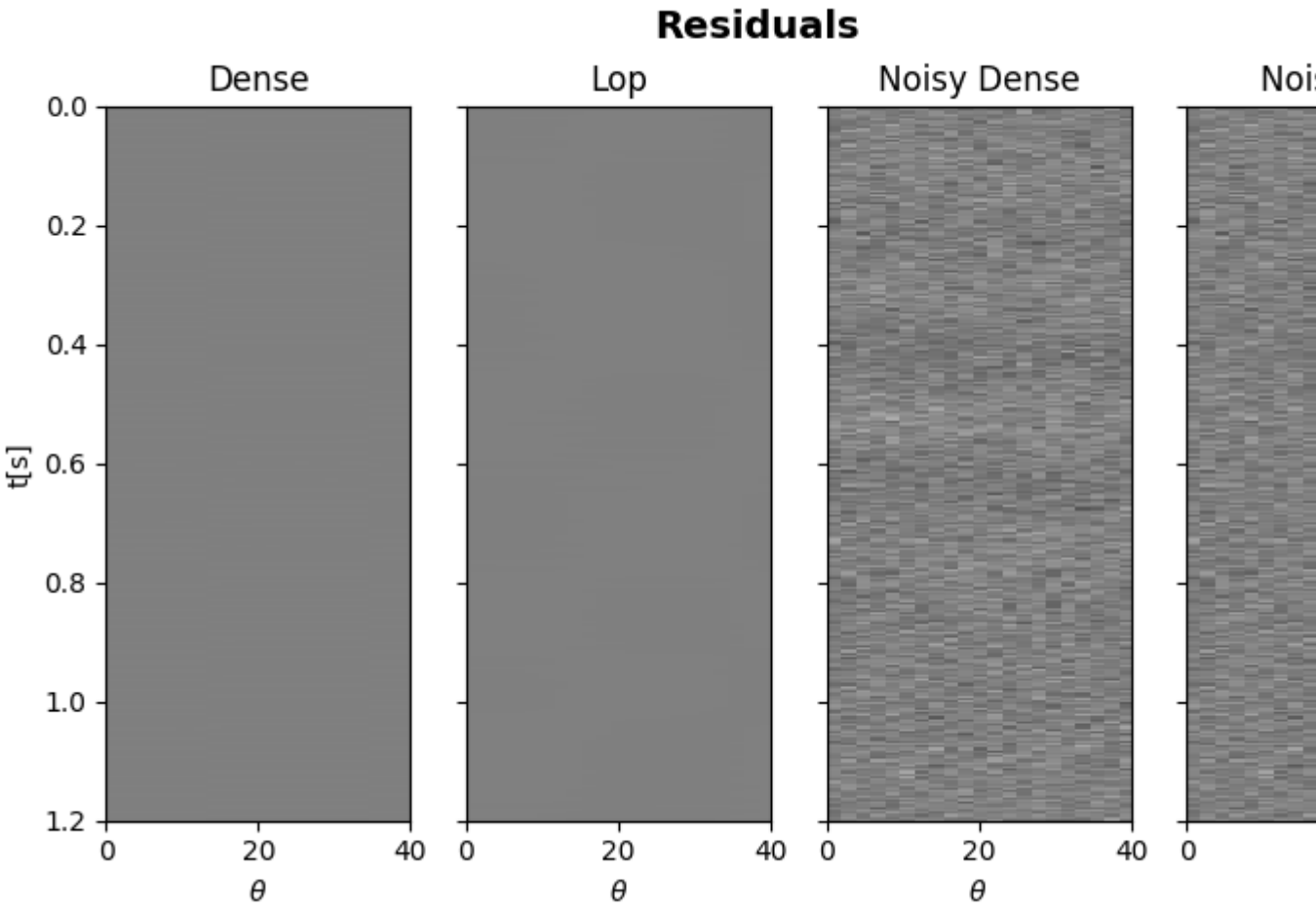
# Residuals
```

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```
fig, axs = plt.subplots(1, 4, figsize=(8, 5), sharey=True)
fig.suptitle('Residuals', fontsize=14,
             fontweight='bold', y=0.95)
im = axs[0].imshow(dPP_dense_res, cmap='gray',
                   extent=(theta[0], theta[-1], t0[-1], t0[0]),
                   vmin=-0.1, vmax=0.1)
axs[0].set_title('Dense')
axs[0].set_xlabel(r'$\theta$')
axs[0].set_ylabel('t[s]')
axs[0].axis('tight')
axs[1].imshow(dPP_res, cmap='gray',
              extent=(theta[0], theta[-1], t0[-1], t0[0]),
              vmin=-0.1, vmax=0.1)
axs[1].set_title('Lop')
axs[1].set_xlabel(r'$\theta$')
axs[1].axis('tight')
axs[2].imshow(dPPn_dense_res, cmap='gray',
              extent=(theta[0], theta[-1], t0[-1], t0[0]),
              vmin=-0.1, vmax=0.1)
axs[2].set_title('Noisy Dense')
axs[2].set_xlabel(r'$\theta$')
axs[2].axis('tight')
axs[3].imshow(dPPn_res, cmap='gray',
              extent=(theta[0], theta[-1], t0[-1], t0[0]),
              vmin=-0.1, vmax=0.1)
axs[3].set_title('Noisy Lop')
axs[3].set_xlabel(r'$\theta$')
axs[3].axis('tight')
plt.tight_layout()
plt.subplots_adjust(top=0.85)
```


Data
Noisy Data
V_P
V_S
ρ
0.0
0.2
0.4
0.6
0.8
1.0
1.2
0
20
40
0
20
40
1500
2000
1250
1500
2250
2500



We move now to a 2d example. First of all the model is loaded and data generated.

```
# model
inputfile = '../testdata/avo/poststack_model.npz'

model = np.load(inputfile)
x, z = model['x'][::6]/1000., model['z'][:300]/1000.
nx, nz = len(x), len(z)
m = 1000*model['model'][:300, ::6]

mvp = m.copy()
mvs = m/2
mrho = m/3+300
m = np.log(np.stack((mvp, mvs, mrho), axis=1))

# smooth model
nsmoothz, nsmoothx = 30, 25
mback = filtfilt(np.ones(nsmoothz)/float(nsmoothz), 1, m, axis=0)
mback = filtfilt(np.ones(nsmoothx)/float(nsmoothx), 1, mback, axis=2)

# dense operator
PPop_dense = \
    pylops.avo.prestack.PrestackLinearModelling(wav, theta, vsvp=vsvp,
```

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```
                                                nt0=nz, spatdims=(nx,),
                                                linearization='akirich',
                                                explicit=True)

# lop operator
PPop = pylops.avo.prestack.PrestackLinearModelling(wav, theta, vsvp=vsvp,
                                                   nt0=nz, spatdims=(nx,),
                                                   linearization='akirich')

# data
dPP = PPop_dense*m.swapaxes(0, 1).flatten()
dPP = dPP.reshape(ntheta, nz, nx).swapaxes(0, 1)
dPPn = dPP + np.random.normal(0, 5e-2, dPP.shape)
```

Finally we perform the same 4 different inversions as in the post-stack tutorial (see *06. Post-stack inversion* for more details).

```
# dense inversion with noise-free data
minv_dense = \
    pylops.avo.prestack.PrestackInversion(dPP, theta, wav, m0=mback,
                                          explicit=True,
                                          simultaneous=False)

# dense inversion with noisy data
minv_dense_noisy = \
    pylops.avo.prestack.PrestackInversion(dPPn, theta, wav, m0=mback,
                                          explicit=True, epsI=4e-2,
                                          simultaneous=False)

# spatially regularized lop inversion with noisy data
minv_lop_reg = \
    pylops.avo.prestack.PrestackInversion(dPPn, theta, wav,
                                          m0=minv_dense_noisy,
                                          explicit=False, epsR=1e1,
                                          **dict(damp=np.sqrt(1e-4),
                                                 iter_lim=20))

# blockiness promoting inversion with noisy data
minv_blocky = \
    pylops.avo.prestack.PrestackInversion(dPPn, theta, wav,
                                          m0=mback,
                                          explicit=False,
                                          epsR=0.4, epsRL1=0.1,
                                          **dict(mu=0.1,
                                                 niter_outer=3,
                                                 niter_inner=3,
                                                 iter_lim=5, damp=1e-3))
```

Let's now visualize the inverted elastic parameters for the different scenarios

```
def plotmodel(axs, m, x, z, vmin, vmax,
              params=('VP', 'VS', 'Rho'),
              cmap='gist_rainbow', title=None):
    """Quick visualization of model
    """
    for ip, param in enumerate(params):
```

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```
        axs[ip].imshow(m[:, ip],
                       extent=(x[0], x[-1], z[-1], z[0]),
                       vmin=vmin, vmax=vmax, cmap=cmap)
        axs[ip].set_title('%s - %s' %(param, title))
        axs[ip].axis('tight')
    plt.setp(axs[1].get_yticklabels(), visible=False)
    plt.setp(axs[2].get_yticklabels(), visible=False)

# data
fig = plt.figure(figsize=(8, 9))
ax1 = plt.subplot2grid((2, 3), (0, 0), colspan=3)
ax2 = plt.subplot2grid((2, 3), (1, 0))
ax3 = plt.subplot2grid((2, 3), (1, 1), sharey=ax2)
ax4 = plt.subplot2grid((2, 3), (1, 2), sharey=ax2)
ax1.imshow(dPP[:, 0], cmap='gray',
           extent=(x[0], x[-1], z[-1], z[0]),
           vmin=-0.4, vmax=0.4)
ax1.vlines([x[nx//5], x[nx//2], x[4*nx//5]], ymin=z[0], ymax=z[-1],
           colors='w', linestyles='--')
ax1.set_xlabel('x [km]')
ax1.set_ylabel('z [km]')
ax1.set_title(r'Stack ($\theta$=0)')
ax1.axis('tight')
ax2.imshow(dPP[:, :, nx//5], cmap='gray',
           extent=(theta[0], theta[-1], z[-1], z[0]),
           vmin=-0.4, vmax=0.4)
ax2.set_xlabel(r'$\theta$')
ax2.set_ylabel('z [km]')
ax2.set_title(r'Gather (x=%.2f)' % x[nx//5])
ax2.axis('tight')
ax3.imshow(dPP[:, :, nx//2], cmap='gray',
           extent=(theta[0], theta[-1], z[-1], z[0]),
           vmin=-0.4, vmax=0.4)
ax3.set_xlabel(r'$\theta$')
ax3.set_title(r'Gather (x=%.2f)' % x[nx//2])
ax3.axis('tight')
ax4.imshow(dPP[:, :, 4*nx//5], cmap='gray',
           extent=(theta[0], theta[-1], z[-1], z[0]),
           vmin=-0.4, vmax=0.4)
ax4.set_xlabel(r'$\theta$')
ax4.set_title(r'Gather (x=%.2f)' % x[4*nx//5])
ax4.axis('tight')
plt.setp(ax3.get_yticklabels(), visible=False)
plt.setp(ax4.get_yticklabels(), visible=False)

# noisy data
fig = plt.figure(figsize=(8, 9))
ax1 = plt.subplot2grid((2, 3), (0, 0), colspan=3)
ax2 = plt.subplot2grid((2, 3), (1, 0))
ax3 = plt.subplot2grid((2, 3), (1, 1), sharey=ax2)
ax4 = plt.subplot2grid((2, 3), (1, 2), sharey=ax2)
ax1.imshow(dPPn[:, 0], cmap='gray',
           extent=(x[0], x[-1], z[-1], z[0]),
           vmin=-0.4, vmax=0.4)
ax1.vlines([x[nx//5], x[nx//2], x[4*nx//5]], ymin=z[0], ymax=z[-1],
           colors='w', linestyles='--')
ax1.set_xlabel('x [km]')
```

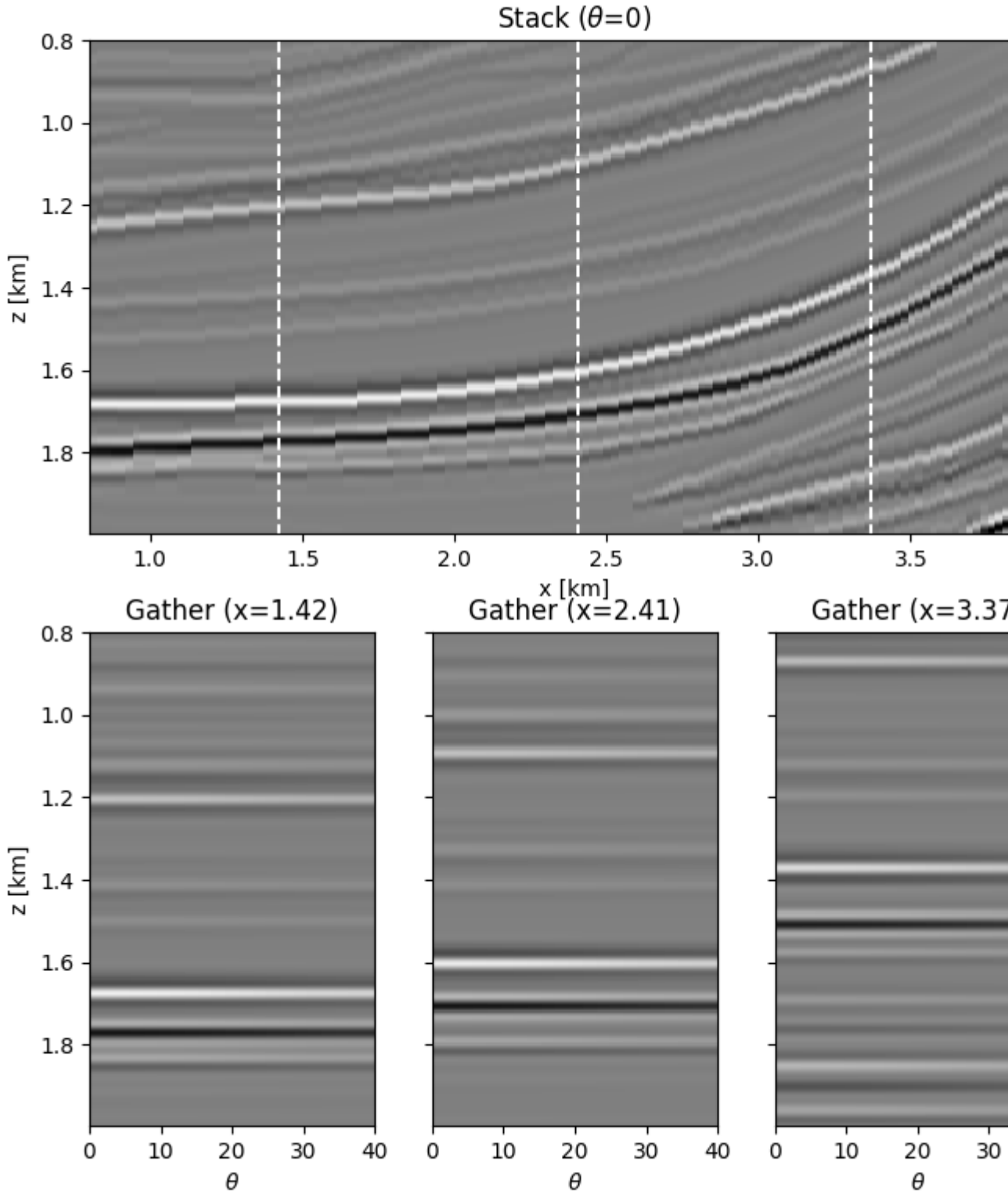
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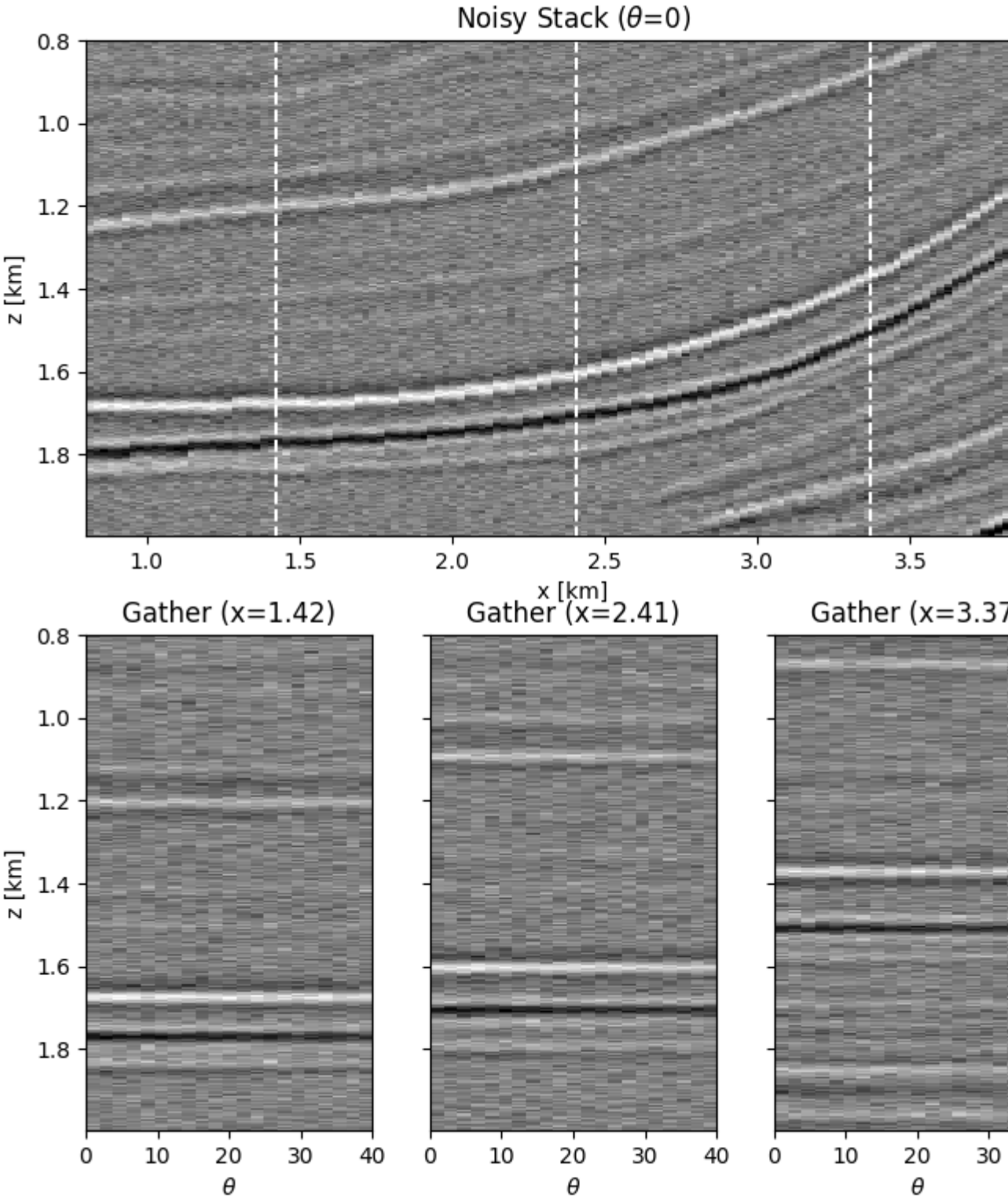
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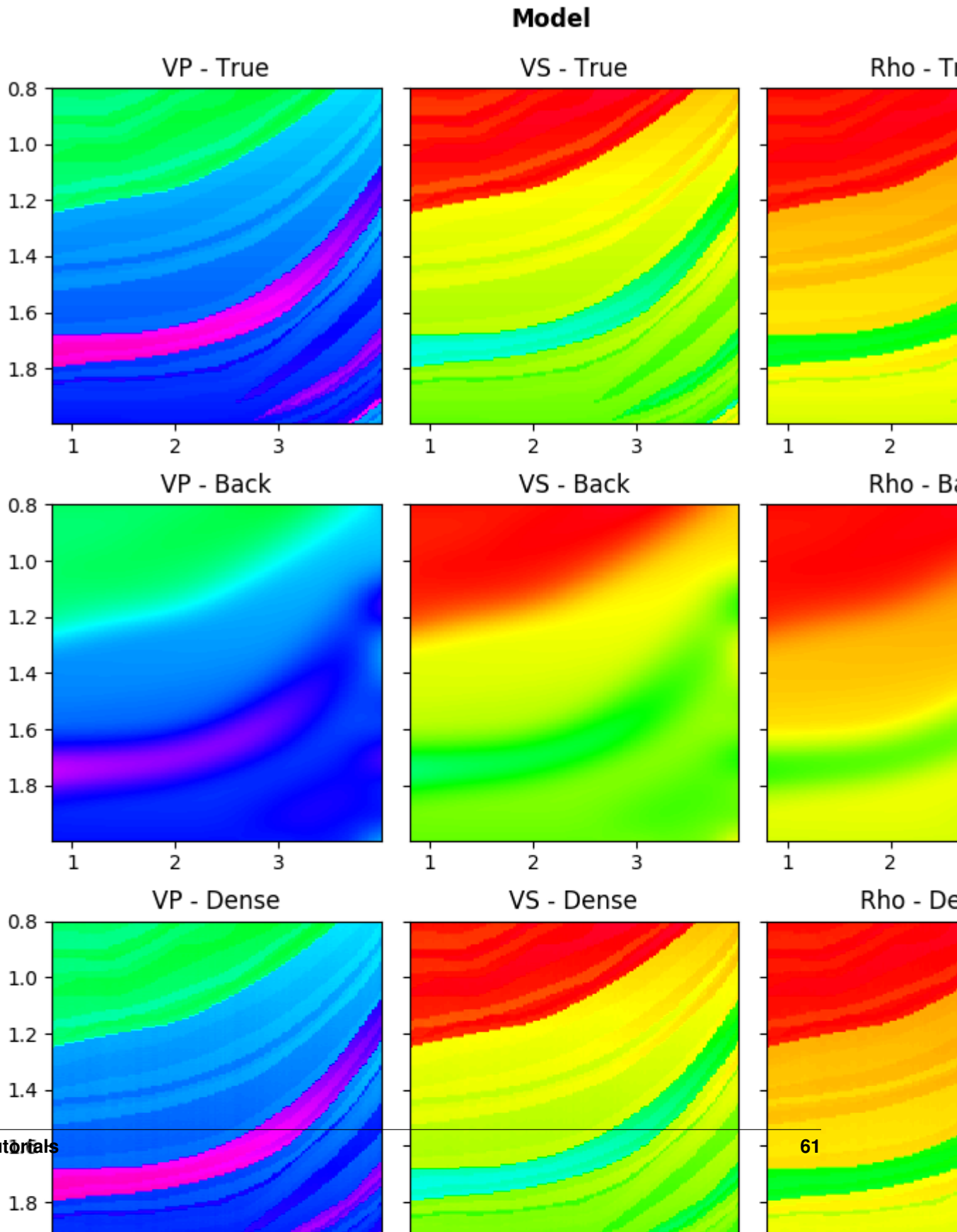
```
ax1.set_ylabel('z [km]')
ax1.set_title(r'Noisy Stack ($\theta$=0)')
ax1.axis('tight')
ax2.imshow(dPPn[:, :, nx//5], cmap='gray',
           extent=(theta[0], theta[-1], z[-1], z[0]),
           vmin=-0.4, vmax=0.4)
ax2.set_xlabel(r'$\theta$')
ax2.set_ylabel('z [km]')
ax2.set_title(r'Gather (x=%.2f)' % x[nx//5])
ax2.axis('tight')
ax3.imshow(dPPn[:, :, nx//2], cmap='gray',
           extent=(theta[0], theta[-1], z[-1], z[0]),
           vmin=-0.4, vmax=0.4)
ax3.set_title(r'Gather (x=%.2f)' % x[nx//2])
ax3.set_xlabel(r'$\theta$')
ax3.axis('tight')
ax4.imshow(dPPn[:, :, 4*nx//5], cmap='gray',
           extent=(theta[0], theta[-1], z[-1], z[0]),
           vmin=-0.4, vmax=0.4)
ax4.set_xlabel(r'$\theta$')
ax4.set_title(r'Gather (x=%.2f)' % x[4*nx//5])
ax4.axis('tight')
plt.setp(ax3.get_yticklabels(), visible=False)
plt.setp(ax4.get_yticklabels(), visible=False)

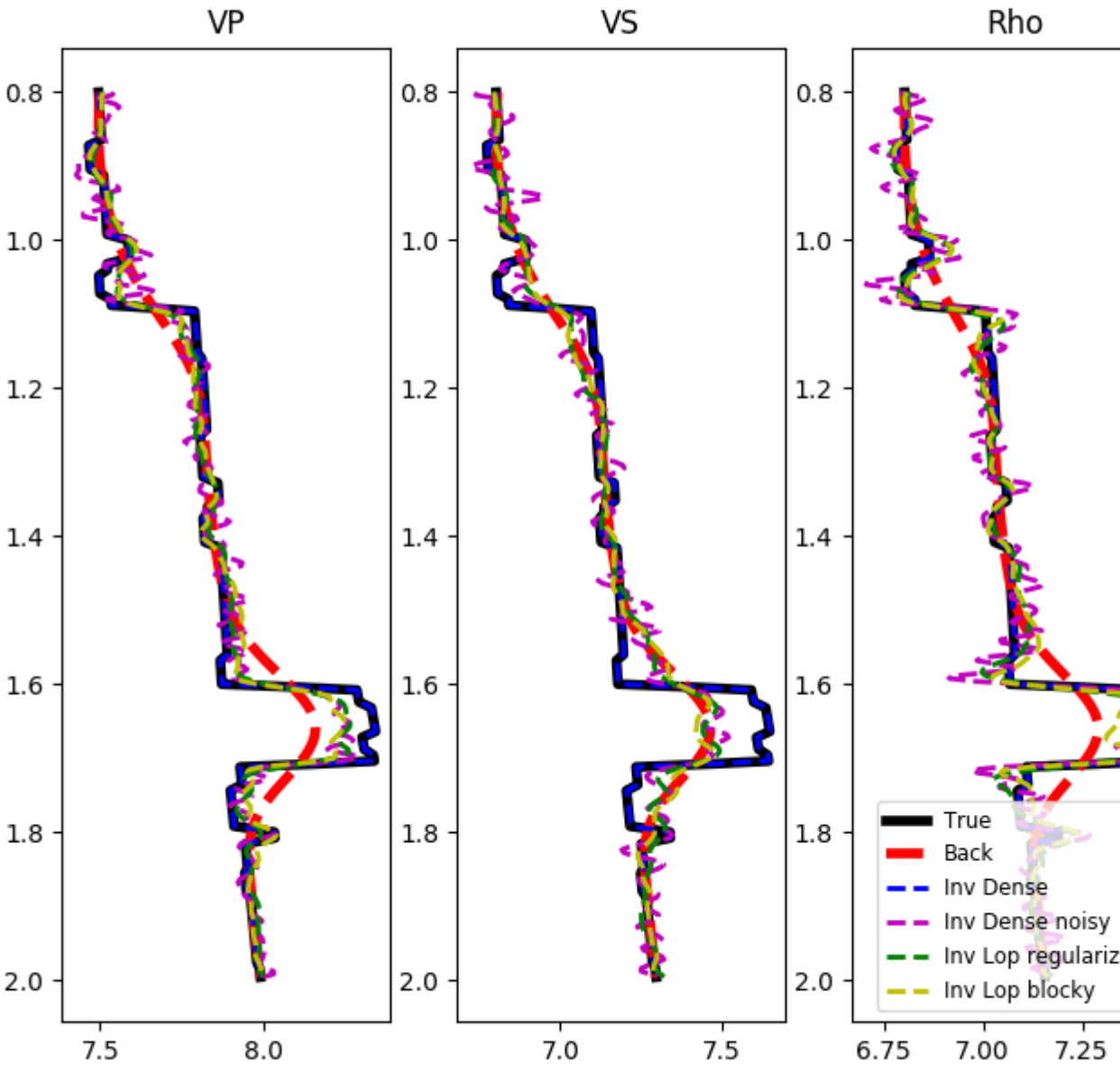
# inverted models
fig, axs = plt.subplots(6, 3, figsize=(8, 19))
fig.suptitle('Model', fontsize=12, fontweight='bold', y=0.95)
plotmodel(axs[0], m, x, z, m.min(),
          m.max(), title='True')
plotmodel(axs[1], mback, x, z, m.min(),
          m.max(), title='Back')
plotmodel(axs[2], minv_dense, x, z,
          m.min(), m.max(), title='Dense')
plotmodel(axs[3], minv_dense_noisy, x, z,
          m.min(), m.max(), title='Dense noisy')
plotmodel(axs[4], minv_lop_reg, x, z,
          m.min(), m.max(), title='Lop regularized')
plotmodel(axs[5], minv_blocky, x, z,
          m.min(), m.max(), title='Lop blocky')
plt.tight_layout()
plt.subplots_adjust(top=0.92)

fig, axs = plt.subplots(1, 3, figsize=(8, 7))
for ip, param in enumerate(['VP', 'VS', 'Rho']):
    axs[ip].plot(m[:, ip, nx//2], z, 'k', lw=4, label='True')
    axs[ip].plot(mback[:, ip, nx//2], z, '--r', lw=4, label='Back')
    axs[ip].plot(minv_dense[:, ip, nx//2], z, '--b', lw=2, label='Inv Dense')
    axs[ip].plot(minv_dense_noisy[:, ip, nx//2], z, '--m', lw=2,
                 label='Inv Dense noisy')
    axs[ip].plot(minv_lop_reg[:, ip, nx//2], z, '--g', lw=2,
                 label='Inv Lop regularized')
    axs[ip].plot(minv_blocky[:, ip, nx // 2], z, '--y', lw=2,
                 label='Inv Lop blocky')
    axs[ip].set_title(param)
    axs[ip].invert_yaxis()
axs[2].legend(loc=8, fontsize='small')
```


Stack (θ=0)
z [km]
x [km]
Gather (x=1.42)
Gather (x=2.41)
Gather (x=3.37
θ


Noisy Stack (θ=0)
z [km]
x [km]
Gather (x=1.42)
Gather (x=2.41)
Gather (x=3.37
θ


Model
VP - True
VS - True
Rho - Tr
VP - Back
VS - Back
Rho - Ba
VP - Dense
VS - Dense
Rho - De
0.8
1.0
1.2
1.4
1.6
1.8
1
2
3



•

While the background model `m0` has been provided in all the examples so far, it is worth showing that the module *pylops.avo.prestack.PrestackInversion* can also produce so-called relative elastic parameters (i.e., variations from an average medium property) when the background model `m0` is not available.

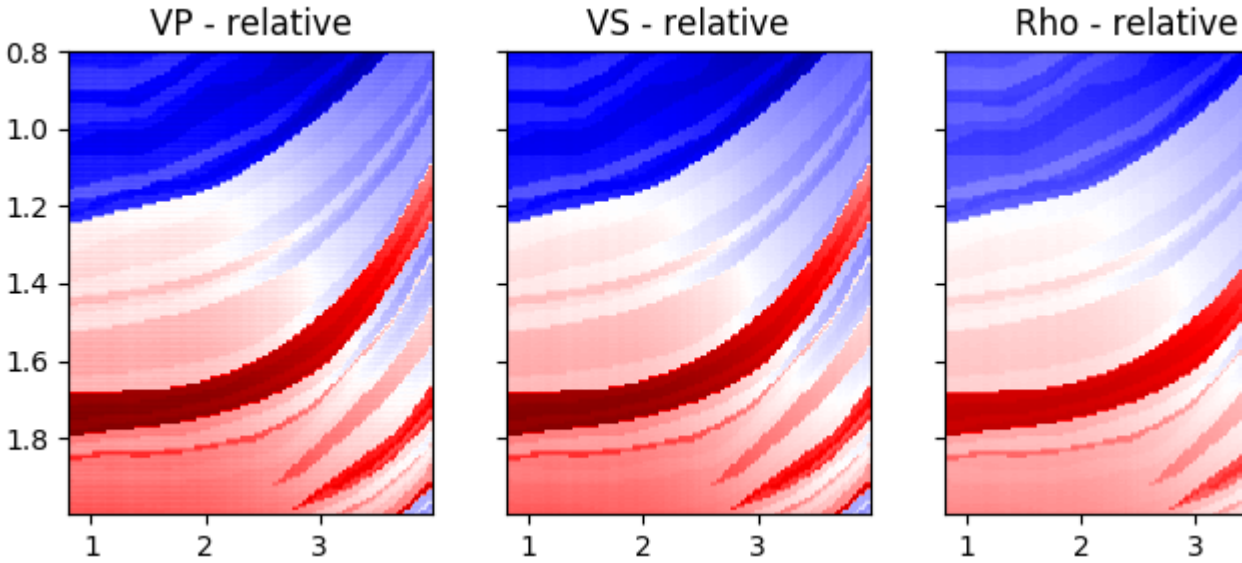
```
dminv = \
    pylops.avo.prestack.PrestackInversion(dPP, theta, wav, m0=None,
                                          explicit=True,
                                          simultaneous=False)
```

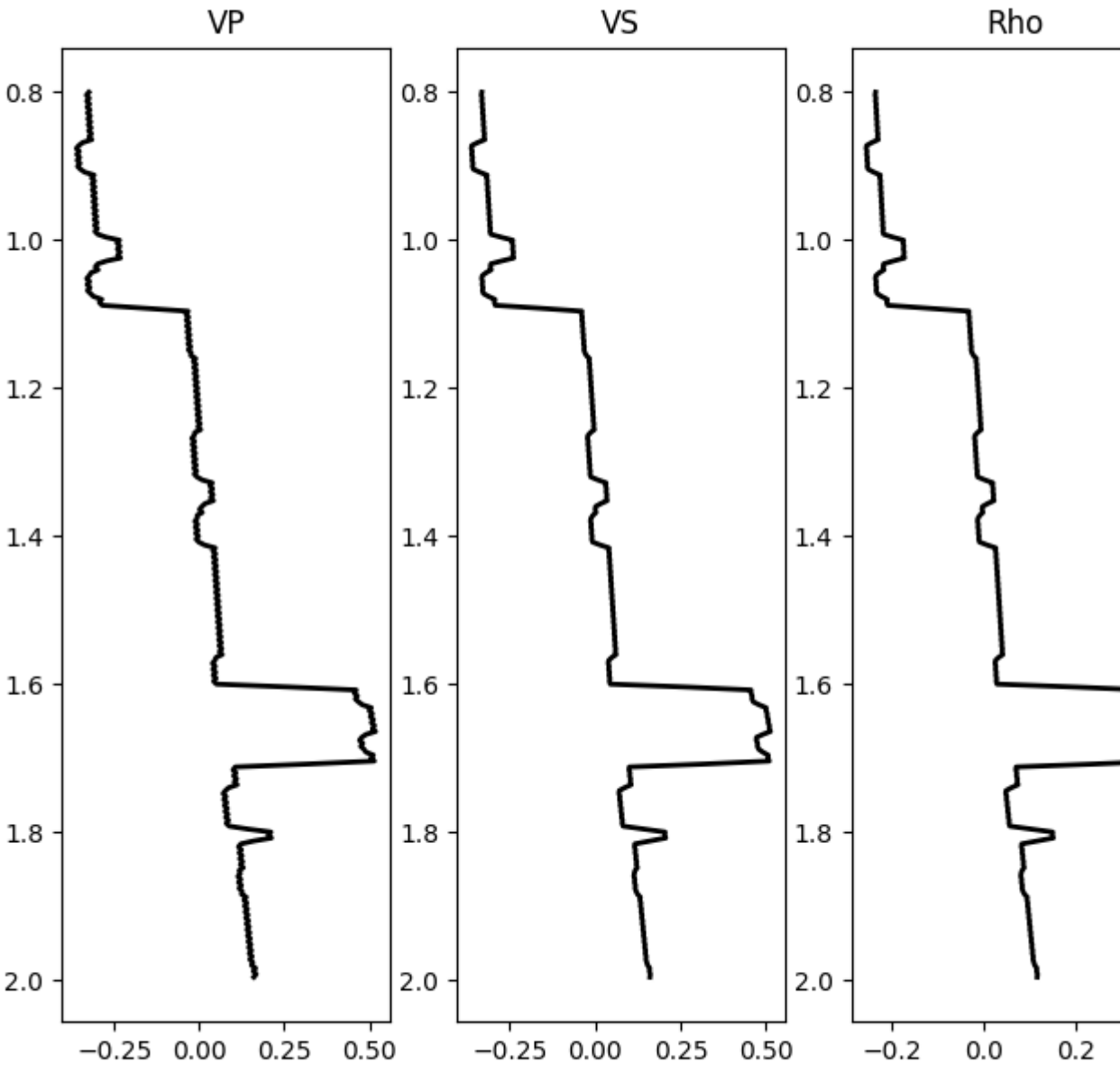
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```
fig, axs = plt.subplots(1, 3, figsize=(8, 3))
plotmodel(axs, dminv, x, z, -dminv.max(), dminv.max(),
          cmap='seismic', title='relative')

fig, axs = plt.subplots(1, 3, figsize=(8, 7))
for ip, param in enumerate(['VP', 'VS', 'Rho']):
    axs[ip].plot(dminv[:, ip, nx//2], z, 'k', lw=2)
    axs[ip].set_title(param)
    axs[ip].invert_yaxis()
```





•

Total running time of the script: (1 minutes 17.086 seconds)

Note: Click *here* to download the full example code

3.4.8 08. Multi-Dimensional Deconvolution

This example shows how to set-up and run the `pylops.waveeqprocessing.MDD` inversion using synthetic data.

```
import warnings
import numpy as np
import matplotlib.pyplot as plt
import pylops

from pylops.utils.tapers import taper3d
from pylops.utils.wavelets import ricker
from pylops.utils.seismicevents import makeaxis, hyperbolic2d

warnings.filterwarnings('ignore')
plt.close('all')
```

Let's start by creating a set of hyperbolic events to be used as our MDC kernel

```
# Input parameters
par = {'ox':-150, 'dx':10, 'nx':31,
       'oy':-250, 'dy':10, 'ny':51,
       'ot':0, 'dt':0.004, 'nt':300,
       'f0': 20, 'nfmax': 200}

t0_m = [0.2]
vrms_m = [700.]
amp_m = [1.]

t0_G = [0.2, 0.5, 0.7]
vrms_G = [800., 1200., 1500.]
amp_G = [1., 0.6, 0.5]

# Taper
tap = taper3d(par['nt'], [par['ny'], par['nx']],
              (5, 5), tapertype='hanning')

# Create axis
t, t2, x, y = makeaxis(par)

# Create wavelet
wav = ricker(t[:41], f0=par['f0'])[0]

# Generate model
m, mwav = hyperbolic2d(x, t, t0_m, vrms_m, amp_m, wav)

# Generate operator
G, Gwav = np.zeros((par['ny'], par['nx'], par['nt'])), \
          np.zeros((par['ny'], par['nx'], par['nt']))
for iy, y0 in enumerate(y):
    G[iy], Gwav[iy] = hyperbolic2d(x-y0, t, t0_G, vrms_G, amp_G, wav)
G, Gwav = G*tap, Gwav*tap

# Add negative part to data and model
m = np.concatenate((np.zeros((par['nx'], par['nt']-1)), m), axis=-1)
mwav = np.concatenate((np.zeros((par['nx'], par['nt']-1)), mwav), axis=-1)
Gwav2 = np.concatenate((np.zeros((par['ny'], par['nx'], par['nt']-1)), Gwav),
                       axis=-1)

# Define MDC linear operator
Gwav_fft = np.fft.rfft(Gwav2, 2*par['nt']-1, axis=-1)
Gwav_fft = Gwav_fft[..., :par['nfmax']]
```

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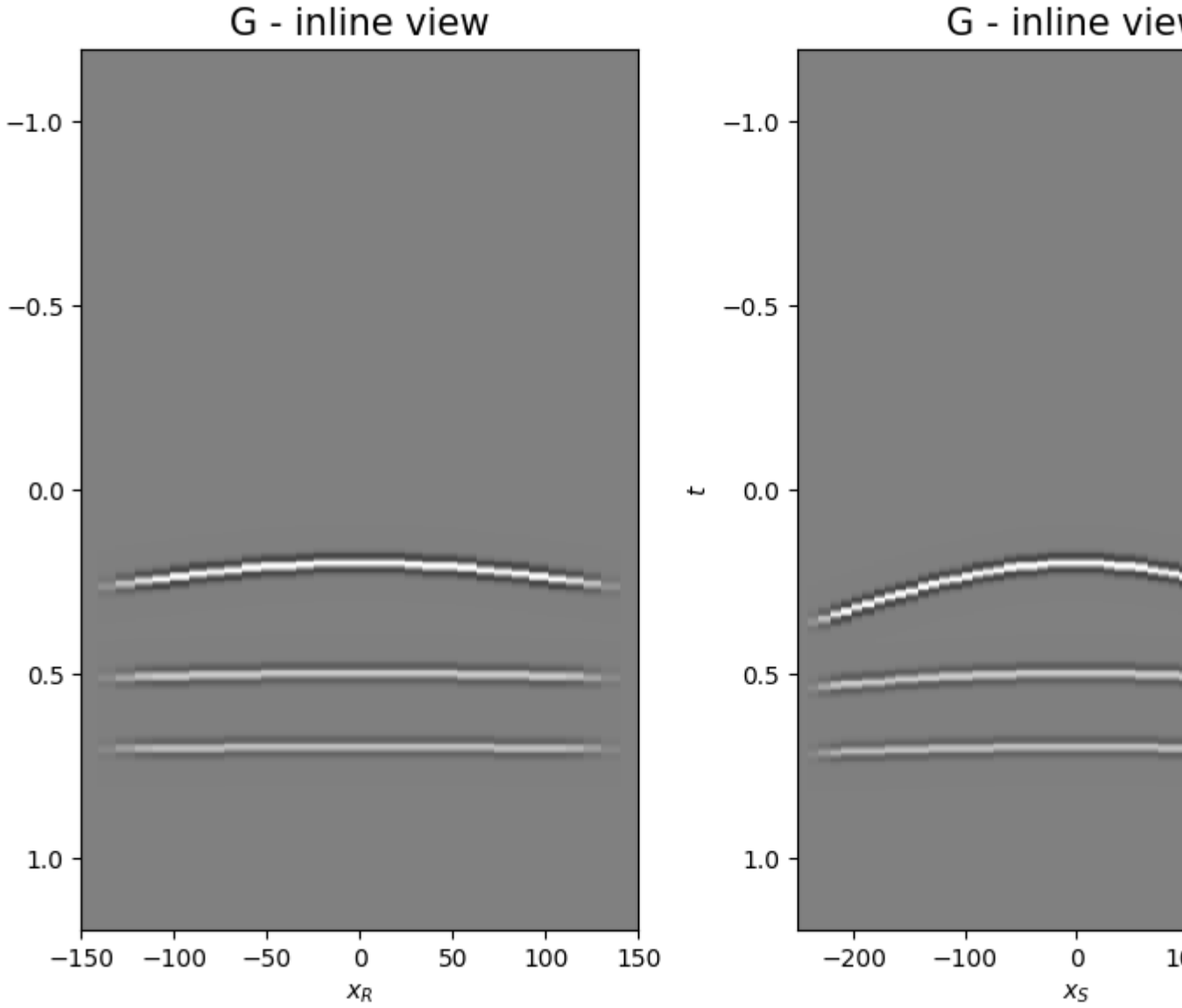
```
MDCop = pylops.waveeqprocessing.MDC(Gwav_fft, nt=2 * par['nt']-1, nv=1,
                                    dt=0.004, dr=1., dtype='float32')

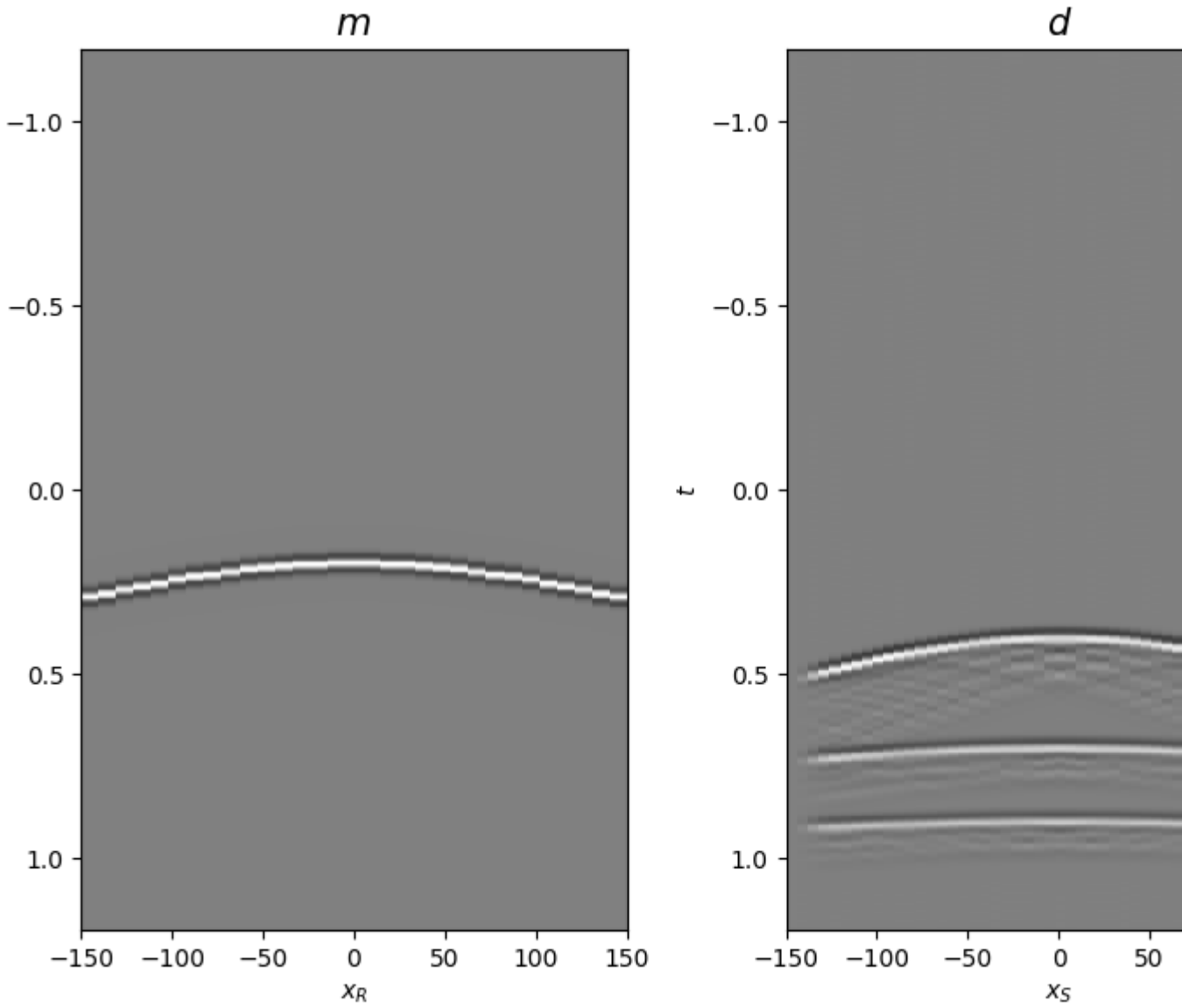
# Create data
d = MDCop*m.flatten()
d = d.reshape(par['ny'], 2*par['nt']-1)
```

Let's display what we have so far: operator, input model, and data

```
fig, axs = plt.subplots(1, 2, figsize=(8, 6))
axs[0].imshow(Gwav2[int(par['ny']/2)].T, aspect='auto',
              interpolation='nearest', cmap='gray',
              vmin=-np.abs(Gwav2.max()), vmax=np.abs(Gwav2.max()),
              extent=(x.min(), x.max(), t2.max(), t2.min()))
axs[0].set_title('G - inline view', fontsize=15)
axs[0].set_xlabel(r'$x_R$')
axs[1].set_ylabel(r'$t$')
axs[1].imshow(Gwav2[:, int(par['nx']/2)].T, aspect='auto',
              interpolation='nearest', cmap='gray',
              vmin=-np.abs(Gwav2.max()), vmax=np.abs(Gwav2.max()),
              extent=(y.min(), y.max(), t2.max(), t2.min()))
axs[1].set_title('G - inline view', fontsize=15)
axs[1].set_xlabel(r'$x_S$')
axs[1].set_ylabel(r'$t$')
fig.tight_layout()

fig, axs = plt.subplots(1, 2, figsize=(8, 6))
axs[0].imshow(mwav.T, aspect='auto', interpolation='nearest', cmap='gray',
              vmin=-np.abs(mwav.max()), vmax=np.abs(mwav.max()),
              extent=(x.min(), x.max(), t2.max(), t2.min()))
axs[0].set_title(r'$m$', fontsize=15)
axs[0].set_xlabel(r'$x_R$')
axs[1].set_ylabel(r'$t$')
axs[1].imshow(d.T, aspect='auto', interpolation='nearest', cmap='gray',
              vmin=-np.abs(d.max()), vmax=np.abs(d.max()),
              extent=(x.min(), x.max(), t2.max(), t2.min()))
axs[1].set_title(r'$d$', fontsize=15)
axs[1].set_xlabel(r'$x_S$')
axs[1].set_ylabel(r'$t$')
fig.tight_layout()
```


G - inline view
G - inline vie
−1.0
−0.5
0.0
0.5
1.0
t
−150
−100
−50
0
50
100
150
x_R
−200
−100
0
x_S



We are now ready to feed our operator to *pylops.waveeqprocessing.MDD* and invert back for our input model

```
minv, madj, psfinv, psfadj = \
    pylops.waveeqprocessing.MDD(Gwav, d[:, par['nt'] - 1:],
                                dt=par['dt'], dr=par['dx'],
                                nfmax=par['nfmax'], wav=wav,
                                twosided=True, add_negative=True,
                                adjoint=True, psf=True,
                                dtype='complex64', dottest=False,
                                **dict(damp=1e-4, iter_lim=20, show=0))

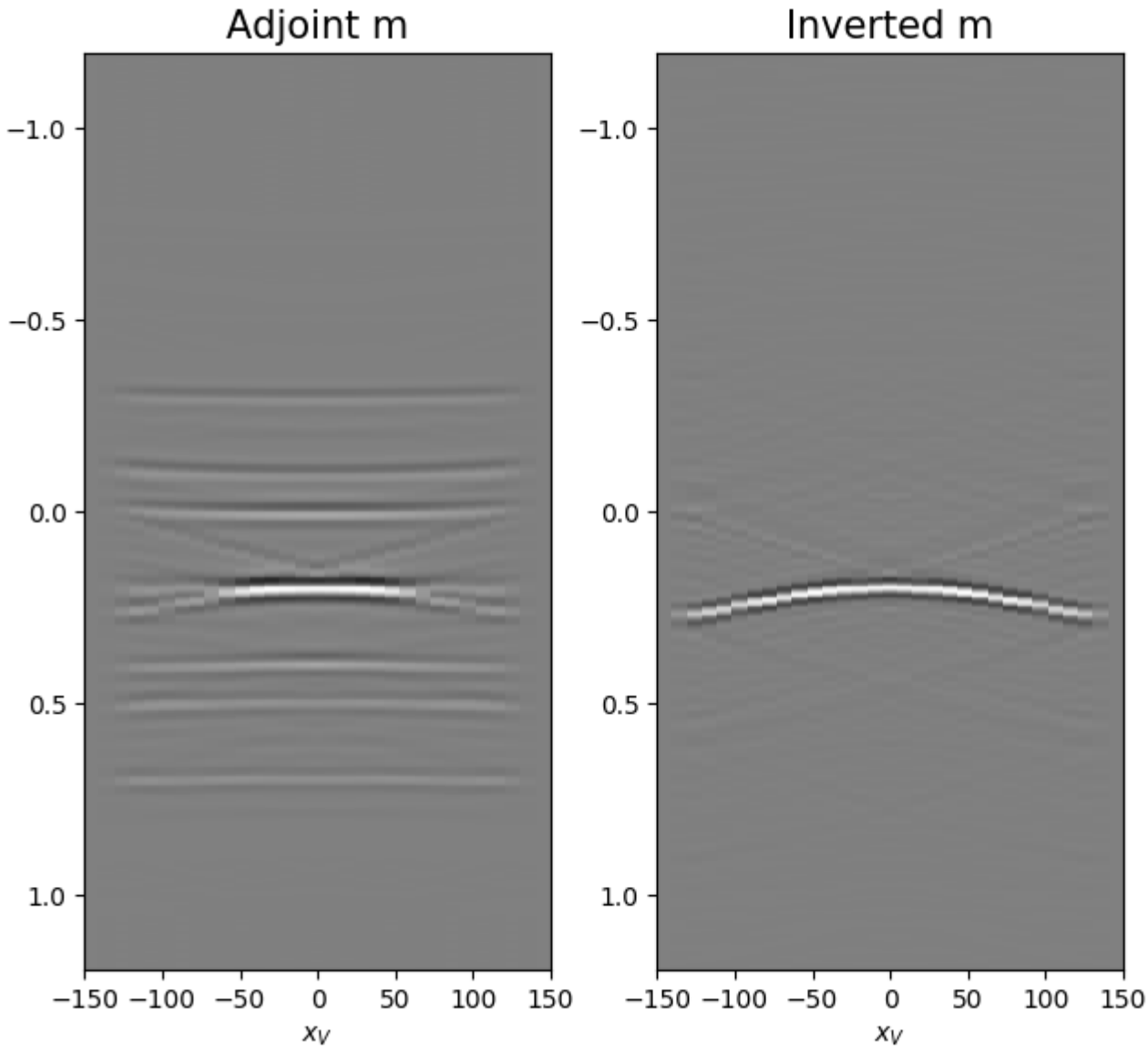
fig = plt.figure(figsize=(8, 6))
ax1 = plt.subplot2grid((1, 5), (0, 0), colspan=2)
ax2 = plt.subplot2grid((1, 5), (0, 2), colspan=2)
ax3 = plt.subplot2grid((1, 5), (0, 4))
ax1.imshow(madj.T, aspect='auto', interpolation='nearest', cmap='gray',
```

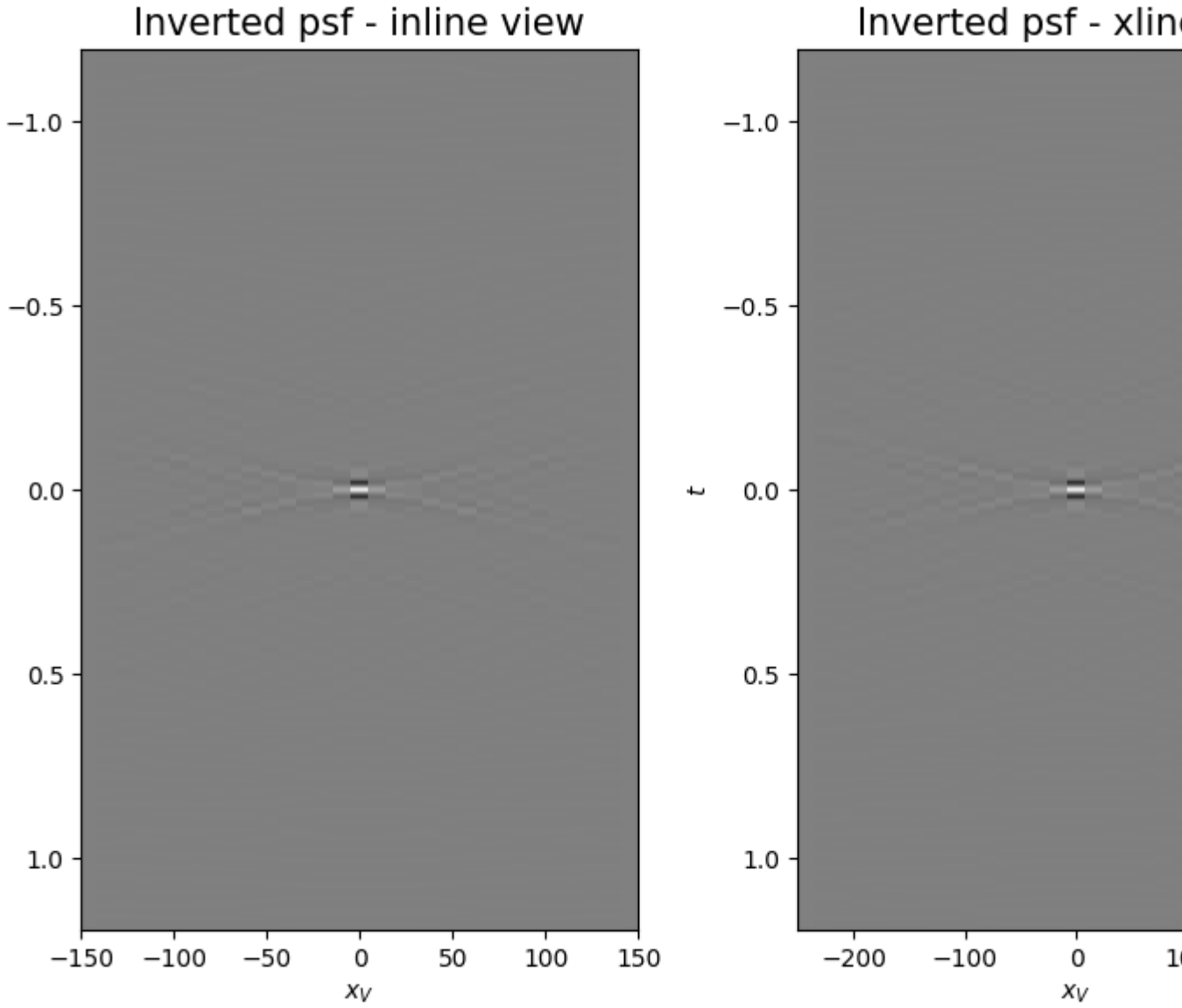
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```
           vmin=-np.abs(madj.max()), vmax=np.abs(madj.max()),
           extent=(x.min(), x.max(), t2.max(), t2.min()))
ax1.set_title('Adjoint m', fontsize=15)
ax1.set_xlabel(r'$x_V$')
axs[1].set_ylabel(r'$t$')
ax2.imshow(minv.T, aspect='auto', interpolation='nearest', cmap='gray',
           vmin=-np.abs(minv.max()), vmax=np.abs(minv.max()),
           extent=(x.min(), x.max(), t2.max(), t2.min()))
ax2.set_title('Inverted m', fontsize=15)
ax2.set_xlabel(r'$x_V$')
axs[1].set_ylabel(r'$t$')
ax3.plot(madj[int(par['nx']/2)]/np.abs(madj[int(par['nx']/2)]).max(),
         t2, 'r', lw=5)
ax3.plot(minv[int(par['nx']/2)]/np.abs(minv[int(par['nx']/2)]).max(),
         t2, 'k', lw=3)
ax3.set_ylim([t2[-1], t2[0]])
fig.tight_layout()

fig, axs = plt.subplots(1, 2, figsize=(8, 6))
axs[0].imshow(psfinv[int(par['nx']/2)].T,
              aspect='auto', interpolation='nearest',
              vmin=-np.abs(psfinv.max()), vmax=np.abs(psfinv.max()),
              cmap='gray', extent=(x.min(), x.max(), t2.max(), t2.min()))
axs[0].set_title('Inverted psf - inline view', fontsize=15)
axs[0].set_xlabel(r'$x_V$')
axs[1].set_ylabel(r'$t$')
axs[1].imshow(psfinv[:, int(par['nx']/2)].T,
              aspect='auto', interpolation='nearest',
              vmin=-np.abs(psfinv.max()), vmax=np.abs(psfinv.max()),
              cmap='gray', extent=(y.min(), y.max(), t2.max(), t2.min()))
axs[1].set_title('Inverted psf - xline view', fontsize=15)
axs[1].set_xlabel(r'$x_V$')
axs[1].set_ylabel(r'$t$')
fig.tight_layout()
```


Adjoint m
Inverted m
−1.0
−0.5
0.0
0.5
1.0
−150 −100 −50 0 50 100 150
x_V



We repeat the same procedure but this time we will add a preconditioning by means of `causality_precond` parameter, which enforces the inverted model to be zero in the negative part of the time axis (as expected by theory). This preconditioning will have the effect of speeding up the convergence of the iterative solver and thus reduce the computation time of the deconvolution

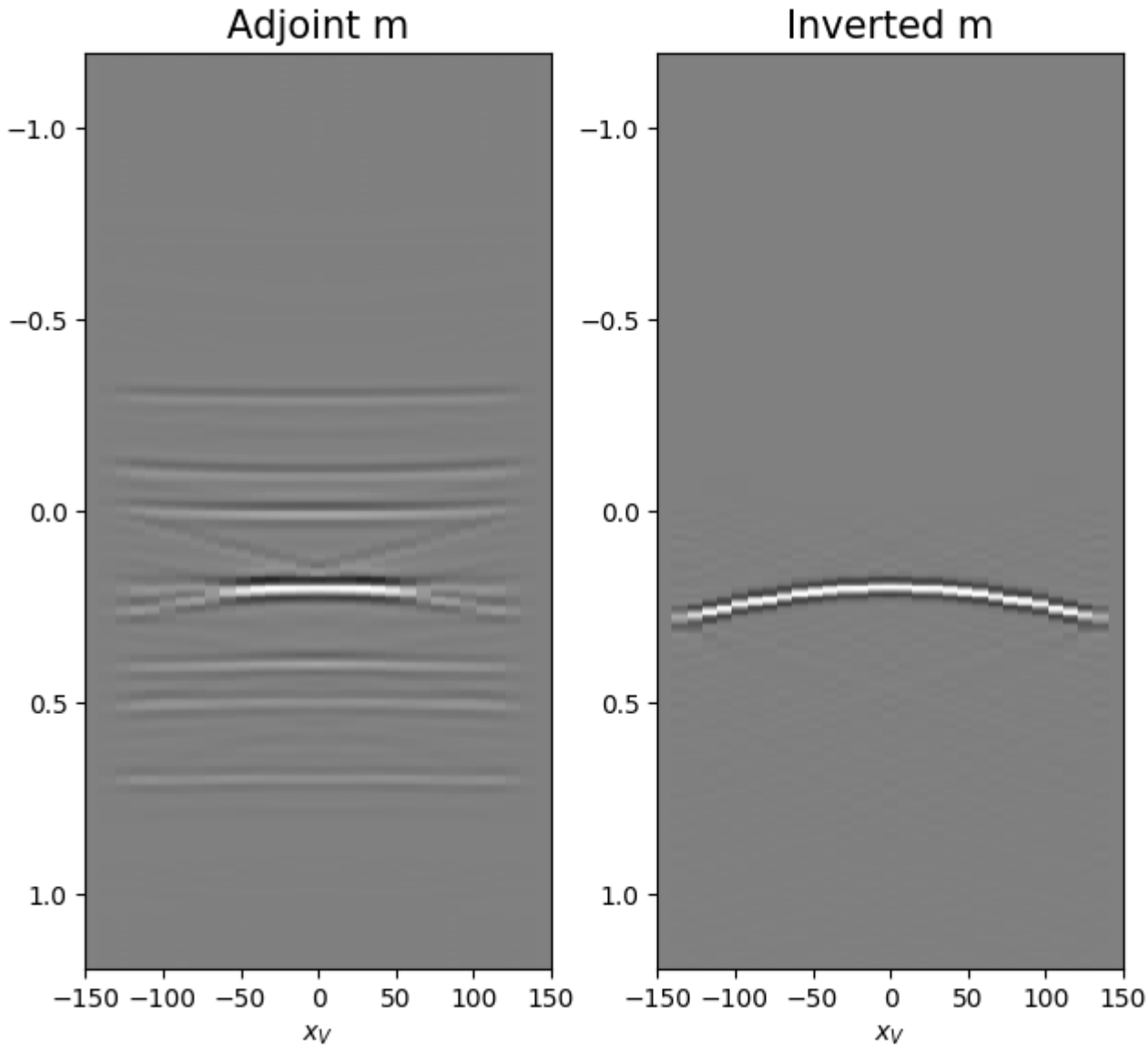
```
minvprec = pylops.waveeqprocessing.MDD(Gwav, d[:, par['nt'] - 1:],
                                       dt=par['dt'], dr=par['dx'],
                                       nfmax=par['nfmax'], wav=wav,
                                       twosided=True, add_negative=True,
                                       adjoint=False, psf=False,
                                       causality_precond=True,
                                       dtype='complex64',
                                       dottest=False,
                                       **dict(damp=1e-4, iter_lim=50, show=0))

# sphinx_gallery_thumbnail_number = 5
```

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```
fig = plt.figure(figsize=(8, 6))
ax1 = plt.subplot2grid((1, 5), (0, 0), colspan=2)
ax2 = plt.subplot2grid((1, 5), (0, 2), colspan=2)
ax3 = plt.subplot2grid((1, 5), (0, 4))
ax1.imshow(madj.T, aspect='auto', interpolation='nearest', cmap='gray',
           vmin=-np.abs(madj.max()), vmax=np.abs(madj.max()),
           extent=(x.min(), x.max(), t2.max(), t2.min()))
ax1.set_title('Adjoint m', fontsize=15)
ax1.set_xlabel(r'$x_V$')
axs[1].set_ylabel(r'$t$')
ax2.imshow(minvprec.T, aspect='auto', interpolation='nearest', cmap='gray',
           vmin=-np.abs(minvprec.max()), vmax=np.abs(minvprec.max()),
           extent=(x.min(), x.max(), t2.max(), t2.min()))
ax2.set_title('Inverted m', fontsize=15)
ax2.set_xlabel(r'$x_V$')
axs[1].set_ylabel(r'$t$')
ax3.plot(madj[int(par['nx']/2)]/np.abs(madj[int(par['nx']/2)]).max(),
         t2, 'r', lw=5)
ax3.plot(minvprec[int(par['nx']/2)]/np.abs(minv[int(par['nx']/2)]).max(),
         t2, 'k', lw=3)
ax3.set_ylim([t2[-1], t2[0]])
fig.tight_layout()
```



Total running time of the script: (0 minutes 13.022 seconds)

Note: Click *here* to download the full example code

3.4.9 09. Marchenko redatuming by inversion

This example shows how to set-up and run the *pylops.waveeqprocessing.Marchenko* inversion using synthetic data.

```
# sphinx_gallery_thumbnail_number = 5
# pylint: disable=C0103
import warnings
import numpy as np
```

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```
import matplotlib.pyplot as plt

from scipy.signal import convolve
from pylops.waveeqprocessing import Marchenko

warnings.filterwarnings('ignore')
plt.close('all')
```

Let's start by defining some input parameters and loading the test data

```
# Input parameters
inputfile = '../testdata/marchenko/input.npz'

vs_zx = [1060, 1200] # virtual source z,x
vel = 2400.0         # velocity
toff = 0.045         # direct arrival time shift
nsmooth = 10         # time window smoothing
nfmax = 1000         # max frequency for MDC (#samples)
niter = 10           # iterations

inputdata = np.load(inputfile)

# Receivers
r = inputdata['r']
nr = r.shape[1]
dr = r[0, 1]-r[0, 0]

# Sources
s = inputdata['s']
ns = s.shape[1]
ds = s[0, 1]-s[0, 0]

# Virtual points
vs = inputdata['vs']

# Density model
rho = inputdata['rho']
z, x = inputdata['z'], inputdata['x']

# Reflection data (R[s, r, t]) and subsurface fields
R = inputdata['R'][:, :, :-100]
R = np.swapaxes(R, 0, 1) # just because of how the data was saved

Gsub = inputdata['Gsub'][:-100]
G0sub = inputdata['G0sub'][:-100]
wav = inputdata['wav']
wav_c = np.argmax(wav)

t = inputdata['t'][:-100]
ot, dt, nt = t[0], t[1]-t[0], len(t)

Gsub = np.apply_along_axis(convolve, 0, Gsub, wav, mode='full')
Gsub = Gsub[wav_c:][:nt]
G0sub = np.apply_along_axis(convolve, 0, G0sub, wav, mode='full')
G0sub = G0sub[wav_c:][:nt]

plt.figure(figsize=(10, 5))
```

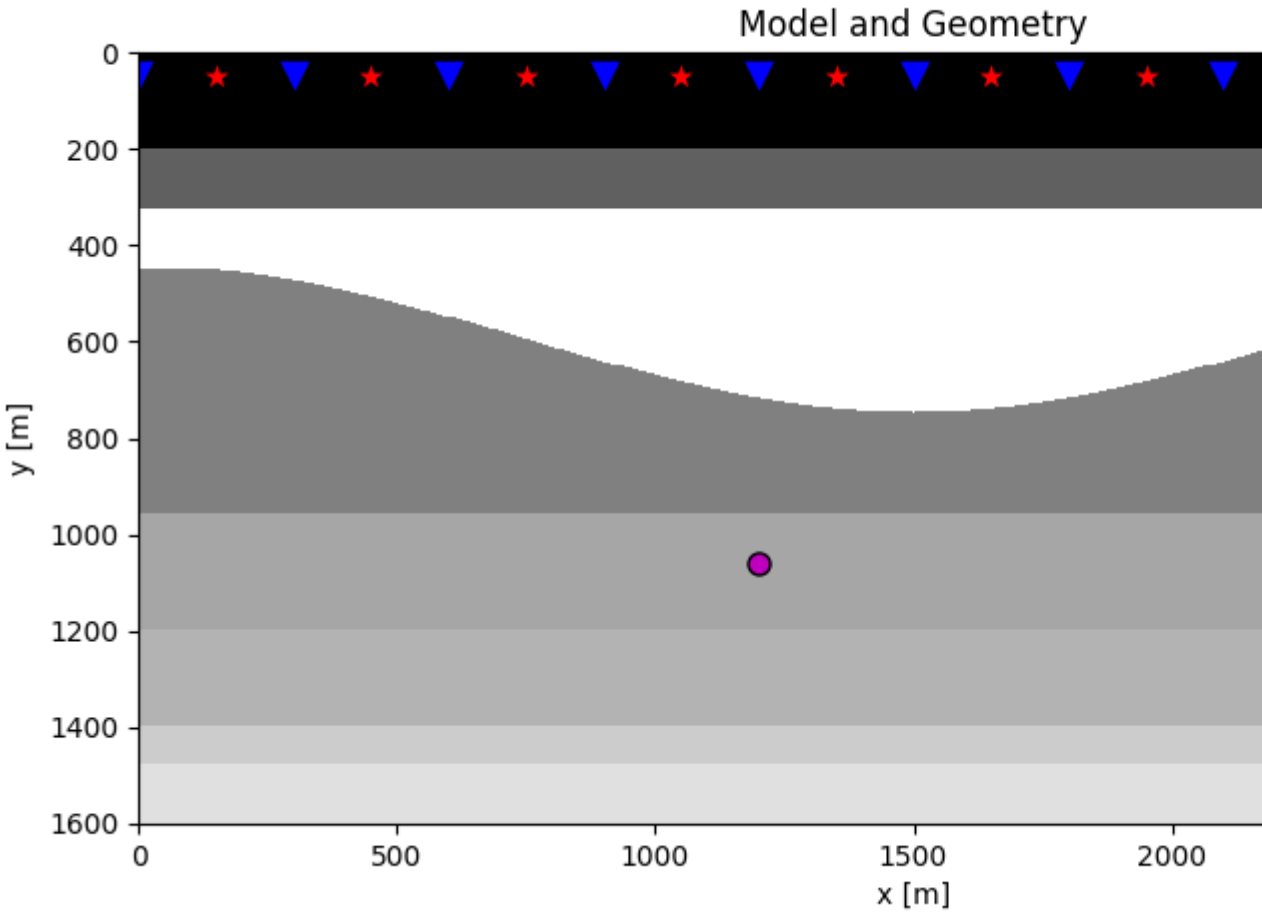
(continues on next page)

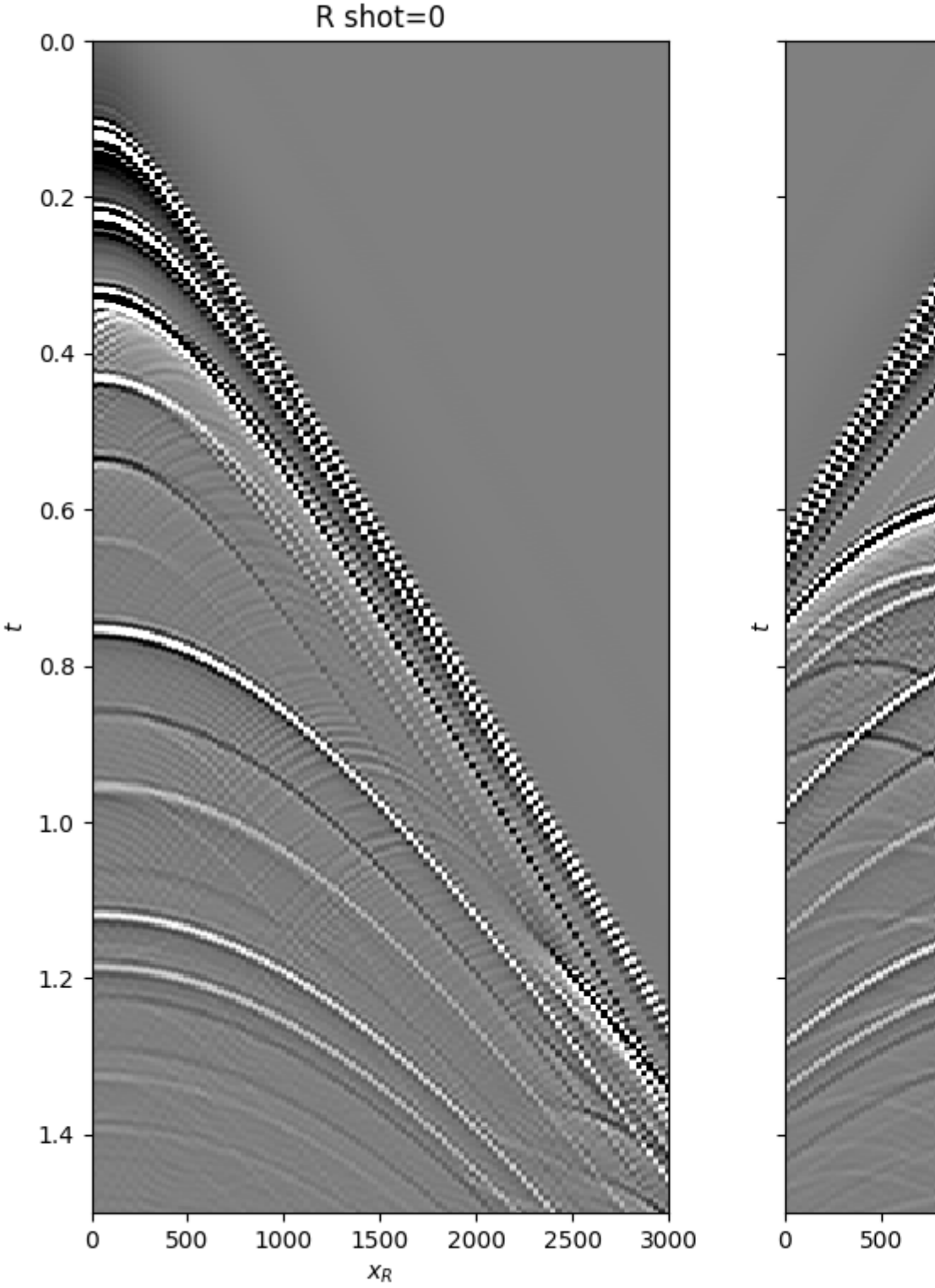
(continued from previous page)

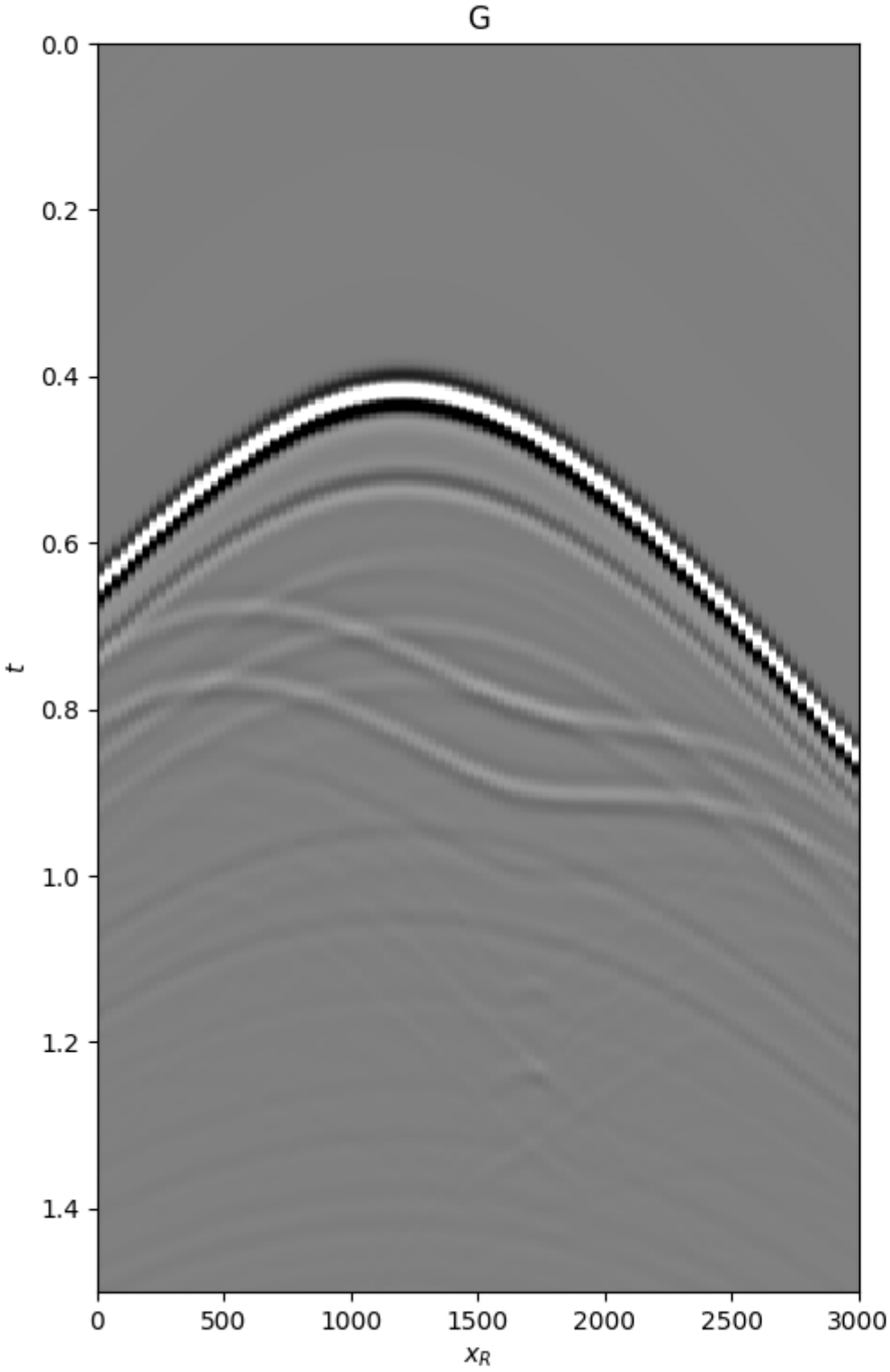
```
plt.imshow(rho, cmap='gray', extent=(x[0], x[-1], z[-1], z[0]))
plt.scatter(s[0, 5::10], s[1, 5::10], marker='*', s=150, c='r', edgecolors='k')
plt.scatter(r[0, ::10], r[1, ::10], marker='v', s=150, c='b', edgecolors='k')
plt.scatter(vs[0], vs[1], marker='.', s=250, c='m', edgecolors='k')
plt.axis('tight')
plt.xlabel('x [m]')
plt.ylabel('y [m]')
plt.title('Model and Geometry')
plt.xlim(x[0], x[-1])

fig, axs = plt.subplots(1, 3, sharey=True, figsize=(15, 9))
axs[0].imshow(R[0].T, cmap='gray', vmin=-1e-2, vmax=1e-2,
              extent=(r[0, 0], r[0, -1], t[-1], t[0]))
axs[0].set_title('R shot=0')
axs[0].set_xlabel(r'$x_R$')
axs[0].set_ylabel(r'$t$')
axs[0].axis('tight')
axs[0].set_ylim(1.5, 0)
axs[1].imshow(R[ns//2].T, cmap='gray', vmin=-1e-2, vmax=1e-2,
              extent=(r[0, 0], r[0, -1], t[-1], t[0]))
axs[1].set_title('R shot=%d' %(ns//2))
axs[1].set_xlabel(r'$x_R$')
axs[1].set_ylabel(r'$t$')
axs[1].axis('tight')
axs[1].set_ylim(1.5, 0)
axs[2].imshow(R[-1].T, cmap='gray', vmin=-1e-2, vmax=1e-2,
              extent=(r[0, 0], r[0, -1], t[-1], t[0]))
axs[2].set_title('R shot=%d' %ns)
axs[2].set_xlabel(r'$x_R$')
axs[2].axis('tight')
axs[2].set_ylim(1.5, 0)

fig, axs = plt.subplots(1, 2, sharey=True, figsize=(12, 9))
axs[0].imshow(Gsub, cmap='gray', vmin=-1e6, vmax=1e6,
              extent=(r[0, 0], r[0, -1], t[-1], t[0]))
axs[0].set_title('G')
axs[0].set_xlabel(r'$x_R$')
axs[0].set_ylabel(r'$t$')
axs[0].axis('tight')
axs[0].set_ylim(1.5, 0)
axs[1].imshow(G0sub, cmap='gray', vmin=-1e6, vmax=1e6,
              extent=(r[0, 0], r[0, -1], t[-1], t[0]))
axs[1].set_title('G0')
axs[1].set_xlabel(r'$x_R$')
axs[1].set_ylabel(r'$t$')
axs[1].axis('tight')
axs[1].set_ylim(1.5, 0)
```


Model and Geometry
0
200
400
600
800
1000
1200
1400
1600
y [m]
0
500
1000
1500
2000
x [m]


R shot=0
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
t
0
500
1000
1500
2000
2500
3000
x_R


G
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
t
0
500
1000
1500
2000
2500
3000
x_R

Let's now create an object of the *pylops.waveeqprocessing.Marchenko* class and apply redatuming for a single subsurface point `vs`.

```
# direct arrival window
trav = np.sqrt((vs[0]-r[0])**2+(vs[1]-r[1])**2)/vel

MarchenkoWM = Marchenko(R, dt=dt, dr=dr, nfmax=nfmax, wav=wav,
                        toff=toff, nsmooth=nsmooth)

f1_inv_minus, f1_inv_plus, p0_minus, g_inv_minus, g_inv_plus = \
    MarchenkoWM.apply_onepoint(trav, G0=G0sub.T, rtm=True, greens=True,
                               dottest=True, **dict(iter_lim=niter, show=True))
g_inv_tot = g_inv_minus + g_inv_plus
```

Out:

```
Dot test passed, v^T(Opu)=491.131956 - u^T(Op^Tv)=491.131956
Dot test passed, v^T(Opu)=464.113195 - u^T(Op^Tv)=464.113195

LSQR            Least-squares solution of  Ax = b
The matrix A has   282598 rows  and   282598 cols
damp = 0.00000000000000e+00   calc_var =        0
atol = 1.00e-08                 conlim = 1.00e+08
btol = 1.00e-08               iter_lim =       10

   Itn      x[0]       r1norm     r2norm   Compatible    LS       Norm A   Cond A
     0  0.00000e+00   3.134e+07  3.134e+07    1.0e+00  3.3e-08
     1  0.00000e+00   1.374e+07  1.374e+07    4.4e-01  9.3e-01   1.1e+00  1.0e+00
     2  0.00000e+00   7.770e+06  7.770e+06    2.5e-01  3.9e-01   1.8e+00  2.2e+00
     3  0.00000e+00   5.750e+06  5.750e+06    1.8e-01  3.3e-01   2.1e+00  3.4e+00
     4  0.00000e+00   3.930e+06  3.930e+06    1.3e-01  3.4e-01   2.5e+00  5.1e+00
     5  0.00000e+00   3.042e+06  3.042e+06    9.7e-02  2.6e-01   2.9e+00  6.8e+00
     6  0.00000e+00   2.423e+06  2.423e+06    7.7e-02  2.2e-01   3.3e+00  8.6e+00
     7  0.00000e+00   1.675e+06  1.675e+06    5.3e-02  2.5e-01   3.6e+00  1.1e+01
     8  0.00000e+00   1.248e+06  1.248e+06    4.0e-02  2.0e-01   3.9e+00  1.3e+01
     9  0.00000e+00   1.004e+06  1.004e+06    3.2e-02  1.5e-01   4.2e+00  1.4e+01
    10  0.00000e+00   7.762e+05  7.762e+05    2.5e-02  1.8e-01   4.4e+00  1.6e+01

LSQR finished
The iteration limit has been reached

istop =       7   r1norm = 7.8e+05   anorm = 4.4e+00   arnorm = 6.1e+05
itn   =      10   r2norm = 7.8e+05   acond = 1.6e+01   xnorm  = 3.6e+07
```

We can now compare the result of Marchenko redatuming via LSQR with standard redatuming

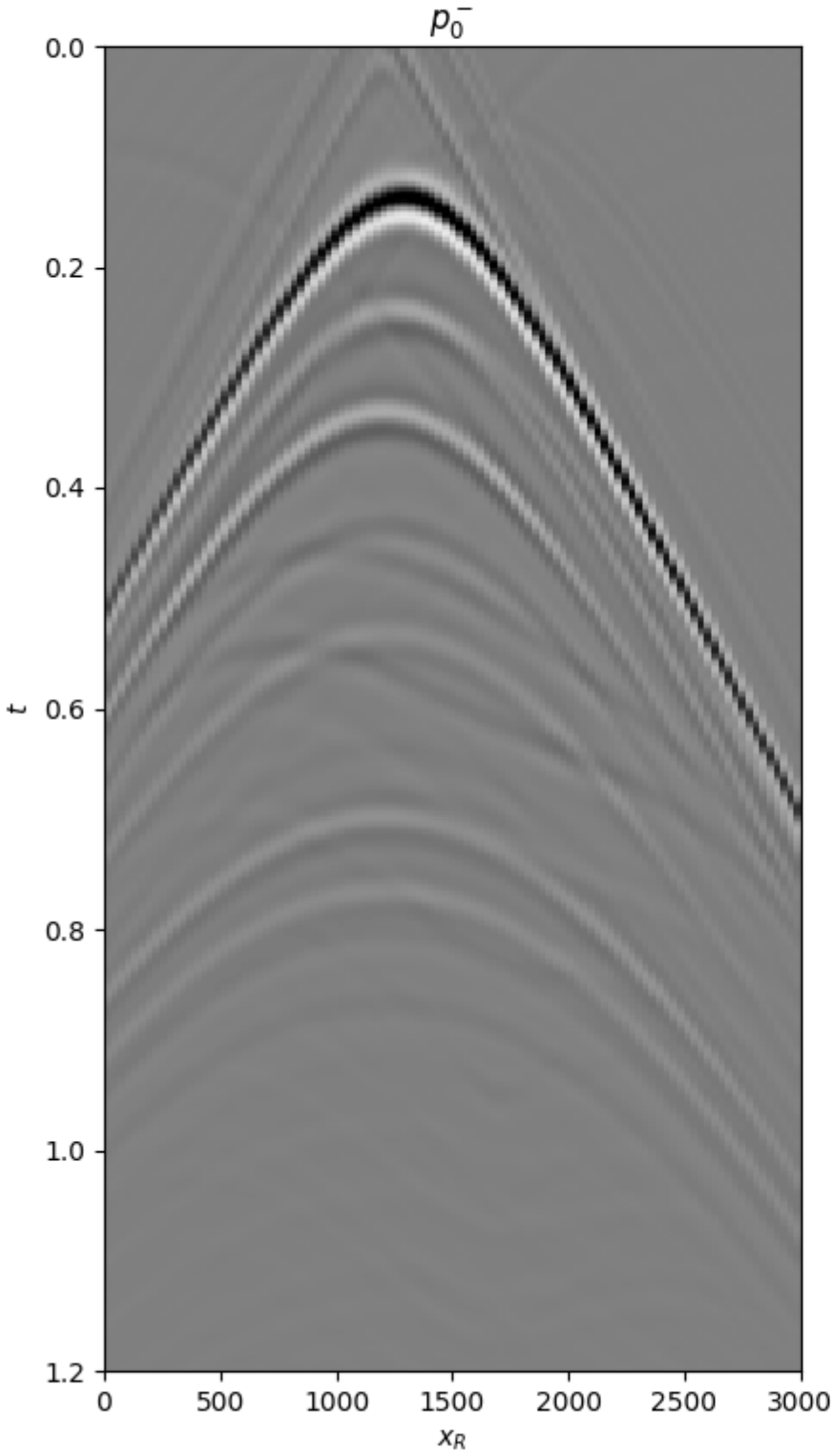
```
fig, axs = plt.subplots(1, 3, sharey=True, figsize=(16, 9))
axs[0].imshow(p0_minus.T, cmap='gray', vmin=-5e5, vmax=5e5,
              extent=(r[0, 0], r[0, -1], t[-1], -t[-1]))
axs[0].set_title(r'$p_0^-$')
axs[0].set_xlabel(r'$x_R$')
axs[0].set_ylabel(r'$t$')
axs[0].axis('tight')
axs[0].set_ylim(1.2, 0)
axs[1].imshow(g_inv_minus.T, cmap='gray', vmin=-5e5, vmax=5e5,
              extent=(r[0, 0], r[0, -1], t[-1], -t[-1]))
axs[1].set_title(r'$g^-$')
axs[1].set_xlabel(r'$x_R$')
```

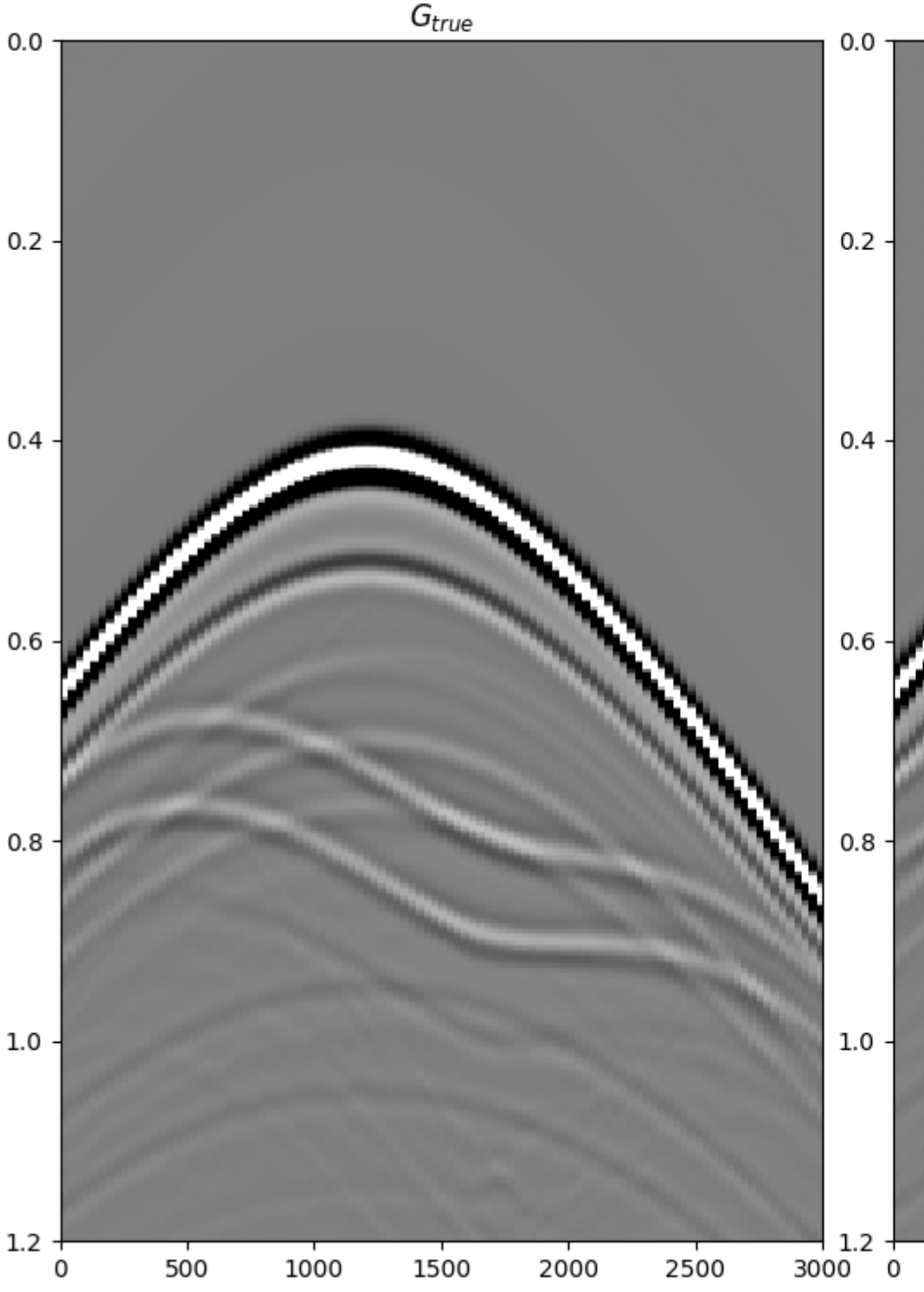
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```
axs[1].set_ylabel(r'$t$')
axs[1].axis('tight')
axs[1].set_ylim(1.2, 0)
axs[2].imshow(g_inv_plus.T, cmap='gray', vmin=-5e5, vmax=5e5,
              extent=(r[0, 0], r[0, -1], t[-1], -t[-1]))
axs[2].set_title(r'$g^+$')
axs[2].set_xlabel(r'$x_R$')
axs[2].set_ylabel(r'$t$')
axs[2].axis('tight')
axs[2].set_ylim(1.2, 0)

fig = plt.figure(figsize=(15, 9))
ax1 = plt.subplot2grid((1, 5), (0, 0), colspan=2)
ax2 = plt.subplot2grid((1, 5), (0, 2), colspan=2)
ax3 = plt.subplot2grid((1, 5), (0, 4))
ax1.imshow(Gsub, cmap='gray', vmin=-5e5, vmax=5e5,
           extent=(r[0, 0], r[0, -1], t[-1], t[0]))
ax1.set_title(r'$G_{true}$')
axs[0].set_xlabel(r'$x_R$')
axs[0].set_ylabel(r'$t$')
ax1.axis('tight')
ax1.set_ylim(1.2, 0)
ax2.imshow(g_inv_tot.T, cmap='gray', vmin=-5e5, vmax=5e5,
           extent=(r[0, 0], r[0, -1], t[-1], -t[-1]))
ax2.set_title(r'$G_{est}$')
axs[1].set_xlabel(r'$x_R$')
axs[1].set_ylabel(r'$t$')
ax2.axis('tight')
ax2.set_ylim(1.2, 0)
ax3.plot(Gsub[:, nr//2]/Gsub.max(), t, 'r', lw=5)
ax3.plot(g_inv_tot[nr//2, nt-1:]/g_inv_tot.max(), t, 'k', lw=3)
ax3.set_ylim(1.2, 0)
```


p_0^-
0.0
0.2
0.4
0.6
0.8
1.0
1.2
t
0
500
1000
1500
2000
2500
3000
x_R


G_{true}
0.0
0.2
0.4
0.6
0.8
1.0
1.2
0
500
1000
1500
2000
2500
3000

Note that Marchenko redatuming can also be applied simultaneously to multiple subsurface points. Use *pylops.waveeqprocessing.Marchenko.apply_multiplepoints* instead of *pylops.waveeqprocessing.Marchenko.apply_onepoint*.

Total running time of the script: (0 minutes 24.561 seconds)

Note: Click *here* to download the full example code

3.4.10 10. Radon filtering

In this example we will be taking advantage of the *pylops.signalprocessing.Radon2D* operator to perform filtering of unwanted events from a seismic data. For those of you not familiar with seismic data, let's imagine that we have a data composed of a certain number of flat events and a parabolic event , we are after a transform that allows us to separate such an event from the others and filter it out. Those of you with a geophysics background may immediately realize this is the case of seismic angle (or offset) gathers after migration and those events with parabolic moveout are generally residual multiples that we would like to suppress prior to performing further analysis of our data.

The Radon transform is actually a very good transform to perform such a separation. We can thus devise a simple workflow that takes our data as input, applies a Radon transform, filters some of the events out and goes back to the original domain.

```
import numpy as np
import matplotlib.pyplot as plt

import pylops
from pylops.utils.wavelets import ricker

plt.close('all')
np.random.seed(0)
```

Let's first create a data composed on 3 linear events and a parabolic event.

```
par = {'ox':0, 'dx':2, 'nx':121,
       'ot':0, 'dt':0.004, 'nt':100,
       'f0': 30}

# linear events
v = 1500
t0 = [0.1, 0.2, 0.3]
theta = [0, 0, 0]
amp = [1., -2, 0.5]

# parabolic event
tp0 = [0.13]
px = [0]
pxx = [5e-7]
ampp = [0.7]

# create axis
taxis, taxis2, xaxis, yaxis = pylops.utils.seismicevents.makeaxis(par)

# create wavelet
wav = ricker(taxis[:41], f0=par['f0'])[0]

# generate model
```

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```
y = pylops.utils.seismicevents.linear2d(xaxis, taxis, v, t0,
                                        theta, amp, wav)[1] + \
    pylops.utils.seismicevents.parabolic2d(xaxis, taxis, tp0,
                                           px, pxx, ampp, wav)[1]
```

We can now create the *pylops.signalprocessing.Radon2D* operator. We also apply its adjoint to the data to obtain a representation of those 3 linear events overlapping to a parabolic event in the Radon domain. Similarly, we feed the operator to a sparse solver like *pylops.optimization.sparsity.FISTA* to obtain a sparse represention of the data in the Radon domain. At this point we try to filter out the unwanted event. We can see how this is much easier for the sparse transform as each event has a much more compact representation in the Radon domain than for the adjoint transform.

```
# radon operator
npx = 61
pxmax = 5e-4
px = np.linspace(-pxmax, pxmax, npx)

Rop = pylops.signalprocessing.Radon2D(taxis, xaxis, px, kind='linear',
                                      interp='nearest',  centeredh=False,
                                      dtype='float64')

# adjoint Radon transform
xadj = Rop.H * y.flatten()
xadj = xadj.reshape(npx, par['nt'])

# sparse Radon transform
xinv, niter, cost = \
    pylops.optimization.sparsity.FISTA(Rop, y.flatten(), 15,
                                       eps=1e1, returninfo=True)
xinv = xinv.reshape(npx, par['nt'])

# filtering
xfilt = np.zeros_like(xadj)
xfilt[npx//2-3:npx//2+4] = xadj[npx//2-3:npx//2+4]

yfilt = Rop * xfilt.flatten()
yfilt = yfilt.reshape(par['nx'], par['nt'])

# filtering on sparse transform
xinvfilt = np.zeros_like(xinv)
xinvfilt[npx//2-3:npx//2+4] = xinv[npx//2-3:npx//2+4]

yinvfilt = Rop * xinvfilt.flatten()
yinvfilt = yinvfilt.reshape(par['nx'], par['nt'])
```

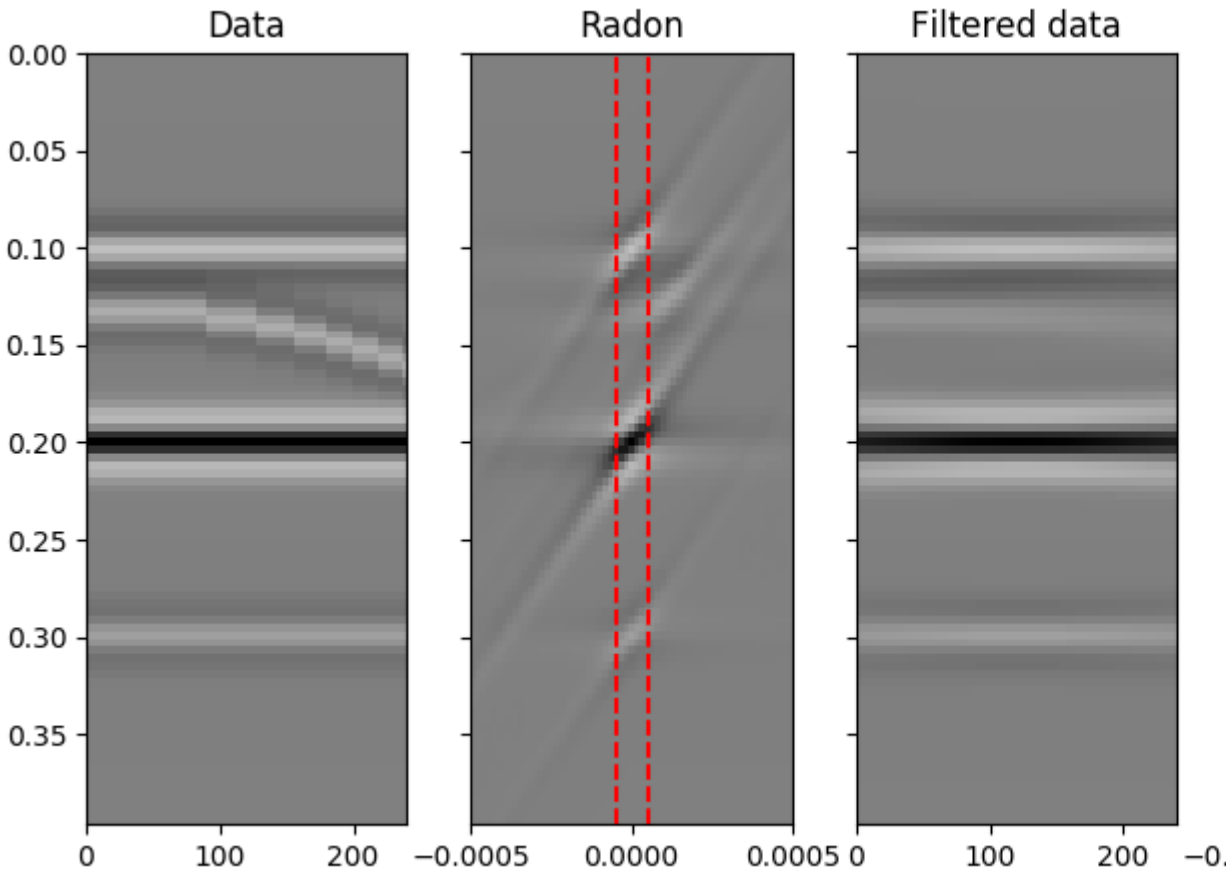
Finally we visualize our results.

```
fig, axs = plt.subplots(1, 5, sharey=True, figsize=(12, 5))
axs[0].imshow(y.T, cmap='gray',
              vmin=-np.abs(y).max(), vmax=np.abs(y).max(),
              extent=(xaxis[0], xaxis[-1], taxis[-1], taxis[0]))
axs[0].set_title('Data')
axs[0].axis('tight')
axs[1].imshow(xadj.T, cmap='gray',
              vmin=-np.abs(xadj).max(), vmax=np.abs(xadj).max(),
              extent=(px[0], px[-1], taxis[-1], taxis[0]))
```

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```
axs[1].axvline(px[npx//2-3], color='r', linestyle='--')
axs[1].axvline(px[npx//2+3], color='r', linestyle='--')
axs[1].set_title('Radon')
axs[1].axis('tight')
axs[2].imshow(yfilt.T, cmap='gray',
              vmin=-np.abs(yfilt).max(), vmax=np.abs(yfilt).max(),
              extent=(xaxis[0], xaxis[-1], taxis[-1], taxis[0]))
axs[2].set_title('Filtered data')
axs[2].axis('tight')
axs[3].imshow(xinv.T, cmap='gray',
              vmin=-np.abs(xinv).max(), vmax=np.abs(xinv).max(),
              extent=(px[0], px[-1], taxis[-1], taxis[0]))
axs[3].axvline(px[npx//2-3], color='r', linestyle='--')
axs[3].axvline(px[npx//2+3], color='r', linestyle='--')
axs[3].set_title('Sparse Radon')
axs[3].axis('tight')
axs[4].imshow(yinvfilt.T, cmap='gray',
              vmin=-np.abs(y).max(), vmax=np.abs(y).max(),
              extent=(xaxis[0], xaxis[-1], taxis[-1], taxis[0]))
axs[4].set_title('Sparse filtered data')
axs[4].axis('tight')
```



As expected, the Radon domain is a suitable domain for this type of filtering and the sparse transform improves the ability to filter out parabolic events with small curvature.

On the other hand, it is important to note that we have not been able to correctlypreserve the amplitudes of each event. This is because the sparse Radon transform can only identify a sparsest response that explain the data within a certain threshold. For this reason an more suitable approach for preserving amplitudes could be to apply a parabolic Raodn transform with the aim of reconstructing only the unwanted event and apply an adaptive subtractionbetween the input data and the reconstructed unwanted event.

Total running time of the script: (0 minutes 34.780 seconds)

Note: Click *here* to download the full example code

3.4.11 11. Seismic regularization

The problem of *seismic data regularization* (or interpolation) is a very simple one to write, yet ill-posed and very hard to solve.

The forward modelling operator is a simple *pylops.Restriction* operator which is applied along the spatial direction(s).

$$\mathbf{y} = \mathbf{R}\mathbf{x}$$

Here $\mathbf{y} = [\mathbf{y}_{R1}^T, \mathbf{y}_{R2}^T, ..., \mathbf{y}_{RN^T}]^T$ where each vector $\mathbf{y}_{Ri}$ contains all time samples recorded in the seismic data at the specific receiver R_i. Similarly, $\mathbf{x} = [\mathbf{x}_{r1}^T, \mathbf{x}_{r2}^T, ..., \mathbf{x}_{rM}^T]$, contains all traces at the regularly and finely sampled receiver locations r_i.

By inverting such an equation we can create a regularized data with densely and regularly spatial direction(s).

```
import numpy as np
import matplotlib.pyplot as plt
from scipy.signal import convolve

import pylops
from pylops.utils.wavelets import ricker
from pylops.utils.seismicevents import makeaxis, linear2d

np.random.seed(0)
plt.close('all')
```

Let's start by creating a very simple 2d data composed of 3 linear events input parameters

```
par = {'ox':0, 'dx':2, 'nx':70,
       'ot':0, 'dt':0.004, 'nt':80,
       'f0': 20}

v = 1500
t0_m = [0.1, 0.2, 0.28]
theta_m = [0, 30, -80]
phi_m = [0]
amp_m = [1., -2, 0.5]

# axis
taxis, t2, xaxis, y = makeaxis(par)
```

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```
# wavelet
wav = ricker(taxis[:41], f0=par['f0'])[0]

# model
_, x = linear2d(xaxis, taxis, v, t0_m, theta_m, amp_m, wav)
```

We can now define the spatial locations along which the data has been sampled. In this specific example we will assume that we have access only to 40% of the 'original' locations.

```
perc_subsampling = 0.6
nxsub = int(np.round(par['nx']*perc_subsampling))

iava = np.sort(np.random.permutation(np.arange(par['nx']))[:nxsub])

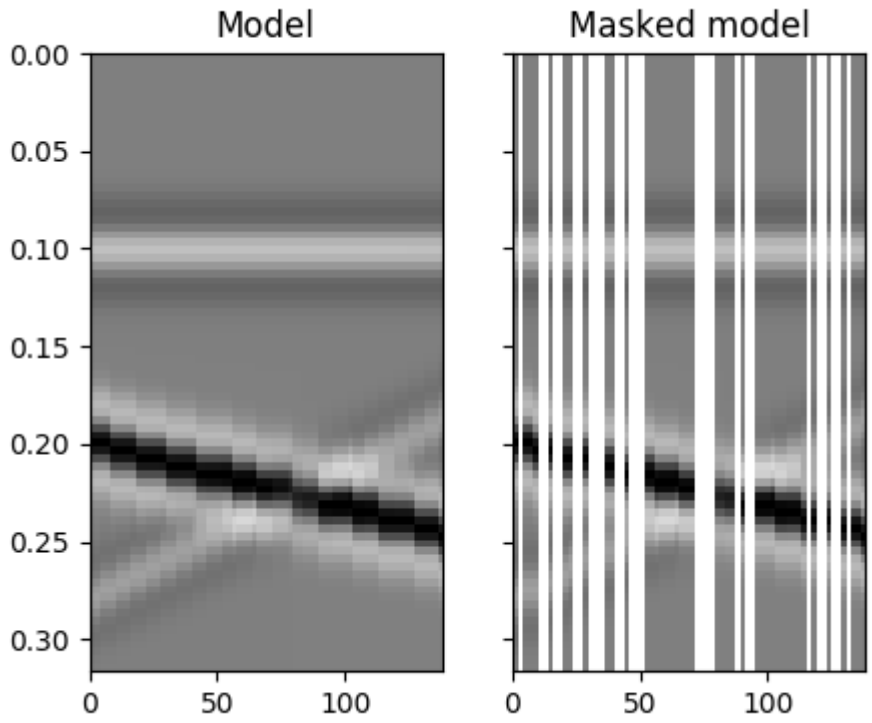
# restriction operator
Rop = pylops.Restriction(par['nx']*par['nt'],
                         iava, dims=(par['nx'], par['nt']),
                         dir=0, dtype='float64')

# data
y = Rop*x.ravel()
y = y.reshape(nxsub, par['nt'])

# mask
ymask = Rop.mask(x.flatten())

# inverse
xinv = Rop / y.ravel()
xinv = xinv.reshape(par['nx'], par['nt'])

fig, axs = plt.subplots(1, 2, sharey=True, figsize=(5, 4))
axs[0].imshow(x.T, cmap='gray', vmin=-2, vmax=2,
              extent=(xaxis[0], xaxis[-1], taxis[-1], taxis[0]))
axs[0].set_title('Model')
axs[0].axis('tight')
axs[1].imshow(ymask.T, cmap='gray', vmin=-2, vmax=2,
              extent=(xaxis[0], xaxis[-1], taxis[-1], taxis[0]))
axs[1].set_title('Masked model')
axs[1].axis('tight')
```



As we can see, inverting the restriction operator is not possible without adding any prior information into the inverse problem. In the following we will consider two possible routes:

- regularized inversion with second derivative along the spatial axis

$$J = ||\mathbf{y} - \mathbf{R}\mathbf{x}||_2 + \epsilon_\nabla^2 ||\nabla \mathbf{x}||_2$$

- sparsity-promoting inversion with `pylops.FFT2` operator used as sparsyfing transform

$$J = ||\mathbf{y} - \mathbf{R}\mathbf{F}^H\mathbf{x}||_2 + \epsilon ||\mathbf{F}^H\mathbf{x}||_1$$

```
# smooth inversion
D2op = pylops.SecondDerivative(par['nx']*par['nt'],
                               dims=(par['nx'], par['nt']),
                               dir=0, dtype='float64')

xsmooth, _, _ = \
    pylops.waveeqprocessing.SeismicInterpolation(y, par['nx'], iava,
                                                 kind='spatial',
                                                 **dict(epsRs=[np.sqrt(0.1)],
                                                        damp=np.sqrt(1e-4),
                                                        iter_lim=50, show=0))

# sparse inversion with FFT2
nfft = 2**8
FFTop = pylops.signalprocessing.FFT2D(dims=[par['nx'], par['nt']],
                                      nffts=[nfft, nfft],
                                      sampling=[par['dx'], par['dt']])
X = FFTop*x.flatten()
```

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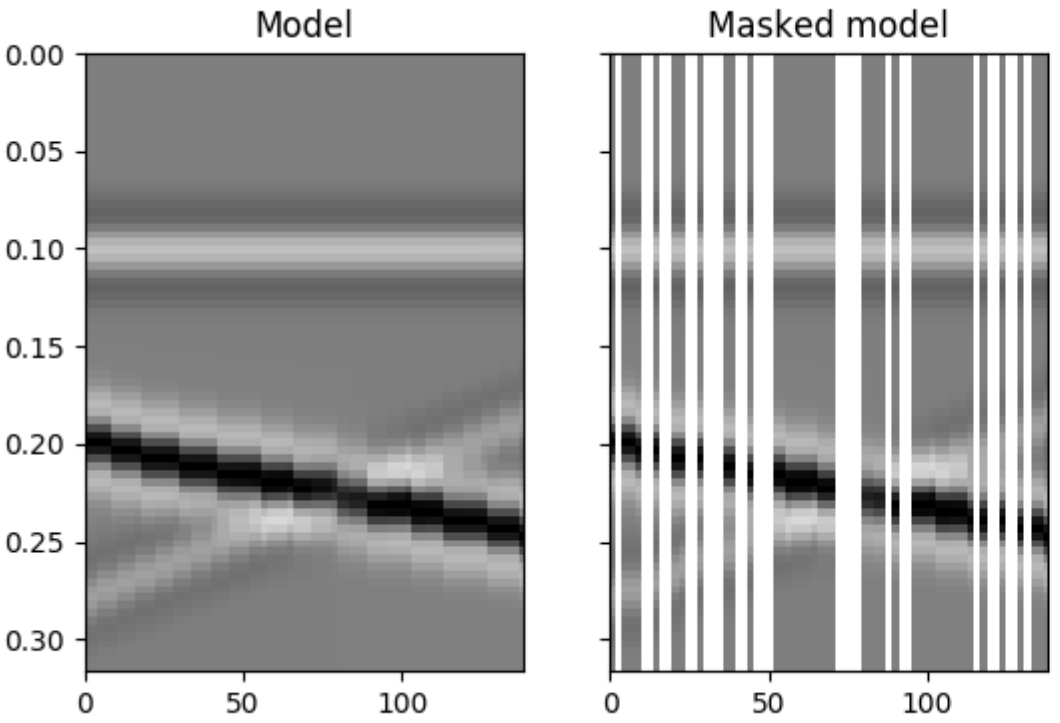
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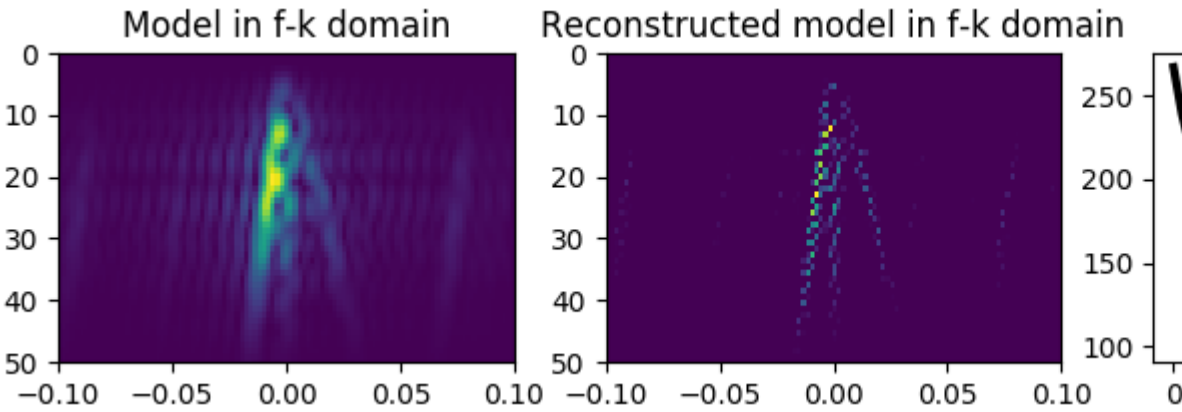
```
X = np.reshape(X, (nfft, nfft))

xl1, Xl1, cost = \
    pylops.waveeqprocessing.SeismicInterpolation(y, par['nx'], iava, kind='fk',
                                                 nffts=(nfft, nfft),
                                                 sampling=(par['dx'],
                                                           par['dt']),
                                                 **dict(niter=50, eps=1e-1,
                                                        returninfo=True))

fig, axs = plt.subplots(1, 4, sharey=True, figsize=(13, 4))
axs[0].imshow(x.T, cmap='gray', vmin=-2, vmax=2,
              extent=(xaxis[0], xaxis[-1], taxis[-1], taxis[0]))
axs[0].set_title('Model')
axs[0].axis('tight')
axs[1].imshow(ymask.T, cmap='gray', vmin=-2, vmax=2,
              extent=(xaxis[0], xaxis[-1], taxis[-1], taxis[0]))
axs[1].set_title('Masked model')
axs[1].axis('tight')
axs[2].imshow(xsmooth.T, cmap='gray', vmin=-2, vmax=2,
              extent=(xaxis[0], xaxis[-1], taxis[-1], taxis[0]))
axs[2].set_title('Smoothed model')
axs[2].axis('tight')
axs[3].imshow(xl1.T, cmap='gray', vmin=-2, vmax=2,
              extent=(xaxis[0], xaxis[-1], taxis[-1], taxis[0]))
axs[3].set_title('L1 model')
axs[3].axis('tight')

fig, axs = plt.subplots(1, 3, figsize=(10, 2))
axs[0].imshow(np.fft.fftshift(np.abs(X[:, :nfft//2-1]), axes=0).T,
              extent=(np.fft.fftshift(FFTop.f1)[0],
                      np.fft.fftshift(FFTop.f1)[-1],
                      FFTop.f2[nfft // 2 - 1], FFTop.f2[0]))
axs[0].set_title('Model in f-k domain')
axs[0].axis('tight')
axs[0].set_xlim(-0.1, 0.1)
axs[0].set_ylim(50, 0)
axs[1].imshow(np.fft.fftshift(np.abs(Xl1[:, :nfft // 2 - 1]), axes=0).T,
              extent=(np.fft.fftshift(FFTop.f1)[0],
                      np.fft.fftshift(FFTop.f1)[-1],
                      FFTop.f2[nfft // 2 - 1], FFTop.f2[0],))
axs[1].set_title('Reconstructed model in f-k domain')
axs[1].axis('tight')
axs[1].set_xlim(-0.1, 0.1)
axs[1].set_ylim(50, 0)
axs[2].plot(cost, 'k', lw=3)
axs[2].set_title('FISTA convergence')
```

- 

- 

We see how adding prior information to the inversion can help improving the estimate of the regularized seismic data. Nevertheless, in both cases the reconstructed data is not perfect. A better sparsyfing transform could in fact be chosen here to be the linear *pylops.signalprocessing.Radon2D* transform in spite of the `pylops.FFT2` transform.

```
npx = 40
pxmax = 1e-3
px = np.linspace(-pxmax, pxmax, npx)
Radop = pylops.signalprocessing.Radon2D(taxis, xaxis, px, engine='numba')

RRop = Rop*Radop

# adjoint
Xadj_fromx = Radop.H*x.flatten()
Xadj_fromx = Xadj_fromx.reshape(npx, par['nt'])
```

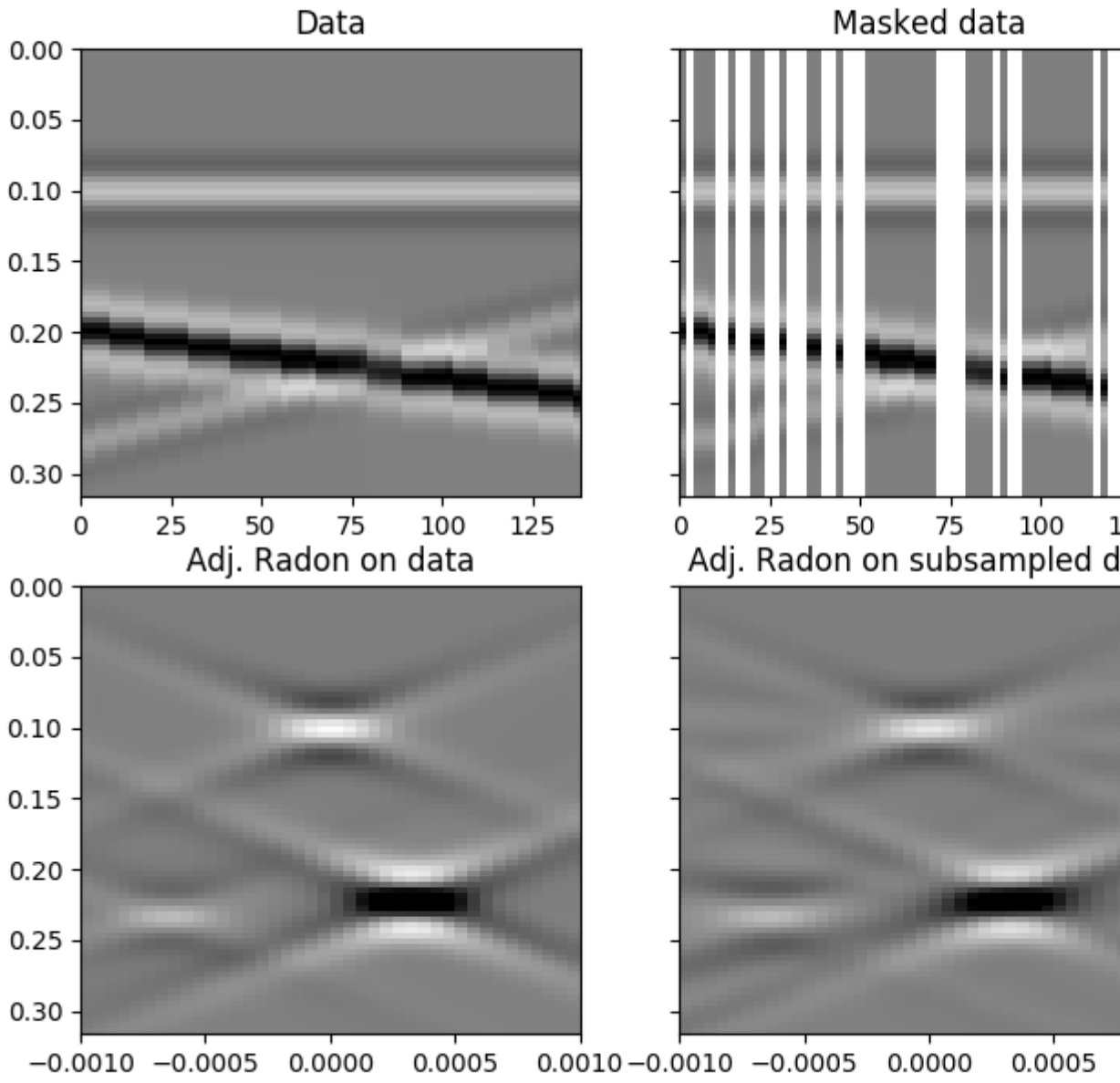
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```
Xadj = RRop.H*y.flatten()
Xadj = Xadj.reshape(npx, par['nt'])

# L1 inverse
xl1, Xl1, cost = \
    pylops.waveeqprocessing.SeismicInterpolation(y, par['nx'], iava,
                                                 kind='radon-linear',
                                                 spataxis=xaxis,
                                                 taxis=taxis, paxis=px,
                                                 centeredh=True,
                                                 **dict(niter=50, eps=1e-1,
                                                        returninfo=True))

fig, axs = plt.subplots(2, 3, sharey=True, figsize=(12, 7))
axs[0][0].imshow(x.T, cmap='gray', vmin=-2, vmax=2,
                 extent=(xaxis[0], xaxis[-1], taxis[-1], taxis[0]))
axs[0][0].set_title('Data', fontsize=12)
axs[0][0].axis('tight')
axs[0][1].imshow(ymask.T, cmap='gray', vmin=-2, vmax=2,
                 extent=(xaxis[0], xaxis[-1], taxis[-1], taxis[0]))
axs[0][1].set_title('Masked data', fontsize=12)
axs[0][1].axis('tight')
axs[0][2].imshow(xl1.T, cmap='gray', vmin=-2, vmax=2,
                 extent=(xaxis[0], xaxis[-1], taxis[-1], taxis[0]))
axs[0][2].set_title('Reconstructed data', fontsize=12)
axs[0][2].axis('tight')
axs[1][0].imshow(Xadj_fromx.T, cmap='gray', vmin=-70, vmax=70,
                 extent=(px[0], px[-1], taxis[-1], taxis[0]))
axs[1][0].set_title('Adj. Radon on data', fontsize=12)
axs[1][0].axis('tight')
axs[1][1].imshow(Xadj.T, cmap='gray', vmin=-50, vmax=50,
                 extent=(px[0], px[-1], taxis[-1], taxis[0]))
axs[1][1].set_title('Adj. Radon on subsampled data', fontsize=12)
axs[1][1].axis('tight')
axs[1][2].imshow(Xl1.T, cmap='gray', vmin=-0.2, vmax=0.2,
                 extent=(px[0], px[-1], taxis[-1], taxis[0]))
axs[1][2].set_title('Inverse Radon on subsampled data', fontsize=12)
axs[1][2].axis('tight')
```



Finally, let's take now a more realistic dataset. We will use once again the linear *pylops.signalprocessing.Radon2D* transform but we will take advantnge of the *pylops.signalprocessing.Sliding2D* operator to perform such a transform locally instead of globally to the entire dataset.

```
inputfile = '../testdata/marchenko/input.npz'
inputdata = np.load(inputfile)

x = inputdata['R'][50, :, ::2]
x = x/np.abs(x).max()
```

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```
taxis, xaxis = inputdata['t'][::2], inputdata['r'][0]

par = {}
par['nx'], par['nt'] = x.shape
par['dx'] = inputdata['r'][0, 1] - inputdata['r'][0, 0]
par['dt'] = inputdata['t'][1] - inputdata['t'][0]

# add wavelet
wav = inputdata['wav'][::2]
wav_c = np.argmax(wav)
x = np.apply_along_axis(convolve, 1, x, wav, mode='full')
x = x[:, wav_c:][:, :par['nt']]

# gain
gain = np.tile((taxis**2)[:, np.newaxis], (1, par['nx'])).T
x = x*gain

# subsampling locations
perc_subsampling = 0.5
Nsub = int(np.round(par['nx']*perc_subsampling))
iava = np.sort(np.random.permutation(np.arange(par['nx']))[:Nsub])

# restriction operator
Rop = pylops.Restriction(par['nx']*par['nt'], iava,
                         dims=(par['nx'], par['nt']),
                         dir=0, dtype='float64')

y = Rop*x.flatten()
xadj = Rop.H*y.flatten()

y = y.reshape(Nsub, par['nt'])
xadj = xadj.reshape(par['nx'], par['nt'])

# apply mask
ymask = Rop.mask(x.flatten())

# sliding windows with radon transform
dx = par['dx']
nwins = 4
nwin = 27
nover = 3
npx = 31
pxmax = 5e-4
px = np.linspace(-pxmax, pxmax, npx)
dimsd = x.shape
dims = (nwins*npx, dimsd[1])

Op = \
    pylops.signalprocessing.Radon2D(taxis, np.linspace(-par['dx']*nwin//2,
                                                       par['dx']*nwin//2,
                                                       nwin),
                                    px, centeredh=True, kind='linear',
                                    engine='numba')
Slidop = pylops.signalprocessing.Sliding2D(Op, dims, dimsd, nwin, nover,
                                           tapertype='cosine', design=True)

# adjoint
```

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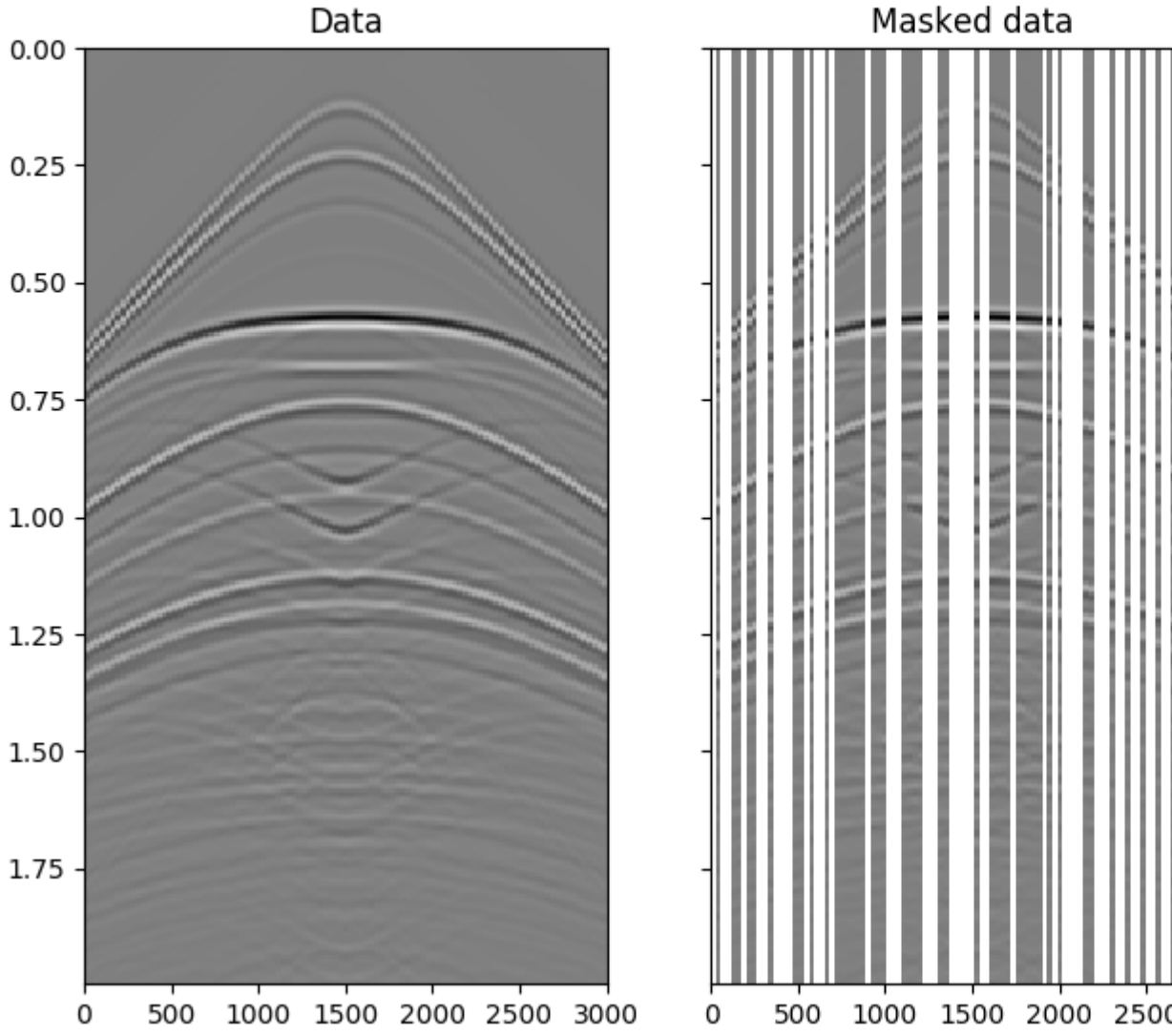
```
RSop = Rop*Slidop

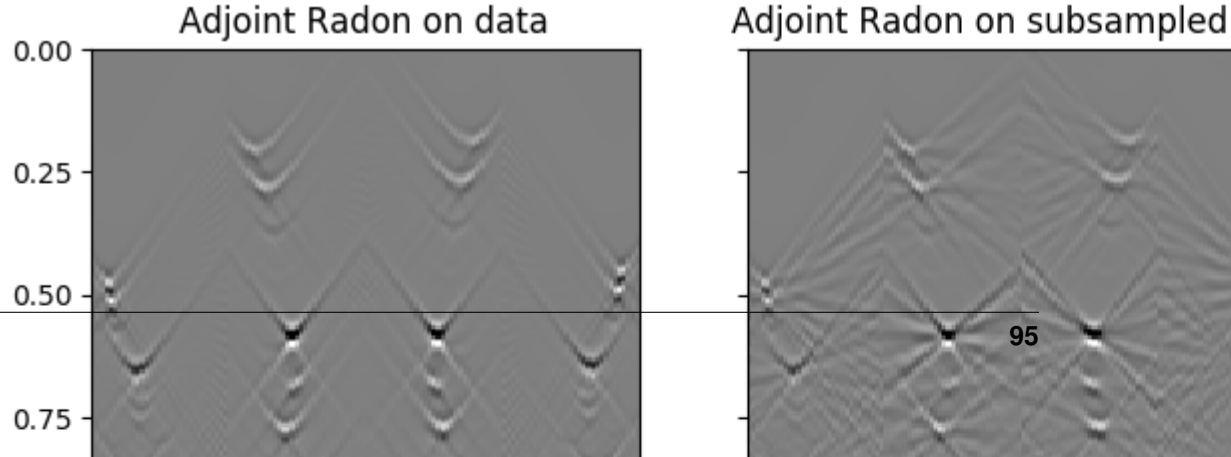
Xadj_fromx = Slidop.H*x.flatten()
Xadj_fromx = Xadj_fromx.reshape(npx*nwins, par['nt'])

Xadj = RSop.H*y.flatten()
Xadj = Xadj.reshape(npx*nwins, par['nt'])

# inverse
xl1, Xl1, _ = \
    pylops.waveeqprocessing.SeismicInterpolation(y, par['nx'], iava,
                                                 kind='sliding',
                                                 spataxis=xaxis,
                                                 taxis=taxis, paxis=px,
                                                 nwins=nwins, nwin=nwin,
                                                 nover=nover,
                                                 **dict(niter=50, eps=1e-2))

fig, axs = plt.subplots(2, 3, sharey=True, figsize=(12, 14))
axs[0][0].imshow(x.T, cmap='gray', vmin=-0.1, vmax=0.1,
                 extent=(xaxis[0], xaxis[-1], taxis[-1], taxis[0]))
axs[0][0].set_title('Data')
axs[0][0].axis('tight')
axs[0][1].imshow(ymask.T, cmap='gray', vmin=-0.1, vmax=0.1,
                 extent=(xaxis[0], xaxis[-1], taxis[-1], taxis[0]))
axs[0][1].set_title('Masked data')
axs[0][1].axis('tight')
axs[0][2].imshow(xl1.T, cmap='gray', vmin=-0.1, vmax=0.1,
                 extent=(xaxis[0], xaxis[-1], taxis[-1], taxis[0]))
axs[0][2].set_title('Reconstructed data')
axs[0][2].axis('tight')
axs[1][0].imshow(Xadj_fromx.T, cmap='gray', vmin=-1, vmax=1,
                 extent=(px[0], px[-1], taxis[-1], taxis[0]))
axs[1][0].set_title('Adjoint Radon on data')
axs[1][0].axis('tight')
axs[1][1].imshow(Xadj.T, cmap='gray', vmin=-0.6, vmax=0.6,
                 extent=(px[0], px[-1], taxis[-1], taxis[0]))
axs[1][1].set_title('Adjoint Radon on subsampled data')
axs[1][1].axis('tight')
axs[1][2].imshow(Xl1.T, cmap='gray', vmin=-0.03, vmax=0.03,
                 extent=(px[0], px[-1], taxis[-1], taxis[0]))
axs[1][2].set_title('Inverse Radon on subsampled data')
axs[1][2].axis('tight')
```


Data
Masked data
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
0
500
1000
1500
2000
2500
3000


Adjoint Radon on data
Adjoint Radon on subsampled
0.00
0.25
0.50
0.75

As expected the linear *pylops.signalprocessing.Radon2D* is able to locally explain events in the input data and leads to a satisfactory recovery. Note that increasing the number of iterations and sliding windows can further refine the result, especially the accuracy of weak events, as shown in this companion notebook.

Total running time of the script: (0 minutes 13.105 seconds)

Note: Click *here* to download the full example code

3.4.12 12. Seismic wavefield decomposition

Multi-component seismic data can be decomposed in their up- and down-going constituents. This task can be accurately achieved by linearly combining the input pressure and particle velocity data in the frequency-wavenumber described in details in *pylops.waveeqprocessing.UpDownComposition2D* and *pylops.waveeqprocessing.WavefieldDecomposition*.

In this tutorial we will consider a simple synthetic data composed of six events (three up-going and three down-going). We will first combine them to create pressure and particle velocity data and then show how we can retrieve their directional constituents both by directly combining the input data as well as by setting an inverse problem. The latter approach results vital in case of spatial aliasing, as applying simple scaled summation in the frequency-wavenumber would result in sub-optimal decomposition due to the superposition of different frequency-wavenumber pairs at some (aliased) locations.

```
import numpy as np
import matplotlib.pyplot as plt
from scipy.signal import filtfilt

import pylops
from pylops.utils.wavelets import ricker
from pylops.utils.seismicevents import makeaxis, hyperbolic2d

np.random.seed(0)
plt.close('all')
```

Let's first the input up- and down-going wavefields

```
par = {'ox':-220,  'dx':5, 'nx':89,
       'ot':0, 'dt':0.004, 'nt':200,
       'f0': 40}

t0_plus = np.array([0.2, 0.5, 0.7])
t0_minus = t0_plus + 0.04
vrms = np.array([1400., 1500., 2000.])
amp = np.array([1., -0.6, 0.5])
vel_sep = 1000.0 # velocity at separation level
rho_sep = 1000.0 # density at separation level

# Create axis
t, t2, x, y = makeaxis(par)

# Create wavelet
wav = ricker(t[:41], f0=par['f0'])[0]

# Create data
_, p_minus = hyperbolic2d(x, t, t0_minus, vrms, amp, wav)
_, p_plus = hyperbolic2d(x, t, t0_plus, vrms, amp, wav)
```

We can now combine them to create pressure and particle velocity data

```
critical = 1.1
ntaper = 51
nfft=2**10

# 2d fft operator
FFTop = pylops.signalprocessing.FFT2D(dims=[par['nx'], par['nt']],
                                      nffts=[nfft, nfft],
                                      sampling=[par['dx'], par['dt']])

#obliquity factor
[Kx, F] = np.meshgrid(FFTop.f1, FFTop.f2, indexing='ij')
k = F/vel_sep
Kz = np.sqrt((k**2-Kx**2).astype(np.complex))
Kz[np.isnan(Kz)] = 0
OBL=rho_sep*(np.abs(F)/Kz)
OBL[Kz==0]=0

mask = np.abs(Kx)<critical*np.abs(F)/vel_sep
OBL *= mask
OBL = filtfilt(np.ones(ntaper)/float(ntaper), 1, OBL, axis=0)
OBL = filtfilt(np.ones(ntaper)/float(ntaper), 1, OBL, axis=1)

# composition operator
UPop = \
    pylops.waveeqprocessing.UpDownComposition2D(par['nt'], par['nx'],
                                                par['dt'], par['dx'],
                                                rho_sep, vel_sep,
                                                nffts=(nfft, nfft),
                                                critical=critical*100.,
                                                ntaper=ntaper,
                                                dtype='complex128')

# wavefield modelling
d = UPop * np.concatenate((p_plus.flatten(), p_minus.flatten())).flatten()
d = np.real(d.reshape(2*par['nx'], par['nt']))
p, vz = d[:par['nx']], d[par['nx']:]

# obliquity scaled vz
VZ = FFTop * vz.flatten()
VZ = VZ.reshape(nfft, nfft)

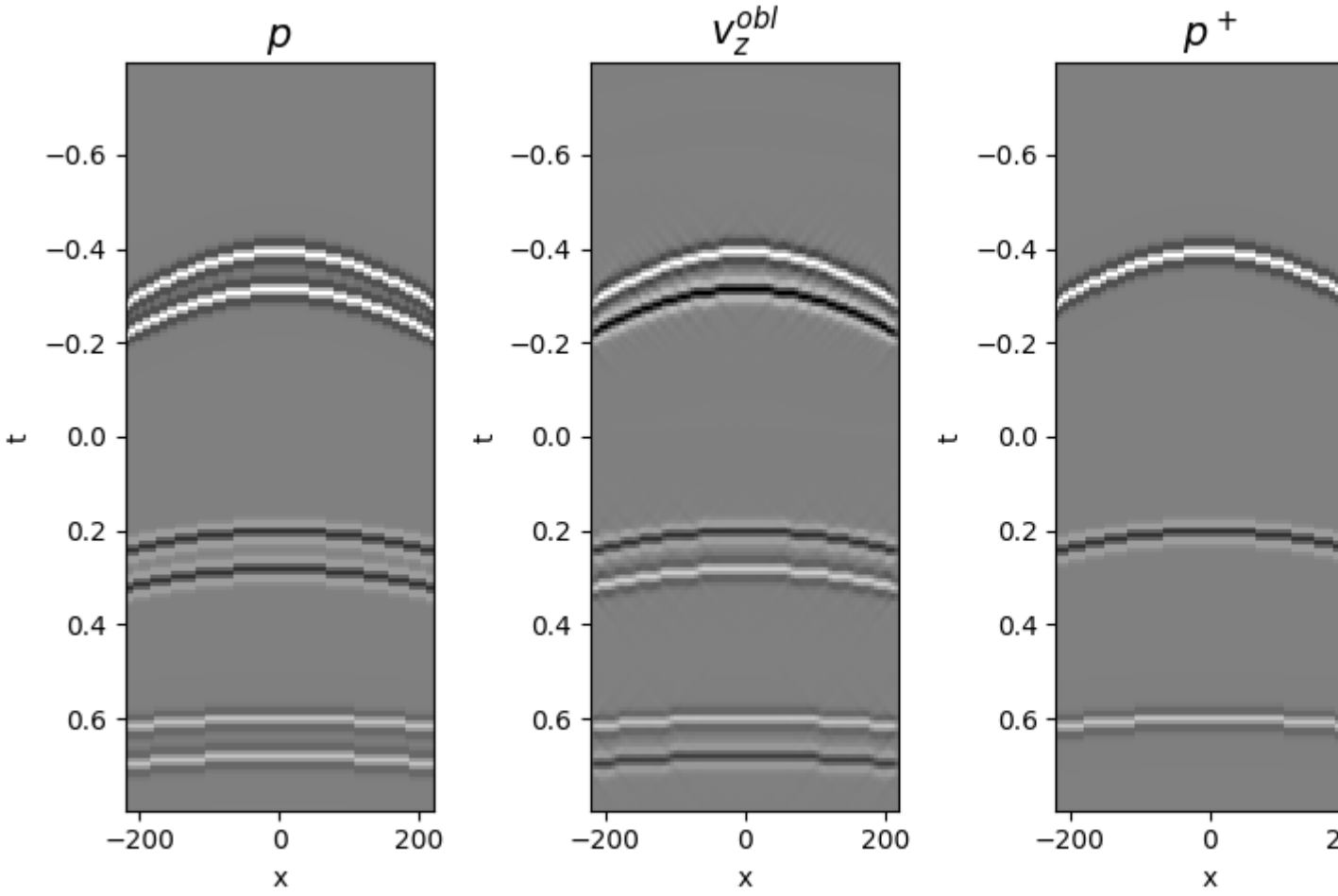
VZ_obl = OBL * VZ
vz_obl = FFTop.H*VZ_obl.flatten()
vz_obl = np.real(vz_obl.reshape(par['nx'], par['nt']))

fig, axs = plt.subplots(1, 4, figsize=(10, 5))
axs[0].imshow(p.T, aspect='auto',
              interpolation='nearest', cmap='gray',
              extent=(x.min(),x.max(),t2.max(),t2.min()),
              vmin=-1, vmax=1)
axs[0].set_title(r'$p$', fontsize=15)
axs[0].set_xlabel('x')
axs[0].set_ylabel('t')
axs[1].imshow(vz_obl.T, aspect='auto',
              interpolation='nearest', cmap='gray',
              extent=(x.min(), x.max(), t2.max(), t2.min()),
```

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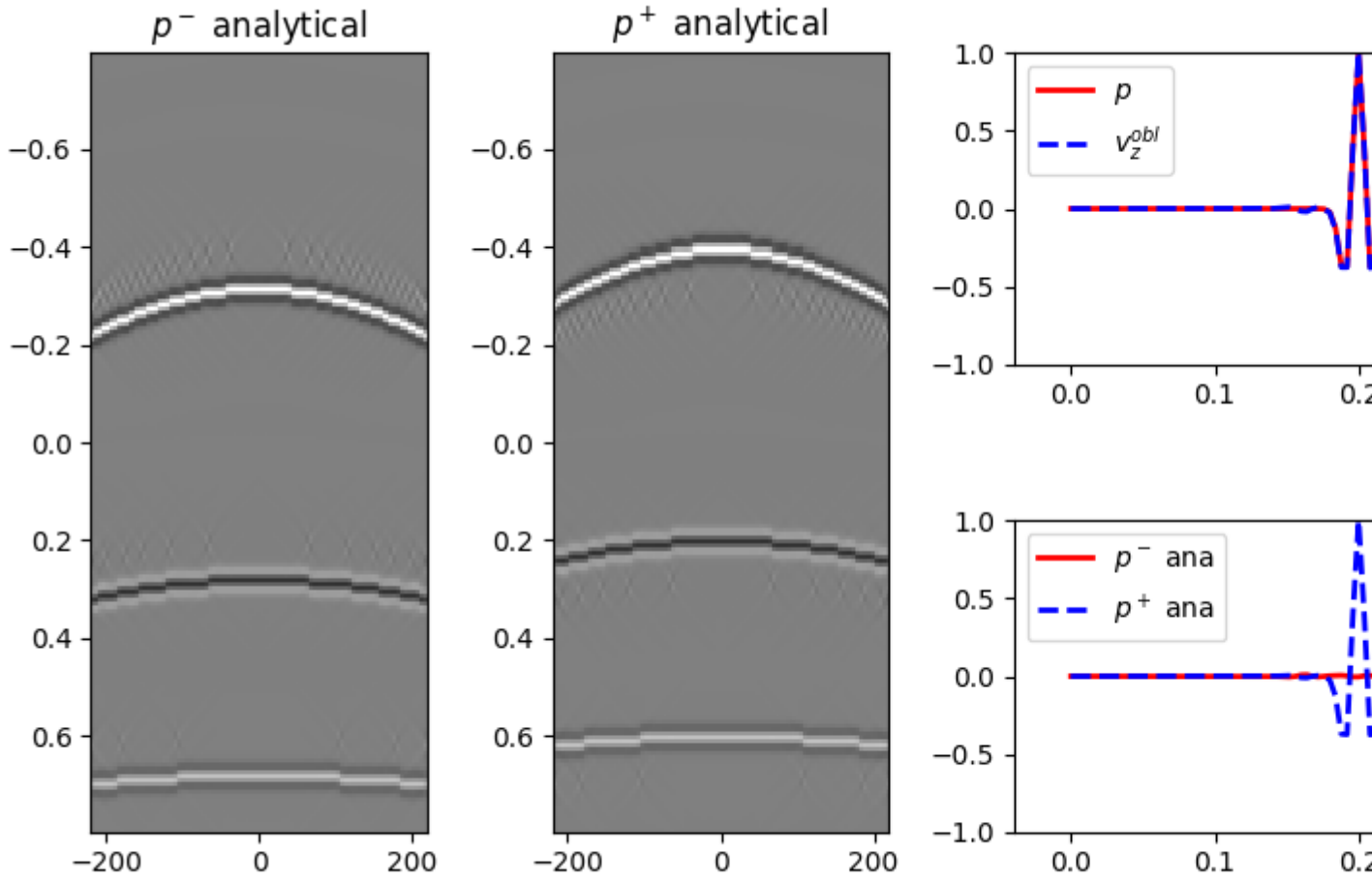
(continued from previous page)

```
              vmin=-1, vmax=1)
axs[1].set_title(r'$v_z^{obl}$', fontsize=15)
axs[1].set_xlabel('x')
axs[1].set_ylabel('t')
axs[2].imshow(p_plus.T, aspect='auto',
              interpolation='nearest', cmap='gray',
              extent=(x.min(), x.max(), t2.max(), t2.min()),
              vmin=-1, vmax=1)
axs[2].set_title(r'$p^+$', fontsize=15)
axs[2].set_xlabel('x')
axs[2].set_ylabel('t')
axs[3].imshow(p_minus.T, aspect='auto',
              interpolation='nearest', cmap='gray',
              extent=(x.min(), x.max(), t2.max(), t2.min()),
              vmin=-1, vmax=1)
axs[3].set_title(r'$p^-$', fontsize=15)
axs[3].set_xlabel('x')
axs[3].set_ylabel('t')
plt.tight_layout()
```



Wavefield separation is first performed using the analytical expression for combining pressure and particle velocity data in the wavenumber-frequency domain

```
pup_sep, pdown_sep = \
    pylops.waveeqprocessing.WavefieldDecomposition(p, vz, par['nt'], par['nx'],
                                                   par['dt'], par['dx'],
                                                   rho_sep, vel_sep,
                                                   nffts=(nfft, nfft),
                                                   kind='analytical',
                                                   critical=critical*100,
                                                   ntaper=ntaper,
                                                   dtype='complex128')
fig = plt.figure(figsize=(12, 5))
axs0 = plt.subplot2grid((2, 5), (0, 0), rowspan=2)
axs1 = plt.subplot2grid((2, 5), (0, 1), rowspan=2)
axs2 = plt.subplot2grid((2, 5), (0, 2), colspan=3)
axs3 = plt.subplot2grid((2, 5), (1, 2), colspan=3)
axs0.imshow(pup_sep.T, cmap='gray',
              extent=(x.min(),x.max(),t2.max(),t2.min()),
              vmin=-1, vmax=1)
axs0.set_title(r'$p^-$ analytical')
axs0.axis('tight')
axs1.imshow(pdown_sep.T, cmap='gray',
              extent=(x.min(),x.max(),t2.max(),t2.min()),
              vmin=-1, vmax=1)
axs1.set_title(r'$p^+$ analytical')
axs1.axis('tight')
axs2.plot(t, p[par['nx']//2], 'r', lw=2, label=r'$p$')
axs2.plot(t, vz_obl[par['nx']//2], '--b', lw=2, label=r'$v_z^{obl}$')
axs2.set_ylim(-1, 1)
axs2.set_title('Data at x=%.2f' % x[par['nx']//2])
axs2.set_xlabel('t [s]')
axs2.legend()
axs3.plot(t, pup_sep[par['nx']//2], 'r', lw=2, label=r'$p^-$ ana')
axs3.plot(t, pdown_sep[par['nx']//2], '--b', lw=2, label=r'$p^+$ ana')
axs3.set_title('Separated wavefields at x=%.2f' % x[par['nx']//2])
axs3.set_xlabel('t [s]')
axs3.set_ylim(-1, 1)
axs3.legend()
plt.tight_layout()
```



We repeat the same exercise but this time we invert the composition operator *pylops.waveeqprocessing.UpDownComposition2D*

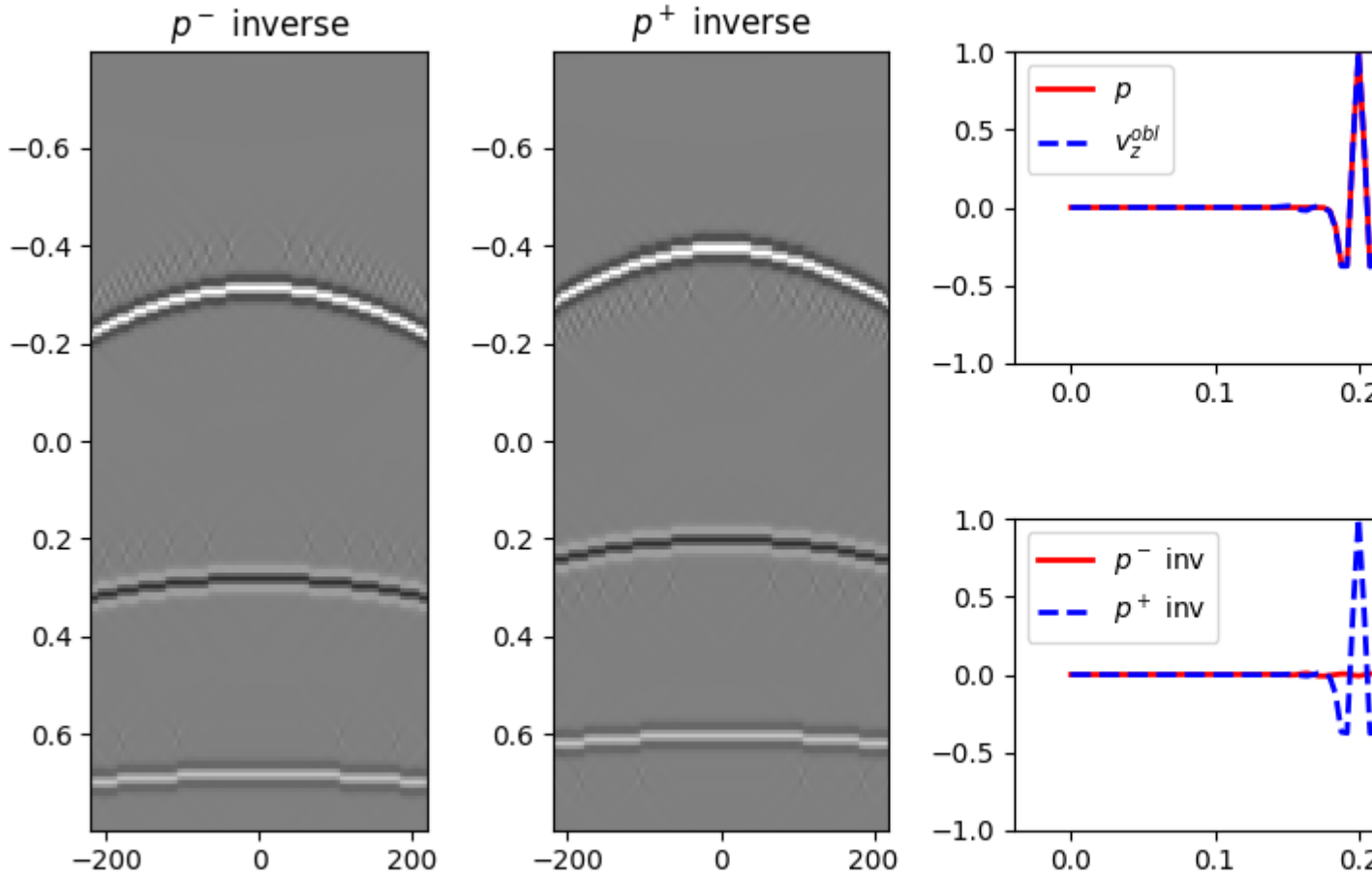
```
pup_inv, pdown_inv = \
    pylops.waveeqprocessing.WavefieldDecomposition(p, vz, par['nt'], par['nx'],
                                                   par['dt'], par['dx'],
                                                   rho_sep, vel_sep,
                                                   nffts=(nfft, nfft),
                                                   kind='inverse',
                                                   critical=critical*100,
                                                   ntaper=ntaper,
                                                   dtype='complex128',
                                                   **dict(damp=1e-10,
                                                          iter_lim=20))

fig = plt.figure(figsize=(12, 5))
axs0 = plt.subplot2grid((2, 5), (0, 0), rowspan=2)
axs1 = plt.subplot2grid((2, 5), (0, 1), rowspan=2)
axs2 = plt.subplot2grid((2, 5), (0, 2), colspan=3)
axs3 = plt.subplot2grid((2, 5), (1, 2), colspan=3)
axs0.imshow(pup_inv.T, cmap='gray',
              extent=(x.min(),x.max(),t2.max(),t2.min()),
              vmin=-1, vmax=1)
```

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```
axs0.set_title(r'$p^-$ inverse')
axs0.axis('tight')
axs1.imshow(pdown_inv.T, cmap='gray',
              extent=(x.min(),x.max(),t2.max(),t2.min()),
              vmin=-1, vmax=1)
axs1.set_title(r'$p^+$ inverse')
axs1.axis('tight')
axs2.plot(t, p[par['nx']//2], 'r', lw=2, label=r'$p$')
axs2.plot(t, vz_obl[par['nx']//2], '--b', lw=2, label=r'$v_z^{obl}$')
axs2.set_ylim(-1, 1)
axs2.set_title('Data at x=%.2f' % x[par['nx']//2])
axs2.set_xlabel('t [s]')
axs2.legend()
axs3.plot(t, pup_inv[par['nx']//2], 'r', lw=2, label=r'$p^-$ inv')
axs3.plot(t, pdown_inv[par['nx']//2], '--b', lw=2, label=r'$p^+$ inv')
axs3.set_title('Separated wavefields at x=%.2f' % x[par['nx']//2])
axs3.set_xlabel('t [s]')
axs3.set_ylim(-1, 1)
axs3.legend()
plt.tight_layout()
```



The up- and down-going constituents have been succesfully separated in both cases. To see more examples, including

applying wavefield separation and regularization simultaneously, head over to these companion notebooks: notebook1 and notebook2

Total running time of the script: (0 minutes 19.325 seconds)

Note: Click *here* to download the full example code

3.4.13 13. Least-squares migration

Seismic migration is the process by which seismic data are manipulated to create an image of the subsurface reflectivity.

While traditionally solved as the adjont of the demigration operator, it is becoming more and more common to solve the underlying inverse problem in the quest for more accurate and detailed subsurface images.

Indipendently of the choice of the modelling operator (i.e., ray-based or full wavefield-based), the demigration/migration process can be expressed as a linear operator of such a kind:

$$d(\mathbf{x_r}, \mathbf{x_s}, t) = w(t) * \int_V G(\mathbf{x}, \mathbf{x_s}, t) G(\mathbf{x_r}, \mathbf{x}, t) m(\mathbf{x}) d\mathbf{x}$$

where $m(\mathbf{x})$ is the reflectivity at every location in the subsurface, $G(\mathbf{x}, \mathbf{x_s}, t)$ and $G(\mathbf{x_r}, \mathbf{x}, t)$ are the Green's functions from source-to-subsurface-to-receiver and finally $w(t)$ is the wavelet. Ultimately, while the Green's functions can be computed in many different ways, solving this system of equations for the reflectivity model is what we generally refer to as Least-squares migration (LSM).

In this tutorial we will consider the most simple scenario where we use an eikonal solver to compute the Green's functions and show how we can use the *pylops.waveeqprocessing.LSM* operator to perform LSM.

```
import numpy as np
import matplotlib.pyplot as plt
from scipy.sparse.linalg import lsqr

import pylops

plt.close('all')
np.random.seed(0)
```

To start we create a simple model with 2 interfaces

```
# Velocity Model
nx, nz = 81, 60
dx, dz = 4, 4
x, z = np.arange(nx)*dx, np.arange(nz)*dz
v0 = 1000 # initial velocity
kv = 0. # gradient
vel = np.outer(np.ones(nx), v0 +kv*z)

# Reflectivity Model
refl = np.zeros((nx, nz))
refl[:, 30] = -1
refl[:, 50] = 0.5

# Receivers
nr = 11
rx = np.linspace(10*dx, (nx-10)*dx, nr)
```

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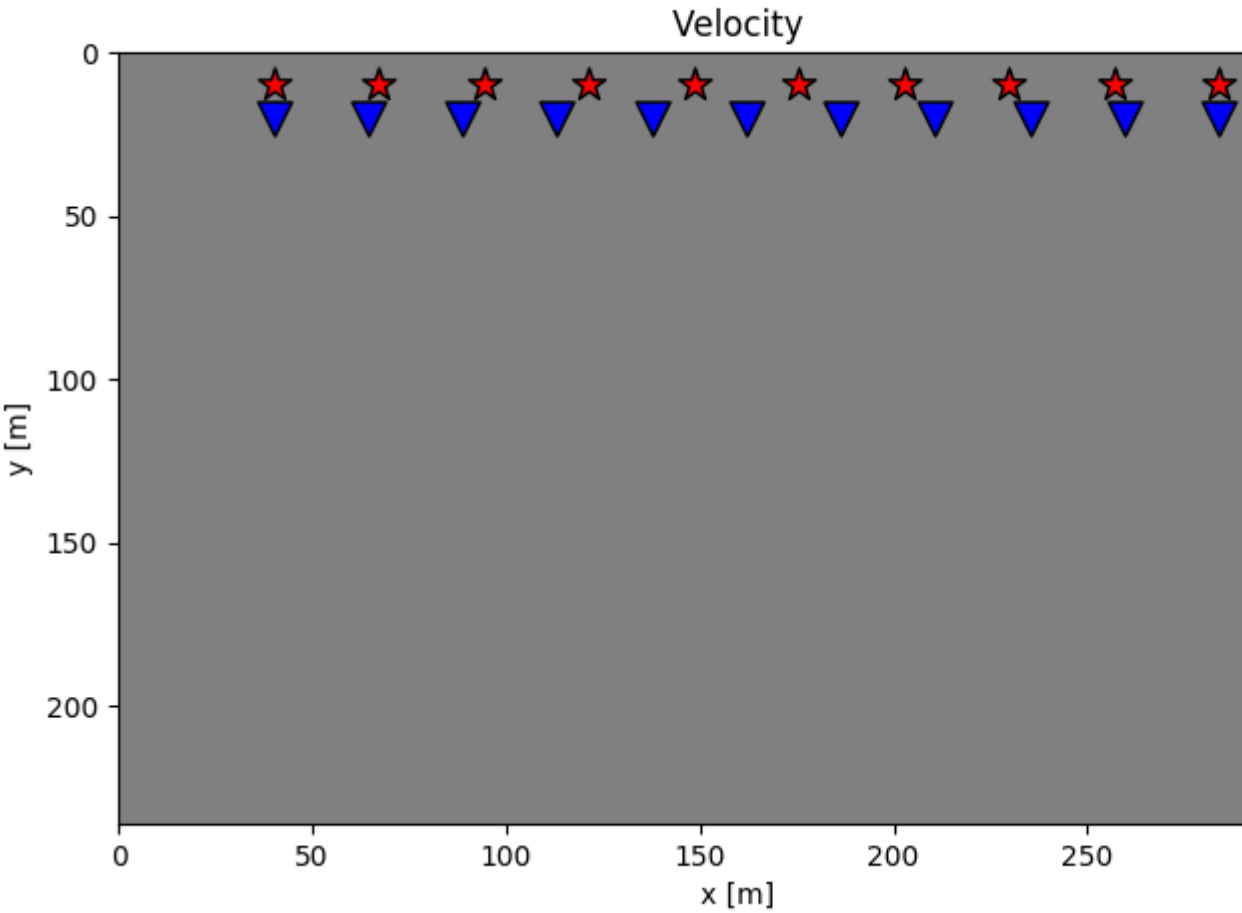
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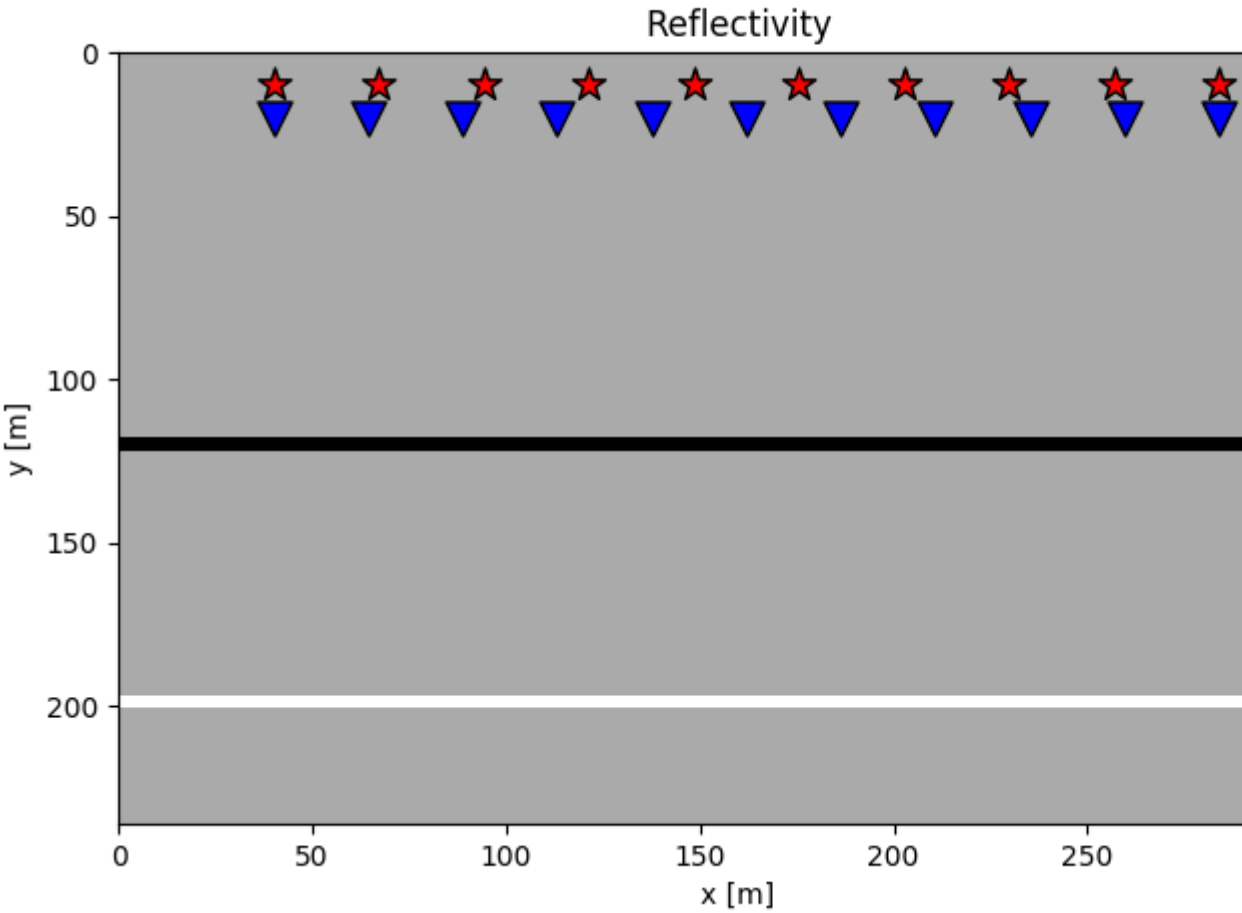
```
rz = 20*np.ones(nr)
recs = np.vstack((rx, rz))
dr = recs[0,1]-recs[0,0]

# Sources
ns = 10
sx = np.linspace(dx*10, (nx-10)*dx, ns)
sz = 10*np.ones(ns)
sources = np.vstack((sx, sz))
ds = sources[0,1]-sources[0,0]

plt.figure(figsize=(10, 5))
im = plt.imshow(vel.T, cmap='gray', extent = (x[0], x[-1], z[-1], z[0]))
plt.scatter(recs[0],  recs[1], marker='v', s=150, c='b', edgecolors='k')
plt.scatter(sources[0], sources[1], marker='*', s=150, c='r', edgecolors='k')
plt.colorbar(im)
plt.axis('tight')
plt.xlabel('x [m]'),plt.ylabel('y [m]')
plt.title('Velocity')
plt.xlim(x[0], x[-1])

plt.figure(figsize=(10, 5))
im = plt.imshow(refl.T, cmap='gray', extent = (x[0], x[-1], z[-1], z[0]))
plt.scatter(recs[0],  recs[1], marker='v', s=150, c='b', edgecolors='k')
plt.scatter(sources[0], sources[1], marker='*', s=150, c='r', edgecolors='k')
plt.colorbar(im)
plt.axis('tight')
plt.xlabel('x [m]'),plt.ylabel('y [m]')
plt.title('Reflectivity')
plt.xlim(x[0], x[-1])
```


Velocity
0
50
100
150
200
y [m]
0
50
100
150
200
250
x [m]



We can now create our LSM object and invert for the reflectivity using two different solvers: `scipy.sparse.linalg.lsqr` (LS solution) and *`pylops.optimization.sparsity.FISTA`* (LS solution with sparse model).

```
nt = 651
dt = 0.004
t = np.arange(nt)*dt
wav, wavt, wavc = pylops.utils.wavelets.ricker(t[:41], f0=20)


lsm = pylops.waveeqprocessing.LSM(z, x, t, sources, recs, v0, wav, wavc,
                                  mode='analytic')

d = lsm.Demop * refl.ravel()
d = d.reshape(ns, nr, nt)

madj = lsm.Demop.H * d.ravel()
madj = madj.reshape(nx, nz)

minv = lsm.solve(d.ravel(), solver=lsqr, **dict(iter_lim=100))
minv = minv.reshape(nx, nz)

minv_sparse = lsm.solve(d.ravel(),
```

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```
                          solver=pylops.optimization.sparsity.FISTA,
                          **dict(eps=1e2, niter=100))
minv_sparse = minv_sparse.reshape(nx, nz)

# demigration
dadj = lsm.Demop * madj.ravel()
dadj = dadj.reshape(ns, nr, nt)

dinv = lsm.Demop * minv.ravel()
dinv = dinv.reshape(ns, nr, nt)

dinv_sparse = lsm.Demop * minv_sparse.ravel()
dinv_sparse = dinv_sparse.reshape(ns, nr, nt)

# sphinx_gallery_thumbnail_number = 2
fig, axs = plt.subplots(2, 2, figsize=(10, 8))
axs[0][0].imshow(refl.T, cmap='gray', vmin=-1, vmax=1)
axs[0][0].axis('tight')
axs[0][0].set_title(r'$m$')
axs[0][1].imshow(madj.T, cmap='gray', vmin=-madj.max(), vmax=madj.max())
axs[0][1].set_title(r'$m_{adj}$')
axs[0][1].axis('tight')
axs[1][0].imshow(minv.T, cmap='gray', vmin=-1, vmax=1)
axs[1][0].axis('tight')
axs[1][0].set_title(r'$m_{inv}$')
axs[1][1].imshow(minv_sparse.T, cmap='gray', vmin=-1, vmax=1)
axs[1][1].axis('tight')
axs[1][1].set_title(r'$m_{FISTA}$')

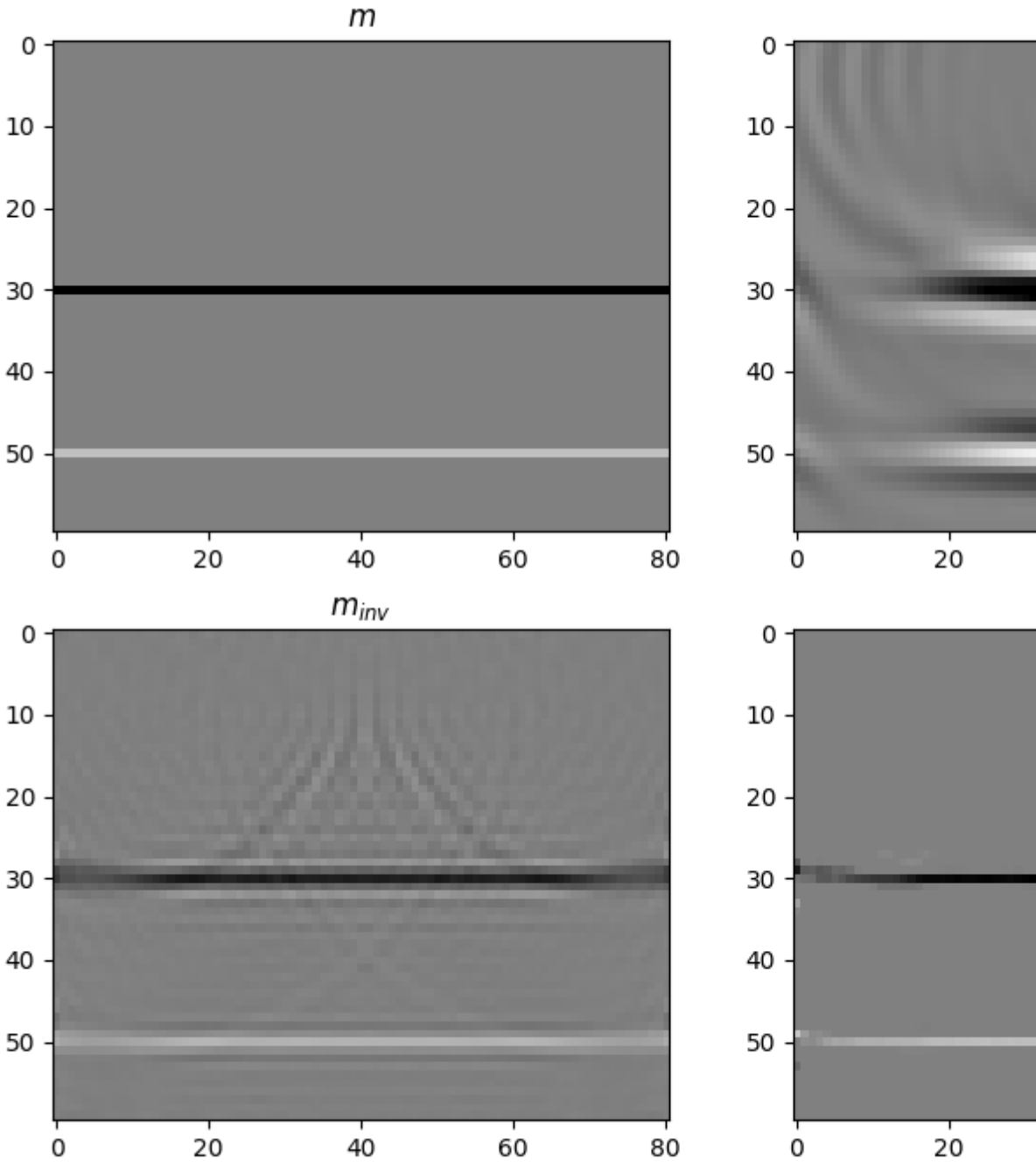
fig, axs = plt.subplots(1, 4, figsize=(10, 4))
axs[0].imshow(d[0, :, :300].T, cmap='gray',
              vmin=-d.max(), vmax=d.max())
axs[0].set_title(r'$d$')
axs[0].axis('tight')
axs[1].imshow(dadj[0, :, :300].T, cmap='gray',
              vmin=-dadj.max(), vmax=dadj.max())
axs[1].set_title(r'$d_{adj}$')
axs[1].axis('tight')
axs[2].imshow(dinv[0, :, :300].T, cmap='gray',
              vmin=-d.max(), vmax=d.max())
axs[2].set_title(r'$d_{inv}$')
axs[2].axis('tight')
axs[3].imshow(dinv_sparse[0, :, :300].T, cmap='gray',
              vmin=-d.max(), vmax=d.max())
axs[3].set_title(r'$d_{fista}$')
axs[3].axis('tight')

fig, axs = plt.subplots(1, 4, figsize=(10, 4))
axs[0].imshow(d[ns//2, :, :300].T, cmap='gray',
              vmin=-d.max(), vmax=d.max())
axs[0].set_title(r'$d$')
axs[0].axis('tight')
axs[1].imshow(dadj[ns//2, :, :300].T, cmap='gray',
              vmin=-dadj.max(), vmax=dadj.max())
axs[1].set_title(r'$d_{adj}$')
axs[1].axis('tight')
axs[2].imshow(dinv[ns//2, :, :300].T, cmap='gray',
```

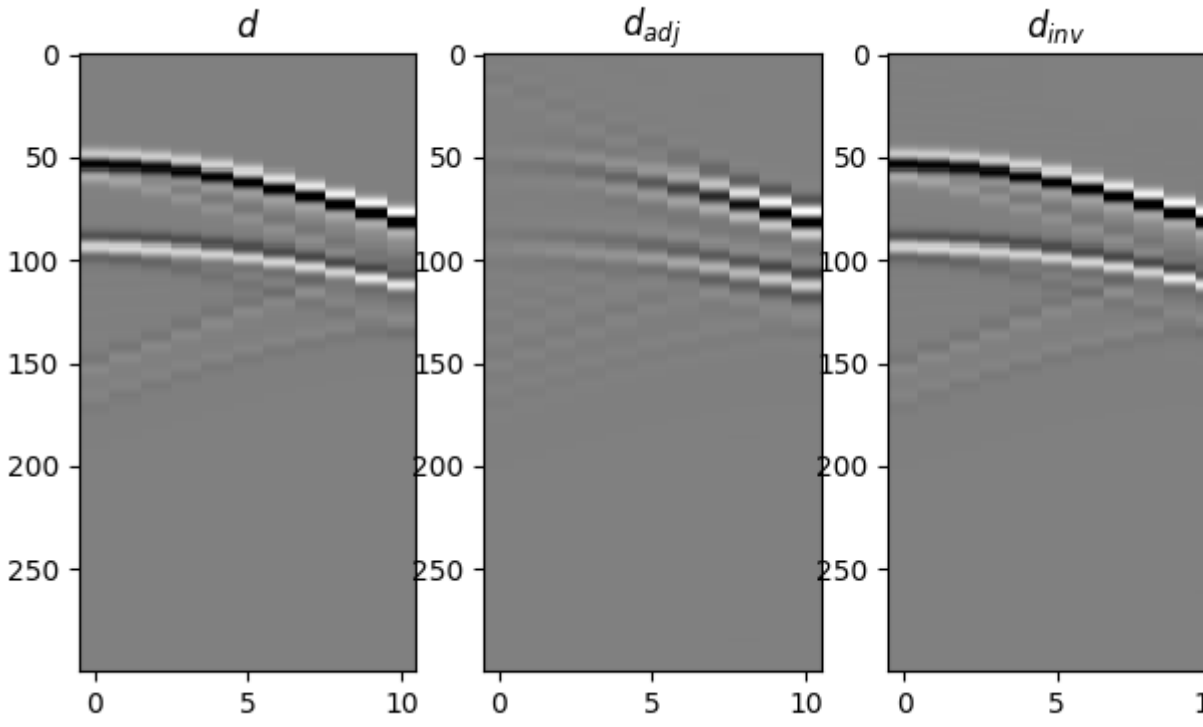
(continues on next page)

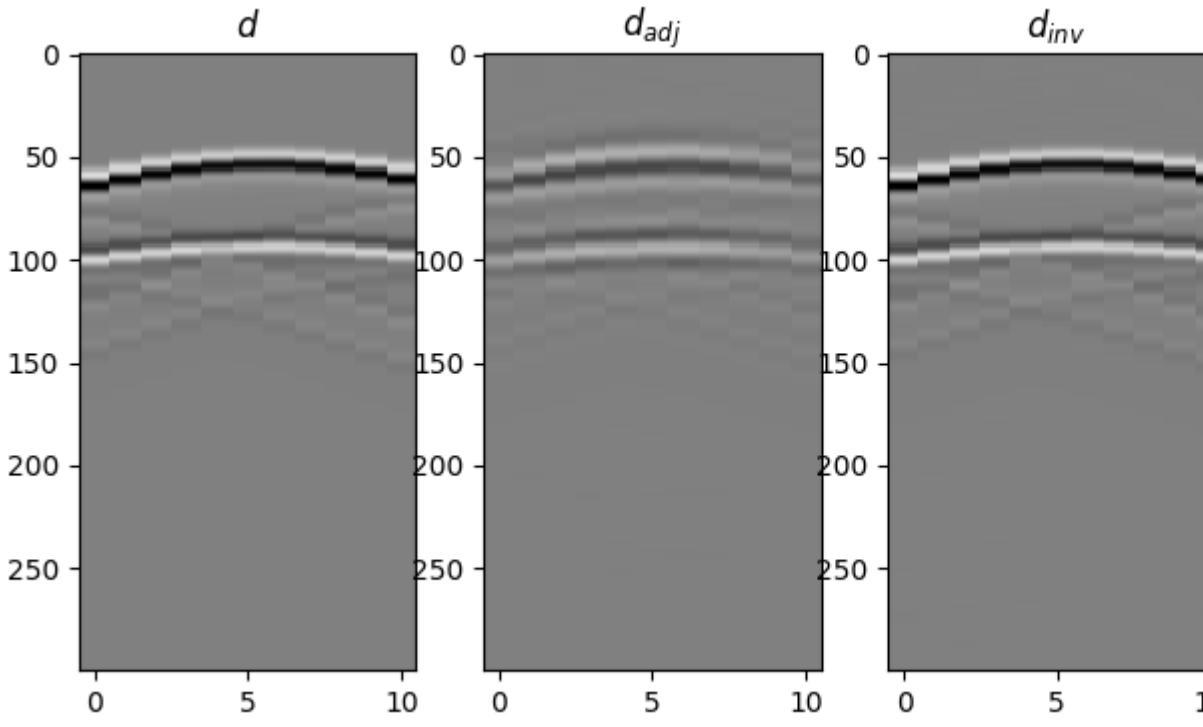
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```
              vmin=-d.max(), vmax=d.max())
axs[2].set_title(r'$d_{inv}$')
axs[2].axis('tight')
axs[3].imshow(dinv_sparse[ns//2, :, :300].T, cmap='gray',
              vmin=-d.max(), vmax=d.max())
axs[3].set_title(r'$d_{fista}$')
axs[3].axis('tight')
```


m
m_{inv}
0
10
20
30
40
50
0
20
40
60
80

•





This was just a short teaser, for a more advanced set of examples of 2D and 3D traveltime-based LSM head over to this notebook.

Total running time of the script: (0 minutes 7.411 seconds)

Note: Click *here* to download the full example code

3.4.14 14. Bayesian Inversion

This tutorial focuses on Bayesian inversion, a special type of inverse problem that aims at incorporating prior information in terms of model and data probabilities in the inversion process.

In this case we will be dealing with the same problem that we discussed in *03. Solvers*, but instead of defining ad-hoc regularization or preconditioning terms we parametrize and model our input signal in the frequency domain in a probabilistic fashion: the central frequency, amplitude and phase of the three sinusoids have gaussian distributions as follows:

$$X(f) = \sum_{i=1}^{3} a_i e^{j\phi_i} \delta(f - f_i)$$

where $f_i \sim N(f_{0,i}, \sigma_{f,i})$, $a_i \sim N(a_{0,i}, \sigma_{a,i})$, and $\phi_i \sim N(\phi_{0,i}, \sigma_{\phi,i})$.

Based on the above definition, we construct some prior models in the frequency domain, convert each of them to the time domain and use such an ensemble to estimate the prior mean $\mu_{\mathbf{x}}$ and model covariance $\mathbf{C}_{\mathbf{x}}$.

We then create our data by sampling the true signal at certain locations and solve the resconstruction problem within a Bayesian framework. Since we are assuming gaussianity in our priors, the equation to obtain the posterion mean can be derived analytically:

$$\mathbf{x} = \mathbf{x_0} + (\mathbf{R}\mathbf{C}_x\mathbf{R}^T + \mathbf{C}_y)^{-1}(\mathbf{y} - \mathbf{R}\mathbf{x_0})$$

```
# sphinx_gallery_thumbnail_number = 2
import numpy as np
import matplotlib.pyplot as plt
import pylops

from scipy.sparse.linalg import lsqr


plt.close('all')
np.random.seed(10)
```

Let's start by creating our true model and prior realizations

```
def prior_realization(f0, a0, phi0, sigmaf, sigmaa, sigmaphi, dt, nt, nfft):
    """Create realization from prior mean and std for amplitude, frequency and
    phase
    """
    f = np.fft.rfftfreq(nfft, dt)
    df = f[1] - f[0]
    ifreqs = [int(np.random.normal(f, sigma)/df)
              for f, sigma in zip(f0, sigmaf)]
    amps = [np.random.normal(a, sigma) for a, sigma in zip(a0, sigmaa)]
    phis = [np.random.normal(phi, sigma) for phi, sigma in zip(phi0, sigmaphi)]

    # input signal in frequency domain
    X = np.zeros(nfft//2+1, dtype='complex128')
```

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```
    X[ifreqs] = np.array(amps).squeeze() * \
                np.exp(1j * np.deg2rad(np.array(phis))).squeeze()

    # input signal in time domain
    FFTop = pylops.signalprocessing.FFT(nt, nfft=nfft, real=True)
    x = FFTop.H*X
    return x

# Priors
nreals = 100
f0 = [5, 3, 8]
sigmaf = [0.5, 1., 0.6]
a0 = [1., 1., 1.]
sigmaa = [0.1, 0.5, 0.6]
phi0 = [-90., 0., 0.]
sigmaphi = [0.1, 0.2, 0.4]
sigmad = 1e-2

# Prior models
nt = 200
nfft = 2**11
dt = 0.004
t = np.arange(nt)*dt
x = prior_realization(f0, a0, phi0, [0, 0, 0], [0, 0, 0],
                      [0, 0, 0], dt, nt, nfft)
xs = \
    np.array([prior_realization(f0, a0, phi0, sigmaf,
                                sigmaa, sigmaphi, dt, nt, nfft)
              for _ in range(nreals)])
```

We have now a set of prior models in time domain. We can easily use sample statistics to estimate the prior mean and covariance. For the covariance, we perform a second step where we average values around the main diagonal for each row and find a smooth, compact filter that we use to define a convolution linear operator that mimics the action of the covariance matrix on a vector

```
x0 = np.average(xs, axis=0)
Cm = ((xs - x0).T @ (xs - x0))/nreals

N = 30 # lenght of decorrelation
diags = np.array([Cm[i, i-N:i+N+1] for i in range(N, nt-N)])
diag_ave = np.average(diags, axis=0)
diag_ave *= np.hamming(2*N+1) # add a taper at the end to avoid edge effects

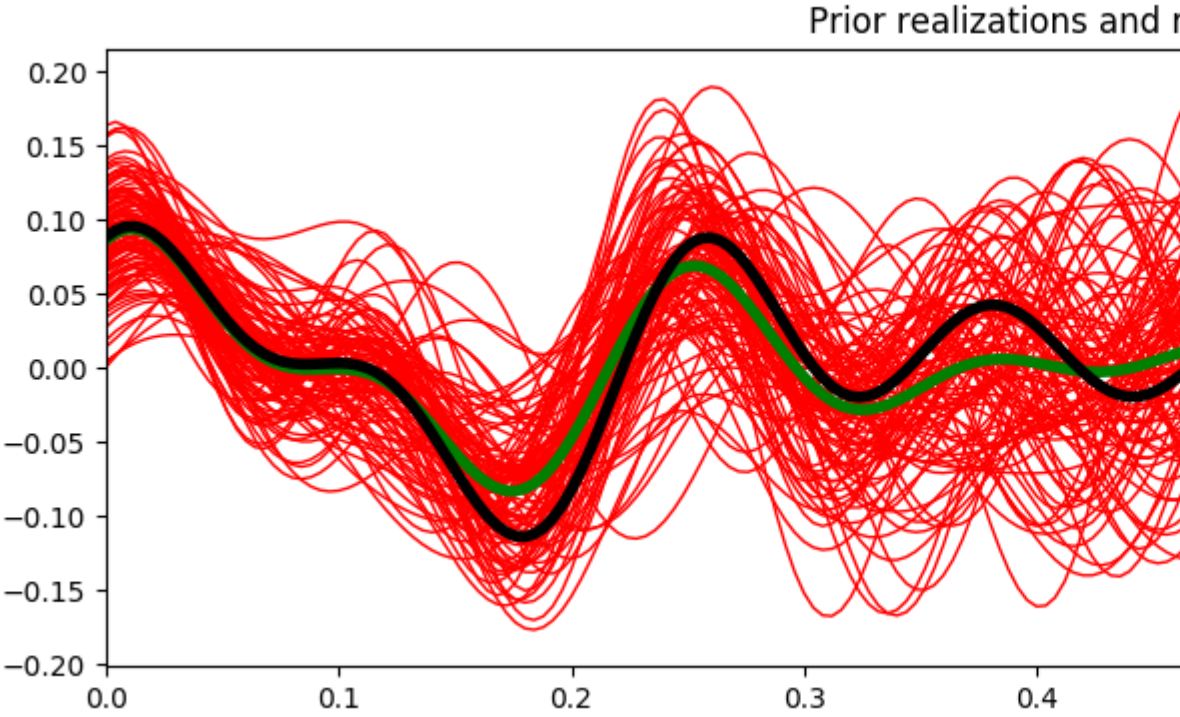
fig, ax = plt.subplots(1, 1, figsize=(12, 4))
ax.plot(t, xs.T, 'r', lw=1)
ax.plot(t, x0, 'g', lw=4)
ax.plot(t, x, 'k', lw=4)
ax.set_title('Prior realizations and mean')
ax.set_xlim(0, 0.8)

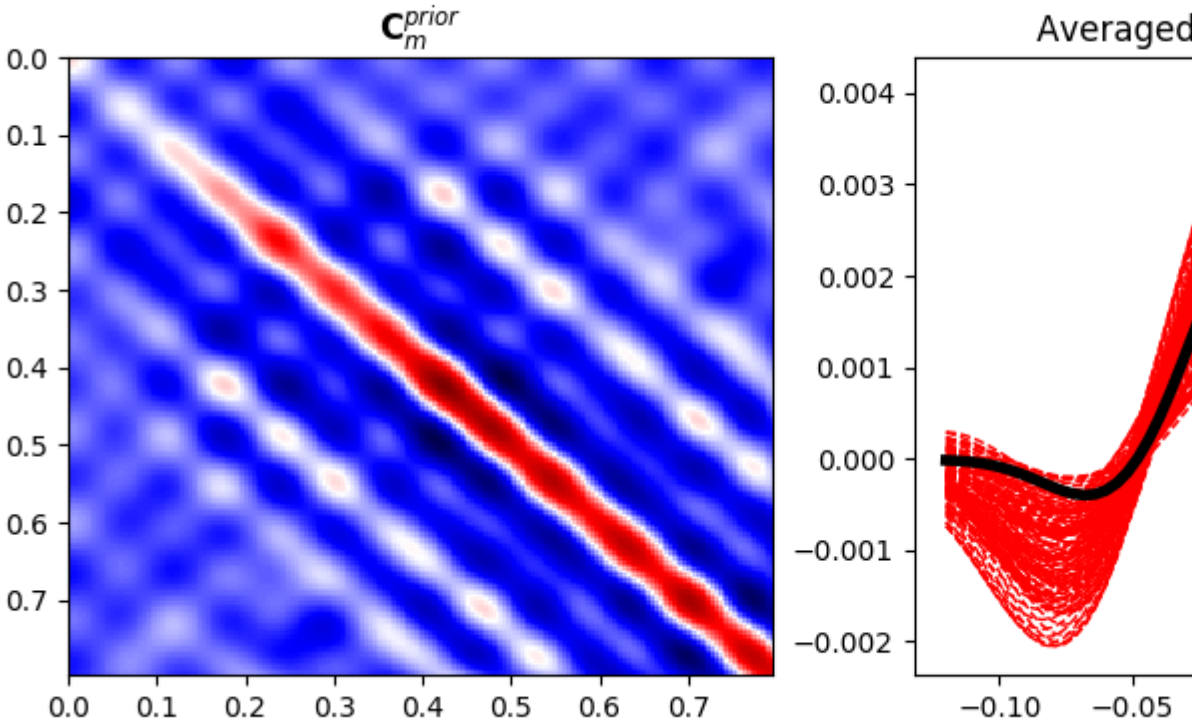
fig, (ax1, ax2) = plt.subplots(1, 2, figsize=(10, 4))
im = ax1.imshow(Cm, interpolation='nearest', cmap='seismic',
                extent=(t[0], t[-1], t[-1], t[0]))
ax1.set_title(r"$\mathbf{C}_m^{prior}$")
ax1.axis('tight')
ax2.plot(np.arange(-N, N + 1) * dt, diags.T, '--r', lw=1)
```

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```
ax2.plot(np.arange(-N, N + 1) * dt, diag_ave, 'k', lw=4)
ax2.set_title("Averaged covariance 'filter'")
```





Let's define now the sampling operator as well as create our covariance matrices in terms of linear operators. This may not be strictly necessary here but shows how even Bayesian-type of inversion can very easily scale to large model and data spaces.

```
# Sampling operator
perc_subsampling = 0.2
ntsub = int(np.round(nt*perc_subsampling))
iava = np.sort(np.random.permutation(np.arange(nt))[:ntsub])
iava[-1] = nt-1 # assume we have the last sample to avoid instability
Rop = pylops.Restriction(nt, iava, dtype='float64')

# Covariance operators
Cm_op = \
    pylops.signalprocessing.Convolve1D(nt, diag_ave, offset=N)
Cd_op = sigmad**2 * pylops.Identity(ntsub)
```

We model now our data and add noise that respects our prior definition

```
n = np.random.normal(0, sigmad, nt)
y = Rop * x
yn = Rop * (x + n)
ymask = Rop.mask(x)
ynmask = Rop.mask(x + n)
```

First we apply the Bayesian inversion equation

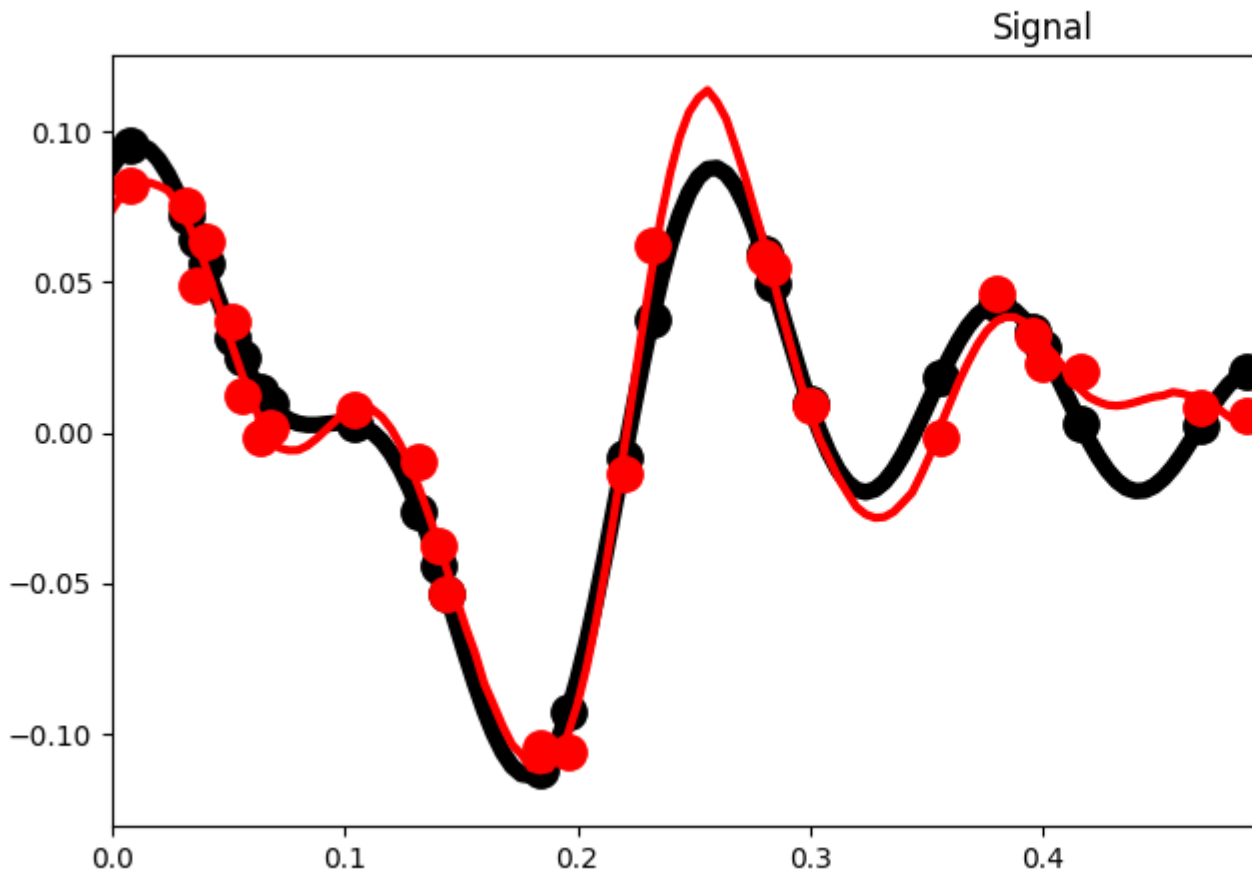
```
xbayes = x0 + Cm_op * Rop.H * (lsqr(Rop * Cm_op * Rop.H + Cd_op,
                                    yn - Rop*x0, iter_lim=400)[0])

# Visualize
```

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```
fig, ax = plt.subplots(1, 1, figsize=(12, 5))
ax.plot(t, x, 'k', lw=6)
ax.plot(t, ymask, '.k', ms=25, label='available samples')
ax.plot(t, ynmask, '.r', ms=25, label='available noisy samples')
ax.plot(t, xbayes, 'r', lw=3, label='Bayesian inverse')
ax.legend()
ax.set_title('Signal')
ax.set_xlim(0, 0.8)
```



So far we have been able to estimate our posterion mean. What about its uncertainties (i.e., posterion covariance)?

In real-life applications it is very difficult (if not impossible) to directly compute the posterior covariance matrix. It is much more useful to create a set of models that sample the posterion probability. We can do that by solving our problem several times using different prior realizations as starting guesses:

```
xpost = [x0 + Cm_op * Rop.H * (lsqr(Rop * Cm_op * Rop.H + Cd_op, yn - Rop * x0, iter_
↪lim=400)[0])
         for x0 in xs[:30]]
xpost = np.array(xpost)

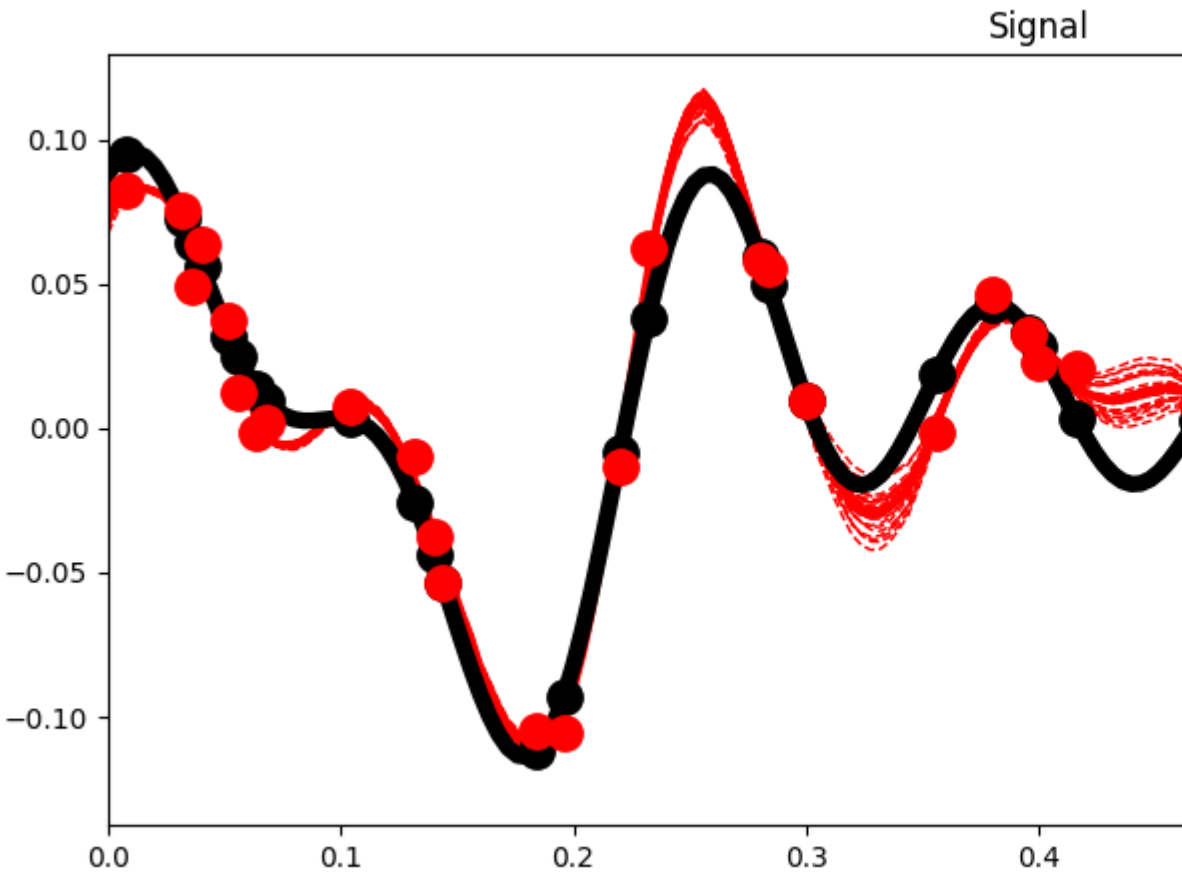
x0post = np.average(xpost, axis=0)
Cm_post = ((xpost - x0post).T @ (xpost - x0post)) / nreals
```

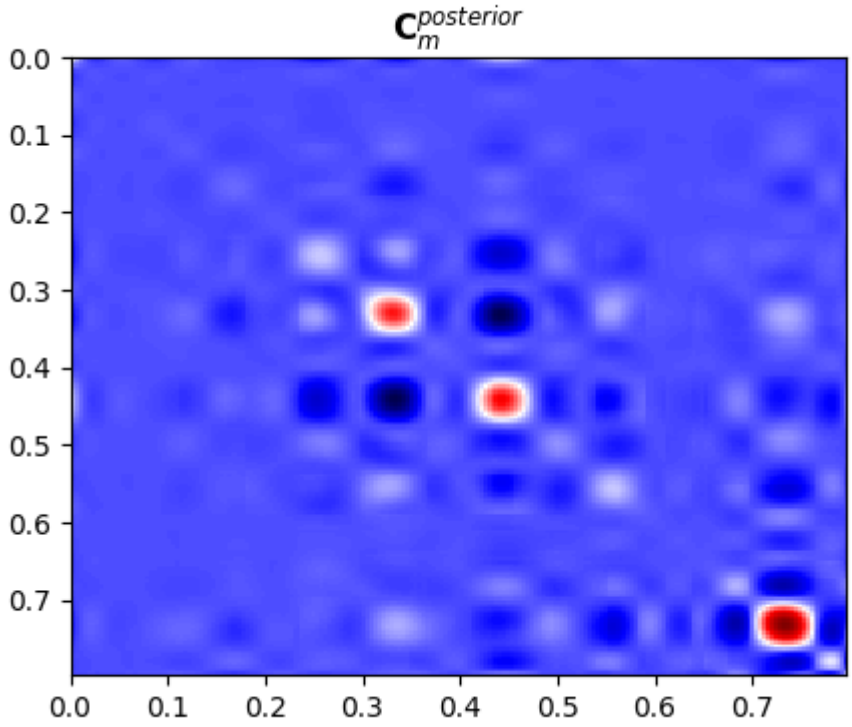
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```
# Visualize
fig, ax = plt.subplots(1, 1, figsize=(12, 5))
ax.plot(t, xpost.T, '--r', lw=1)
ax.plot(t, x0post, 'r', lw=3, label='bayesian inverse')
ax.plot(t, x, 'k', lw=6)
ax.plot(t, ymask, '.k', ms=25, label='available samples')
ax.plot(t, ynmask, '.r', ms=25, label='available noisy samples')
ax.legend()
ax.set_title('Signal')
ax.set_xlim(0, 0.8)

fig, ax = plt.subplots(1, 1, figsize=(5, 4))
im = ax.imshow(Cm_post, interpolation='nearest', cmap='seismic',
               extent=(t[0], t[-1], t[-1], t[0]))
ax.set_title(r"$\mathbf{C}_m^{posterior}$")
ax.axis('tight')
```





-

Total running time of the script: (0 minutes 2.003 seconds)

3.5 Frequenty Asked Questions

None so far…

3.6 PyLops API

The Application Programming Interface (API) of PyLops can be loosely seen as composed of a stack of three main layers:

- *Linear operators*: building blocks for the setting up of inverse problems
- *Solvers*: interfaces to a variety of solvers, providing an easy way to augment an inverse problem with additional regularization and/or preconditioning term
- *Applications*: high-level interfaces allowing users to easily setup and solve specific problems (while hiding the non-needed details - i.e., creation and setup of linear operators and solvers).

3.6.1 Linear operators

Templates

LinearOperator([Op, explicit])	Common interface for performing matrix-vector products.
FunctionOperator(f, *args, **kwargs)	Function Operator.

pylops.LinearOperator

class pylops.**LinearOperator**(*Op=None*, *explicit=False*)

Common interface for performing matrix-vector products.

This class is an overload of the `scipy.sparse.linalg.LinearOperator` class. It adds functionalities by overloading standard operators such as `__truediv__` as well as creating convenience methods such as `eigs`, `cond`, and `conj`.

Note: End users of PyLops should not use this class directly but simply use operators that are already implemented. This class is meant for developers and it has to be used as the parent class of any new operator developed within PyLops. Find more details regarding implementation of new operators at *Implementing new operators*.

Parameters

Op [`scipy.sparse.linalg.LinearOperator` or `scipy.sparse.linalg._ProductLinearOperator` or `scipy.sparse.linalg._SumLinearOperator`] Operator

explicit [`bool`] Operator contains a matrix that can be solved explicitly (`True`) or not (`False`)

Methods

`__init__`(self[, Op, explicit])	Initialize this LinearOperator.
`adjoint`(self)	Hermitian adjoint.
`cond`(self, **kwargs_eig)	Condition number of linear operator.
`conj`(self)	Complex conjugate operator
`div`(self, y[, niter])	Solve the linear problem $\mathbf{y} = \mathbf{A}\mathbf{x}$.
`dot`(self, x)	Matrix-matrix or matrix-vector multiplication.
`eigs`(self[, neigs, symmetric, niter])	Most significant eigenvalues of linear operator.
`matmat`(self, X)	Matrix-matrix multiplication.
`matvec`(self, x)	Matrix-vector multiplication.
`rmatvec`(self, x)	Adjoint matrix-vector multiplication.
`transpose`(self)	Transpose this linear operator.

div(*self*, *y*, *niter=100*)

Solve the linear problem $\mathbf{y} = \mathbf{A}\mathbf{x}$.

Overloading of operator `/` to improve expressivity of *Pylops* when solving inverse problems.

Parameters

y [`np.ndarray`] Data

niter [`int`, optional] Number of iterations (to be used only when `explicit=False`)

Returns

xest [np.ndarray] Estimated model

eigs(*self*, *neigs=None*, *symmetric=False*, *niter=None*, ***kwargs_eig*)
Most significant eigenvalues of linear operator.

Return an estimate of the most significant eigenvalues of the linear operator. If the operator has rectangular shape (shape[0]!=shape[1]), eigenvalues are first computed for the square operator $\mathbf{A^H A}$ and the square-root values are returned.

Parameters

neigs [int] Number of eigenvalues to compute (if None, return all). Note that for explicit=False, only $N - 1$ eigenvalues can be computed where N is the size of the operator in the model space

symmetric [bool, optional] Operator is symmetric (True) or not (False). User should set this parameter to True only when it is guaranteed that the operator is real-symmetric or complex-hermitian matrices

niter [int, optional] Number of iterations for eigenvalue estimation

****kwargs_eig** Arbitrary keyword arguments for scipy.sparse.linalg.eigs or scipy.sparse.linalg.eigsh

Returns

eigenvalues [numpy.ndarray] Operator eigenvalues.

Notes

Eigenvalues are estimated using scipy.sparse.linalg.eigs (explicit=True) or scipy.sparse.linalg.eigsh (explicit=False).

This is a port of ARPACK [1], a Fortran package which provides routines for quickly finding eigenvalues/eigenvectors of a matrix. As ARPACK requires only left-multiplication by the matrix in question, eigenvalues/eigenvectors can also be estimated for linear operators when the dense matrix is not available.

cond(*self*, ***kwargs_eig*)
Condition number of linear operator.

Return an estimate of the condition number of the linear operator as the ratio of the largest and lowest estimated eigenvalues.

Parameters

****kwargs_eig** Arbitrary keyword arguments for scipy.sparse.linalg.eigs or scipy.sparse.linalg.eigsh

Returns

eigenvalues [numpy.ndarray] Operator eigenvalues.

Notes

The condition number of a matrix (or linear operator) can be estimated as the ratio of the largest and lowest estimated eigenvalues:

$$k = \frac{\lambda_{max}}{\lambda_{min}}$$

The condition number provides an indication of the rate at which the solution of the inversion of the linear operator A will change with respect to a change in the data y.

Thus, if the condition number is large, even a small error in y may cause a large error in x. On the other hand, if the condition number is small then the error in x is not much bigger than the error in y. A problem with a low condition number is said to be *well-conditioned*, while a problem with a high condition number is said to be *ill-conditioned*.

conj(*self*)
Complex conjugate operator

Returns

eigenvalues [*pylops.LinearOperator*] Complex conjugate operator

Examples using `pylops.LinearOperator`

- sphx_glr_gallery_plot_sliding.py
- sphx_glr_gallery_plot_linearregr.py
- sphx_glr_gallery_plot_regr.py
- *01. The LinearOpeator*

pylops.FunctionOperator

class pylops.**FunctionOperator**(*f*, **args*, ***kwargs*)
Function Operator.

Simple wrapper to functions for forward f and adjoint fc multiplication.

Functions f and fc are such that $f : \mathbb{F}^m \to \mathbb{F}^n$ and $fc : \mathbb{F}^n \to \mathbb{F}^m$ where $\mathbb{F}$ is the appropriate underlying type (e.g., $\mathbb{R}$ for real or $\mathbb{C}$ for complex)

FunctionOperator can be called in the following ways: `FunctionOperator(f, n)`, `FunctionOperator(f, n, m)`, `FunctionOperator(f, fc, n)`, and `FunctionOperator(f, fc, n, m)`.

The first two methods can only be used for forward modelling and will return *NotImplementedError* if the adjoint is called. The first and third method assume the matrix (or matrices) to be square. All methods can be called with the *dtype* keyword argument.

Parameters

f [`callable`] Function for forward multiplication.

fc [`callable`, optional] Function for adjoint multiplication.

n [`int`, optional] Number of rows (length of data vector).

m [`int`, optional] Number of columns (length of model vector).

dtype [`str`, optional] Type of elements in input array.

Examples

```
>>> from pylops.basicoperators import FunctionOperator
>>> def forward(v):
...     return np.array([2*v[0], 3*v[1]])
...
>>> A = FunctionOperator(forward, 2)
```

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```
>>> A
<2x2 FunctionOperator with dtype=float64>
>>> A.matvec(np.ones(2))
array([2.,  3.])
>>> A @ np.ones(2)
array([2.,  3.])
```

Attributes

shape [tuple] Operator shape $[n \times m]$

explicit [bool] Operator contains a matrix that can be solved explicitly (True) or not (False)

Methods

__init__(self, f, *args, **kwargs)	Initialize this LinearOperator.
adjoint(self)	Hermitian adjoint.
cond(self, **kwargs_eig)	Condition number of linear operator.
conj(self)	Complex conjugate operator
div(self, y[, niter])	Solve the linear problem $\mathbf{y} = \mathbf{A}\mathbf{x}$.
dot(self, x)	Matrix-matrix or matrix-vector multiplication.
eigs(self[, neigs, symmetric, niter])	Most significant eigenvalues of linear operator.
matmat(self, X)	Matrix-matrix multiplication.
matvec(self, x)	Matrix-vector multiplication.
rmatvec(self, x)	Adjoint matrix-vector multiplication.
transpose(self)	Transpose this linear operator.

Basic operators

MatrixMult(A[, dims, dtype])	Matrix multiplication.
Identity(N[, M, dtype, inplace])	Identity operator.
Zero(N[, M, dtype])	Zero operator.
Diagonal(diag[, dims, dir, dtype])	Diagonal operator.
Transpose(dims, axes[, dtype])	Transpose operator.
Flip(N[, dims, dir, dtype])	Flip along an axis.
Roll(N[, dims, dir, shift, dtype])	Roll along an axis.
Symmetrize(N[, dims, dir, dtype])	Symmetrize along an axis.
Restriction(M, iava[, dims, dir, dtype, inplace])	Restriction (or sampling) operator.
Regression(taxis, order[, dtype])	Polynomial regression.
LinearRegression(taxis[, dtype])	Linear regression.
CausalIntegration(N[, dims, dir, sampling, ...])	Causal integration.
Spread(dims, dimsd[, table, dtable, fh, ...])	Spread operator.
VStack(ops[, dtype])	Vertical stacking.
HStack(ops[, dtype])	Horizontal stacking.
Block(ops[, dtype])	Block operator.
BlockDiag(ops[, dtype])	Block-diagonal operator.
Kronecker(Op1, Op2[, dtype])	Kronecker operator.

pylops.MatrixMult

class pylops.**MatrixMult**(*A*, *dims=None*, *dtype='float64'*)

Matrix multiplication.

Simple wrapper to `numpy.dot` and `numpy.vdot` for an input matrix $\mathbf{A}$.

Parameters

A [`numpy.ndarray` or `scipy.sparse` matrix] Matrix.

dims [`tuple`, optional] Number of samples for each other dimension of model (model/data will be reshaped and `A` applied multiple times to each column of the model/data).

dtype [`str`, optional] Type of elements in input array.

Attributes

shape [`tuple`] Operator shape

explicit [`bool`] Operator contains a matrix that can be solved explicitly (`True`) or not (`False`)

Methods

`__init__`(self, A[, dims, dtype])	Initialize this LinearOperator.
`adjoint`(self)	Hermitian adjoint.
`cond`(self, **kwargs_eig)	Condition number of linear operator.
`conj`(self)	Complex conjugate operator
`div`(self, y[, niter])	Solve the linear problem $\mathbf{y} = \mathbf{A}\mathbf{x}$.
`dot`(self, x)	Matrix-matrix or matrix-vector multiplication.
`eigs`(self[, neigs, symmetric, niter])	Most significant eigenvalues of linear operator.
`inv`(self)	Return the inverse of $\mathbf{A}$.
`matmat`(self, X)	Matrix-matrix multiplication.
`matvec`(self, x)	Matrix-vector multiplication.
`rmatvec`(self, x)	Adjoint matrix-vector multiplication.
`transpose`(self)	Transpose this linear operator.

inv(*self*)

Return the inverse of $\mathbf{A}$.

Returns

Ainv [`numpy.ndarray`] Inverse matrix.

Examples using `pylops.MatrixMult`

- sphx_glr_gallery_plot_ista.py
- sphx_glr_gallery_plot_matrixmult.py
- sphx_glr_gallery_plot_stacking.py
- *02. The Dot-Test*

pylops.Identity

class pylops.**Identity**(*N*, *M=None*, *dtype='float64'*, *inplace=True*)

Identity operator.

Simply move model to data in forward model and viceversa in adjoint mode if $M = N$. If $M > N$ removes last $M - N$ elements from model in forward and pads with 0 in adjoint. If $N > M$ removes last $N - M$ elements from data in adjoint and pads with 0 in forward.

Parameters

N [int] Number of samples in data (and model, if M is not provided).

M [int, optional] Number of samples in model.

dtype [str, optional] Type of elements in input array.

inplace [bool, optional] Work inplace (True) or make a new copy (False). By default, data is a reference to the model (in forward) and model is a reference to the data (in adjoint).

Notes

For $M = N$, an *Identity* operator simply moves the model $\mathbf{x}$ to the data $\mathbf{y}$ in forward mode and viceversa in adjoint mode:

$$y_i = x_i \quad \forall i = 1, 2, ..., N$$

or in matrix form:

$$\mathbf{y} = \mathbf{I}\mathbf{x} = \mathbf{x}$$

and

$$\mathbf{x} = \mathbf{I}\mathbf{y} = \mathbf{y}$$

For $M > N$, the *Identity* operator takes the first M elements of the model $\mathbf{x}$ into the data $\mathbf{y}$ in forward mode

$$y_i = x_i \quad \forall i = 1, 2, ..., N$$

and all the elements of the data $\mathbf{y}$ into the first M elements of model in adjoint mode (other elements are 0):

$$x_i = y_i \quad \forall i = 1, 2, ..., M$$
$$x_i = 0 \quad \forall i = M + 1, ..., N$$

Attributes

shape [tuple] Operator shape

explicit [bool] Operator contains a matrix that can be solved explicitly (True) or not (False)

Methods

__init__(self, N[, M, dtype, inplace])	Initialize this LinearOperator.
adjoint(self)	Hermitian adjoint.
cond(self, **kwargs_eig)	Condition number of linear operator.

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Table 6 – continued from previous page

`conj`(self)	Complex conjugate operator
`div`(self, y[, niter])	Solve the linear problem $\mathbf{y} = \mathbf{A}\mathbf{x}$.
`dot`(self, x)	Matrix-matrix or matrix-vector multiplication.
`eigs`(self[, neigs, symmetric, niter])	Most significant eigenvalues of linear operator.
`matmat`(self, X)	Matrix-matrix multiplication.
`matvec`(self, x)	Matrix-vector multiplication.
`rmatvec`(self, x)	Adjoint matrix-vector multiplication.
`transpose`(self)	Transpose this linear operator.

Examples using `pylops.Identity`

- sphx_glr_gallery_plot_identity.py
- sphx_glr_gallery_plot_stacking.py
- sphx_glr_gallery_plot_tvreg.py
- *14. Bayesian Inversion*

pylops.Zero

class pylops.**Zero**(*N*, *M=None*, *dtype='float64'*)

Zero operator.

Transform model into array of zeros of size N in forward and transform data into array of zeros of size N in adjoint.

Parameters

N [int] Number of samples in data (and model in M is not provided).

M [int, optional] Number of samples in model.

dtype [str, optional] Type of elements in input array.

Notes

An *Zero* operator simply creates a null data vector $\mathbf{y}$ in forward mode:

$$\mathbf{0x} = \mathbf{0}_N$$

and a null model vector $\mathbf{x}$ in forward mode:

$$\mathbf{0y} = \mathbf{0}_M$$

Attributes

shape [tuple] Operator shape

explicit [bool] Operator contains a matrix that can be solved explicitly (`True`) or not (`False`)

Methods

`__init__`(self, N[, M, dtype])	Initialize this LinearOperator.
`adjoint`(self)	Hermitian adjoint.
`cond`(self, **kwargs_eig)	Condition number of linear operator.
`conj`(self)	Complex conjugate operator
`div`(self, y[, niter])	Solve the linear problem $\mathbf{y} = \mathbf{A}\mathbf{x}$.
`dot`(self, x)	Matrix-matrix or matrix-vector multiplication.
`eigs`(self[, neigs, symmetric, niter])	Most significant eigenvalues of linear operator.
`matmat`(self, X)	Matrix-matrix multiplication.
`matvec`(self, x)	Matrix-vector multiplication.
`rmatvec`(self, x)	Adjoint matrix-vector multiplication.
`transpose`(self)	Transpose this linear operator.

Examples using `pylops.Zero`

- sphx_glr_gallery_plot_zero.py

pylops.Diagonal

class pylops.**Diagonal**(*diag*, *dims=None*, *dir=0*, *dtype='float64'*)

Diagonal operator.

Applies element-wise multiplication of the input vector with the vector `diag` in forward and with its complex conjugate in adjoint mode.

This operator can also broadcast; in this case the input vector is reshaped into its dimensions `dims` and the element-wise multiplication with `diag` is perfomed on the direction `dir`. Note that the vector `diag` will need to have size equal to `dims[dir]`.

Parameters

diag [`numpy.ndarray`] Vector to be used for element-wise multiplication.

dims [`list`, optional] Number of samples for each dimension (`None` if only one dimension is available)

dir [`int`, optional] Direction along which multiplication is applied.

dtype [`str`, optional] Type of elements in input array.

Notes

Element-wise multiplication between the model $\mathbf{x}$ and/or data $\mathbf{y}$ vectors and the array $\mathbf{d}$ can be expressed as

$$y_i = d_i x_i \quad \forall i = 1, 2, ..., N$$

This is equivalent to a matrix-vector multiplication with a matrix containing the vector $\mathbf{d}$ along its main diagonal.

For real-valued `diag`, the Diagonal operator is self-adjoint as the adjoint of a diagonal matrix is the diagonal matrix itself. For complex-valued `diag`, the adjoint is equivalent to the element-wise multiplication with the complex conjugate elements of `diag`.

Attributes

shape [`tuple`] Operator shape

explicit [`bool`] Operator contains a matrix that can be solved explicitly (`True`) or not (`False`)

Methods

`__init__`(self, diag[, dims, dir, dtype])	Initialize this LinearOperator.
`adjoint`(self)	Hermitian adjoint.
`cond`(self, **kwargs_eig)	Condition number of linear operator.
`conj`(self)	Complex conjugate operator
`div`(self, y[, niter])	Solve the linear problem $\mathbf{y} = \mathbf{A}\mathbf{x}$.
`dot`(self, x)	Matrix-matrix or matrix-vector multiplication.
`eigs`(self[, neigs, symmetric, niter])	Most significant eigenvalues of linear operator.
`matmat`(self, X)	Matrix-matrix multiplication.
`matrix`(self)	Return diagonal matrix as dense `numpy.ndarray`
`matvec`(self, x)	Matrix-vector multiplication.
`rmatvec`(self, x)	Adjoint matrix-vector multiplication.
`transpose`(self)	Transpose this linear operator.

matrix(*self*)
Return diagonal matrix as dense `numpy.ndarray`

Returns

densemat [`numpy.ndarray`] Dense matrix.

Examples using `pylops.Diagonal`

- sphx_glr_gallery_plot_diagonal.py
- *01. The LinearOpeator*
- *02. The Dot-Test*

pylops.Transpose

class pylops.**Transpose**(*dims*, *axes*, *dtype='float64'*)
Transpose operator.

Transpose axes of a multi-dimensional array. This operator works with flattened input model (or data), which are however multi-dimensional in nature and will be reshaped and treated as such in both forward and adjoint modes.

Parameters

dims [`tuple`, optional] Number of samples for each dimension (`None` if only one dimension is available)

axes [`tuple`, optional] Direction along which transposition is applied

dtype [`str`, optional] Type of elements in input array

Raises

ValueError If `axes` contains repeated dimensions (or a dimension is missing)

Notes

The Transpose operator reshapes the input model into a multi-dimensional array of size `dims` and transposes (or swaps) its axes as defined in `axes`.

Similarly, in adjoint mode the data is reshaped into a multi-dimensional array whose size is a permuted version of `dims` defined by `axes`. The array is then rearragned into the original model dimensions `dims`.

Attributes

shape [`tuple`] Operator shape

explicit [`bool`] Operator contains a matrix that can be solved explicitly (`True`) or not (`False`)

Methods

`__init__`(self, dims, axes[, dtype])	Initialize this LinearOperator.
`adjoint`(self)	Hermitian adjoint.
`cond`(self, **kwargs_eig)	Condition number of linear operator.
`conj`(self)	Complex conjugate operator
`div`(self, y[, niter])	Solve the linear problem $\mathbf{y} = \mathbf{A}\mathbf{x}$.
`dot`(self, x)	Matrix-matrix or matrix-vector multiplication.
`eigs`(self[, neigs, symmetric, niter])	Most significant eigenvalues of linear operator.
`matmat`(self, X)	Matrix-matrix multiplication.
`matvec`(self, x)	Matrix-vector multiplication.
`rmatvec`(self, x)	Adjoint matrix-vector multiplication.
`transpose`(self)	Transpose this linear operator.

Examples using `pylops.Transpose`

- sphx_glr_gallery_plot_transpose.py

pylops.Flip

class `pylops.`**`Flip`**(*N*, *dims=None*, *dir=0*, *dtype='float64'*)

Flip along an axis.

Flip a multi-dimensional array along a specified direction `dir`.

Parameters

N [`int`] Number of samples in model.

dims [`list`, optional] Number of samples for each dimension (`None` if only one dimension is available)

dir [`int`, optional] Direction along which flipping is applied.

dtype [`str`, optional] Type of elements in input array.

Notes

The Flip operator flips the input model (and data) along any chosen direction. For simplicity, given a one dimensional array, in forward mode this is equivalent to:

$$y[i] = x[N - i] \quad \forall i = 0, 1, 2, ..., N - 1$$

where N is the lenght of the input model. As this operator is self-adjoint, x and y in the equation above are simply swapped in adjoint mode.

Attributes

shape [tuple] Operator shape

explicit [bool] Operator contains a matrix that can be solved explicitly (True) or not (False)

Methods

__init__(self, N[, dims, dir, dtype])	Initialize this LinearOperator.
adjoint(self)	Hermitian adjoint.
cond(self, **kwargs_eig)	Condition number of linear operator.
conj(self)	Complex conjugate operator
div(self, y[, niter])	Solve the linear problem $\mathbf{y} = \mathbf{A}\mathbf{x}$.
dot(self, x)	Matrix-matrix or matrix-vector multiplication.
eigs(self[, neigs, symmetric, niter])	Most significant eigenvalues of linear operator.
matmat(self, X)	Matrix-matrix multiplication.
matvec(self, x)	Matrix-vector multiplication.
rmatvec(self, x)	Adjoint matrix-vector multiplication.
transpose(self)	Transpose this linear operator.

Examples using pylops.Flip

- sphx_glr_gallery_plot_flip.py

pylops.Roll

class pylops.**Roll**(*N*, *dims=None*, *dir=0*, *shift=1*, *dtype='float64'*)

Roll along an axis.

Roll a multi-dimensional array along a specified direction dir for a chosen number of samples (shift).

Parameters

N [int] Number of samples in model.

dims [list, optional] Number of samples for each dimension (None if only one dimension is available)

dir [int, optional] Direction along which rolling is applied.

shift [int, optional] Number of samples by which elements are shifted

dtype [str, optional] Type of elements in input array.

Notes

The Roll operator is a thin wrapper around `numpy.roll` and shifts elements in a multi-dimensional array along a specified direction for a chosen number of samples.

Attributes

shape [`tuple`] Operator shape

explicit [`bool`] Operator contains a matrix that can be solved explicitly (`True`) or not (`False`)

Methods

`__init__`(self, N[, dims, dir, shift, dtype])	Initialize this LinearOperator.
`adjoint`(self)	Hermitian adjoint.
`cond`(self, **kwargs_eig)	Condition number of linear operator.
`conj`(self)	Complex conjugate operator
`div`(self, y[, niter])	Solve the linear problem $\mathbf{y} = \mathbf{A}\mathbf{x}$.
`dot`(self, x)	Matrix-matrix or matrix-vector multiplication.
`eigs`(self[, neigs, symmetric, niter])	Most significant eigenvalues of linear operator.
`matmat`(self, X)	Matrix-matrix multiplication.
`matvec`(self, x)	Matrix-vector multiplication.
`rmatvec`(self, x)	Adjoint matrix-vector multiplication.
`transpose`(self)	Transpose this linear operator.

Examples using `pylops.Roll`

- sphx_glr_gallery_plot_roll.py

pylops.Symmetrize

class `pylops.`**`Symmetrize`**(*N*, *dims=None*, *dir=0*, *dtype='float64'*)

Symmetrize along an axis.

Symmetrize a multi-dimensional array along a specified direction `dir`.

Parameters

N [`int`] Number of samples in model. Symmetric data has $2N - 1$ samples

dims [`list`, optional] Number of samples for each dimension (`None` if only one dimension is available)

dir [`int`, optional] Direction along which symmetrization is applied

dtype [`str`, optional] Type of elements in input array

Notes

The Symmetrize operator constructs a symmetric array given an input model in forward mode, by pre-pending the input model in reversed order.

For simplicity, given a one dimensional array, the forward operation can be expressed as:

$$y[i] = \begin{cases} x[i-N], & i \geq N \\ x[N-i], & \text{otherwise} \end{cases}$$

for $i = 0, 1, 2, ..., 2N-2$, where N is the lenght of the input model.

In adjoint mode, the Symmetrize operator assigns the sums of the elements in position $N-i$ and $N+i$ to position i as follows:

apart from the central sample where $x[0] = y[N]$.

Attributes

shape [tuple] Operator shape

explicit [bool] Operator contains a matrix that can be solved explicitly (True) or not (False)

Methods

__init__(self, N[, dims, dir, dtype])	Initialize this LinearOperator.
adjoint(self)	Hermitian adjoint.
cond(self, **kwargs_eig)	Condition number of linear operator.
conj(self)	Complex conjugate operator
div(self, y[, niter])	Solve the linear problem $\mathbf{y} = \mathbf{A}\mathbf{x}$.
dot(self, x)	Matrix-matrix or matrix-vector multiplication.
eigs(self[, neigs, symmetric, niter])	Most significant eigenvalues of linear operator.
matmat(self, X)	Matrix-matrix multiplication.
matvec(self, x)	Matrix-vector multiplication.
rmatvec(self, x)	Adjoint matrix-vector multiplication.
transpose(self)	Transpose this linear operator.

Examples using pylops.Symmetrize

- sphx_glr_gallery_plot_symmetrize.py
- sphx_glr_gallery_plot_wavest.py

pylops.Restriction

class pylops.**Restriction**(*M*, *iava*, *dims=None*, *dir=0*, *dtype='float64'*, *inplace=True*)

Restriction (or sampling) operator.

Extract subset of values from input vector at locations iava in forward mode and place those values at locations iava in an otherwise zero vector in adjoint mode.

Parameters

M [int] Number of samples in model.

iava [list or numpy.ndarray] Integer indices of available samples for data selection.

dims [list] Number of samples for each dimension (None if only one dimension is available)

dir [int, optional] Direction along which restriction is applied.

dtype [str, optional] Type of elements in input array.

inplace [bool, optional] Work inplace (True) or make a new copy (False). By default, data is a reference to the model (in forward) and model is a reference to the data (in adjoint).

See also:

pylops.signalprocessing.Interp Interpolation operator

Notes

Extraction (or *sampling*) of a subset of N values at locations `iava` from an input (or model) vector $\mathbf{x}$ of size M can be expressed as:

$$y_i = x_{l_i} \quad \forall i = 1, 2, ..., M$$

where $\mathbf{l} = [l_1, l_2, ..., l_M]$ is a vector containing the indeces of the original array at which samples are taken.

Conversely, in adjoint mode the available values in the data vector $\mathbf{y}$ are placed at locations $\mathbf{l} = [l_1, l_2, ..., l_M]$ in the model vector:

$$x_{l_i} = y_i \quad \forall i = 1, 2, ..., M$$

and $x_j = 0 j \neq l_i$ (i.e., at all other locations in input vector).

Attributes

shape [tuple] Operator shape

explicit [bool] Operator contains a matrix that can be solved explicitly (True) or not (False)

Methods

__init__(self, M, iava[, dims, dir, dtype, ...])	Initialize this LinearOperator.
adjoint(self)	Hermitian adjoint.
cond(self, **kwargs_eig)	Condition number of linear operator.
conj(self)	Complex conjugate operator
div(self, y[, niter])	Solve the linear problem $\mathbf{y} = \mathbf{A}\mathbf{x}$.
dot(self, x)	Matrix-matrix or matrix-vector multiplication.
eigs(self[, neigs, symmetric, niter])	Most significant eigenvalues of linear operator.
mask(self, x)	Apply mask to input signal returning a signal of same size with values at `iava` locations and `0` at other locations
matmat(self, X)	Matrix-matrix multiplication.
matvec(self, x)	Matrix-vector multiplication.
rmatvec(self, x)	Adjoint matrix-vector multiplication.
transpose(self)	Transpose this linear operator.

mask (*self*, *x*)

Apply mask to input signal returning a signal of same size with values at `iava` locations and `0` at other locations

Parameters

x [numpy.ndarray] Input array (can be either flattened or not)

Returns

y [numpy.ma.core.MaskedArray] Masked array.

Examples using `pylops.Restriction`

- sphx_glr_gallery_plot_restriction.py
- sphx_glr_gallery_plot_tvreg.py
- *03. Solvers*
- *05. 2D Interpolation*
- *11. Seismic regularization*
- *14. Bayesian Inversion*

pylops.Regression

class pylops.**Regression**(*taxis*, *order*, *dtype='float64'*)

Polynomial regression.

Creates an operator that applies polynomial regression to a set of points. Values along the t-axis must be provided while initializing the operator. The coefficients of the polynomial regression form the model vector to be provided in forward mode, while the values of the regression curve shall be provided in adjoint mode.

Parameters

taxis [numpy.ndarray] Elements along the t-axis.

order [int] Order of the regressed polynomial.

dtype [str, optional] Type of elements in input array.

Raises

TypeError If `t` is not numpy.ndarray.

See also:

LinearRegression Linear regression

Notes

The Regression operator solves the following problem:

$$y_i = \sum_{n=0}^{order} x_n t_i^n \qquad \forall i = 1, 2, ..., N$$

where N represents the order of the chosen polynomial. We can express this problem in a matrix form

$$\mathbf{y} = \mathbf{A}\mathbf{x}$$

where

$$\mathbf{y} = [y_1, y_2, ..., y_N]^T, \qquad \mathbf{x} = [x_0, x_1, ..., x_{order}]^T$$

and

$$\mathbf{A} = \begin{bmatrix} 1 & t_1 & t_1^2 & .. & t_1^{order} \\ 1 & t_2 & t_2^2 & .. & t_1^{order} \\ .. & .. & .. & .. & .. \\ 1 & t_N & t_N^2 & .. & t_N^{order} \end{bmatrix}$$

Attributes

shape [tuple] Operator shape

explicit [bool] Operator contains a matrix that can be solved explicitly (True) or not (False)

Methods

__init__(self, taxis, order[, dtype])	Initialize this LinearOperator.
adjoint(self)	Hermitian adjoint.
apply(self, t, x)	Return values along y-axis given certain t location(s) along t-axis and regression coefficients x
cond(self, **kwargs_eig)	Condition number of linear operator.
conj(self)	Complex conjugate operator
div(self, y[, niter])	Solve the linear problem $\mathbf{y} = \mathbf{A}\mathbf{x}$.
dot(self, x)	Matrix-matrix or matrix-vector multiplication.
eigs(self[, neigs, symmetric, niter])	Most significant eigenvalues of linear operator.
matmat(self, X)	Matrix-matrix multiplication.
matvec(self, x)	Matrix-vector multiplication.
rmatvec(self, x)	Adjoint matrix-vector multiplication.
transpose(self)	Transpose this linear operator.

apply(*self*, *t*, *x*)
Return values along y-axis given certain t location(s) along t-axis and regression coefficients x

Parameters

taxis [numpy.ndarray] Elements along the t-axis.

x [numpy.ndarray] Regression coefficients

dtype [str, optional]

Returns

———-

y [numpy.ndarray] Values along y-axis

Examples using pylops.Regression

- sphx_glr_gallery_plot_regr.py

pylops.LinearRegression

pylops.**LinearRegression**(*taxis*, *dtype='float64'*)
Linear regression.

Creates an operator that applies linear regression to a set of points. Values along the t-axis must be provided while initializing the operator. Intercept and gradient form the model vector to be provided in forward mode, while the values of the regression line curve shall be provided in adjoint mode.

Parameters

taxis [numpy.ndarray] Elements along the t-axis.

dtype [str, optional] Type of elements in input array.

Raises

TypeError If t is not numpy.ndarray.

See also:

Regression Polynomial regression

Notes

The LinearRegression operator solves the following problem:

$$y_i = x_0 + x_1 t_i \qquad \forall i = 1, 2, ..., N$$

We can express this problem in a matrix form

$$\mathbf{y} = \mathbf{A}\mathbf{x}$$

where

$$\mathbf{y} = [y_1, y_2, ..., y_N]^T, \qquad \mathbf{x} = [x_0, x_1]^T$$

and

$$\mathbf{A} = \begin{bmatrix} 1 & t_1 \\ 1 & t_2 \\ .. & .. \\ 1 & t_N \end{bmatrix}$$

Note that this is a particular case of the *pylops.Regression* operator and it is in fact just a lazy call of that operator with order=1.

Attributes

shape [tuple] Operator shape

explicit [bool] Operator contains a matrix that can be solved explicitly (True) or not (False)

Examples using pylops.LinearRegression

- sphx_glr_gallery_plot_linearregr.py

pylops.CausalIntegration

class pylops.**CausalIntegration**(*N*, *dims=None*, *dir=-1*, *sampling=1*, *halfcurrent=True*, *dtype='float64'*)

Causal integration.

Apply causal integration to a multi-dimensional array along dir axis.

Parameters

N [int] Number of samples in model.

dims [list, optional] Number of samples for each dimension (None if only one dimension is available)

dir [int, optional] Direction along which smoothing is applied.

sampling [float, optional] Sampling step dx.

halfcurrent [float, optional] Add half of current value (True) or the entire value (False)

dtype [str, optional] Type of elements in input array.

Notes

The CausalIntegration operator applies a causal integration to any chosen direction of a multi-dimensional array.

For simplicity, given a one dimensional array, the causal integration is:

$$y(t) = \int x(t)dt$$

which can be discretised as :

$$y[i] = \sum_{j=0}^{i} x[j]dt$$

or

$$y[i] = (\sum_{j=0}^{i-1} x[j] + 0.5x[i])dt$$

where dt is the sampling interval. In our implementation, the choice to add $x[i]$ or just $0.5x[i]$ is made by selecting the halfcurrent parameter.

Note that the integral of a signal has no unique solution, as any constant c can be added to y, for example if $x(t) = t^2$ the resulting integration is:

$$y(t) = \int t^2 dt = \frac{t^3}{3} + c$$

If we apply a first derivative to y we in fact obtain:

$$x(t) = \frac{dy}{dt} = t^2$$

no matter the choice of c.

Attributes

shape [tuple] Operator shape

explicit [bool] Operator contains a matrix that can be solved explicitly (True) or not (False)

Methods

`__init__`(self, N[, dims, dir, sampling, ...])	Initialize this LinearOperator.
`adjoint`(self)	Hermitian adjoint.
`cond`(self, **kwargs_eig)	Condition number of linear operator.
`conj`(self)	Complex conjugate operator
`div`(self, y[, niter])	Solve the linear problem $\mathbf{y} = \mathbf{A}\mathbf{x}$.
`dot`(self, x)	Matrix-matrix or matrix-vector multiplication.
`eigs`(self[, neigs, symmetric, niter])	Most significant eigenvalues of linear operator.
`matmat`(self, X)	Matrix-matrix multiplication.
`matvec`(self, x)	Matrix-vector multiplication.
`rmatvec`(self, x)	Adjoint matrix-vector multiplication.
`transpose`(self)	Transpose this linear operator.

Examples using `pylops.CausalIntegration`

- sphx_glr_gallery_plot_causalintegration.py

pylops.Spread

class pylops.**Spread**(*dims*, *dimsd*, *table=None*, *dtable=None*, *fh=None*, *interp=False*, *engine='numpy'*, *dtype='float64'*)

Spread operator.

Spread values from the input model vector arranged as a 2-dimensional array of size $[n_{x0} \times n_{t0}]$ into the data vector of size $[n_x \times n_t]$. Spreading is performed along parametric curves provided as look-up table of pre-computed indices (`table`) or computed on-the-fly using a function handle (`fh`).

In adjont mode, values from the data vector are instead stacked along the same parametric curves.

Parameters

dims [`tuple`] Dimensions of model vector (vector will be reshaped internally into a two-dimensional array of size $[n_{x0} \times n_{t0}]$, where the first dimension is the spreading/stacking direction)

dimsd [`tuple`] Dimensions of model vector (vector will be reshaped internal into a two-dimensional array of size $[n_x \times n_t]$)

table [`np.ndarray`, optional] Look-up table of indeces of size $[n_{x0} \times n_{t0} \times n_x]$ (if `None` use function handle `fh`)

dtable [`np.ndarray`, optional] Look-up table of decimals remainders for linear interpolation of size $[n_{x0} \times n_{t0} \times n_x]$ (if `None` use function handle `fh`)

fh [`np.ndarray`, optional] Function handle that returns an index (and a fractional value in case of `interp=True`) to be used for spreading/stacking given indices in $x0$ and t axes (if `None` use look-up table `table`)

interp [`bool`, optional] Apply linear interpolation (`True`) or nearest interpolation (`False`) during stacking/spreading along parametric curve. To be used only if `engine='numba'`, inferred directly from the number of outputs of `fh` for `engine='numpy'`

engine [`str`, optional] Engine used for fft computation (`numpy` or `numba`). Note that `numba` can only be used when providing a look-up table

dtype [`str`, optional] Type of elements in input array.

Raises

KeyError If `engine` is neither `numpy` nor `numba`

NotImplementedError If both `table` and `fh` are not provided

ValueError If `table` has shape different from $[n_{x0} \times n_t 0 \times n_x]$

Notes

The Spread operator applies the following linear transform in forward mode to the model vector after reshaping it into a 2-dimensional array of size $[n_x \times n_t]$:

$$m(x0, t_0) \rightarrow d(x, t = f(x0, x, t_0))$$

where $f(x0, x, t)$ is a mapping function that returns a value t given values $x0$, x, and t_0.

In adjoint mode, the model is reconstructed by means of the following stacking operation:

$$m(x0, t_0) = \int d(x, t = f(x0, x, t_0))dx$$

Note that `table` (or `fh`) must return integer numbers representing indices in the axis t. However it also possible to perform linear interpolation as part of the spreading/stacking process by providing the decimal part of the mapping function ($t - \lfloor t \rfloor$) either in `dtable` input parameter or as second value in the return of `fh` function.

Attributes

shape [`tuple`] Operator shape

explicit [`bool`] Operator contains a matrix that can be solved explicitly (`True`) or not (`False`)

Methods

`__init__`(self, dims, dimsd[, table, dtable, ...])	Initialize this LinearOperator.
`adjoint`(self)	Hermitian adjoint.
`cond`(self, **kwargs_eig)	Condition number of linear operator.
`conj`(self)	Complex conjugate operator
`div`(self, y[, niter])	Solve the linear problem $\mathbf{y} = \mathbf{Ax}$.
`dot`(self, x)	Matrix-matrix or matrix-vector multiplication.
`eigs`(self[, neigs, symmetric, niter])	Most significant eigenvalues of linear operator.
`matmat`(self, X)	Matrix-matrix multiplication.
`matvec`(self, x)	Matrix-vector multiplication.
`rmatvec`(self, x)	Adjoint matrix-vector multiplication.
`transpose`(self)	Transpose this linear operator.

pylops.VStack

class pylops.**VStack**(*ops*, *dtype=None*)

Vertical stacking.

Stack a set of N linear operators vertically.

Parameters

ops [`list`] Linear operators to be stacked

dtype [str, optional] Type of elements in input array.

Notes

A vertical stack of N linear operators is created such as its application in forward mode leads to

$$\begin{bmatrix} \mathbf{L}_1 \\ \mathbf{L}_2 \\ ... \\ \mathbf{L}_N \end{bmatrix} \mathbf{x} = \begin{bmatrix} \mathbf{L}_1\mathbf{x} \\ \mathbf{L}_2\mathbf{x} \\ ... \\ \mathbf{L}_N\mathbf{x} \end{bmatrix} = \begin{bmatrix} \mathbf{y}_1 \\ \mathbf{y}_2 \\ ... \\ \mathbf{y}_N \end{bmatrix}$$

while its application in adjoint mode leads to

$$\begin{bmatrix} \mathbf{L}_1^H & \mathbf{L}_2^H & ... & \mathbf{L}_N^H \end{bmatrix} \begin{bmatrix} \mathbf{y}_1 \\ \mathbf{y}_2 \\ ... \\ \mathbf{y}_N \end{bmatrix} = \mathbf{L}_1^H\mathbf{y}_1 + \mathbf{L}_2^H\mathbf{y}_2 + ... + \mathbf{L}_N^H\mathbf{y}_N$$

Attributes

shape [tuple] Operator shape

explicit [bool] Operator contains a matrix that can be solved explicitly (True) or not (False)

Methods

__init__(self, ops[, dtype])	Initialize this LinearOperator.
adjoint(self)	Hermitian adjoint.
cond(self, **kwargs_eig)	Condition number of linear operator.
conj(self)	Complex conjugate operator
div(self, y[, niter])	Solve the linear problem $\mathbf{y} = \mathbf{A}\mathbf{x}$.
dot(self, x)	Matrix-matrix or matrix-vector multiplication.
eigs(self[, neigs, symmetric, niter])	Most significant eigenvalues of linear operator.
matmat(self, X)	Matrix-matrix multiplication.
matvec(self, x)	Matrix-vector multiplication.
rmatvec(self, x)	Adjoint matrix-vector multiplication.
transpose(self)	Transpose this linear operator.

Examples using `pylops.VStack`

- sphx_glr_gallery_plot_stacking.py

pylops.HStack

class pylops.**HStack**(*ops*, *dtype='float64'*)

Horizontal stacking.

Stack a set of N linear operators horizontally.

Parameters

ops [list] Linear operators to be stacked

dtype [str, optional] Type of elements in input array.

Notes

An horizontal stack of N linear operators is created such as its application in forward mode leads to

$$\begin{bmatrix} \mathbf{L}_1 & \mathbf{L}_2 & ... & \mathbf{L}_N \end{bmatrix} \begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \\ ... \\ \mathbf{x}_N \end{bmatrix} = \mathbf{L}_1\mathbf{x}_1 + \mathbf{L}_2\mathbf{x}_2 + ... + \mathbf{L}_N\mathbf{x}_N$$

while its application in adjoint mode leads to

$$\begin{bmatrix} \mathbf{L}_1^H \\ \mathbf{L}_2^H \\ ... \\ \mathbf{L}_N^H \end{bmatrix} \mathbf{y} = \begin{bmatrix} \mathbf{L}_1^H\mathbf{y} \\ \mathbf{L}_2^H\mathbf{y} \\ ... \\ \mathbf{L}_N^H\mathbf{y} \end{bmatrix} = \begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \\ ... \\ \mathbf{x}_N \end{bmatrix}$$

Attributes

shape [tuple] Operator shape

explicit [bool] Operator contains a matrix that can be solved explicitly (True) or not (False)

Methods

__init__(self, ops[, dtype])	Initialize this LinearOperator.
adjoint(self)	Hermitian adjoint.
cond(self, **kwargs_eig)	Condition number of linear operator.
conj(self)	Complex conjugate operator
div(self, y[, niter])	Solve the linear problem $\mathbf{y} = \mathbf{A}\mathbf{x}$.
dot(self, x)	Matrix-matrix or matrix-vector multiplication.
eigs(self[, neigs, symmetric, niter])	Most significant eigenvalues of linear operator.
matmat(self, X)	Matrix-matrix multiplication.
matvec(self, x)	Matrix-vector multiplication.
rmatvec(self, x)	Adjoint matrix-vector multiplication.
transpose(self)	Transpose this linear operator.

Examples using `pylops.HStack`

- sphx_glr_gallery_plot_stacking.py

pylops.Block

pylops.**Block**(*ops*, *dtype=None*)

Block operator.

Create a block operator from N lists of M linear operators each.

Parameters

ops [list] List of lists of operators to be combined in block fashion

dtype [str, optional] Type of elements in input array.

Notes

In mathematics, a block or a partitioned matrix is a matrix that is interpreted as being broken into sections called blocks or submatrices. Similarly a block operator is composed of N sets of M linear operators each such that its application in forward mode leads to

$$\begin{bmatrix} \mathbf{L_{1,1}} & \mathbf{L_{1,2}} & ... & \mathbf{L_{1,M}} \\ \mathbf{L_{2,1}} & \mathbf{L_{2,2}} & ... & \mathbf{L_{2,M}} \\ ... & ... & ... & ... \\ \mathbf{L_{N,1}} & \mathbf{L_{N,2}} & ... & \mathbf{L_{N,M}} \end{bmatrix} \begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \\ ... \\ \mathbf{x}_M \end{bmatrix} = \begin{bmatrix} \mathbf{L_{1,1}}\mathbf{x}_1 + \mathbf{L_{1,2}}\mathbf{x}_2 + \mathbf{L_{1,M}}\mathbf{x}_M \\ \mathbf{L_{2,1}}\mathbf{x}_1 + \mathbf{L_{2,2}}\mathbf{x}_2 + \mathbf{L_{2,M}}\mathbf{x}_M \\ ... \\ \mathbf{L_{N,1}}\mathbf{x}_1 + \mathbf{L_{N,2}}\mathbf{x}_2 + \mathbf{L_{N,M}}\mathbf{x}_M \end{bmatrix}$$

while its application in adjoint mode leads to

$$\begin{bmatrix} \mathbf{L_{1,1}}^H & \mathbf{L_{2,1}}^H & ... & \mathbf{L_{N,1}}^H \\ \mathbf{L_{1,2}}^H & \mathbf{L_{2,2}}^H & ... & \mathbf{L_{N,2}}^H \\ ... & ... & ... & ... \\ \mathbf{L_{1,M}}^H & \mathbf{L_{2,M}}^H & ... & \mathbf{L_{N,M}}^H \end{bmatrix} \begin{bmatrix} \mathbf{y}_1 \\ \mathbf{y}_2 \\ ... \\ \mathbf{y}_N \end{bmatrix} = \begin{bmatrix} \mathbf{L_{1,1}}^H\mathbf{y}_1 + \mathbf{L_{2,1}}^H\mathbf{y}_2 + \mathbf{L_{N,1}}^H\mathbf{y}_N \\ \mathbf{L_{1,2}}^H\mathbf{y}_1 + \mathbf{L_{2,2}}^H\mathbf{y}_2 + \mathbf{L_{N,2}}^H\mathbf{y}_N \\ ... \\ \mathbf{L_{1,M}}^H\mathbf{y}_1 + \mathbf{L_{2,M}}^H\mathbf{y}_2 + \mathbf{L_{N,M}}^H\mathbf{y}_N \end{bmatrix}$$

Attributes

shape [tuple] Operator shape

explicit [bool] Operator contains a matrix that can be solved explicitly (True) or not (False)

Examples using `pylops.Block`

- sphx_glr_gallery_plot_stacking.py

pylops.BlockDiag

class pylops.**BlockDiag**(*ops*, *dtype=None*)

Block-diagonal operator.

Create a block-diagonal operator from N linear operators.

Parameters

ops [list] Linear operators to be stacked

dtype [str, optional] Type of elements in input array.

Notes

A block-diagonal operator composed of N linear operators is created such as its application in forward mode leads to

$$\begin{bmatrix} \mathbf{L_1} & \mathbf{0} & ... & \mathbf{0} \\ \mathbf{0} & \mathbf{L_2} & ... & \mathbf{0} \\ ... & ... & ... & ... \\ \mathbf{0} & \mathbf{0} & ... & \mathbf{L_N} \end{bmatrix} \begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \\ ... \\ \mathbf{x}_N \end{bmatrix} = \begin{bmatrix} \mathbf{L_1}\mathbf{x}_1 \\ \mathbf{L_2}\mathbf{x}_2 \\ ... \\ \mathbf{L_N}\mathbf{x}_N \end{bmatrix}$$

while its application in adjoint mode leads to

$$\begin{bmatrix} \mathbf{L_1}^H & \mathbf{0} & ... & \mathbf{0} \\ \mathbf{0} & \mathbf{L_2}^H & ... & \mathbf{0} \\ ... & ... & ... & ... \\ \mathbf{0} & \mathbf{0} & ... & \mathbf{L_N}^H \end{bmatrix} \begin{bmatrix} \mathbf{y}_1 \\ \mathbf{y}_2 \\ ... \\ \mathbf{y}_N \end{bmatrix} = \begin{bmatrix} \mathbf{L_1}^H\mathbf{y}_1 \\ \mathbf{L_2}^H\mathbf{y}_2 \\ ... \\ \mathbf{L_N}^H\mathbf{y}_N \end{bmatrix}$$

Attributes

shape [tuple] Operator shape

explicit [bool] Operator contains a matrix that can be solved explicitly (True) or not (False)

Methods

__init__(self, ops[, dtype])	Initialize this LinearOperator.
adjoint(self)	Hermitian adjoint.
cond(self, **kwargs_eig)	Condition number of linear operator.
conj(self)	Complex conjugate operator
div(self, y[, niter])	Solve the linear problem $\mathbf{y} = \mathbf{A}\mathbf{x}$.
dot(self, x)	Matrix-matrix or matrix-vector multiplication.
eigs(self[, neigs, symmetric, niter])	Most significant eigenvalues of linear operator.
matmat(self, X)	Matrix-matrix multiplication.
matvec(self, x)	Matrix-vector multiplication.
rmatvec(self, x)	Adjoint matrix-vector multiplication.
transpose(self)	Transpose this linear operator.

Examples using `pylops.BlockDiag`

- sphx_glr_gallery_plot_stacking.py

pylops.Kronecker

class pylops.**Kronecker**(*Op1*, *Op2*, *dtype='float64'*)

Kronecker operator.

Perform Krocker product of two operators. Note that the combined operator is never created explicitely, rather the product of this operator with the model vector is performed in forward mode, or the product of the adjoint of this operator and the data vector in adjoint mode.

Parameters

Op1 [*pylops.LinearOperator*] First operator

Op2 [*pylops.LinearOperator*] Second operator

dtype [str, optional] Type of elements in input array.

Notes

The Kronecker product denoted with :math‘otimes‘ is an operation on two operators $\mathbf{Op_1}$ and $\mathbf{Op_2}$ of sizes $[n_1 \times m_1]$ and $[n_2 \times m_2]$ respectively, resulting in a block matrix of size $[n_1 n_2 \times m_1 m_2]$.

$$\mathbf{Op_1} \otimes \mathbf{Op_2} = \begin{bmatrix} Op_1^{1,1}\mathbf{Op_2} & ... & Op_1^{1,m_1}\mathbf{Op_2} \\ ... & ... & ... \\ Op_1^{n_1,1}\mathbf{Op_2} & ... & Op_1^{n_1,m_1}\mathbf{Op_2} \end{bmatrix}$$

The application of the resulting matrix to a vector $\mathbf{x}$ of size $[m_1 m_2 \times 1]$ is equivalent to the application of the second operator $\mathbf{Op_2}$ to the columns of a matrix of size $[m_2 \times m_1]$ obtained by reshaping the input vector $\mathbf{x}$, followed by the application of the first operator to the transposed matrix produced by the first operator. Similarly, in adjoint mode the two operators are applied in reversed order to a reshaped matrix of size $[n_1 \times n_2]$.

Attributes

shape [tuple] Operator shape

explicit [bool] Operator contains a matrix that can be solved explicitly (True) or not (False)

Methods

__init__(self, Op1, Op2[, dtype])	Initialize this LinearOperator.
adjoint(self)	Hermitian adjoint.
cond(self, **kwargs_eig)	Condition number of linear operator.
conj(self)	Complex conjugate operator
div(self, y[, niter])	Solve the linear problem $\mathbf{y} = \mathbf{A}\mathbf{x}$.
dot(self, x)	Matrix-matrix or matrix-vector multiplication.
eigs(self[, neigs, symmetric, niter])	Most significant eigenvalues of linear operator.
matmat(self, X)	Matrix-matrix multiplication.
matvec(self, x)	Matrix-vector multiplication.
rmatvec(self, x)	Adjoint matrix-vector multiplication.
transpose(self)	Transpose this linear operator.

Examples using `pylops.Kronecker`

- sphx_glr_gallery_plot_stacking.py

Smoothing and derivatives

Smoothing1D(nsmooth, dims[, dir, dtype])	1D Smoothing.
Smoothing2D(nsmooth, dims[, nodir, dtype])	2D Smoothing.
FirstDerivative(N[, dims, dir, sampling, ...])	First derivative.
SecondDerivative(N[, dims, dir, sampling, ...])	Second derivative.
Laplacian(dims[, dirs, weights, sampling, ...])	Laplacian.

pylops.Smoothing1D

pylops.**Smoothing1D**(*nsmooth*, *dims*, *dir=0*, *dtype='float64'*)

1D Smoothing.

Apply smoothing to model (and data) along a specific direction of a multi-dimensional array depending on the choice of dir.

Parameters

nsmooth [int] Lenght of smoothing operator (must be odd)

dims [tuple or int] Number of samples for each dimension

dir [int, optional] Direction along which smoothing is applied

dtype [str, optional] Type of elements in input array.

Notes

The Smoothing1D operator is a special type of convolutional operator that convolves the input model (or data) with a constant filter of size n_{smooth}:

$$\mathbf{f} = [1/n_{smooth}, 1/n_{smooth}, ..., 1/n_{smooth}]$$

When applied to the first direction:

$$y[i,j,k] = 1/n_{smooth} \sum_{l=-(n_{smooth}-1)/2}^{(n_{smooth}-1)/2} x[l,j,k]$$

Similarly when applied to the second direction:

$$y[i,j,k] = 1/n_{smooth} \sum_{l=-(n_{smooth}-1)/2}^{(n_{smooth}-1)/2} x[i,l,k]$$

and the third direction:

$$y[i,j,k] = 1/n_{smooth} \sum_{l=-(n_{smooth}-1)/2}^{(n_{smooth}-1)/2} x[i,j,l]$$

Note that since the filter is symmetrical, the *Smoothing1D* operator is self-adjoint.

Attributes

shape [`tuple`] Operator shape

explicit [`bool`] Operator contains a matrix that can be solved explicitly (`True`) or not (`False`)

Examples using `pylops.Smoothing1D`

- sphx_glr_gallery_plot_smoothing1d.py
- sphx_glr_gallery_plot_causalintegration.py
- sphx_glr_gallery_plot_wavest.py
- *03. Solvers*

pylops.Smoothing2D

`pylops.`**`Smoothing2D`**(*nsmooth*, *dims*, *nodir=None*, *dtype='float64'*)

2D Smoothing.

Apply smoothing to model (and data) along two directions of a multi-dimensional array depending on the choice of `nodir`.

Parameters

nsmooth [`tuple` or `list`] Lenght of smoothing operatorin 1st and 2nd dimensions (must be odd)

dims [`tuple`] Number of samples for each dimension

nodir [`int`, optional] Direction along which smoothing is NOT applied (set to None for 2d arrays)

dtype [str, optional] Type of elements in input array.

See also:

lops.signalprocessing.Convolve2D 2D convolution

Notes

The 2D Smoothing operator is a special type of convolutional operator that convolves the input model (or data) with a constant 2d filter of size $n_{smooth,1}$ x $n_{smooth,2}$:

Its application to a two dimensional input signal is:

$$y[i,j] = 1/(n_{smooth,1} * n_{smooth,2}) \sum_{l=-(n_{smooth,1}-1)/2}^{(n_{smooth,1}-1)/2} \sum_{m=-(n_{smooth,2}-1)/2}^{(n_{smooth,2}-1)/2} x[l,m]$$

Note that since the filter is symmetrical, the *Smoothing2D* operator is self-adjoint.

Attributes

shape [tuple] Operator shape

explicit [bool] Operator contains a matrix that can be solved explicitly (True) or not (False)

Examples using pylops.Smoothing2D

- sphx_glr_gallery_plot_smoothing2d.py
- sphx_glr_gallery_plot_causalintegration.py

pylops.FirstDerivative

class pylops.**FirstDerivative**(*N*, *dims=None*, *dir=0*, *sampling=1.0*, *edge=False*, *dtype='float64'*)

First derivative.

Apply second-order centered first derivative.

Parameters

N [int] Number of samples in model.

dims [tuple, optional] Number of samples for each dimension (None if only one dimension is available)

dir [int, optional] Direction along which smoothing is applied.

sampling [float, optional] Sampling step dx.

edge [bool, optional] Use reduced order derivative at edges (True) or ignore them (False)

dtype [str, optional] Type of elements in input array.

Notes

The FirstDerivative operator applies a first derivative to any chosen direction of a multi-dimensional array.

For simplicity, given a one dimensional array, the second-order centered first derivative is:

$$y[i] = (0.5x[i+1] - 0.5x[i-1])/dx$$

Attributes

shape [tuple] Operator shape

explicit [bool] Operator contains a matrix that can be solved explicitly (True) or not (False)

Methods

__init__(self, N[, dims, dir, sampling, ...])	Initialize this LinearOperator.
adjoint(self)	Hermitian adjoint.
cond(self, **kwargs_eig)	Condition number of linear operator.
conj(self)	Complex conjugate operator
div(self, y[, niter])	Solve the linear problem $\mathbf{y} = \mathbf{A}\mathbf{x}$.
dot(self, x)	Matrix-matrix or matrix-vector multiplication.
eigs(self[, neigs, symmetric, niter])	Most significant eigenvalues of linear operator.
matmat(self, X)	Matrix-matrix multiplication.
matvec(self, x)	Matrix-vector multiplication.
rmatvec(self, x)	Adjoint matrix-vector multiplication.
transpose(self)	Transpose this linear operator.

Examples using pylops.FirstDerivative

- sphx_glr_gallery_plot_causalintegration.py
- sphx_glr_gallery_plot_derivative.py
- sphx_glr_gallery_plot_stacking.py
- sphx_glr_gallery_plot_tvreg.py
- *04. Image deblurring*

pylops.SecondDerivative

class pylops.**SecondDerivative**(*N*, *dims=None*, *dir=0*, *sampling=1*, *edge=False*, *dtype='float64'*)

Second derivative.

Apply second-order second derivative.

Parameters

N [int] Number of samples in model.

dims [tuple, optional] Number of samples for each dimension (None if only one dimension is available)

dir [int, optional] Direction along which smoothing is applied.

sampling [float, optional] Sampling step dx.

edge [bool, optional] Use reduced order derivative at edges (True) or ignore them (False)

dtype [str, optional] Type of elements in input array.

Notes

The SecondDerivative operator applies a second derivative to any chosen direction of a multi-dimensional array. For simplicity, given a one dimensional array, the second-order centered first derivative is:

$$y[i] = (x[i+1] - 2x[i] + x[i-1])/dx^2$$

Attributes

shape [tuple] Operator shape

explicit [bool] Operator contains a matrix that can be solved explicitly (True) or not (False)

Methods

__init__(self, N[, dims, dir, sampling, ...])	Initialize this LinearOperator.
adjoint(self)	Hermitian adjoint.
cond(self, **kwargs_eig)	Condition number of linear operator.
conj(self)	Complex conjugate operator
div(self, y[, niter])	Solve the linear problem $\mathbf{y} = \mathbf{A}\mathbf{x}$.
dot(self, x)	Matrix-matrix or matrix-vector multiplication.
eigs(self[, neigs, symmetric, niter])	Most significant eigenvalues of linear operator.
matmat(self, X)	Matrix-matrix multiplication.
matvec(self, x)	Matrix-vector multiplication.
rmatvec(self, x)	Adjoint matrix-vector multiplication.
transpose(self)	Transpose this linear operator.

Examples using pylops.SecondDerivative

- sphx_glr_gallery_plot_causalintegration.py
- sphx_glr_gallery_plot_derivative.py
- sphx_glr_gallery_plot_stacking.py
- sphx_glr_gallery_plot_tvreg.py
- sphx_glr_gallery_plot_wavest.py
- *03. Solvers*
- *11. Seismic regularization*

pylops.Laplacian

pylops.**Laplacian**(*dims*, *dirs=(0, 1)*, *weights=(1, 1)*, *sampling=(1, 1)*, *edge=False*, *dtype='float64'*)

Laplacian.

Apply second-order centered laplacian operator to a multi-dimensional array (at least 2 dimensions are required)

Parameters

dims [tuple] Number of samples for each dimension.

dirs [tuple, optional] Directions along which laplacian is applied.

weights [tuple, optional] Weight to apply to each direction (real laplacian operator if `weights=[1,1]`)

sampling [tuple, optional] Sampling steps `dx` and `dy` for each direction

edge [bool, optional] Use reduced order derivative at edges (`True`) or ignore them (`False`)

dtype [str, optional] Type of elements in input array.

Returns

l2op [*pylops.LinearOperator*] Laplacian linear operator

Notes

The Laplacian operator applies a second derivative along two directions of a multi-dimensional array.

For simplicity, given a two dimensional array, the Laplacian is:

$$y[i,j] = (x[i+1,j] + x[i-1,j] + x[i,j-1] + x[i,j+1] - 4x[i,j])/(dx * dy)$$

Examples using `pylops.Laplacian`

- sphx_glr_gallery_plot_causalintegration.py
- sphx_glr_gallery_plot_derivative.py
- *05. 2D Interpolation*

Signal processing

FFT(dims[, dir, nfft, sampling, real, ...])	One dimensional Fast-Fourier Transform.
FFT2D(dims[, dirs, nffts, sampling, dtype])	Two dimensional Fast-Fourier Transform.
FFTND(dims[, dirs, nffts, sampling, dtype])	N-dimensional Fast-Fourier Transform.
Convolve1D(N, h[, offset, dims, dir, dtype, ...])	1D convolution operator.
Convolve2D(N, h, dims[, offset, nodir, ...])	2D convolution operator.
ConvolveND(N, h, dims[, offset, dirs, ...])	ND convolution operator.
Interp(M, iava[, dims, dir, kind, dtype])	Interpolation operator.
Radon2D(taxis, haxis, pxaxis[, kind, ...])	Two dimensional Radon transform.
Radon3D(taxis, hyaxis, hxaxis, pyaxis, pxaxis)	Three dimensional Radon transform.
Sliding2D(Op, dims, dimsd, nwin, nover[, ...])	2D Sliding transform operator.
Sliding3D(Op, dims, dimsd, nwin, nover, nop)	3D Sliding transform operator.
Fredholm1(G[, nz, saveGt, usematmul, dtype])	Fredholm integral of first kind.

pylops.signalprocessing.FFT

pylops.signalprocessing.**FFT**(*dims*, *dir=0*, *nfft=None*, *sampling=1.0*, *real=False*, *fftshift=False*, *engine='numpy'*, *dtype='complex128'*, ***kwargs_fftw*)

One dimensional Fast-Fourier Transform.

Apply Fast-Fourier Transform (FFT) along a specific direction `dir` of a multi-dimensional array of size `dim`.

Note that the FFT operator is an overload to either the numpy `numpy.fft.fft` (or `numpy.fft.rfft` for real models) in forward mode and to the numpy `numpy.fft.ifft` (or `numpy.fft.irfft` for real models) in adjoint mode, or to the `pyfftw.FFTW` class.

In both cases, scaling is properly taken into account to guarantee that the operator is passing the dot-test.

Note: For a real valued input signal, it is possible to store the values of the Fourier transform at positive frequencies only as values at negative frequencies are simply their complex conjugates. However as the operation of removing the negative part of the frequency axis in forward mode and adding the complex conjugates in adjoint mode is nonlinear, the Linear Operator FTT with `real=True` is not expected to pass the dot-test. It is thus *only* advised to use this flag when a forward and adjoint FFT is used in the same chained operator (e.g., `FFT.H*Op*FFT`) such as in `pylops.waveeqprocessing.mdd.MDC`.

Parameters

dims [`tuple`] Number of samples for each dimension

dir [`int`, optional] Direction along which FFT is applied.

nfft [`int`, optional] Number of samples in Fourier Transform (same as input if `nfft=None`)

sampling [`float`, optional] Sampling step `dt`.

real [`bool`, optional] Model to which fft is applied has real numbers (`True`) or not (`False`). Used to enforce that the output of adjoint of a real model is real.

fftshift [`bool`, optional] Apply fftshift/ifftshift (`True`) or not (`False`)

engine [`str`, optional] Engine used for fft computation (`numpy` or `fftw`)

dtype [`str`, optional] Type of elements in input array.

****kwargs_fftw** Arbitrary keyword arguments for `pyfftw.FTTW`

Raises

ValueError If `dims` is not provided and if `dir` is bigger than `len(dims)`

NotImplementedError If `engine` is neither `numpy` nor `numba`

Notes

The FFT operator applies the forward Fourier transform to a signal $d(t)$ in forward mode:

$$D(f) = F(d) = \int d(t) e^{-j2\pi ft} dt$$

Similarly, the inverse Fourier transform is applied to the Fourier spectrum $D(f)$ in adjoint mode:

$$d(t) = F^{-1}(D) = \int D(f) e^{j2\pi ft} df$$

Both operators are effectively discretized and solved by a fast iterative algorithm known as Fast Fourier Transform.

Note that the FFT operator is a special operator in that the adjoint is also the inverse of the forward mode. Moreover, in case of real signal in time domain, the Fourier transform in Hermitian.

Attributes

shape [`tuple`] Operator shape

explicit [`bool`] Operator contains a matrix that can be solved explicitly (True) or not (False)

Examples using `pylops.signalprocessing.FFT`

- sphx_glr_gallery_plot_fft.py
- *02. The Dot-Test*
- *03. Solvers*
- *14. Bayesian Inversion*

pylops.signalprocessing.FFT2D

class pylops.signalprocessing.**FFT2D**(*dims*, *dirs=(0, 1)*, *nffts=(None, None)*, *sampling=(1.0, 1.0)*, *dtype='complex128'*)

Two dimensional Fast-Fourier Transform.

Apply two dimensional Fast-Fourier Transform (FFT) to any pair of axes of a multi-dimensional array depending on the choice of `dirs`. Note that the FFT2D operator is a simple overload to the numpy `numpy.fft.fft2` in forward mode and to the numpy `numpy.fft.ifft2` in adjoint mode, however scaling is taken into account differently to guarantee that the operator is passing the dot-test.

Parameters

dims [`tuple`] Number of samples for each dimension

dirs [`tuple`, optional] Pair of directions along which FFT2D is applied

nffts [`tuple`, optional] Number of samples in Fourier Transform for each direction (same as input if `nffts=(None, None)`)

sampling [`tuple`, optional] Sampling steps `dy` and `dx`

dtype [`str`, optional] Type of elements in input array

Raises

ValueError If `dims` has less than two elements, and if `dirs`, `nffts`, or `sampling` has more or less than two elements.

Notes

The FFT2D operator applies the two-dimensional forward Fourier transform to a signal $d(y, x)$ in forward mode:

$$D(k_y, k_x) = F(d) = \int \int d(y, x) e^{-j2\pi k_y y} e^{-j2\pi k_x x} dy dx$$

Similarly, the two-dimensional inverse Fourier transform is applied to the Fourier spectrum $D(k_y, k_x)$ in adjoint mode:

$$d(y, x) = F^{-1}(D) = \int \int D(k_y, k_x) e^{j2\pi k_y y} e^{j2\pi k_x x} dk_y dk_x$$

Both operators are effectively discretized and solved by a fast iterative algorithm known as Fast Fourier Transform.

Attributes

shape [`tuple`] Operator shape

explicit [`bool`] Operator contains a matrix that can be solved explicitly (True) or not (False)

Methods

`__init__`(self, dims[, dirs, nffts, ...])	Initialize this LinearOperator.
`adjoint`(self)	Hermitian adjoint.
`cond`(self, **kwargs_eig)	Condition number of linear operator.
`conj`(self)	Complex conjugate operator
`div`(self, y[, niter])	Solve the linear problem $\mathbf{y} = \mathbf{A}\mathbf{x}$.
`dot`(self, x)	Matrix-matrix or matrix-vector multiplication.
`eigs`(self[, neigs, symmetric, niter])	Most significant eigenvalues of linear operator.
`matmat`(self, X)	Matrix-matrix multiplication.
`matvec`(self, x)	Matrix-vector multiplication.
`rmatvec`(self, x)	Adjoint matrix-vector multiplication.
`transpose`(self)	Transpose this linear operator.

Examples using `pylops.signalprocessing.FFT2D`

- sphx_glr_gallery_plot_fft.py
- sphx_glr_gallery_plot_tvreg.py
- *11. Seismic regularization*
- *12. Seismic wavefield decomposition*

pylops.signalprocessing.FFTND

class pylops.signalprocessing.**FFTND**(*dims*, *dirs=(0, 1, 2)*, *nffts=(None, None, None)*, *sampling=(1.0, 1.0, 1.0)*, *dtype='complex128'*)

N-dimensional Fast-Fourier Transform.

Apply n-dimensional Fast-Fourier Transform (FFT) to any n axes of a multi-dimensional array depending on the choice of `dirs`. Note that the FFTND operator is a simple overload to the numpy `numpy.fft.fftn` in forward mode and to the numpy `numpy.fft.ifftn` in adjoint mode, however scaling is taken into account differently to guarantee that the operator is passing the dot-test.

Parameters

dims [`tuple`] Number of samples for each dimension

dirs [`tuple`, optional] Directions along which FFTND is applied

nffts [`tuple`, optional] Number of samples in Fourier Transform for each direction (same as input if `nffts=(None, None, None, ..., None)`)

sampling [`tuple`, optional] Sampling steps in each direction

dtype [`str`, optional] Type of elements in input array

Raises

ValueError If `dims`, `dirs`, `nffts`, or `sampling` have less than three elements and if the dimension of `dirs`, `nffts`, and `sampling` is not the same

Notes

The FFTND operator applies the n-dimensional forward Fourier transform to a multi-dimensional array. Without loss of generality we consider here a three-dimensional signal $d(z, y, x)$. The FFTND in forward mode is:

$$D(k_z, k_y, k_x) = F(d) = \int\int d(z, y, x) e^{-j2\pi k_z z} e^{-j2\pi k_y y} e^{-j2\pi k_x x} dz dy dx$$

Similarly, the three-dimensional inverse Fourier transform is applied to the Fourier spectrum $D(k_z, k_y, k_x)$ in adjoint mode:

$$d(z, y, x) = F^{-1}(D) = \int\int D(k_z, k_y, k_x) e^{j2\pi k_z z} e^{j2\pi k_y y} e^{j2\pi k_x x} dk_z dk_y dk_x$$

Both operators are effectively discretized and solved by a fast iterative algorithm known as Fast Fourier Transform.

Attributes

shape [tuple] Operator shape

explicit [bool] Operator contains a matrix that can be solved explicitly (True) or not (False)

Methods

__init__(self, dims[, dirs, nffts, ...])	Initialize this LinearOperator.
adjoint(self)	Hermitian adjoint.
cond(self, **kwargs_eig)	Condition number of linear operator.
conj(self)	Complex conjugate operator
div(self, y[, niter])	Solve the linear problem $\mathbf{y} = \mathbf{A}\mathbf{x}$.
dot(self, x)	Matrix-matrix or matrix-vector multiplication.
eigs(self[, neigs, symmetric, niter])	Most significant eigenvalues of linear operator.
matmat(self, X)	Matrix-matrix multiplication.
matvec(self, x)	Matrix-vector multiplication.
rmatvec(self, x)	Adjoint matrix-vector multiplication.
transpose(self)	Transpose this linear operator.

Examples using pylops.signalprocessing.FFTND

- sphx_glr_gallery_plot_fft.py

pylops.signalprocessing.Convolve1D

class pylops.signalprocessing.**Convolve1D**(*N*, *h*, *offset=0*, *dims=None*, *dir=0*, *dtype='float64'*, *method='direct'*)

1D convolution operator.

Apply one-dimensional convolution with a compact filter to model (and data) along a specific direction of a multi-dimensional array depending on the choice of dir.

Parameters

N [int] Number of samples in model.

h [numpy.ndarray] 1d compact filter to be convolved to input signal

offset [int] Index of the center of the compact filter

dims [tuple] Number of samples for each dimension (None if only one dimension is available)

dir [int, optional] Direction along which convolution is applied

method [str, optional] Method used to calculate the convolution (direct or fft). Note that fft approach is always used if dims=None.

dtype [str, optional] Type of elements in input array.

Raises

ValueError If offset is bigger than len(h) - 1

Notes

The Convolve1D operator applies convolution between the input signal $x(t)$ and a compact filter kernel $h(t)$ in forward model:

$$y(t) = \int_{-\inf}^{\inf} h(t-\tau)x(\tau)d\tau$$

This operation can be discretized as follows

$$y[n] = \sum_{m=-\inf}^{\inf} h[n-m]x[m]$$

as well as performed in the frequency domain.

$$Y(f) = F(h(t)) * F(x(t))$$

Convolve1D operator uses scipy.signal.convolve that automatically chooses the best domain for the operation to be carried out for one dimensional inputs. The fft implementation scipy.signal.fftconvolve is however enforced for signals in 2 or more dimensions as this routine efficently operates on multi-dimensional arrays.

As the adjoint of convolution is correlation, Convolve1D operator applies correlation in the adjoint mode.

In time domain:

$$x(t) = \int_{-\inf}^{\inf} h(t+\tau)x(\tau)d\tau$$

or in frequency domain:

$$y(t) = F^{-1}(H(f)^* * X(f))$$

Attributes

shape [tuple] Operator shape

explicit [bool] Operator contains a matrix that can be solved explicitly (True) or not (False)

Methods

`__init__`(self, N, h[, offset, dims, dir, ...])	Initialize this LinearOperator.
`adjoint`(self)	Hermitian adjoint.
`cond`(self, **kwargs_eig)	Condition number of linear operator.
`conj`(self)	Complex conjugate operator
`div`(self, y[, niter])	Solve the linear problem $\mathbf{y} = \mathbf{A}\mathbf{x}$.
`dot`(self, x)	Matrix-matrix or matrix-vector multiplication.
`eigs`(self[, neigs, symmetric, niter])	Most significant eigenvalues of linear operator.
`matmat`(self, X)	Matrix-matrix multiplication.
`matvec`(self, x)	Matrix-vector multiplication.
`rmatvec`(self, x)	Adjoint matrix-vector multiplication.
`transpose`(self)	Transpose this linear operator.

Examples using `pylops.signalprocessing.Convolve1D`

- sphx_glr_gallery_plot_convolve.py
- sphx_glr_gallery_plot_ista.py
- *14. Bayesian Inversion*

pylops.signalprocessing.Convolve2D

`pylops.signalprocessing.`**`Convolve2D`**(*N*, *h*, *dims*, *offset=(0, 0)*, *nodir=None*, *dtype='float64'*, *method='fft'*)

2D convolution operator.

Apply two-dimensional convolution with a compact filter to model (and data) along a pair of specific directions of a two or three-dimensional array depending on the choice of `nodir`.

Parameters

N [`int`] Number of samples in model

h [`numpy.ndarray`] 2d compact filter to be convolved to input signal

dims [`list`] Number of samples for each dimension

offset [`tuple`, optional] Indeces of the center of the compact filter

nodir [`int`, optional] Direction along which convolution is NOT applied (set to None for 2d arrays)

dtype [`str`, optional] Type of elements in input array.

method [`str`, optional] Method used to calculate the convolution (`direct` or `fft`).

Returns

cop [*`pylops.LinearOperator`*] Convolve2D linear operator

Notes

The Convolve2D operator applies two-dimensional convolution between the input signal $d(t, x)$ and a compact filter kernel $h(t, x)$ in forward model:

$$y(t, x) = \int_{-\inf}^{\inf} \int_{-\inf}^{\inf} h(t - \tau, x - \chi) d(\tau, \chi) d\tau d\chi$$

This operation can be discretized as follows

$$y[i,n] = \sum_{j=-\inf}^{\inf} \sum_{m=-\inf}^{\inf} h[i-j, n-m] d[j,m]$$

as well as performed in the frequency domain.

$$Y(f, k_x) = F(h(t,x)) * F(d(t,x))$$

Convolve2D operator uses `scipy.signal.convolve2d` that automatically chooses the best domain for the operation to be carried out.

As the adjoint of convolution is correlation, Convolve2D operator applies correlation in the adjoint mode.

In time domain:

$$y(t,x) = \int_{-\inf}^{\inf} \int_{-\inf}^{\inf} h(t+\tau, x+\chi) d(\tau,\chi) d\tau d\chi$$

or in frequency domain:

$$y(t,x) = F^{-1}(H(f,k_x)^* * X(f,k_x))$$

Examples using `pylops.signalprocessing.Convolve2D`

- sphx_glr_gallery_plot_convolve.py
- *04. Image deblurring*

pylops.signalprocessing.ConvolveND

class pylops.signalprocessing.**ConvolveND**(*N*, *h*, *dims*, *offset=(0, 0, 0)*, *dirs=None*, *method='fft'*, *dtype='float64'*)

ND convolution operator.

Apply n-dimensional convolution with a compact filter to model (and data) along a set of directions `dirs` of a n-dimensional array.

Parameters

N [`int`] Number of samples in model

h [`numpy.ndarray`] nd compact filter to be convolved to input signal

dims [`list`] Number of samples for each dimension

offset [`tuple`, optional] Indices of the center of the compact filter

dirs [`tuple`, optional] Directions along which convolution is applied (set to `None` for filter of same dimension as input vector)

method [`str`, optional] Method used to calculate the convolution (`direct` or `fft`).

dtype [`str`, optional] Type of elements in input array.

Notes

The ConvolveND operator applies n-dimensional convolution between the input signal $d(x_1, x_2, ..., x_N)$ and a compact filter kernel $h(x_1, x_2, ..., x_N)$ in forward model. This is a straighforward extension to multiple dimensions of *pylops.signalprocessing.Convolve2D* operator.

Attributes

shape [tuple] Operator shape

explicit [bool] Operator contains a matrix that can be solved explicitly (True) or not (False)

Methods

__init__(self, N, h, dims[, offset, dirs, ...])	Initialize this LinearOperator.
adjoint(self)	Hermitian adjoint.
cond(self, **kwargs_eig)	Condition number of linear operator.
conj(self)	Complex conjugate operator
div(self, y[, niter])	Solve the linear problem $\mathbf{y} = \mathbf{A}\mathbf{x}$.
dot(self, x)	Matrix-matrix or matrix-vector multiplication.
eigs(self[, neigs, symmetric, niter])	Most significant eigenvalues of linear operator.
matmat(self, X)	Matrix-matrix multiplication.
matvec(self, x)	Matrix-vector multiplication.
rmatvec(self, x)	Adjoint matrix-vector multiplication.
transpose(self)	Transpose this linear operator.

Examples using `pylops.signalprocessing.ConvolveND`

- sphx_glr_gallery_plot_convolve.py

pylops.signalprocessing.Interp

pylops.signalprocessing.**Interp**(*M*, *iava*, *dims=None*, *dir=0*, *kind='linear'*, *dtype='float64'*)

Interpolation operator.

Apply different kind of interpolations given a subset of values from input vector at locations iava.

Nearest neighbour interpolation is a thin wrapper around *pylops.Restriction* at np.round(iava) locations.

Linear interpolation extracts values from input vector at locations np.floor(iava) and np.floor(iava)+1 and linearly combines them in forward mode, places weighted versions of the interpolated values at locations np.floor(iava) and np.floor(iava)+1 in an otherwise zero vector in adjoint mode.

Note: the vector iava should contain unique values. If the same index is repeated twice an error will be raised. This also applies when values beyond the last element of the input array for *linear interpolation* as those values are forced to be just before this element.

Parameters

M [int] Number of samples in model.

iava [list or numpy.ndarray] Floating indices of locations of available samples for interpolation.

dims [list] Number of samples for each dimension (None if only one dimension is available)

dir [int, optional] Direction along which restriction is applied.

kind [str, optional] Kind of interpolation (nearest and linear are currently supported)

dtype [str, optional] Type of elements in input array.

Returns

op [*pylops.LinearOperator*] Linear intepolation operator

iava [list or numpy.ndarray] Corrected indices of locations of available samples (samples at M-1 or beyond are forced to be at M-1-eps)

Raises

ValueError If the vector iava contains repeated values.

NotImplementedError If kind is not nearest or linear

See also:

pylops.Restriction Restriction operator

Notes

Linear interpolation of a subset of N values at locations iava from an input (or model) vector $\mathbf{x}$ of size M can be expressed as:

$$y_i = (1 - w_i) x_{l_i^l} + w_i x_{l_i^r} \quad \forall i = 1, 2, ..., M$$

where $\mathbf{l^l} = [\lfloor l_1 \rfloor, \lfloor l_2 \rfloor, ..., \lfloor l_M \rfloor]$ and $\mathbf{l^r} = [\lfloor l_1 \rfloor + 1, \lfloor l_2 \rfloor + 1, ..., \lfloor l_M \rfloor + 1]$ are vectors containing the indeces of the original array at which samples are taken, and $\mathbf{w} = [l_1 - \lfloor l_1 \rfloor, l_2 - \lfloor l_2 \rfloor, ..., l_M - \lfloor l_M \rfloor]$ are the linear interpolation weights.

This operator can be implemented by simply summing two *pylops.Restriction* operators which are weighted using pylops.basicoperators.Diagonal operators.

Examples using pylops.signalprocessing.Interp

- sphx_glr_gallery_plot_restriction.py

pylops.signalprocessing.Radon2D

pylops.signalprocessing.**Radon2D**(*taxis*, *haxis*, *pxaxis*, *kind='linear'*, *centeredh=True*, *interp=True*, *onthefly=False*, *engine='numpy'*, *dtype='float64'*)

Two dimensional Radon transform.

Apply two dimensional Radon forward (and adjoint) transform to a 2-dimensional array of size $[n_{px} \times n_t]$ (and $[n_x \times n_t]$).

In forward mode this entails to spreading the model vector along parametric curves (lines, parabolas, or hyperbolas depending on the choice of kind), while stacking values in the data vector along the same parametric curves is performed in adjoint mode.

Parameters

taxis [np.ndarray] Time axis

haxis [np.ndarray] Spatial axis

pxaxis [np.ndarray] Axis of scanning variable p_x of parametric curve

kind [str, optional] Curve to be used for stacking/spreading (linear, parabolic, and hyperbolic are currently supported)

centeredh [bool, optional] Assume centered spatial axis (True) or not (False)

interp [bool, optional] Apply linear interpolation (True) or nearest interpolation (False) during stacking/spreading along parametric curve

onthefly [bool, optional] Compute stacking parametric curves on-the-fly as part of forward and adjoint modelling (True) or at initialization and store them in look-up table (False). Using a look-up table is computationally more efficient but increases the memory burden

engine [str, optional] Engine used for computation (numpy or numba)

dtype [str, optional] Type of elements in input array.

Returns

r2op [*pylops.LinearOperator*] Radon operator

Raises

KeyError If engine is neither numpy nor numba

NotImplementedError If kind is not linear, parabolic, or hyperbolic

See also:

pylops.signalprocessing.Radon3D Three dimensional Radon transform

pylops.Spread Spread operator

Notes

The Radon2D operator applies the following linear transform in adjoint mode to the data after reshaping it into a 2-dimensional array of size $[n_x \times n_t]$ in adjoint mode:

$$m(p_x, t_0) = \int d(x, t = f(p_x, x, t))dx$$

where $f(p_x, x, t) = t_0 + p_x * x$ where $p_x = sin(\theta)/v$ in linear mode, $f(p_x, x, t) = t_0 + p_x * x^2$ in parabolic mode, and $f(p_x, x, t) = \sqrt{t_0^2 + x^2/p_x^2}$ in hyperbolic mode.

As the adjoint operator can be interpreted as a repeated summation of sets of elements of the model vector along chosen parametric curves, the forward is implemented as spreading of values in the data vector along the same parametric curves. This operator is actually a thin wrapper around the *pylops.Spread* operator.

Examples using pylops.signalprocessing.Radon2D

- sphx_glr_gallery_plot_sliding.py
- sphx_glr_gallery_plot_radon.py
- *10. Radon filtering*

- *11. Seismic regularization*

pylops.signalprocessing.Radon3D

pylops.signalprocessing.**Radon3D**(*taxis*, *hyaxis*, *hxaxis*, *pyaxis*, *pxaxis*, *kind='linear'*, *centeredh=True*, *interp=True*, *onthefly=False*, *engine='numpy'*, *dtype='float64'*)

Three dimensional Radon transform.

Apply three dimensional Radon forward (and adjoint) transform to a 3-dimensional array of size $[n_{py} \times n_{px} \times n_t]$ (and $[n_y \times n_x \times n_t]$).

In forward mode this entails to spreading the model vector along parametric curves (lines, parabolas, or hyperbolas depending on the choice of `kind`), while stacking values in the data vector along the same parametric curves is performed in adjoint mode.

Parameters

taxis [`np.ndarray`] Time axis

hxaxis [`np.ndarray`] Fast patial axis

hyaxis [`np.ndarray`] Slow spatial axis

pyaxis [`np.ndarray`] Axis of scanning variable p_y of parametric curve

pxaxis [`np.ndarray`] Axis of scanning variable p_x of parametric curve

kind [`str`, optional] Curve to be used for stacking/spreading (`linear`, `parabolic`, and `hyperbolic` are currently supported)

centeredh [`bool`, optional] Assume centered spatial axis (`True`) or not (`False`)

interp [`bool`, optional] Apply linear interpolation (`True`) or nearest interpolation (`False`) during stacking/spreading along parametric curve

onthefly [`bool`, optional] Compute stacking parametric curves on-the-fly as part of forward and adjoint modelling (`True`) or at initialization and store them in look-up table (`False`). Using a look-up table is computationally more efficient but increases the memory burden

engine [`str`, optional] Engine used for computation (`numpy` or `numba`)

dtype [`str`, optional] Type of elements in input array.

Returns

r3op [*pylops.LinearOperator*] Radon operator

Raises

KeyError If `engine` is neither `numpy` nor `numba`

NotImplementedError If `kind` is not `linear`, `parabolic`, or `hyperbolic`

See also:

pylops.signalprocessing.Radon2D Two dimensional Radon transform

pylops.Spread Spread operator

Notes

The Radon3D operator applies the following linear transform in adjoint mode to the data after reshaping it into a 3-dimensional array of size $[n_y \times n_x \times n_t]$ in adjoint mode:

$$m(p_y, p_x, t_0) = \int d(y, x, t = f(p_y, p_x, y, x, t))dxdy$$

where $f(p_y, p_x, y, x, t) = t_0 + p_y * y + p_x * x$ in linear mode, $f(p_y, p_x, y, x, t) = t_0 + p_y * y^2 + p_x * x^2$ in parabolic mode, and $f(p_y, p_x, y, x, t) = \sqrt{t_0^2 + y^2/p_y^2 + x^2/p_x^2}$ in hyperbolic mode.

As the adjoint operator can be interpreted as a repeated summation of sets of elements of the model vector along chosen parametric curves, the forward is implemented as spreading of values in the data vector along the same parametric curves. This operator is actually a thin wrapper around the *pylops.Spread* operator.

Examples using `pylops.signalprocessing.Radon3D`

- sphx_glr_gallery_plot_sliding.py
- sphx_glr_gallery_plot_radon.py

pylops.signalprocessing.Sliding2D

`pylops.signalprocessing.`**`Sliding2D`**(*Op*, *dims*, *dimsd*, *nwin*, *nover*, *tapertype='hanning'*, *design=False*)

2D Sliding transform operator.

Apply a transform operator `Op` repeatedly to patches of the model vector in forward mode and patches of the data vector in adjoint mode. More specifically, in forward mode the model vector is divided into patches each patch is transformed, and patches are then recombined in a sliding window fashion. Both model and data should be 2-dimensional arrays in nature as they are internally reshaped and interpreted as 2-dimensional arrays. Each patch contains in fact a portion of the array in the first dimension (and the entire second dimension).

This operator can be used to perform local, overlapping transforms (e.g., `pylops.signalprocessing.FFT2` or *pylops.signalprocessing.Radon2D*) of 2-dimensional arrays.

Note: The shape of the model has to be consistent with the number of windows for this operator not to return an error. As the number of windows depends directly on the choice of `nwin` and `nover`, it is recommended to use `design=True` if unsure about the choice `dims` and use the number of windows printed on screen to define such input parameter.

Parameters

Op [*pylops.LinearOperator*] Transform operator

dims [`tuple`] Shape of 2-dimensional model. Note that `dims[0]` should be multiple of the model size of the transform in the first dimension

dimsd [`tuple`] Shape of 2-dimensional data

nwin [`int`] Number of samples of window

nover [`int`] Number of samples of overlapping part of window

tapertype [`str`, optional] Type of taper (`hanning`, `cosine`, `cosinesquare` or `None`)

design [bool, optional] Print number of sliding window (True) or not (False)

Returns

Sop [*pylops.LinearOperator*] Sliding operator

Raises

ValueError Identified number of windows is not consistent with provided model shape (dims).

Examples using `pylops.signalprocessing.Sliding2D`

- sphx_glr_gallery_plot_sliding.py
- *11. Seismic regularization*

pylops.signalprocessing.Sliding3D

pylops.signalprocessing.**Sliding3D**(*Op*, *dims*, *dimsd*, *nwin*, *nover*, *nop*, *tapertype='hanning'*, *design=False*)

3D Sliding transform operator.

Apply a transform operator Op repeatedly to patches of the model vector in forward mode and patches of the data vector in adjoint mode. More specifically, in forward mode the model vector is divided into patches each patch is transformed, and patches are then recombined in a sliding window fashion. Both model and data should be 3-dimensional arrays in nature as they are internally reshaped and interpreted as 3-dimensional arrays. Each patch contains in fact a portion of the array in the first and second dimensions (and the entire third dimension).

This operator can be used to perform local, overlapping transforms (e.g., pylops.signalprocessing.FFTN or *pylops.signalprocessing.Radon3D*) of 3-dimensional arrays.

Note: The shape of the model has to be consistent with the number of windows for this operator not to return an error. As the number of windows depends directly on the choice of nwin and nover, it is recommended to use design=True if unsure about the choice dims and use the number of windows printed on screen to define such input parameter.

Parameters

Op [*pylops.LinearOperator*] Transform operator

dims [tuple] Shape of 3-dimensional model. Note that dims[0] and dims[1] should be multiple of the model sizes of the transform in the first and second dimensions

dimsd [tuple] Shape of 3-dimensional data

nwin [tuple] Number of samples of window

nover [tuple] Number of samples of overlapping part of window

nop [tuple] Number of samples in axes of transformed domain associated to spatial axes in the data

tapertype [str, optional] Type of taper (hanning, cosine, cosinesquare or None)

design [bool, optional] Print number sliding window (True) or not (False)

Returns

Sop [*pylops.LinearOperator*] Sliding operator

Raises

ValueError Identified number of windows is not consistent with provided model shape (`dims`).

Examples using `pylops.signalprocessing.Sliding3D`

- sphx_glr_gallery_plot_sliding.py

pylops.signalprocessing.Fredholm1

class pylops.signalprocessing.**Fredholm1**(*G*, *nz=1*, *saveGt=True*, *usematmul=True*, *dtype='float64'*)

Fredholm integral of first kind.

Implement a multi-dimensional Fredholm integral of first kind. Note that if the integral is two dimensional, this can be directly implemented using `pylops.basicoperators.MatrixMult`. A multi-dimensional Fredholm integral can be performed as a `pylops.basicoperators.BlockDiag` operator of a series of `pylops.basicoperators.MatrixMult`. However, here we take advantage of the structure of the kernel and perform it in a more efficient manner.

Parameters

G [`numpy.ndarray`] Multi-dimensional convolution kernel of size $[n_{slice} \times n_x \times n_y]$

nz [`numpy.ndarray`, optional] Additional dimension of model

saveGt [`bool`, optional] Save `G` and `G^H` to speed up the computation of adjoint (`True`) or create `G^H` on-the-fly (`False`) Note that `saveGt=True` will double the amount of required memory

usematmul [`bool`, optional] Use `numpy.matmul` (`True`) or for-loop with `numpy.dot` (`False`). As it is not possible to define which approach is more performant (this is highly dependent on the size of `G` and input arrays as well as the hardware used in the compution), we advise users to time both methods for their specific problem prior to making a choice.

dtype [`str`, optional] Type of elements in input array.

Notes

A multi-dimensional Fredholm integral of first kind can be expressed as

$$d(sl, x, z) = \int G(sl, x, y) m(sl, y, z) dy \quad \forall sl = 1, n_{slice}$$

on the other hand its adjoin is expressed as

$$m(sl, y, z) = \int G^*(sl, y, x) d(sl, x, z) dx \quad \forall sl = 1, n_{slice}$$

In discrete form, this operator can be seen as a block-diagonal matrix multiplication:

$$\begin{bmatrix} \mathbf{G}_{sl1} & \mathbf{0} & ... & \mathbf{0} \\ \mathbf{0} & \mathbf{G}_{sl2} & ... & \mathbf{0} \\ ... & ... & ... & ... \\ \mathbf{0} & \mathbf{0} & ... & \mathbf{G}_{slN} \end{bmatrix} \begin{bmatrix} \mathbf{m}_{sl1} \\ \mathbf{m}_{sl2} \\ ... \\ \mathbf{m}_{slN} \end{bmatrix}$$

Attributes

shape [tuple] Operator shape

explicit [bool] Operator contains a matrix that can be solved explicitly (True) or not (False)

Methods

__init__(self, G[, nz, saveGt, usematmul, dtype])	Initialize this LinearOperator.
adjoint(self)	Hermitian adjoint.
cond(self, **kwargs_eig)	Condition number of linear operator.
conj(self)	Complex conjugate operator
div(self, y[, niter])	Solve the linear problem $\mathbf{y} = \mathbf{A}\mathbf{x}$.
dot(self, x)	Matrix-matrix or matrix-vector multiplication.
eigs(self[, neigs, symmetric, niter])	Most significant eigenvalues of linear operator.
matmat(self, X)	Matrix-matrix multiplication.
matvec(self, x)	Matrix-vector multiplication.
rmatvec(self, x)	Adjoint matrix-vector multiplication.
transpose(self)	Transpose this linear operator.

Wave-Equation processing

UpDownComposition2D(nt, nr, dt, dr, rho, vel)	2D Up-down wavefield composition.
MDC(G, nt, nv[, dt, dr, twosided, fast, ...])	Multi-dimensional convolution.
Demigration(z, x, t, srcs, recs, vel, wav, ...)	Demigration operator.

pylops.waveeqprocessing.UpDownComposition2D

pylops.waveeqprocessing.**UpDownComposition2D**(*nt*, *nr*, *dt*, *dr*, *rho*, *vel*, *nffts=(None, None)*, *critical=100.0*, *ntaper=10*, *scaling=1.0*, *dtype='complex128'*)

2D Up-down wavefield composition.

Apply multi-component seismic wavefield composition from its up- and down-going constituents. This input model required by the operator should be created by flattening the concatenated separated wavefields of size $[n_r \times n_t]$ along the spatial axis.

Similarly, the data is also a concatenation of flattened pressure and vertical particle velocity wavefields.

Parameters

nt [int] Number of samples along the time axis

nr [int] Number of samples along the receiver axis

dt [float] Sampling along the time axis

dr [float] Sampling along the receiver array

rho [float] Density along the receiver array (must be constant)

vel [float] Velocity along the receiver array (must be constant)

nffts [tuple, optional] Number of samples along the wavenumber and frequency axes

critical [float, optional] Percentage of angles to retain in obliquity factor. For example, if critical=100 only angles below the critical angle :math‘frac{f(k_x)}{vel}‘ will be retained

ntaper [float, optional] Number of samples of taper applied to obliquity factor around critical angle

scaling [float, optional] Scaling to apply to the operator (see Notes for more details)

dtype [str, optional] Type of elements in input array.

Returns

UDop [*pylops.LinearOperator*] Up-down wavefield composition operator

See also:

WavefieldDecomposition Wavefield decomposition

Notes

Multi-component seismic data ($p(x,t)$ and $v_z(x,t)$) can be synthesized in the frequency-wavenumber domain as the superposition of the up- and downgoing constituents of the pressure wavefield ($p^-(x,t)$ and $p^+(x,t)$) as follows [1]:

$$\begin{bmatrix} \mathbf{p}(k_x,\omega) \\ \mathbf{v_z}(k_x,\omega) \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ \frac{k_z}{\omega\rho} & -\frac{k_z}{\omega\rho} \end{bmatrix} \begin{bmatrix} \mathbf{p}^+(k_x,\omega) \\ \mathbf{p}^-(k_x,\omega) \end{bmatrix}$$

which we can write in a compact matrix-vector notation as:

$$\begin{bmatrix} \mathbf{p} \\ s*\mathbf{v_z} \end{bmatrix} = \begin{bmatrix} \mathbf{F} & 0 \\ 0 & s*\mathbf{F} \end{bmatrix} \mathbf{W} \begin{bmatrix} \mathbf{F}^H & 0 \\ 0 & \mathbf{F}^H \end{bmatrix} \mathbf{p}^{\pm}$$

where $\mathbf{F}$ is the 2-dimensional FFT (`pylops.signalprocessing.FFT2`), $\mathbf{W}$ is a weighting matrix implemented via `pylops.basicprocessing.Diagonal`, and s is a scaling factor that is applied to both the particle velocity data and to the operator has shown above. Such a scaling is required to balance out the different dynamic range of pressure and particle velocity when solving the wavefield separation problem as an inverse problem.

As the operator is effectively obtained by chaining basic PyLops operators the adjoint is automatically implemented for this operator.

Examples using `pylops.waveeqprocessing.UpDownComposition2D`

- *12. Seismic wavefield decomposition*

pylops.waveeqprocessing.MDC

pylops.waveeqprocessing.**MDC**(*G*, *nt*, *nv*, *dt=1.0*, *dr=1.0*, *twosided=True*, *fast=None*, *dtype=None*, *fftengine='numpy'*, *transpose=True*, *saveGt=True*, *conj=False*)

Multi-dimensional convolution.

Apply multi-dimensional convolution between two datasets. If `transpose=True`, model and data should be provided after flattening 2- or 3-dimensional arrays of size $[n_r(\times n_{vs}) \times n_t]$ and $[n_s(\times n_{vs}) \times n_t]$ (or $2*n_t-1$ for `twosided=True`), respectively. If `transpose=False`, model and data should be provided after flattening 2- or 3-dimensional arrays of size $[n_t \times n_r(\times n_{vs})]$ and $[n_t \times n_s(\times n_{vs})]$ (or $2*n_t-1$ for `twosided=True`), respectively.

Warning: A new implementation of MDC is provided in v1.5.0. This currently affects only the inner working of the operator and end-users can use the operator in the same way as they used to do with the previous one. Nevertheless, it is now reccomended to use the operator with `transpose=False`, as this behaviour will become default in version v2.0.0 and the behaviour with `transpose=True` will be deprecated.

Parameters

G [`numpy.ndarray`] Multi-dimensional convolution kernel in frequency domain of size $[n_s \times n_r \times n_{fmax}]$ if `transpose=True` or size $[n_{fmax} \times n_s \times n_r]$ if `transpose=False`

nt [`int`] Number of samples along time axis

nv [`int`] Number of samples along virtual source axis

dt [`float`, optional] Sampling of time integration axis

dr [`float`, optional] Sampling of receiver integration axis

twosided [`bool`, optional] MDC operator has both negative and positive time (`True`) or only positive (`False`)

fast [`bool`, optional] *Deprecated*, will be removed in v2.0.0

dtype [`str`, optional] *Deprecated*, will be removed in v2.0.0

fftengine [`str`, optional] Engine used for fft computation (`numpy` or `fftw`)

transpose [`bool`, optional] Transpose `G` and inputs such that time/frequency is placed in first dimension. This allows back-compatibility with v1.4.0 and older but will be removed in v2.0.0 where time/frequency axis will be required to be in first dimension for efficiency reasons.

saveGt [`bool`, optional] Save `G` and `G^H` to speed up the computation of adjoint of *`pylops.signalprocessing.Fredholm1`* (`True`) or create `G^H` on-the-fly (`False`) Note that `saveGt=True` will be faster but double the amount of required memory

conj [`str`, optional] Perform Fredholm integral computation with complex conjugate of `G`

See also:

MDD Multi-dimensional deconvolution

Notes

The so-called multi-dimensional convolution (MDC) is a chained operator [1]. It is composed of a forward Fourier transform, a multi-dimensional integration, and an inverse Fourier transform:

$$y(t, s, v) = F^{-1}\Big(\int_S G(f, s, r) F(x(t, r, v)) dr \Big)$$

This operation can be discretized and performed by means of a linear operator

$$\mathbf{D} = \mathbf{F}^H \mathbf{G} \mathbf{F}$$

where $\mathbf{F}$ is the Fourier transform applied along the time axis and $\mathbf{G}$ is the multi-dimensional convolution kernel.

Examples using `pylops.waveeqprocessing.MDC`

- sphx_glr_gallery_plot_mdc.py
- *08. Multi-Dimensional Deconvolution*

pylops.waveeqprocessing.Demigration

`pylops.waveeqprocessing.`**`Demigration`**(*z*, *x*, *t*, *srcs*, *recs*, *vel*, *wav*, *wavcenter*, *y=None*, *trav=None*, *mode='eikonal'*)

Demigration operator.

Seismic demigration/migration operator.

Parameters

z [`numpy.ndarray`] Depth axis

x [`numpy.ndarray`] Spatial axis

t [`numpy.ndarray`] Time axis for data

srcs [`numpy.ndarray`] Sources in array of size $[2/3 \times n_s]$

recs [`numpy.ndarray`] Receivers in array of size $[2/3 \times n_r]$

vel [`numpy.ndarray` or `float`] Velocity model of size $[(n_y \times) n_x \times n_z]$ (or constant)

wav [`numpy.ndarray`] Wavelet

wavcenter [`int`] Index of wavelet center

y [`numpy.ndarray`] Additional spatial axis (for 3-dimensional problems)

mode [`str`, optional] Computation mode (`analytic`, `eikonal` or `byot`, see Notes for more details)

trav [`numpy.ndarray`, optional] Traveltime table of size $[(n_y *) n_x * n_z \times n_r]$ (to be provided if `mode='byot'`)

Returns

demop [*`pylops.LinearOperator`*] Demigration/Migration operator

Raises

NotImplementedError If `mode` is neither `analytic`, `eikonal`, or `byot`

Notes

The demigration operator synthetizes seismic data given from a propagation velocity model v and a reflectivity model m. In forward mode:

$$d(\mathbf{x_r}, \mathbf{x_s}, t) = w(t) * \int_V G(\mathbf{x}, \mathbf{x_s}, t) G(\mathbf{x_r}, \mathbf{x}, t) m(\mathbf{x}) d\mathbf{x}$$

where $m(\mathbf{x})$ is the model and it represents the reflectivity at every location in the subsurface, $G(\mathbf{x}, \mathbf{x_s}, t)$ and $G(\mathbf{x_r}, \mathbf{x}, t)$ are the Green's functions from source-to-subsurface-to-receiver and finally $w(t)$ is the wavelet. Depending on the choice of `mode` the Green's function will be computed and applied differently:

- `mode=analytic` or `mode=eikonal`: traveltime curves between source to receiver pairs are computed for every subsurface point and Green's functions are implemented from traveltime look-up tables, placing the reflectivity values at corresponding source-to-receiver time in the data.

- `byot`: bring your own table. Traveltime table provided directly by user using `trav` input parameter. Green's functions are then implemented in the same way as previous options.

The adjoint of the demigration operator is a *migration* operator which projects data in the model domain creating an image of the subsurface reflectivity.

Geophysicical subsurface characterization

avo.AVOLinearModelling(theta[, vsvp, nt0, ...])	AVO Linearized modelling.
poststack.PoststackLinearModelling(wav, nt0)	Post-stack linearized seismic modelling operator.
prestack.PrestackLinearModelling(wav, theta)	Pre-stack linearized seismic modelling operator.
prestack.PrestackWaveletModelling(m, theta, nwav)	Pre-stack linearized seismic modelling operator for wavelet.

pylops.avo.avo.AVOLinearModelling

class pylops.avo.avo.**AVOLinearModelling**(*theta*, *vsvp=0.5*, *nt0=1*, *spatdims=None*, *linearization='akirich'*, *dtype='float64'*)

AVO Linearized modelling.

Create operator to be applied to a combination of elastic parameters for generation of seismic pre-stack reflectivity.

Parameters

theta [np.ndarray] Incident angles in degrees

vsvp [np.ndarray or float] VS/VP ratio

nt0 [int, optional] number of samples (if vsvp is a scalar)

spatdims [int or tuple, optional] Number of samples along spatial axis (or axes) (None if only one dimension is available)

linearization [str, optional] choice of linearization, akirich: Aki-Richards, fatti: Fatti

dtype [str, optional] Type of elements in input array.

Raises

NotImplementedError If linearization is not an implemented linearization

Notes

The AVO linearized operator performs a linear combination of three (or two) elastic parameters arranged in input vector $\mathbf{m}$ of size $n_{t0} \times N$ to create the so-called seismic reflectivity:

$$r(t, \theta, x, y) = \sum_{i=1}^{N} G_i(t, \theta) m_i(t, x, y) \qquad \forall \quad t, \theta$$

where $N = 2/3$. Note that the reflectivity can be in 1d, 2d or 3d and spatdims contains the dimensions of the spatial axis (or axes) x and y.

Attributes

shape [tuple] Operator shape

explicit [bool] Operator contains a matrix that can be solved explicitly (True) or not (False)

Methods

__init__(self, theta[, vsvp, nt0, spatdims, ...])	Initialize this LinearOperator.
adjoint(self)	Hermitian adjoint.
cond(self, **kwargs_eig)	Condition number of linear operator.
conj(self)	Complex conjugate operator
div(self, y[, niter])	Solve the linear problem $\mathbf{y} = \mathbf{A}\mathbf{x}$.
dot(self, x)	Matrix-matrix or matrix-vector multiplication.
eigs(self[, neigs, symmetric, niter])	Most significant eigenvalues of linear operator.
matmat(self, X)	Matrix-matrix multiplication.
matvec(self, x)	Matrix-vector multiplication.
rmatvec(self, x)	Adjoint matrix-vector multiplication.
transpose(self)	Transpose this linear operator.

Examples using `pylops.avo.avo.AVOLinearModelling`

- sphx_glr_gallery_plot_avo.py

pylops.avo.poststack.PoststackLinearModelling

pylops.avo.poststack.**PoststackLinearModelling**(*wav*, *nt0*, *spatdims=None*, *explicit=False*, *sparse=False*)

Post-stack linearized seismic modelling operator.

Create operator to be applied to an acoustic impedance trace (or stack of traces) for generation of band-limited seismic post-stack data. The input model and data have shape $[n_{t0}(\times n_x \times n_y)]$.

Parameters

wav [np.ndarray] Wavelet in time domain (must have odd number of elements and centered to zero). If 1d, assume stationary wavelet for the entire time axis. If 2d, use as non-stationary wavelet (user must provide one wavelet per time sample in an array of size $[n_{t0} \times n_{wav}]$ where n_{wav} is the lenght of each wavelet)

nt0 [int] Number of samples along time axis

spatdims [int or tuple, optional] Number of samples along spatial axis (or axes) (None if only one dimension is available)

explicit [bool, optional] Create a chained linear operator (False, preferred for large data) or a MatrixMult linear operator with dense matrix (True, preferred for small data)

sparse [bool, optional] Create a sparse matrix (True) or dense (False) when explicit=True

Returns

Pop [LinearOperator] post-stack modelling operator.

Raises

ValueError If wav is 2dimensional but does not contain nt0 wavelets

Notes

Post-stack seismic modelling is the process of constructing seismic post-stack data from a profile of acoustic impedance in time (or depth) domain. This can be easily achieved using the following forward model:

$$d(t, \theta = 0) = w(t) * \frac{dln(AI(t))}{dt}$$

where $AI(t)$ is the acoustic impedance profile and $w(t)$ is the time domain seismic wavelet. In compact form:

$$\mathbf{d} = \mathbf{W}\mathbf{D}\mathbf{a}\mathbf{i}$$

On the other hand, post-stack inversion aims at recovering the impedance profile from the band-limited seismic stack data.

Examples using `pylops.avo.poststack.PoststackLinearModelling`

- *06. Post-stack inversion*

pylops.avo.prestack.PrestackLinearModelling

`pylops.avo.prestack.`**`PrestackLinearModelling`**(*wav*, *theta*, *vsvp=0.5*, *nt0=1*, *spatdims=None*, *linearization='akirich'*, *explicit=False*)

Pre-stack linearized seismic modelling operator.

Create operator to be applied to elastic property profiles for generation of band-limited seismic angle gathers from a linearized version of the Zoeppritz equation.

Parameters

wav [`np.ndarray`] Wavelet in time domain (must had odd number of elements and centered to zero)

theta [`np.ndarray`] Incident angles in degrees

vsvp [`float` or `np.ndarray`] VS/VP ratio (constant or time/depth variant)

nt0 [`int`, optional] number of samples (if `vsvp` is a scalar)

spatdims [`int` or `tuple`, optional] Number of samples along spatial axis (or axes) (`None` if only one dimension is available)

linearization [`str`, optional] choice of linearization, `akirich`: Aki-Richards, `fatti`: Fatti

explicit [`bool`, optional] Create a chained linear operator (`False`, preferred for large data) or a `MatrixMult` linear operator with dense matrix (`True`, preferred for small data)

Returns

Preop [`LinearOperator`] pre-stack modelling operator.

Raises

NotImplementedError If `linearization` is not an implemented linearization

Notes

Pre-stack seismic modelling is the process of constructing seismic pre-stack data from three (or two) profiles of elastic parameters in time (or depth) domain arranged in an input vector $\mathbf{m}$ of size $nt0 \times N$. This can be easily achieved using the following forward model:

$$d(t, \theta) = w(t) * \sum_{i=1}^{N} G_i(t, \theta) m_i(t)$$

where $w(t)$ is the time domain seismic wavelet. In compact form:

$$\mathbf{d} = \mathbf{G}\mathbf{m}$$

On the other hand, pre-stack inversion aims at recovering the different profiles of elastic properties from the band-limited seismic pre-stack data.

Examples using `pylops.avo.prestack.PrestackLinearModelling`

- sphx_glr_gallery_plot_prestack.py
- *07. Pre-stack (AVO) inversion*

pylops.avo.prestack.PrestackWaveletModelling

`pylops.avo.prestack.`**`PrestackWaveletModelling`**(*m*, *theta*, *nwav*, *wavc=None*, *vsvp=0.5*, *linearization='akirich'*)

Pre-stack linearized seismic modelling operator for wavelet.

Create operator to be applied to a wavelet for generation of band-limited seismic angle gathers using a linearized version of the Zoeppritz equation.

Parameters

m [`np.ndarray`] elastic parameter profles of size $[n_{t0} \times N]$ where $N = 3/2$

theta [`int`] Incident angles in degrees

nwav [`np.ndarray`] Number of samples of wavelet to be applied/estimated

wavc [`int`, optional] Index of the center of the wavelet

vsvp [`np.ndarray` or `float`, optional] VS/VP ratio

linearization [`str`, optional] choice of linearization, `akirich`: Aki-Richards, `fatti`: Fatti

Returns

Mconv [`LinearOperator`] pre-stack modelling operator for wavelet estimation.

Raises

NotImplementedError If `linearization` is not an implemented linearization

Notes

Pre-stack seismic modelling for wavelet estimate is the process of constructing seismic reflectivities using three (or two) profiles of elastic parameters in time (or depth) domain arranged in an input vector $\mathbf{m}$ of size $nt0 \times N$:

$$d(t, \theta) = \sum_{i=1}^{N} G_i(t, \theta) m_i(t) * w(t)$$

where $w(t)$ is the time domain seismic wavelet. In compact form:

$$\mathbf{d} = \mathbf{G}\mathbf{w}$$

On the other hand, pre-stack wavelet estimation aims at recovering the wavelet given knowledge of the band-limited seismic pre-stack data and the elastic parameter profiles.

Examples using `pylops.avo.prestack.PrestackWaveletModelling`

- sphx_glr_gallery_plot_wavest.py

3.6.2 Solvers

Least-squares

leastsquares.NormalEquationsInversion(Op, ...)	Inversion of normal equations.
leastsquares.RegularizedInversion(Op, Regs, data)	Regularized inversion.
leastsquares.PreconditionedInversion(Op, P, data)	Preconditioned inversion.

pylops.optimization.leastsquares.NormalEquationsInversion

pylops.optimization.leastsquares.**NormalEquationsInversion**(*Op*, *Regs*, *data*, *Weight=None*, *dataregs=None*, *epsI=0*, *epsRs=None*, *x0=None*, *returninfo=False*, ***kwargs_cg*)

Inversion of normal equations.

Solve the regularized normal equations for a system of equations given the operator `Op`, a data weighting operator `Weight` and a list of regularization terms `Regs`

Parameters

Op [*pylops.LinearOperator*] Operator to invert

Regs [`list`] Regularization operators (`None` to avoid adding regularization)

data [`numpy.ndarray`] Data

Weight [*pylops.LinearOperator*, optional] Weight operator

dataregs [list, optional] Regularization data (must have the same number of elements as Regs)

epsI [float, optional] Tikhonov damping

epsRs [list, optional] Regularization dampings (must have the same number of elements as Regs)

x0 [numpy.ndarray, optional] Initial guess

returninfo [bool, optional] Return info of CG solver

****kwargs_cg** Arbitrary keyword arguments for scipy.sparse.linalg.cg solver

Returns

xinv [numpy.ndarray] Inverted model.

istop [int] Convergence information:

0: successful exit

>0: convergence to tolerance not achieved, number of iterations

<0: illegal input or breakdown

See also:

RegularizedInversion Regularized inversion

PreconditionedInversion Preconditioned inversion

Notes

Solve the following normal equations for a system of regularized equations given the operator $\mathbf{Op}$, a data weighting operator $\mathbf{W}$, a list of regularization terms $\mathbf{R_i}$, the data $\mathbf{d}$ and regularization damping factors ϵ_I and ϵ_{R_i}:

$$(\mathbf{Op}^T\mathbf{WOp} + \sum_i \epsilon_{R_i}^2 \mathbf{R}_i^T\mathbf{R}_i + \epsilon_I^2\mathbf{I})\mathbf{x} = \mathbf{Op}^T\mathbf{Wd} + \sum_i \epsilon_{R_i}^2 \mathbf{R}_i^T\mathbf{d}_{R_i}$$

Examples using `pylops.optimization.leastsquares.NormalEquationsInversion`

- *03. Solvers*
- *04. Image deblurring*
- *05. 2D Interpolation*

pylops.optimization.leastsquares.RegularizedInversion

pylops.optimization.leastsquares.**RegularizedInversion**(*Op*, *Regs*, *data*, *Weight=None*, *dataregs=None*, *epsRs=None*, *x0=None*, *returninfo=False*, ***kwargs_lsqr*)

Regularized inversion.

Solve a system of regularized equations given the operator Op, a data weighting operator Weight, and a list of regularization terms Regs.

Parameters

Op [*pylops.LinearOperator*] Operator to invert

Regs [list] Regularization operators (None to avoid adding regularization)

data [numpy.ndarray] Data

Weight [*pylops.LinearOperator*, optional] Weight operator

dataregs [list, optional] Regularization data (if None a zero data will be used for every regularization operator in Regs)

epsRs [list, optional] Regularization dampings

x0 [numpy.ndarray, optional] Initial guess

returninfo [bool, optional] Return info of LSQR solver

****kwargs_lsqr** Arbitrary keyword arguments for scipy.sparse.linalg.lsqr solver

Returns

xinv [numpy.ndarray] Inverted model $\mathbf{Op}$

istop [int] Gives the reason for termination

1 means $\mathbf{x}$ is an approximate solution to $\mathbf{d} = \mathbf{Opx}$

2 means $\mathbf{x}$ approximately solves the least-squares problem

itn [int] Iteration number upon termination

r1norm [float] $||\mathbf{r}||_2$, where $\mathbf{r} = \mathbf{d} - \mathbf{Opx}$

r2norm [float] $\sqrt{\mathbf{r}^T\mathbf{r} + \epsilon^2\mathbf{x}^T\mathbf{x}}$. Equal to r1norm if $\epsilon = 0$

See also:

RegularizedOperator Regularized operator

NormalEquationsInversion Normal equations inversion

PreconditionedInversion Preconditioned inversion

Notes

Solve the following system of regularized equations given the operator $\mathbf{Op}$, a data weighting operator $\mathbf{W}^{1/2}$, a list of regularization terms $\mathbf{R_i}$, the data $\mathbf{d}$ and regularization damping factors ϵ_I: and ϵ_{R_i}:

$$\begin{bmatrix} \mathbf{W}^{1/2}\mathbf{Op} \\ \epsilon_{R_1}\mathbf{R}_1 \\ ... \\ \epsilon_{R_N}\mathbf{R}_N \end{bmatrix} \mathbf{x} = \begin{bmatrix} \mathbf{W}^{1/2}\mathbf{d} \\ \epsilon_{R_1}\mathbf{d}_{R_1} \\ ... \\ \epsilon_{R_N}\mathbf{d}_{R_N} \end{bmatrix}$$

where the Weight provided here is equivalent to the square-root of the weight in *pylops.optimization.leastsquares.NormalEquationsInversion*. Note that this system is solved using the scipy.sparse.linalg.lsqr and an initial guess x0 can be provided to this solver, despite the original solver does not allow so.

Examples using `pylops.optimization.leastsquares.RegularizedInversion`

- sphx_glr_gallery_plot_ista.py
- sphx_glr_gallery_plot_tvreg.py
- sphx_glr_gallery_plot_wavest.py
- *03. Solvers*
- *05. 2D Interpolation*

pylops.optimization.leastsquares.PreconditionedInversion

pylops.optimization.leastsquares.**PreconditionedInversion**(*Op*, *P*, *data*, *x0=None*, *returninfo=False*, ***kwargs_lsqr*)

Preconditioned inversion.

Solve a system of preconditioned equations given the operator `Op` and a preconditioner `P`.

Parameters

Op [*pylops.LinearOperator*] Operator to invert

P [*pylops.LinearOperator*] Preconditioner

data [`numpy.ndarray`] Data

x0 [`numpy.ndarray`] Initial guess

returninfo [`bool`] Return info of LSQR solver

****kwargs_lsqr** Arbitrary keyword arguments for `scipy.sparse.linalg.lsqr` solver

Returns

xinv [`numpy.ndarray`] Inverted model.

istop [`int`] Gives the reason for termination

`1` means $\mathbf{x}$ is an approximate solution to $\mathbf{d} = \mathbf{Opx}$

`2` means $\mathbf{x}$ approximately solves the least-squares problem

itn [`int`] Iteration number upon termination

r1norm [`float`] $||\mathbf{r}||_2$, where $\mathbf{r} = \mathbf{d} - \mathbf{Opx}$

r2norm [`float`] $\sqrt{\mathbf{r}^T\mathbf{r} + \epsilon^2\mathbf{x}^T\mathbf{x}}$. Equal to `r1norm` if $\epsilon = 0$

See also:

RegularizedInversion Regularized inversion

NormalEquationsInversion Normal equations inversion

Notes

Solve the following system of preconditioned equations given the operator $\mathbf{Op}$, a preconditioner $\mathbf{P}$, the data $\mathbf{d}$

$$\mathbf{d} = \mathbf{Op}(\mathbf{Pp})$$

where $\mathbf{p}$ is the solution in the preconditioned space and $\mathbf{x} = \mathbf{Pp}$ is the solution in the original space.

Examples using `pylops.optimization.leastsquares.PreconditionedInversion`

- sphx_glr_gallery_plot_wavest.py
- *03. Solvers*

Sparsity

sparsity.IRLS(Op, data, nouter[, threshR, ...])	Iteratively reweighted least squares.
sparsity.ISTA(Op, data, niter[, eps, alpha, ...])	Iterative Soft Thresholding Algorithm (ISTA).
sparsity.FISTA(Op, data, niter[, eps, ...])	Fast Iterative Soft Thresholding Algorithm (FISTA).
sparsity.SPGL1(Op, data[, SOp, tau, sigma, x0])	Spectral Projected-Gradient for L1 norm.
sparsity.SplitBregman(Op, RegsL1, data[, ...])	Split Bregman for mixed L2-L1 norms.

pylops.optimization.sparsity.IRLS

pylops.optimization.sparsity.**IRLS**(*Op*, *data*, *nouter*, *threshR=False*, *epsR=1e-10*, *epsI=1e-10*, *x0=None*, *tolIRLS=1e-10*, *returnhistory=False*, ***kwargs_cg*)

Iteratively reweighted least squares.

Solve an optimization problem with $L1$ cost function given the operator `Op` and data `y`. The cost function is minimized by iteratively solving a weighted least squares problem with the weight at iteration i being based on the data residual at iteration $i + 1$.

The IRLS solver is robust to *outliers* since the L1 norm given less weight to large residuals than L2 norm does.

Parameters

Op [*pylops.LinearOperator*] Operator to invert

data [numpy.ndarray] Data

nouter [int] Number of outer iterations

threshR [bool, optional] Apply thresholding in creation of weight (`True`) or damping (`False`)

epsR [float, optional] Damping to be applied to residuals for weighting term

espI [float, optional] Tikhonov damping

x0 [numpy.ndarray, optional] Initial guess

tolIRLS [float, optional] Tolerance. Stop outer iterations if difference between inverted model at subsequent iterations is smaller than `tolIRLS`

returnhistory [bool, optional] Return history of inverted model for each outer iteration of IRLS

****kwargs_cg** Arbitrary keyword arguments for `scipy.sparse.linalg.cg` solver

Returns

xinv [numpy.ndarray] Inverted model

nouter [int] Number of effective outer iterations

xinv_hist [numpy.ndarray, optional] History of inverted model

rw_hist [numpy.ndarray, optional] History of weights

Notes

Solves the following optimization problem for the operator $\mathbf{Op}$ and the data $\mathbf{d}$:

$$J = ||\mathbf{d} - \mathbf{Op}\mathbf{x}||_1$$

by a set of outer iterations which require to repeateadly solve a weighted least squares problem of the form:

$$\mathbf{x}^{(i+1)} = \arg\min_{\mathbf{x}} ||\mathbf{d} - \mathbf{Op}\mathbf{x}||_{2,\mathbf{R}^{(i)}} + \epsilon_I^2||\mathbf{x}||$$

where $\mathbf{R}^{(i)}$ is a diagonal weight matrix whose diagonal elements at iteration i are equal to the absolute inverses of the residual vector $\mathbf{r}^{(i)} = \mathbf{y} - \mathbf{Op}\mathbf{x}^{(i)}$ at iteration i. More specifically the j-th element of the diagonal of $\mathbf{R}^{(i)}$ is

$$R_{j,j}^{(i)} = \frac{1}{|r_j^{(i)}| + \epsilon_R}$$

or

$$R_{j,j}^{(i)} = \frac{1}{max(|r_j^{(i)}|, \epsilon_R)}$$

depending on the choice `threshR`. In either case, ϵ_R is the user-defined stabilization/thresholding factor [1].

Examples using `pylops.optimization.sparsity.IRLS`

- sphx_glr_gallery_plot_linearregr.py
- sphx_glr_gallery_plot_regr.py

pylops.optimization.sparsity.ISTA

pylops.optimization.sparsity.**ISTA**(*Op*, *data*, *niter*, *eps=0.1*, *alpha=None*, *eigsiter=None*, *eigstol=0*, *tol=1e-10*, *monitorres=False*, *returninfo=False*, *show=False*)

Iterative Soft Thresholding Algorithm (ISTA).

Solve an optimization problem with $L1$ regularization function given the operator `Op` and data `y`. The operator can be real or complex, and should ideally be either square $N = M$ or underdetermined $N < M$.

Parameters

Op [*pylops.LinearOperator*] Operator to invert

data [numpy.ndarray] Data

niter [int] Number of iterations

eps [float, optional] Sparsity damping

alpha [float, optional] Step size ($\alpha \leq 1/\lambda_{max}(\mathbf{Op}^H\mathbf{Op})$ guarantees convergence. If `None`, estimated to satisfy the condition, otherwise the condition will not be checked)

eigsiter [float, optional] Number of iterations for eigenvalue estimation if `alpha=None`

eigstol [float, optional] Tolerance for eigenvalue estimation if `alpha=None`

tol [float, optional] Tolerance. Stop iterations if difference between inverted model at subsequent iterations is smaller than `tol`

monitorres [bool, optional] Monitor that residual is decreasing

returninfo [bool, optional] Return info of CG solver

show [bool, optional] Display iterations log

Returns

xinv [numpy.ndarray] Inverted model

niter [int] Number of effective iterations

cost [numpy.ndarray, optional] History of cost function

Raises

ValueError If `monitorres=True` and residual increases

See also:

FISTA Fast Iterative Soft Thresholding Algorithm (FISTA).

Notes

Solves the following optimization problem for the operator $\mathbf{Op}$ and the data $\mathbf{d}$:

$$J = ||\mathbf{d} - \mathbf{Opx}||_2^2 + \epsilon||\mathbf{x}||_1$$

using the Iterative Soft Thresholding Algorithm (ISTA) [1]. This is a very simple iterative algorithm which applies the following step:

$$\mathbf{x}^{(i+1)} = soft(\mathbf{x}^{(i)} + \alpha\mathbf{Op}^H(\mathbf{d} - \mathbf{Opx}^{(i)})), \epsilon\alpha/2)$$

where $\epsilon\alpha/2$ is the threshold and $soft()$ is the so-called soft-thresholding rule.

Examples using `pylops.optimization.sparsity.ISTA`

- sphx_glr_gallery_plot_ista.py
- *03. Solvers*

pylops.optimization.sparsity.FISTA

pylops.optimization.sparsity.**FISTA**(*Op*, *data*, *niter*, *eps=0.1*, *alpha=None*, *eigsiter=None*, *eigstol=0*, *tol=1e-10*, *returninfo=False*, *show=False*)

Fast Iterative Soft Thresholding Algorithm (FISTA).

Solve an optimization problem with $L1$ regularization function given the operator `Op` and data `y`. The operator can be real or complex, and should ideally be either square $N = M$ or underdetermined $N < M$.

Parameters

Op [*pylops.LinearOperator*] Operator to invert

data [numpy.ndarray] Data

niter [int] Number of iterations

eps [float, optional] Sparsity damping

alpha [float, optional] Step size ($\alpha \leq 1/\lambda_{max}(\mathbf{Op}^H\mathbf{Op})$ guarantees convergence. If None, estimated to satisfy the condition, otherwise the condition will not be checked)

eigsiter [int, optional] Number of iterations for eigenvalue estimation if alpha=None

eigstol [float, optional] Tolerance for eigenvalue estimation if alpha=None

tol [float, optional] Tolerance. Stop iterations if difference between inverted model at subsequent iterations is smaller than tol

returninfo [bool, optional] Return info of FISTA solver

show [bool, optional] Display iterations log

Returns

xinv [numpy.ndarray] Inverted model

niter [int] Number of effective iterations

cost [numpy.ndarray, optional] History of cost function

See also:

ISTA Iterative Soft Thresholding Algorithm (FISTA).

Notes

Solves the following optimization problem for the operator $\mathbf{Op}$ and the data $\mathbf{d}$:

$$J = ||\mathbf{d} - \mathbf{Op}\mathbf{x}||_2^2 + \epsilon||\mathbf{x}||_1$$

using the Fast Iterative Soft Thresholding Algorithm (FISTA) [1]. This is a modified version of ISTA solver with improved convergence properties and limitied additional computational cost.

Examples using pylops.optimization.sparsity.FISTA

- sphx_glr_gallery_plot_ista.py
- *03. Solvers*
- *10. Radon filtering*
- *13. Least-squares migration*

pylops.optimization.sparsity.SPGL1

pylops.optimization.sparsity.**SPGL1**(*Op*, *data*, *SOp=None*, *tau=0*, *sigma=0*, *x0=None*, ***kwargs_spgl1*)

Spectral Projected-Gradient for L1 norm.

Solve a constrained system of equations given the operator Op and a sparsyfing transform SOp aiming to retrive a model that is sparse in the sparsyfing domain.

This is a simple wrapper to spgl1.spgl1 which is a porting of the well-known SPGL1 MATLAB solver into Python. In order to be able to use this solver you need to have installed the spgl1 library.

Parameters

Op [*pylops.LinearOperator*] Operator to invert

data [numpy.ndarray] Data

SOp [*pylops.LinearOperator*] Sparsyfing transform

tau [float] Non-negative LASSO scalar. If different from 0, SPGL1 will solve LASSO problem

sigma [list] BPDN scalar. If different from 0, SPGL1 will solve BPDN problem

x0 [numpy.ndarray] Initial guess

****kwargs_spgl1** Arbitrary keyword arguments for spgl1.spgl1 solver

Returns

xinv [numpy.ndarray] Inverted model in original domain.

pinv [numpy.ndarray] Inverted model in sparse domain.

info [dict] Dictionary with the following information:

tau, final value of tau (see sigma above)

rnorm, two-norm of the optimal residual

rgap, relative duality gap (an optimality measure)

gnorm, Lagrange multiplier of (LASSO)

stat, 1: found a BPDN solution, 2: found a BP solution; exit based on small gradient, 3: found a BP solution; exit based on small residual, 4: found a LASSO solution, 5: error, too many iterations, 6: error, linesearch failed, 7: error, found suboptimal BP solution, 8: error, too many matrix-vector products.

niters, number of iterations

nProdA, number of multiplications with A

nProdAt, number of multiplications with A'

n_newton, number of Newton steps

time_project, projection time (seconds)

time_matprod, matrix-vector multiplications time (seconds)

time_total, total solution time (seconds)

niters_lsqr, number of lsqr iterations (if subspace_min=True)

xnorm1, L1-norm model solution history through iterations

rnorm2, L2-norm residual history through iterations

lambdaa, Lagrange multiplier history through iterations

Raises

ModuleNotFoundError If the spgl1 library is not installed

Notes

Solve different variations of sparsity-promoting inverse problem by imposing sparsity in the retrieved model [1].

The first problem is called *basis pursuit denoise (BPDN)* and its cost function is

$$||\mathbf{x}||_1 \quad subj.to \quad ||\mathbf{Op}\mathbf{S}^H\mathbf{x} - \mathbf{b}||_2 <= \sigma,$$

while the second problem is the *l1-regularized least-squares or LASSO* problem and its cost function is

$$||\mathbf{Op}\mathbf{S}^H\mathbf{x} - \mathbf{b}||_2 \quad subj.to \quad ||\mathbf{x}||_1 <= \tau$$

Examples using `pylops.optimization.sparsity.SPGL1`

- *03. Solvers*

pylops.optimization.sparsity.SplitBregman

pylops.optimization.sparsity.**SplitBregman**(*Op*, *RegsL1*, *data*, *niter_outer=3*, *niter_inner=5*, *RegsL2=None*, *dataregsL2=None*, *mu=1.0*, *epsRL1s=None*, *epsRL2s=None*, *tol=1e-10*, *tau=1.0*, *x0=None*, *restart=False*, *show=False*, ***kwargs_lsqr*)

Split Bregman for mixed L2-L1 norms.

Solve an unconstrained system of equations with mixed L2-L1 regularization terms given the operator `Op`, a list of L1 regularization terms `RegsL1`, and an optional list of L2 regularization terms `RegsL2`.

Parameters

Op [*pylops.LinearOperator*] Operator to invert

RegsL1 [`list`] L1 regularization operators

data [`numpy.ndarray`] Data

niter_outer [`int`] Number of iterations of outer loop

niter_inner [`int`] Number of iterations of inner loop

RegsL2 [`list`] Additional L2 regularization operators (if `None`, L2 regularization is not added to the problem)

dataregsL2 [`list`, optional] L2 Regularization data (must have the same number of elements of `RegsL2` or equal to `None` to use a zero data for every regularization operator in `RegsL2`)

mu [`float`, optional] Data term damping

epsRL1s [`list`] L1 Regularization dampings (must have the same number of elements as `RegsL1`)

epsRL2s [`list`] L2 Regularization dampings (must have the same number of elements as `RegsL2`)

tol [`float`, optional] Tolerance. Stop outer iterations if difference between inverted model at subsequent iterations is smaller than `tol`

tau [`float`, optional] Scaling factor in the Bregman update (must be close to 1)

x0 [`numpy.ndarray`, optional] Initial guess

restart [bool, optional] The unconstrained inverse problem in inner loop is initialized with the initial guess (True) or with the last estimate (False)

show [bool, optional] Display iterations log

****kwargs_lsqr** Arbitrary keyword arguments for scipy.sparse.linalg.lsqr solver

Returns

xinv [numpy.ndarray] Inverted model

itn_out [int] Iteration number of outer loop upon termination

Notes

Solve the following system of unconstrained, regularized equations given the operator $\mathbf{Op}$ and a set of mixed norm (L2 and L1) regularization terms $\mathbf{R_{L2,i}}$ and $\mathbf{R_{L1,i}}$, respectively:

$$J = \mu/2||\mathbf{d} - \mathbf{Op}\mathbf{x}|||_2 + \sum_i \epsilon_{R_{L2,i}} ||\mathbf{d}_{\mathbf{R_{L2,i}}} - \mathbf{R_{L2,i}}\mathbf{x}|||_2 + \sum_i ||\mathbf{R_{L1,i}}\mathbf{x}|||_1$$

where μ and $\epsilon_{R_{L2,i}}$ are the damping factors used to weight the different terms of the cost function.

The generalized Split Bergman algorithm is used to solve such cost function: the algorithm is composed of a sequence of unconstrained inverse problems and Bregman updates. Note that the L1 terms are not weighted in the original cost function but are first converted into constraints and then re-inserted in the cost function with Lagrange multipliers $\epsilon_{R_{L1,i}}$, which effectively act as damping factors for those terms. See [1] for detailed derivation.

The scipy.sparse.linalg.lsqr solver and a fast shrinkage algorithm are used within the inner loop to solve the unconstrained inverse problem, and the same procedure is repeated niter_outer times until convergence.

Examples using pylops.optimization.sparsity.SplitBregman

- sphx_glr_gallery_plot_tvreg.py
- *04. Image deblurring*

3.6.3 Applications

Wave-Equation processing

SeismicInterpolation(data, nrec, iava[, ...])	Seismic interpolation (or regularization).
WavefieldDecomposition(p, vz, nt, nr, dt, ...)	Up-down wavefield decomposition.
MDD(G, d[, dt, dr, nfmax, wav, twosided, ...])	Multi-dimensional deconvolution.
Marchenko(R[, R1, dt, nt, dr, nfmax, wav, ...])	Marchenko redatuming
LSM(z, x, t, srcs, recs, vel, wav, wavcenter)	Least-squares Migration (LSM).

pylops.waveeqprocessing.SeismicInterpolation

pylops.waveeqprocessing.**SeismicInterpolation**(*data*, *nrec*, *iava*, *iava1=None*, *kind='fk'*, *nffts=None*, *sampling=None*, *spataxis=None*, *spat1axis=None*, *taxis=None*, *paxis=None*, *p1axis=None*, *centeredh=True*, *nwins=None*, *nwin=None*, *nover=None*, *design=False*, *engine='numba'*, *dottest=False*, ***kwargs_solver*)

Seismic interpolation (or regularization).

Interpolate seismic data from irregular to regular spatial grid. Depending on the size of the input `data`, interpolation is either 2- or 3-dimensional. In case of 3-dimensional interpolation, data can be irregularly sampled in either one or both spatial directions.

Parameters

data [`np.ndarray`] Irregularly sampled seismic data of size $[n_{r_y} (\times n_{r_x} \times n_t)]$

nrec [`int` or `tuple`] Number of elements in the regularly sampled (reconstructed) spatial array, n_{R_y} for 2-dimensional data and (n_{R_y}, n_{R_x}) for 3-dimensional data

iava [`list` or `numpy.ndarray`] Integer (or floating) indices of locations of available samples in first dimension of regularly sampled spatial grid of interpolated signal. The `pylops.basicoperators.Restriction` operator is used in case of integer indices, while the `pylops.signalprocessing.Interp` operator is used in case of floating indices.

iava1 [`list` or `numpy.ndarray`, optional] Integer (or floating) indices of locations of available samples in second dimension of regularly sampled spatial grid of interpolated signal. Can be used only in case of 3-dimensional data.

kind [`str`, optional] Type of inversion: `fk` (default), `spatial`, `radon-linear`, `radon-parabolic` or , `radon-hyperbolic` and `sliding`

nffts [`int` or `tuple`, optional] nffts : `tuple`, optional Number of samples in Fourier Transform for each direction. Required if `kind='fk'`

sampling [`tuple`, optional] Sampling steps `dy` (, `dx`) and `dt`. Required if `kind='fk'` or `kind='radon-lin'`

spataxis [`np.ndarray`, optional] First spatial axis. Required for `kind='radon-lin'`, can also be provided instead of `sampling` for `kind='fk'`

spat1axis [`np.ndarray`, optional] Second spatial axis. Required for `kind='radon-lin'`, can also be provided instead of `sampling` for `kind='fk'`

timeaxis [`np.ndarray`, optional] Time axis. Required for `kind='radon-lin'`, can also be provided instead of `sampling` for `kind='fk'`

paxis [`np.ndarray`, optional] First Radon axis. Required for `kind='radon-linear'`, `kind='radon-parabolic'`, `kind='radon-hyperbolic'` and `kind='sliding'`

p1axis [`np.ndarray`, optional] Second Radon axis. Required for `kind='radon-linear'`, `kind='radon-parabolic'` and `kind='radon-hyperbolic'` and `kind='sliding'`

centeredh [`bool`, optional] Assume centered spatial axis (`True`) or not (`False`). Required for `kind='radon-linear'`, `kind='radon-parabolic'` and

kind='radon-hyperbolic'

nwins [int or tuple, optional] Number of windows. Required for kind='sliding'

nwin [int or tuple, optional] Number of samples of window. Required for kind='sliding'

nover [int or tuple, optional] Number of samples of overlapping part of window. Required for kind='sliding'

design [bool, optional] Print number of sliding window (True) or not (False) when using kind='sliding'

engine [str, optional] Engine used for Radon computations (numpy or numba)

dottest [bool, optional] Apply dot-test

****kwargs_solver** Arbitrary keyword arguments for *pylops.optimization.leastsquares.RegularizedInversion* solver if kind='spatial' or *pylops.optimization.sparsity.FISTA* solver otherwise

Returns

recdata [np.ndarray] Reconstructed data of size $[n_{R_y} (\times n_{R_x} \times n_t)]$

recprec [np.ndarray] Reconstructed data in the sparse or preconditioned domain in case of kind='fk', kind='radon-linear', kind='radon-parabolic', kind='radon-hyperbolic' and kind='sliding'

cost [np.ndarray] Cost function norm

Raises

KeyError If kind is neither spatial, fl, radon-linear, radon-parabolic, radon-hyperbolic nor sliding

Notes

The problem of seismic data interpolation (or regularization) can be formally written as

$$\mathbf{y} = \mathbf{R}\mathbf{x}$$

where a restriction or interpolation operator is applied along the spatial direction(s). Here $\mathbf{y} = [\mathbf{y}_{R1}^T, \mathbf{y}_{R2}^T, ..., \mathbf{y}_{RN^T}]^T$ where each vector $\mathbf{y}_{Ri}$ contains all time samples recorded in the seismic data at the specific receiver R_i. Similarly, $\mathbf{x} = [\mathbf{x}_{r1}^T, \mathbf{x}_{r2}^T, ..., \mathbf{x}_{rM}^T]$, contains all traces at the regularly and finely sampled receiver locations r_i.

Several alternative approaches can be taken to solve such a problem. They mostly differ in the choice of the regularization (or preconditining) used to mitigate the ill-posedness of the problem:

- spatial: least-squares inversion in the original time-space domain with an additional spatial smoothing regularization term, corresponding to the cost function $J = ||\mathbf{y} - \mathbf{R}\mathbf{x}||_2 + \epsilon_\nabla \nabla ||\mathbf{x}||_2$ where ∇ is a second order space derivative implemented via pylops.basicoperators.SecondDerivative in 2-dimensional case and pylops.basicoperators.Laplacian in 3-dimensional case
- fk: L1 inversion in frequency-wavenumber preconditioned domain corresponding to the cost function $J = ||\mathbf{y} - \mathbf{R}\mathbf{F}\mathbf{x}||_2$ where $\mathbf{F}$ is frequency-wavenumber transform implemented via *pylops.signalprocessing.FFT2D* in 2-dimensional case and *pylops.signalprocessing.FFTND* in 3-dimensional case

- `radon-linear`: L1 inversion in linear Radon preconditioned domain using the same cost function as `fk` but with **F** being a Radon transform implemented via *pylops.signalprocessing.Radon2D* in 2-dimensional case and *pylops.signalprocessing.Radon3D* in 3-dimensional case
- `radon-parabolic`: L1 inversion in parabolic Radon preconditioned domain
- `radon-hyperbolic`: L1 inversion in hyperbolic Radon preconditioned domain
- `sliding`: L1 inversion in sliding-linear Radon preconditioned domain using the same cost function as `fk` but with **F** being a sliding Radon transform implemented via *pylops.signalprocessing.Sliding2D* in 2-dimensional case and *pylops.signalprocessing.Sliding3D* in 3-dimensional case

Examples using `pylops.waveeqprocessing.SeismicInterpolation`

- *11. Seismic regularization*

pylops.waveeqprocessing.WavefieldDecomposition

pylops.waveeqprocessing.**WavefieldDecomposition**(*p*, *vz*, *nt*, *nr*, *dt*, *dr*, *rho*, *vel*, *nffts=(None, None)*, *critical=100.0*, *ntaper=10*, *scaling=1.0*, *kind='inverse'*, *restriction=None*, *sptransf=None*, *solver=<function lsqr at 0x7ff8eca880d0>*, *dottest=False*, *dtype='complex128'*, ***kwargs_solver*)

Up-down wavefield decomposition.

Apply seismic wavefield decomposition from its multi-component (pressure and vertical particle velocity) data.

Parameters

p [`np.ndarray`] Pressure data of of size $[n_r \times n_t]$ (or $[n_{r,sub} \times n_t]$ in case a `restriction` operator is provided, and $n_{r,sub}$ must agree with the size of the output of this an operator)

vz [`np.ndarray`] Vertical particle velocity data of size $[n_r \times n_t]$ (or $[n_{r,sub} \times n_t]$)

nt [`int`] Number of samples along the time axis

nr [`np.ndarray`] Number of samples along the receiver axis of the separated pressure consituents

dt [`float`] Sampling along the time axis

dr [`float`] Sampling along the receiver array of the separated pressure consituents

rho [`float`] Density along the receiver array (must be constant)

vel [`float`] Velocity along the receiver array (must be constant)

nffts [`tuple`, optional] Number of samples along the wavenumber and frequency axes

critical [`float`, optional] Percentage of angles to retain in obliquity factor. For example, if `critical=100` only angles below the critical angle $\frac{f(k_x)}{vel}$ will be retained

ntaper [`float`, optional] Number of samples of taper applied to obliquity factor around critical angle

kind [`str`, optional] Type of separation: `inverse` (default) or `analytical`

scaling [`float`, optional] Scaling to apply to the particle velocity data at the

restriction [*pylops.LinearOperator*, optional] Restriction operator

sptransf [*pylops.LinearOperator*, optional] Sparsifying operator

solver [float, optional] Function handle of solver to be used if `kind='inverse'`

dottest [bool, optional] Apply dot-test

dtype [str, optional] Type of elements in input array.

****kwargs_solver** Arbitrary keyword arguments for chosen `solver`

Returns

pup [np.ndarray] Up-going wavefield

pdown [np.ndarray] Down-going wavefield

Raises

KeyError If `kind` is neither `analytical` nor `inverse`

Notes

Up- and down-going components of seismic data ($p^-(x,t)$ and $p^+(x,t)$) can be estimated from multi-component data ($p(x,t)$ and $v_z(x,t)$) by computing the following expression [1]:

$$\begin{bmatrix} \mathbf{p}^+(k_x,\omega) \\ \mathbf{p}^-(k_x,\omega) \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 1 & \frac{\omega\rho}{k_z} \\ 1 & -\frac{\omega\rho}{k_z} \end{bmatrix} \begin{bmatrix} \mathbf{p}(k_x,\omega) \\ \mathbf{v_z}(k_x,\omega) \end{bmatrix}$$

if `kind='analytical'` or alternatively by solving the equation in `ptcpy.waveeqprocessing.UpDownComposition2D` as an inverse problem, if `kind='inverse'`.

The latter approach has several advantages as data regularization can be included as part of the separation process allowing the input data to be aliased. This is obtained by solving the following problem:

$$\begin{bmatrix} \mathbf{p} \\ s * \mathbf{v_z} \end{bmatrix} = \begin{bmatrix} \mathbf{RF} & 0 \\ 0 & s * \mathbf{RF} \end{bmatrix} \mathbf{W} \begin{bmatrix} \mathbf{F}^H\mathbf{S} & 0 \\ 0 & \mathbf{F}^H\mathbf{S} \end{bmatrix} \mathbf{p}^{\pm}$$

where $\mathbf{R}$ is a `ptcpy.basicoperators.Restriction` operator and $\mathbf{S}$ is sparsyfing transform operator (e.g., `ptcpy.signalprocessing.Radon2D`).

Examples using `pylops.waveeqprocessing.WavefieldDecomposition`

- *12. Seismic wavefield decomposition*

pylops.waveeqprocessing.MDD

pylops.waveeqprocessing.**MDD**(*G*, *d*, *dt=0.004*, *dr=1.0*, *nfmax=None*, *wav=None*, *twosided=True*, *causality_precond=False*, *adjoint=False*, *psf=False*, *dtype='float64'*, *dottest=False*, *saveGt=True*, *add_negative=True*, ***kwargs_lsqr*)

Multi-dimensional deconvolution.

Solve multi-dimensional deconvolution problem using `scipy.sparse.linalg.lsqr` iterative solver.

Parameters

G [numpy.ndarray] Multi-dimensional convolution kernel in time domain of size $[n_s \times n_r \times n_t]$ for `twosided=False` or `twosided=True` and `add_negative=True` (with only positive times) or size $[n_s \times n_r \times 2 * n_t - 1]$ for `twosided=True` and `add_negative=False` (with both positive and negative times)

d [numpy.ndarray] Data in time domain $[n_s(\times n_v s) \times n_t]$

dt [float, optional] Sampling of time integration axis

dr [float, optional] Sampling of receiver integration axis

nfmax [int, optional] Index of max frequency to include in deconvolution process

wav [numpy.ndarray, optional] Wavelet to convolve to the inverted model and psf. If `None`, the outputs of the inversion are returned directly

twosided [bool, optional] MDC operator and data both negative and positive time (`True`) or only positive (`False`)

add_negative [bool, optional] Add negative side to MDC operator and data (`True`) or not (`False`)- operator and data are already provided with both positive and negative sides. To be used only with `twosided=True`.

causality_precond [bool, optional] Apply causality mask (`True`) or not (`False`)

adjoint [bool, optional] Compute and return adjoint(s)

psf [bool, optional] Compute and return Point Spread Function (PSF) and its inverse

dtype [bool, optional] Type of elements in input array.

dottest [bool, optional] Apply dot-test

saveGt [bool, optional] Save `G` and `G^H` to speed up the computation of adjoint of *pylops.signalprocessing.Fredholm1* (`True`) or create `G^H` on-the-fly (`False`) Note that `saveGt=True` will be faster but double the amount of required memory

****kwargs_lsqr** Arbitrary keyword arguments for `scipy.sparse.linalg.lsqr` solver

Returns

minv [numpy.ndarray] Inverted model of size $[n_r(\times n_{vs}) \times n_t]$ for `twosided=False` or $[n_r(\times n_v s) \times 2 * n_t - 1]$ for `twosided=True`

madj [numpy.ndarray] Adjoint model of size $[n_r(\times n_{vs}) \times n_t]$ for `twosided=False` or $[n_r(\times n_r) \times 2 * n_t - 1]$ for `twosided=True`

psfinv [numpy.ndarray] Inverted psf of size $[n_r \times n_r \times n_t]$ for `twosided=False` or $[n_r \times n_r \times 2 * n_t - 1]$ for `twosided=True`

psfadj [numpy.ndarray] Adjoint psf of size $[n_r \times n_r \times n_t]$ for `twosided=False` or $[n_r \times n_r \times 2 * n_t - 1]$ for `twosided=True`

See also:

MDC Multi-dimensional convolution

Notes

Multi-dimensional deconvolution (MDD) is a mathematical ill-solved problem, well-known in the image processing and geophysical community [1].

MDD aims at removing the effects of a Multi-dimensional Convolution (MDC) kernel or the so-called blurring operator or point-spread function (PSF) from a given data. It can be written as

$$\mathbf{d} = \mathbf{D}\mathbf{m}$$

or, equivalently, by means of its normal equation

$$\mathbf{m} = (\mathbf{D}^H\mathbf{D})^{-1}\mathbf{D}^H\mathbf{d}$$

where $\mathbf{D}^H\mathbf{D}$ is the PSF.

Examples using `pylops.waveeqprocessing.MDD`

- *08. Multi-Dimensional Deconvolution*

pylops.waveeqprocessing.Marchenko

class pylops.waveeqprocessing.**Marchenko**(*R*, *R1=None*, *dt=0.004*, *nt=None*, *dr=1.0*, *nfmax=None*, *wav=None*, *toff=0.0*, *nsmooth=10*, *dtype='float64'*, *saveRt=True*)

Marchenko redatuming

Solve multi-dimensional Marchenko redatuming problem using `scipy.sparse.linalg.lsqr` iterative solver.

Parameters

R [`numpy.ndarray`] Multi-dimensional reflection response in time or frequency domain of size $[n_s \times n_r \times n_t/n_{fmax}]$. If provided in time, *R* should not be of complex type. Note that the reflection response should have already been multiplied by 2.

R1 [`bool`, optional] *Deprecated*, will be removed in v2.0.0. Simply kept for back-compatibility with previous implementation

dt [`float`, optional] Sampling of time integration axis

nt [`float`, optional] Number of samples in time (not required if `R` is in time)

dr [`float`, optional] Sampling of receiver integration axis

nfmax [`int`, optional] Index of max frequency to include in deconvolution process

wav [`numpy.ndarray`, optional] Wavelet to apply to direct arrival when created using `trav`

toff [`float`, optional] Time-offset to apply to traveltime

nsmooth [`int`, optional] Number of samples of smoothing operator to apply to window

dtype [`bool`, optional] Type of elements in input array.

saveRt [`bool`, optional] Save `R` and `R^H` to speed up the computation of adjoint of *`pylops.signalprocessing.Fredholm1`* (`True`) or create `R^H` on-the-fly (`False`) Note that `saveRt=True` will be faster but double the amount of required memory

Raises

TypeError If `t` is not `numpy.ndarray`.

See also:

MDC Multi-dimensional convolution

MDD Multi-dimensional deconvolution

Notes

Marchenko redatuming is a method that allows to produce correct subsurface-to-surface responses given the availability of a reflection data and a macro-velocity model [R1c4066d82e61-1].

The Marchenko equations can be written in a compact matrix form [R1c4066d82e61-2] and solved by means of iterative solvers such as LSQR:

$$\begin{bmatrix} \Theta \mathbf{R} \mathbf{f}_\mathbf{d}^+ \\ \mathbf{0} \end{bmatrix} = \mathbf{I} - \begin{bmatrix} \mathbf{0} & \Theta \mathbf{R} \\ \Theta \mathbf{R}^* & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{f}^- \\ \mathbf{f}_\mathbf{m}^+ \end{bmatrix}$$

Finally the subsurface Green's functions can be obtained applying the following operator to the retrieved focusing functions

$$\begin{bmatrix} -\mathbf{g}^- \\ \mathbf{g}^{+*} \end{bmatrix} = \mathbf{I} - \begin{bmatrix} \mathbf{0} & \mathbf{R} \\ \mathbf{R}^* & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{f}^- \\ \mathbf{f}^+ \end{bmatrix}$$

Here $\mathbf{R}$ is the monopole-to-particle velocity seismic response (already multiplied by 2).

Attributes

ns [int] Number of samples along source axis

nr [int] Number of samples along receiver axis

shape [tuple] Operator shape

explicit [bool] Operator contains a matrix that can be solved explicitly (True) or not (False)

Methods

__init__(self, R[, R1, dt, nt, dr, nfmax, ...])	Initialize self.
apply_multiplepoints(self, trav[, G0, nfft, ...])	Marchenko redatuming for multiple points
apply_onepoint(self, trav[, G0, nfft, rtm, ...])	Marchenko redatuming for one point

apply_onepoint(*self*, *trav*, *G0=None*, *nfft=None*, *rtm=False*, *greens=False*, *dottest=False*, *fast=None*, ***kwargs_lsqr*)

Marchenko redatuming for one point

Solve the Marchenko redatuming inverse problem for a single point given its direct arrival traveltime curve (trav) and waveform (G0).

Parameters

trav [numpy.ndarray] Traveltime of first arrival from subsurface point to surface receivers of size $[n_r \times 1]$

G0 [numpy.ndarray, optional] Direct arrival in time domain of size $[n_r \times n_t]$ (if None, create arrival using trav)

nfft [int, optional] Number of samples in fft when creating the analytical direct wave

rtm [bool, optional] Compute and return rtm redatuming

greens [bool, optional] Compute and return Green's functions

dottest [bool, optional] Apply dot-test

fast [bool] *Deprecated*, will be removed in v2.0.0

****kwargs_lsqr** Arbitrary keyword arguments for `scipy.sparse.linalg.lsqr` solver

Returns

f1_inv_minus [`numpy.ndarray`] Inverted upgoing focusing function of size $[n_r \times n_t]$

f1_inv_plus [`numpy.ndarray`] Inverted downgoing focusing function of size $[n_r \times n_t]$

p0_minus [`numpy.ndarray`] Single-scattering standard redatuming upgoing Green's function of size $[n_r \times n_t]$

g_inv_minus [`numpy.ndarray`] Inverted upgoing Green's function of size $[n_r \times n_t]$

g_inv_plus [`numpy.ndarray`] Inverted downgoing Green's function of size $[n_r \times n_t]$

apply_multiplepoints (*self*, *trav*, *G0=None*, *nfft=None*, *rtm=False*, *greens=False*, *dottest=False*, ***kwargs_lsqr*)

Marchenko redatuming for multiple points

Solve the Marchenko redatuming inverse problem for multiple points given their direct arrival traveltime curves (`trav`) and waveforms (`G0`).

Parameters

trav [`numpy.ndarray`] Traveltime of first arrival from subsurface points to surface receivers of size $[n_r \times n_{vs}]$

G0 [`numpy.ndarray`, optional] Direct arrival in time domain of size $[n_r \times n_{vs} \times n_t]$ (if None, create arrival using `trav`)

nfft [`int`, optional] Number of samples in fft when creating the analytical direct wave

rtm [`bool`, optional] Compute and return rtm redatuming

greens [`bool`, optional] Compute and return Green's functions

dottest [`bool`, optional] Apply dot-test

****kwargs_lsqr** Arbitrary keyword arguments for `scipy.sparse.linalg.lsqr` solver

Returns

f1_inv_minus [`numpy.ndarray`] Inverted upgoing focusing function of size $[n_r \times n_{vs} \times n_t]$

f1_inv_plus [`numpy.ndarray`] Inverted downgoing focusing functionof size $[n_r \times n_{vs} \times n_t]$

p0_minus [`numpy.ndarray`] Single-scattering standard redatuming upgoing Green's function of size $[n_r \times n_{vs} \times n_t]$

g_inv_minus [`numpy.ndarray`] Inverted upgoing Green's function of size $[n_r \times n_{vs} \times n_t]$

g_inv_plus [`numpy.ndarray`] Inverted downgoing Green's function of size $[n_r \times n_{vs} \times n_t]$

Examples using `pylops.waveeqprocessing.Marchenko`

- *09. Marchenko redatuming by inversion*

pylops.waveeqprocessing.LSM

class pylops.waveeqprocessing.**LSM**(*z*, *x*, *t*, *srcs*, *recs*, *vel*, *wav*, *wavcenter*, *y=None*, *mode='eikonal'*, *dottest=False*)

Least-squares Migration (LSM).

Solve seismic migration as inverse problem given smooth velocity model `vel` and an acquisition setup identified by sources (`src`) and receivers (`recs`)

Parameters

z [`numpy.ndarray`] Depth axis

x [`numpy.ndarray`] Spatial axis

t [`numpy.ndarray`] Time axis for data

srcs [`numpy.ndarray`] Sources in array of size $[2/3 \times n_s]$

recs [`numpy.ndarray`] Receivers in array of size $[2/3 \times n_r]$

vel [`numpy.ndarray` or `float`] Velocity model of size $[(n_y \times) n_x \times n_z]$ (or constant)

wav [`numpy.ndarray`] Wavelet

wavcenter [`int`] Index of wavelet center

y [`numpy.ndarray`] Additional spatial axis (for 3-dimensional problems)

mode [`numpy.ndarray`, optional] Computation mode (`eikonal`, `analytic` - only for constant velocity)

dottest [`bool`, optional] Apply dot-test

See also:

pylops.waveeqprocessing.Demigration Demigration operator

Notes

Inverting a demigration operator is generally referred in the literature as least-squares migration (LSM) as historically a least-squares cost function has been used for this purpose. In practice any other cost function could be used, for examples if `solver='pylops.optimization.sparsity.FISTA'` a sparse representation of reflectivity is produced as result of the inversion.

Finally, it is worth noting that in the first iteration of an iterative scheme aimed at inverting the demigration operator, a projection of the recorded data in the model domain is performed and an approximate (band-limited) image of the subsurface is created. This process is referred to in the literature as *migration*.

Attributes

Demop [*pylops.LinearOperator*] Demigration operator

Methods

`__init__`(self, z, x, t, srcs, recs, vel, ...)	Initialize self.
solve(self, d[, solver])	Solve least-squares migration equations with chosen `solver`

solve (*self*, *d*, *solver=<function lsqr at 0x7ff8eca880d0>*, ***kwargs_solver*)
Solve least-squares migration equations with chosen `solver`

Parameters

d [`numpy.ndarray`] Input data of size $[n_s \times n_r \times n_t]$

solver [`func`, optional] Solver to be used for inversion

****kwargs_solver** Arbitrary keyword arguments for chosen `solver`

Returns

minv [`np.ndarray`] Inverted reflectivity model of size $[(n_y \times) n_x \times n_z]$

Examples using `pylops.waveeqprocessing.LSM`

- *13. Least-squares migration*

Geophysical subsurface characterization

poststack.PoststackInversion(data, wav[, ...])		Post-stack linearized seismic inversion.
prestack.PrestackInversion(data, wav)	theta,	Pre-stack linearized seismic inversion.

pylops.avo.poststack.PoststackInversion

pylops.avo.poststack.**PoststackInversion**(*data*, *wav*, *m0=None*, *explicit=False*, *simultaneous=False*, *epsI=None*, *epsR=None*, *dottest=False*, *epsRL1=None*, ***kwargs_solver*)
Post-stack linearized seismic inversion.

Invert post-stack seismic operator to retrieve an acoustic impedance profile from band-limited seismic post-stack data. Depending on the choice of input parameters, inversion can be trace-by-trace with explicit operator or global with either explicit or linear operator.

Parameters

data [`np.ndarray`] Band-limited seismic post-stack data of size $[n_{t0} (\times n_x \times n_y)]$

wav [`np.ndarray`] Wavelet in time domain (must have odd number of elements and centered to zero). If 1d, assume stationary wavelet for the entire time axis. If 2d of size $[n_{t0} \times n_h]$ use as non-stationary wavelet

m0 [`np.ndarray`, optional] Background model of size $[n_{t0} (\times n_x \times n_y)]$

explicit [`bool`, optional] Create a chained linear operator (`False`, preferred for large data) or a `MatrixMult` linear operator with dense matrix (`True`, preferred for small data)

simultaneous [`bool`, optional] Simultaneously invert entire data (`True`) or invert trace-by-trace (`False`) when using `explicit` operator (note that the entire data is always inverted when working with linear operator)

epsI [`float`, optional] Damping factor for Tikhonov regularization term

epsR [`float`, optional] Damping factor for additional Laplacian regularization term

dottest [`bool`, optional] Apply dot-test

epsRL1 [float, optional] Damping factor for additional blockiness regularization term

****kwargs_solver** Arbitrary keyword arguments for `scipy.linalg.lstsq` solver (if `explicit=True` and `epsR=None`) or `scipy.sparse.linalg.lsqr` solver (if `explicit=False` and/or `epsR` is not `None`)

Returns

minv [np.ndarray] Inverted model of size $[n_{t0}(\times n_x \times n_y)]$

datar [np.ndarray] Residual data (i.e., data - background data) of size $[n_{t0}(\times n_x \times n_y)]$

Notes

The cost function and solver used in the seismic post-stack inversion module depends on the choice of `explicit`, `simultaneous`, `epsI`, and `epsR` parameters:

- `explicit=True`, `epsI=None` and `epsR=None`: the explicit solver `scipy.linalg.lstsq` is used if `simultaneous=False` (or the iterative solver `scipy.sparse.linalg.lsqr` is used if `simultaneous=True`)
- `explicit=True` with `epsI` and `epsR=None`: the regularized normal equations $\mathbf{W}^T\mathbf{d} = (\mathbf{W}^T\mathbf{W} + \epsilon_I^2\mathbf{I})\mathbf{A}\mathbf{I}$ are instead fed into the `scipy.linalg.lstsq` solver if `simultaneous=False` (or the iterative solver `scipy.sparse.linalg.lsqr` if `simultaneous=True`)
- `explicit=False` and `epsR=None`: the iterative solver `scipy.sparse.linalg.lsqr` is used
- `explicit=False` with `epsR` and `epsRL1=None`: the iterative solver *`pylops.optimization.leastsquares.RegularizedInversion`* is used to solve the spatially regularized problem.
- `explicit=False` with `epsR` and `epsRL1`: the iterative solver *`pylops.optimization.sparsity.SplitBregman`* is used to solve the blockiness-promoting (in vertical direction) and spatially regularized (in additional horizontal directions) problem.

Note that the convergence of iterative solvers such as `scipy.sparse.linalg.lsqr` can be very slow for this type of operator. It is suggested to take a two steps approach with first a trace-by-trace inversion using the explicit operator, followed by a regularized global inversion using the outcome of the previous inversion as initial guess.

Examples using `pylops.avo.poststack.PoststackInversion`

- *06. Post-stack inversion*

pylops.avo.prestack.PrestackInversion

pylops.avo.prestack.**PrestackInversion**(*data*, *theta*, *wav*, *m0=None*, *linearization='akirich'*, *explicit=False*, *simultaneous=False*, *epsI=None*, *epsR=None*, *dottest=False*, *returnres=False*, *epsRL1=None*, ***kwargs_solver*)

Pre-stack linearized seismic inversion.

Invert pre-stack seismic operator to retrieve a set of elastic property profiles from band-limited seismic pre-stack data (i.e., angle gathers). Depending on the choice of input parameters, inversion can be trace-by-trace with explicit operator or global with either explicit or linear operator.

Parameters

data [np.ndarray] Band-limited seismic post-stack data of size $[n_{t0} \times n_\theta(\times n_x \times n_y)]$

theta [np.ndarray] Incident angles in degrees

wav [np.ndarray] Wavelet in time domain (must had odd number of elements and centered to zero)

m0 [np.ndarray, optional] Background model of size $[n_{t0} \times n_m (\times n_x \times n_y)]$

linearization [str, optional] choice of linearization, akirich: Aki-Richards, fatti: Fatti (required only when m0 is None)

explicit [bool, optional] Create a chained linear operator (False, preferred for large data) or a MatrixMult linear operator with dense matrix (True, preferred for small data)

simultaneous [bool, optional] Simultaneously invert entire data (True) or invert trace-by-trace (False) when using explicit operator (note that the entire data is always inverted when working with linear operator)

epsI [float or list, optional] Damping factor(s) for Tikhonov regularization term. If a list of n_m elements is provided, the regularization term will have different strenght for each elastic property

epsR [float, optional] Damping factor for additional Laplacian regularization term

dottest [bool, optional] Apply dot-test

returnres [bool, optional] Return residuals

epsRL1 [float, optional] Damping factor for additional blockiness regularization term

****kwargs_solver** Arbitrary keyword arguments for scipy.linalg.lstsq solver (if explicit=True and epsR=None) or scipy.sparse.linalg.lsqr solver (if explicit=False and/or epsR is not None))

Returns

minv [np.ndarray] Inverted model of size $[n_{t0} \times n_m (\times n_x \times n_y)]$

datar [np.ndarray] Residual data (i.e., data - background data) of size $[n_{t0} \times n_\theta (\times n_x \times n_y)]$

Notes

The different choices of cost functions and solvers used in the seismic pre-stack inversion module follow the same convention of the seismic post-stack inversion module.

Refer to *pylops.avo.poststack.PoststackInversion* for more details.

Examples using `pylops.avo.prestack.PrestackInversion`

- *07. Pre-stack (AVO) inversion*

3.7 PyLops Utilities

Alongside with its *Linear Operators* and *Solvers*, PyLops contains also a number of auxiliary routines performing universal tasks that are used by several operators or simply within one or more *Tutorials* for the preparation of input data and subsequent visualization of results.

3.7.1 Shared

Dot-test

dottest(Op, nr, nc[, tol, complexflag, ...])	Dot test.

pylops.utils.dottest

pylops.utils.**dottest**(*Op*, *nr*, *nc*, *tol=1e-06*, *complexflag=0*, *raiseerror=True*, *verb=False*)

Dot test.

Generate random vectors $\mathbf{u}$ and $\mathbf{v}$ and perform dot-test to verify the validity of forward and adjoint operators. This test can help to detect errors in the operator implementation.

Parameters

Op [*pylops.LinearOperator*] Linear operator to test.

nr [int] Number of rows of operator (i.e., elements in data)

nc [int] Number of columns of operator (i.e., elements in model)

tol [float, optional] Dottest tolerance

complexflag [bool, optional] generate random vectors with real (0) or complex numbers (1: only model, 2: only data, 3:both)

raiseerror [bool, optional] Raise error or simply return False when dottest fails

verb [bool, optional] Verbosity

Raises

ValueError If dot-test is not verified within chosen tolerance.

Notes

A dot-test is mathematical tool used in the development of numerical linear operators.

More specifically, a correct implementation of forward and adjoint for a linear operator should verify the following *equality* within a numerical tolerance:

$$(\mathbf{Op} * \mathbf{u})^H * \mathbf{v} = \mathbf{u}^H * (\mathbf{Op}^H * \mathbf{v})$$

Examples using pylops.utils.dottest

- *02. The Dot-Test*

3.7.2 Others

Synthetics

seismicevents.makeaxis(par)	Create axes t, x, and y axes
seismicevents.linear2d(x, t, v, t0, theta, ...)	Linear 2D events

Continued on next page

Table 40 – continued from previous page

seismicevents.parabolic2d(x, t, t0, px, pxx, ...)	Parabolic 2D events
seismicevents.hyperbolic2d(x, t, t0, vrms, ...)	Hyperbolic 2D events
seismicevents.linear3d(x, y, t, v, t0, ...)	Linear 3D events
seismicevents.hyperbolic3d(x, y, t, t0, ...)	Hyperbolic 3D events

pylops.utils.seismicevents.makeaxis

pylops.utils.seismicevents.**makeaxis**(*par*)

Create axes t, x, and y axes

Create space and time axes from dictionary containing initial values (ot, ox, oy), sampling steps (dt, dx, dy) and number of elements (nt, nx, ny) for each axis

Parameters

par [dict] Dictionary containing initial values, sampling steps, and number of elements

Returns

t [numpy.ndarray] time axis

t2 [numpy.ndarray] double time axis (symmetric to zero)

x [numpy.ndarray] x axis

y [numpy.ndarray] y axis (None, if oy, dy, ny are not provided)

Examples

```
>>> par = {'ox':0, 'dx':2, 'nx':60,
>>>        'oy':0, 'dy':2, 'ny':100,
>>>        'ot':0, 'dt':4, 'nt':400}
>>> # Create axis
>>> t, t2, x, y = makeaxis(par)
```

Examples using pylops.utils.seismicevents.makeaxis

- sphx_glr_gallery_plot_sliding.py
- sphx_glr_gallery_plot_mdc.py
- sphx_glr_gallery_plot_seismicevents.py
- *08. Multi-Dimensional Deconvolution*
- *10. Radon filtering*
- *11. Seismic regularization*
- *12. Seismic wavefield decomposition*

pylops.utils.seismicevents.linear2d

pylops.utils.seismicevents.**linear2d**(*x*, *t*, *v*, *t0*, *theta*, *amp*, *wav*)

Linear 2D events

Create 2d linear events given propagation velocity, intercept time, angle, and amplitude of each event

Parameters

x [numpy.ndarray] space axis

t [numpy.ndarray] time axis

v [float] propagation velocity

t0 [tuple or float] intercept time at $x = 0$ of each linear event

theta [tuple or float] angle (in degrees) of each linear event

amp [tuple or float] amplitude of each linear event

wav [numpy.ndarray] wavelet to be applied to data

Returns

d [numpy.ndarray] data without wavelet of size $[n_x \times n_t]$

dwav [numpy.ndarray] data with wavelet of size $[n_x \times n_t]$

Notes

Each event is created using the following relation:

$$t_i(x) = t_{0,i} + p_{x,i}x$$

where $p_{x,i} = sin(\theta_i)/v$

Examples using pylops.utils.seismicevents.linear2d

- sphx_glr_gallery_plot_seismicevents.py
- *10. Radon filtering*
- *11. Seismic regularization*

pylops.utils.seismicevents.parabolic2d

pylops.utils.seismicevents.**parabolic2d**(*x*, *t*, *t0*, *px*, *pxx*, *amp*, *wav*)

Parabolic 2D events

Create 2d parabolic events given intercept time, slowness, curvature, and amplitude of each event

Parameters

x [numpy.ndarray] space axis

t [numpy.ndarray] time axis

t0 [tuple or float] intercept time at $x = 0$ of each parabolic event

px [tuple or float] slowness of each parabolic event

pxx [tuple or float] curvature of each parabolic event

amp [tuple or float] amplitude of each parabolic event

wav [numpy.ndarray] wavelet to be applied to data

Returns

d [numpy.ndarray] data without wavelet of size $[n_x \times n_t]$

dwav [numpy.ndarray] data with wavelet of size $[n_x \times n_t]$

Notes

Each event is created using the following relation:

$$t_i(x) = t_{0,i} + p_{x,i}x + p_{xx,i}x^2$$

Examples using `pylops.utils.seismicevents.parabolic2d`

- sphx_glr_gallery_plot_sliding.py
- sphx_glr_gallery_plot_seismicevents.py
- *10. Radon filtering*

pylops.utils.seismicevents.hyperbolic2d

pylops.utils.seismicevents.**hyperbolic2d**(*x*, *t*, *t0*, *vrms*, *amp*, *wav*)

Hyperbolic 2D events

Create 2d hyperbolic events given intercept time, root-mean-square velocity, and amplitude of each event

Parameters

x [numpy.ndarray] space axis

t [numpy.ndarray] time axis

t0 [tuple or float] intercept time at $x = 0$ of each of hyperbolic event

vrms [tuple or float] root-mean-square velocity of each hyperbolic event

amp [tuple or float] amplitude of each hyperbolic event

wav [numpy.ndarray] wavelet to be applied to data

Returns

d [numpy.ndarray] data without wavelet of size $[n_x \times n_t]$

dwav [numpy.ndarray] data with wavelet of size $[n_x \times n_t]$

Notes

Each event is created using the following relation:

$$t_i(x) = \sqrt{t_{0,i}^2 + x^2/v_{rms,i}^2}$$

Examples using `pylops.utils.seismicevents.hyperbolic2d`

- sphx_glr_gallery_plot_mdc.py
- sphx_glr_gallery_plot_seismicevents.py
- *08. Multi-Dimensional Deconvolution*
- *12. Seismic wavefield decomposition*

pylops.utils.seismicevents.linear3d

pylops.utils.seismicevents.**linear3d**(*x*, *y*, *t*, *v*, *t0*, *theta*, *phi*, *amp*, *wav*)

Linear 3D events

Create 3d linear events given propagation velocity, intercept time, angles, and amplitude of each event.

Parameters

x [numpy.ndarray] space axis in x direction

y [numpy.ndarray] space axis in y direction

t [numpy.ndarray] time axis

v [float] propagation velocity

t0 [tuple or float] intercept time at $x = 0$ of each linear event

theta [tuple or float] angle in x direction (in degrees) of each linear event

phi [tuple or float] angle in y direction (in degrees) of each linear event

amp [tuple or float] amplitude of each linear event

wav [numpy.ndarray] wavelet to be applied to data

Returns

d [numpy.ndarray] data without wavelet of size $[n_y \times n_x \times n_t]$

dwav [numpy.ndarray] data with wavelet of size $[n_y \times n_x \times n_t]$

Notes

Each event is created using the following relation:

$$t_i(x, y) = t_{0,i} + p_{x,i}x + p_{y,i}y$$

where $p_{x,i} = sin(\theta_i)cos(\phi_i)/v$ and $p_{x,i} = sin(\theta_i)sin(\phi_i)/v$.

Examples using `pylops.utils.seismicevents.linear3d`

- sphx_glr_gallery_plot_seismicevents.py

pylops.utils.seismicevents.hyperbolic3d

pylops.utils.seismicevents.**hyperbolic3d**(*x*, *y*, *t*, *t0*, *vrms_x*, *vrms_y*, *amp*, *wav*)

Hyperbolic 3D events

Create 3d hyperbolic events given intercept time, root-mean-square velocities, and amplitude of each event

Parameters

x [numpy.ndarray] space axis in x direction

y [numpy.ndarray] space axis in y direction

t [numpy.ndarray] time axis

t0 [tuple or float] intercept time at $x = 0$ of each of hyperbolic event

vrms_x [tuple or float] root-mean-square velocity in x direction for each hyperbolic event

vrms_y [tuple or float] root-mean-square velocity in y direction for each hyperbolic event

amp [tuple or float] amplitude of each hyperbolic event

wav [numpy.ndarray] wavelet to be applied to data

Returns

d [numpy.ndarray] data without wavelet of size $[n_y \times n_x \times n_t]$

dwav [numpy.ndarray] data with wavelet of size $[n_y \times n_x \times n_t]$

Notes

Each event is created using the following relation:

$$t_i(x, y) = \sqrt{t_{0,i}^2 + x^2/v_{rms_x,i}^2 + y^2/v_{rms_y,i}^2}$$

Note that velocities do not have a physical meaning here (compared to the corresponding *pylops.utils.seismicevents.hyperbolic2d*), they rather simply control the curvature of the hyperboloid along the spatial axes.

Examples using pylops.utils.seismicevents.hyperbolic3d

- sphx_glr_gallery_plot_sliding.py
- sphx_glr_gallery_plot_seismicevents.py

Signal-processing

signalprocessing.convmtx(h, n)	Convolution matrix
signalprocessing.nonstationary_convmtx(H, n)	Convolution matrix from a bank of filters

pylops.utils.signalprocessing.convmtx

pylops.utils.signalprocessing.**convmtx**(*h*, *n*)

Convolution matrix

Equivalent of MATLAB's convmtx function . Makes a dense convolution matrix $\mathbf{C}$ such that the dot product `np.dot(C, x)` is the convolution of the filter h and the input signal x.

Parameters

h [`np.ndarray`] Convolution filter (1D array)

n [`int`] Number of columns (if $len(h) < n$) or rows (if $len(h) \geq n$) of convolution matrix

Returns

C [`np.ndarray`] Convolution matrix of size $len(h) + n - 1 \times n$ (if $len(h) < n$) or $n \times len(h) + n - 1$ (if $len(h) \geq n$)

pylops.utils.signalprocessing.nonstationary_convmtx

pylops.utils.signalprocessing.**nonstationary_convmtx**(*H*, *n*, *hc=0*, *pad=(0, 0)*)

Convolution matrix from a bank of filters

Makes a dense convolution matrix $\mathbf{C}$ such that the dot product `np.dot(C, x)` is the nonstationary convolution of the bank of filters $H = [h_1, h_2, h_n]$ and the input signal x.

Parameters

H [`np.ndarray`] Convolution filters (2D array of shape $[n_{filters} \times n_h]$

n [`int`] Number of columns of convolution matrix

hc [`np.ndarray`, optional] Index of center of first filter

pad [`np.ndarray`] Zero-padding to apply to the bank of filters before and after the provided values (use it to avoid wrap-around or pass filters with enough padding)

Returns

C [`np.ndarray`] Convolution matrix

Tapers

tapers.taper2d(nt, nmask, ntap[, tapertype])	2D taper
tapers.taper3d(nt, nmask, ntap[, tapertype])	3D taper

pylops.utils.tapers.taper2d

pylops.utils.tapers.**taper2d**(*nt*, *nmask*, *ntap*, *tapertype='hanning'*)

2D taper

Create 2d mask of size $[n_{mask} \times n_t]$ with tapering of size `ntap` along the first dimension

Parameters

nt [`int`] Number of time samples of mask along second dimension

nmask [`int`] Number of space samples of mask along first dimension

ntap [`int`] Number of samples of tapering at edges of first dimension

tapertype [`str`, optional] Type of taper (`hanning`, `cosine`, `cosinesquare` or `None`)

Returns

taper [numpy.ndarray] 2d mask with tapering along first dimension of size $[n_{mask} \times n_t]$

Examples using `pylops.utils.tapers.taper2d`

- sphx_glr_gallery_plot_tapers.py

pylops.utils.tapers.taper3d

pylops.utils.tapers.**taper3d**(*nt*, *nmask*, *ntap*, *tapertype='hanning'*)

3D taper

Create 2d mask of size $[n_{mask}[0] \times n_{mask}[1] \times n_t]$ with tapering of size `ntap` along the first and second dimension

Parameters

nt [int] Number of time samples of mask along third dimension

nmask [tuple] Number of space samples of mask along first dimension

ntap [tuple] Number of samples of tapering at edges of first dimension

tapertype [int] Type of taper (`hanning`, `cosine`, `cosinesquare` or `None`)

Returns

taper [numpy.ndarray] 2d mask with tapering along first dimension of size $[n_{mask,0} \times n_{mask,1} \times n_t]$

Examples using `pylops.utils.tapers.taper3d`

- sphx_glr_gallery_plot_mdc.py
- sphx_glr_gallery_plot_tapers.py
- *08. Multi-Dimensional Deconvolution*

Wavelets

wavelets.ricker(t[, f0])	Ricker wavelet
wavelets.gaussian(t[, std])	Gaussian wavelet

pylops.utils.wavelets.ricker

pylops.utils.wavelets.**ricker**(*t*, *f0=10*)

Ricker wavelet

Create a Ricker wavelet given time axis `t` and central frequency `f_0`

Parameters

t [numpy.ndarray] Time axis (positive part including zero sample)

f0 [float, optional] Central frequency

Returns

w [numpy.ndarray] Wavelet

t [numpy.ndarray] Symmetric time axis

wcenter [int] Index of center of wavelet

Examples using `pylops.utils.wavelets.ricker`

- sphx_glr_gallery_plot_sliding.py
- sphx_glr_gallery_plot_avo.py
- sphx_glr_gallery_plot_convolve.py
- sphx_glr_gallery_plot_ista.py
- sphx_glr_gallery_plot_mdc.py
- sphx_glr_gallery_plot_prestack.py
- sphx_glr_gallery_plot_seismicevents.py
- sphx_glr_gallery_plot_wavest.py
- *06. Post-stack inversion*
- *07. Pre-stack (AVO) inversion*
- *08. Multi-Dimensional Deconvolution*
- *10. Radon filtering*
- *11. Seismic regularization*
- *12. Seismic wavefield decomposition*
- *13. Least-squares migration*

pylops.utils.wavelets.gaussian

pylops.utils.wavelets.**gaussian**(*t*, *std=1*)

Gaussian wavelet

Create a Gaussian wavelet given time axis `t` and standard deviation `std` using `scipy.signal.windows.gaussian`.

Parameters

t [numpy.ndarray] Time axis (positive part including zero sample)

std [float, optional] Standard deviation of gaussian

Returns

w [numpy.ndarray] Wavelet

t [numpy.ndarray] Symmetric time axis

wcenter [int] Index of center of wavelet

Geophysicical Reservoir characterization

avo.zoeppritz_scattering(vp1, vs1, rho1, ...)	Zoeppritz solution.
avo.zoeppritz_element(vp1, vs1, rho1, vp0, ...)	Single element of Zoeppritz solution.
avo.zoeppritz_pp(vp1, vs1, rho1, vp0, vs0, ...)	PP reflection coefficient from the Zoeppritz scattering matrix.
avo.approx_zoeppritz_pp(vp1, vs1, rho1, vp0, ...)	PP reflection coefficient from the approximate Zoeppritz equation.
avo.akirichards(theta, vsvp[, n])	Three terms Aki-Richards approximation.
avo.fatti(theta, vsvp[, n])	Three terms Fatti approximation.

pylops.avo.avo.zoeppritz_scattering

pylops.avo.avo.**zoeppritz_scattering**(*vp1*, *vs1*, *rho1*, *vp0*, *vs0*, *rho0*, *theta1*)

Zoeppritz solution.

Calculates the angle dependent p-wave reflectivity of an interface between two media for a set of incident angles.

Parameters

vp1 [float] P-wave velocity of the upper medium

vs1 [float] S-wave velocity of the upper medium

rho1 [float] Density of the upper medium

vp0 [float] P-wave velocity of the lower medium

vs0 [float] S-wave velocity of the lower medium

rho0 [float] Density of the lower medium

theta1 [np.ndarray or float] Incident angles in degrees

Returns

zoep [np.ndarray] 4×4 matrix representing the scattering matrix for the incident angle theta1

See also:

zoeppritz_element Single reflectivity element of Zoeppritz solution

zoeppritz_PP PP reflectivity element of Zoeppritz solution

pylops.avo.avo.zoeppritz_element

pylops.avo.avo.**zoeppritz_element**(*vp1*, *vs1*, *rho1*, *vp0*, *vs0*, *rho0*, *theta1*, *element='PdPu'*)

Single element of Zoeppritz solution.

Simple wrapper to pylops.avo.avo.scattering_matrix, returning any mode reflection coefficient from the Zoeppritz scattering matrix for specific combination of incident and reflected wave and a set of incident angles

Parameters

vp1 [float] P-wave velocity of the upper medium

vs1 [float] S-wave velocity of the upper medium

rho1 [float] Density of the upper medium

vp0 [float] P-wave velocity of the lower medium

vs0 [float] S-wave velocity of the lower medium

rho0 [float] Density of the lower medium

theta1 [np.ndarray or float] Incident angles in degrees

element [str, optional] specific choice of incident and reflected wave combining any two of the following strings: Pd P-wave downgoing, Sd S-wave downgoing, Pu P-wave upgoing, Su S-wave upgoing (e.g., PdPu)

Returns

refl [np.ndarray] reflectivity values for all input angles for specific combination of incident and reflected wave.

See also:

zoeppritz_scattering Zoeppritz solution

zoeppritz_PP PP reflectivity element of Zoeppritz solution

pylops.avo.avo.zoeppritz_pp

pylops.avo.avo.**zoeppritz_pp**(*vp1*, *vs1*, *rho1*, *vp0*, *vs0*, *rho0*, *theta1*)

PP reflection coefficient from the Zoeppritz scattering matrix.

Simple wrapper to pylops.avo.avo.scattering_matrix, returning the PP reflection coefficient from the Zoeppritz scattering matrix for a set of incident angles

Parameters

vp1 [float] P-wave velocity of the upper medium

vs1 [float] S-wave velocity of the upper medium

rho1 [float] Density of the upper medium

vp0 [float] P-wave velocity of the lower medium

vs0 [float] S-wave velocity of the lower medium

rho0 [float] Density of the lower medium

theta1 [np.ndarray or float] Incident angles in degrees

Returns

PPrefl [np.ndarray] PP reflectivity values for all input angles.

See also:

zoeppritz_scattering Zoeppritz solution

zoeppritz_element Single reflectivity element of Zoeppritz solution

pylops.avo.avo.approx_zoeppritz_pp

pylops.avo.avo.**approx_zoeppritz_pp**(*vp1*, *vs1*, *rho1*, *vp0*, *vs0*, *rho0*, *theta1*)

PP reflection coefficient from the approximate Zoeppritz equation.

Approximate calculation of PP reflection from the Zoeppritz scattering matrix for a set of incident angles [1].

Parameters

vp1 [np.ndarray or list or tuple] P-wave velocity of the upper medium

vs1 [np.ndarray or list or tuple] S-wave velocity of the upper medium

rho1 [np.ndarray or list or tuple] Density of the upper medium

vp0 [np.ndarray or list or tuple] P-wave velocity of the lower medium

vs0 [np.ndarray or list or tuple] S-wave velocity of the lower medium

rho0 [np.ndarray or list or tuple] Density of the lower medium

theta1 [np.ndarray or float] Incident angles in degrees

Returns

PPrefl [np.ndarray] PP reflectivity values for all input angles.

See also:

zoeppritz_scattering Zoeppritz solution

zoeppritz_element Single reflectivity element of Zoeppritz solution

zoeppritz_PP PP reflectivity element of Zoeppritz solution

pylops.avo.avo.akirichards

pylops.avo.avo.**akirichards**(*theta*, *vsvp*, *n=1*)

Three terms Aki-Richards approximation.

Computes the coefficients of the of three terms Aki-Richards approximation for a set of angles and a constant or variable VS/VP ratio.

Parameters

theta [np.ndarray] Incident angles in degrees

vsvp [np.ndarray or float] VS/VP ratio

n [int, optional] number of samples (if vsvp is a scalare)

Returns

G1 [np.ndarray] first coefficient of three terms Aki-Richards approximation $[n_{theta} \times n_{vsvp}]$

G2 [np.ndarray] second coefficient of three terms Aki-Richards approximation $[n_{theta} \times n_{vsvp}]$

G3 [np.ndarray] third coefficient of three terms Aki-Richards approximation $[n_{theta} \times n_{vsvp}]$

Notes

The three terms Aki-Richards approximation is used to compute the reflection coefficient as linear combination of contrasts in V_P, V_S, and ρ. More specifically:

$$R(\theta) = G_1(\theta)\frac{\Delta V_P}{\bar{V_P}} + G_2(\theta)\frac{\Delta V_S}{\bar{V_S}} + G_3(\theta)\frac{\Delta \rho}{\bar{\rho}}$$

where $G_1(\theta) = \frac{1}{2cos^2\theta}$, $G_2(\theta) = -4(V_S/V_P)^2 sin^2\theta$, $G_3(\theta) = 0.5 - 2(V_S/V_P)^2 sin^2\theta$, $\frac{\Delta V_P}{\bar{V_P}} = 2\frac{V_{P,2}-V_{P,1}}{V_{P,2}+V_{P,1}}$, $\frac{\Delta V_S}{\bar{V_S}} = 2\frac{V_{S,2}-V_{S,1}}{V_{S,2}+V_{S,1}}$, and $\frac{\Delta \rho}{\bar{\rho}} = 2\frac{\rho_2-\rho_1}{\rho_2+\rho_1}$.

pylops.avo.avo.fatti

pylops.avo.avo.**fatti**(*theta*, *vsvp*, *n=1*)

Three terms Fatti approximation.

Computes the coefficients of the of three terms Fatti approximation for a set of angles and a constant or variable VS/VP ratio.

Parameters

theta [np.ndarray] Incident angles in degrees

vsvp [np.ndarray or float] VS/VP ratio

n [int, optional] number of samples (if vsvp is a scalare)

Returns

G1 [np.ndarray] first coefficient of three terms Smith-Gidlow approximation [$n_{theta} \times n_{vsvp}$]

G2 [np.ndarray] second coefficient of three terms Smith-Gidlow approximation [$n_{theta} \times n_{vsvp}$]

G3 [np.ndarray] third coefficient of three terms Smith-Gidlow approximation [$n_{theta} \times n_{vsvp}$]

Notes

The three terms Fatti approximation is used to compute the reflection coefficient as linear combination of contrasts in AI, SI, and ρ. More specifically:

$$R(\theta) = G_1(\theta)\frac{\Delta AI}{\bar{AI}} + G_2(\theta)\frac{\Delta SI}{\bar{SI}} + G_3(\theta)\frac{\Delta \rho}{\bar{\rho}}$$

where $G_1(\theta) = 0.5(1 + tan^2\theta)$, $G_2(\theta) = -4(V_S/V_P)^2 sin^2\theta$, $G_3(\theta) = 0.5(4(V_S/V_P)^2 sin^2\theta - tan^2\theta)$, $\frac{\Delta AI}{\bar{AI}} = 2\frac{AI_2-AI_1}{AI_2+AI_1}$. $\frac{\Delta SI}{\bar{SI}} = 2\frac{SI_2-SI_1}{SI_2+SI_1}$. $\frac{\Delta \rho}{\bar{\rho}} = 2\frac{\rho_2-\rho_1}{\rho_2+\rho_1}$.

3.8 Implementing new operators

Users are welcome to create new operators and add them to the PyLops library.

In this tutorial, we will go through the key steps in the definition of an operator, using the *pylops.Diagonal* as an example. This is a very simple operator that applies a diagonal matrix to the model in forward mode and to the data in adjoint mode.

3.8.1 Creating the operator

The first thing we need to do is to create a new file with the name of the operator we would like to implement. Note that as the operator will be a class, we need to follow the UpperCaseCamelCase convention both for the class itself and for the filename.

Once we have created the file, we will start by importing the modules that will be needed by the operator. While this varies from operator to operator, you will always need to import the *pylops.LinearOperator* class, which will be used as *parent* class for any of our operators:

```
from pylops import LinearOperator
```

This class is a child of the scipy.sparse.linalg.LinearOperator class itself which implements the same methods of its parent class as well as an additional method for quick inversion: such method can be easily accessed by using \ between the operator and the data (e.g., A\y).

After that we define our new object:

```
class Diagonal(LinearOperator):
```

followed by a numpydoc docstring (starting with r""" and ending with """) containing the documentation of the operator. Such docstring should contain at least a short description of the operator, a Parameters section with a detailed description of the input parameters and a Notes section providing a mathematical explanation of the operator. Take a look at some of the core operators of PyLops to get a feeling of the level of details of the mathematical explanation.

We then need to create the __init__ where the input parameters are passed and saved as members of our class. While the input parameters change from operator to operator, it is always required to create three members, the first called shape with a tuple containing the dimensions of the operator in the data and model space, the second called dtype with the data type object (np.dtype) of the model and data, and the third called explicit with a boolean (True or False) identifying if the operator can be inverted by a direct solver or requires an iterative solver. This member is True if the operator has also a member A that contains the matrix to be inverted like for example in the *pylops.MatrixMult* operator, and it will be False otherwise. In this case we have another member called d which is equal to the input vector containing the diagonal elements of the matrix we want to multiply to the model and data.

```
def __init__(self, d, dtype=None):
    self.d = d.flatten()
    self.shape = (len(self.d), len(self.d))
    self.dtype = np.dtype(dtype)
    self.explicit = False
```

We can then move onto writing the *forward mode* in the method _matvec. In other words, we will need to write the piece of code that will implement the following operation $\mathbf{y} = \mathbf{A}\mathbf{x}$. Such method is always composed of two inputs (the object itself self and the input model x). In our case the code to be added to the forward is very simple, we will just need to apply element-wise multiplication between the model $\mathbf{x}$ and the elements along the diagonal contained in the array $\mathbf{d}$. We will finally need to return the result of this operation:

```
def _matvec(self, x):
    return self.d*x
```

Finally we need to implement the *adjoint mode* in the method _rmatvec. In other words, we will need to write the piece of code that will implement the following operation $\mathbf{x} = \mathbf{A}^H\mathbf{y}$. Such method is also composed of two inputs (the object itself self and the input data y). In our case the code to be added to the forward is the same as the one from the forward (but this will obviously be different from operator to operator):

```
def _rmatvec(self, x):
    return self.d*x
```

And that's it, we have implemented our first linear operator!

3.8.2 Testing the operator

Being able to write an operator is not yet a guarantee of the fact that the operator is correct, or in other words that the adjoint code is actually the *adjoint* of the forward code. Luckily for us, a simple test can be performed to check the validity of forward and adjoint operators, the so called *dot-test*.

We can generate random vectors $\mathbf{u}$ and $\mathbf{v}$ and verify the the following *equality* within a numerical tolerance:

$$(\mathbf{A} * \mathbf{u})^H * \mathbf{v} = \mathbf{u}^H * (\mathbf{A}^H * \mathbf{v})$$

The method *pylops.utils.dottest* implements such a test for you, all you need to do is create a new test within an existing `test_*.py` file in the `pytests` folder (or in a new file).

Generally a test file will start with a number of dictionaries containing different parameters we would like to use in the testing of one or more operators. The test itself starts with a *decorator* that contains a list of all (or some) of dictionaries that will would like to use for our specific operator, followed by the definition of the test

```
@pytest.mark.parametrize("par", [(par1),(par2)])
def test_Diagonal(par):
```

At this point we can first of all create the operator and run the *pylops.utils.dottest* preceded by the `assert` command. Moreover, the forward and adjoint methods should tested towards expected outputs or even better, when the operator allows it (i.e., operator is invertible), a small inversion should be run and the inverted model tested towards the input model.

```
"""Dot-test and inversion for diagonal operator
"""
d = np.arange(par['nx']) + 1.

Dop = Diagonal(d)
assert dottest(Dop, par['nx'], par['nx'],
               complexflag=0 if par['imag'] == 1 else 3)

x = np.ones(par['nx'])
xlsqr = lsqr(Dop, Dop * x, damp=1e-20, iter_lim=300, show=0)[0]
assert_array_almost_equal(x, xlsqr, decimal=4)
```

3.8.3 Documenting the operator

Once the operator has been created, we can add it to the documentation of PyLops. To do so, simply add the name of the operator within the `index.rst` file in `docs/source/api` directory.

Moreover, in order to facilitate the user of your operator by other users, a simple example should be provided as part of the Sphinx-gallery of the documentation of the PyLops library. The directory `examples` containes several scripts that can be used as template.

3.8.4 Final checklist

Before submitting your new operator for review, use the following **checklist** to ensure that your code adheres to the guidelines of PyLops:

- you have created a new file containing a single class (or a function when the new operator is a simple combination of existing operators - see *pylops.Laplacian* for an example of such operator) and added to a new or existing directory within the `pylops` package.
- the new class contains at least `__init__`, `_matvec` and `_matvec` methods.
- the new class (or function) has a numpydoc docstring documenting at least the input `Parameters` and with a `Notes` section providing a mathematical explanation of the operator
- a new test has been added to an existing `test_*.py` file within the `pytests` folder. The test should verify that the new operator passes the *pylops.utils.dottest*. Moreover it is advisable to create a small toy example where the operator is applied in forward mode and the resulting data is inverted using `\` from *pylops.LinearOperator*.
- the new operator is used within at least one *example* (in `examples` directory) or one *tutorial* (in `tutorials` directory).

3.9 Contributing

Contributions are welcome and greatly appreciated!

The best way to get in touch with the core developers and mantainers is to join the PyLops slack channel as well as open new *Issues* directly from the github repo.

Moreover, take a look at the *Roadmap* page for a list of current ideas for improvements and additions to the PyLops library.

3.9.1 Types of Contributions

Report Bugs

Report bugs at https://github.com/equinor/pylops/issues.

If you are playing with the PyLops library and find a bug, please report it including:

- Your operating system name and version.
- Any details about your Python environment.
- Detailed steps to reproduce the bug.

Propose New Operators or Features

Open an issue at https://github.com/equinor/pylops/issues with tag *enhancement*.

If you are proposing a new operator or a new feature:

- Explain in detail how it should work.
- Keep the scope as narrow as possible, to make it easier to implement.

Implement Operators or Features

Look through the Git issues for operator or feature requests. Anything tagged with *enhancement* is open to whoever wants to implement it.

Add Examples or improve Documentation

Writing new operators is not the only way to get involved and contribute. Create examples with existing operators as well as improving the documentation of existing operators is as important as making new operators and very much encouraged.

3.9.2 Getting Started to contribute

Ready to contribute?

1. Fork the *PyLops* repo.
2. Clone your fork locally:

```
>>  git clone https://github.com/your_name_here/pylops.git
```

3. Follow the installation instructions for *developers* that you find in *Installation* page. Ensure that you are able to *pass all the tests before moving forward*.
4. Add the main repository to the list of your remotes (this will be important to ensure you pull the latest changes before tyring to merge your local changes):

```
>>  git remote add upstream https://github.com/equinor/pylops
```

5. Create a branch for local development:

```
>>  git checkout -b name-of-your-branch
```

Now you can make your changes locally.

6. When you're done making changes, check that your code follows the guidelines for *Implementing new operators* and that the both old and new tests pass successfully:

```
>>  make tests
```

7. Commit your changes and push your branch to GitLab:

```
>>  git add .
>> git commit -m "Your detailed description of your changes."
>> git push origin name-of-your-branch
```

Remember to add `-u` when pushing the branch for the first time.

8. Submit a pull request through the GitHub website.

3.9.3 Pull Request Guidelines

Before you submit a pull request, check that it meets these guidelines:

1. The pull request should include new tests for all the core routines that have been developed.
2. If the pull request adds functionality, the docs should be updated accordingly.
3. Ensure that the updated code passes all tests.

3.10 Changelog

3.10.1 Version 1.6.0

Released on: 10/08/2019

- Added *pylops.signalprocessing.ConvolveND* operator
- Added *pylops.utils.signalprocessing.nonstationary_convmtx* to create matrix for non-stationary convolution
- Added possibility to perform seismic modelling (and inversion) with non-stationary wavelet in *pylops.avo.poststack.PoststackLinearModelling*
- Create private methods for `pylops.basicoperators.Block`, *pylops.avo.poststack.PoststackLinearModelling*, *pylops.waveeqprocessing.MDC* to allow calling different operators (e.g., from pylops-distributed or pylops-gpu) within the method

3.10.2 Version 1.5.0

Released on: 30/06/2019

- Added `conj` method to *pylops.LinearOperator*
- Added *pylops.Kronecker*, *pylops.Roll*, and *pylops.Transpose* operators
- Added *pylops.signalprocessing.Fredholm1* operator
- Added *pylops.optimization.sparsity.SPGL1* and *pylops.optimization.sparsity.SplitBregman* solvers
- Sped up *pylops.signalprocessing.Convolve1D* using `scipy.signal.fftconvolve` for multi-dimensional signals
- Changes in implementation of *pylops.waveeqprocessing.MDC* and *pylops.waveeqprocessing.Marchenko* to take advantage of primitives operators
- Added `epsRL1` option to *pylops.avo.poststack.PoststackInversion* and *pylops.avo.prestack.PrestackInversion* to include TV-regularization terms by means of *pylops.optimization.sparsity.SplitBregman* solver

3.10.3 Version 1.4.0

Released on: 01/05/2019

- Added `numba` engine to *pylops.Spread* and *pylops.signalprocessing.Radon2D* operators
- Added *pylops.signalprocessing.Radon3D* operator
- Added *pylops.signalprocessing.Sliding2D* and *pylops.signalprocessing.Sliding3D* operators
- Added *pylops.signalprocessing.FFTND* operator
- Added *pylops.signalprocessing.Radon3D* operator
- Added `niter` option to *pylops.LinearOperator.eigs* method
- Added `show` option to *pylops.optimization.sparsity.ISTA* and *pylops.optimization.sparsity.FISTA* solvers

- Added `pylops.waveeqprocessing.seismicinterpolation`, `pylops.waveeqprocessing.waveeqdecomposition` and `pylops.waveeqprocessing.lsm` submodules
- Added tests for `engine` in various operators
- Added documentation regarding usage of `pylops` Docker container

3.10.4 Version 1.3.0

Released on: 24/02/2019

- Added `fftw` engine to *pylops.signalprocessing.FFT* operator
- Added *pylops.optimization.sparsity.ISTA* and *pylops.optimization.sparsity.FISTA* sparse solvers
- Added possibility to broadcast (handle multi-dimensional arrays) to *pylops.Diagonal* and `pylops..Restriction` operators
- Added *pylops.signalprocessing.Interp* operator
- Added *pylops.Spread* operator
- Added *pylops.signalprocessing.Radon2D* operator

3.10.5 Version 1.2.0

Released on: 13/01/2019

- Added *pylops.LinearOperator.eigs* and *pylops.LinearOperator.cond* methods to estimate estimate eigenvalues and conditioning number using scipy wrapping of ARPACK
- Modified default `dtype` for all operators to be `float64` (or `complex128`) to be consistent with default dtypes used by numpy (and scipy) for real and complex floating point numbers.
- Added *pylops.Flip* operator
- Added *pylops.Symmetrize* operator
- Added *pylops.Block* operator
- Added *pylops.Regression* operator performing polynomial regression and modified *pylops.LinearRegression* to be a simple wrapper of *pylops.Regression* when `order=1`
- Modified *pylops.MatrixMult* operator to work with both numpy ndarrays and scipy sparse matrices
- Added *pylops.avo.prestack.PrestackInversion* routine
- Added possibility to have a data weight via `Weight` input parameter to *pylops.optimization.leastsquares.NormalEquationsInversion* and *pylops.optimization.leastsquares.RegularizedInversion* solvers
- Added *pylops.optimization.sparsity.IRLS* solver

3.10.6 Version 1.1.0

Released on: 13/12/2018

- Added *pylops.CausalIntegration* operator

3.10.7 Version 1.0.1

Released on: 09/12/2018

- Changed module from `lops` to `pylops` for consistency with library name (and pip install).
- Removed quickplots from utilities and `matplotlib` from requirements of *PyLops*.

3.10.8 Version 1.0.0

Released on: 04/12/2018

- First official release.

3.11 Roadmap

This roadmap is aimed at providing an high-level overview on the bug fixes, improvements and new functionality that are planned for the PyLops library.

Any of the fixes/additions mentioned in the roadmap are directly linked to a *Github Issue* that provides more details onto the reason and initial thoughts for the implementation of such a fix/addition.

Striked tasks have been completed and related github issue closed with more details regarding how this task has been carried out.

3.11.1 Library structure

- Create a child repository and python library called `geolops` (just a suggestion) where geoscience-related operators and examples are moved across, keeping the core `pylops` library very generic and multi-purpose - Issue #22.

3.11.2 Code cleaning

- Change all `np.flatten()` into `np.ravel()` - Issue #24.
- Fix all `if: return ... else: ...` statements to enforce a single return with the same number of outputs - Issue #26.
- Protected attributes and @property attributes in linear operator classes? - Issue #27.

3.11.3 Code optimization

- Investigate speed-up given by decorating `_matvec` and `_rmatvec` methods with numba `@jit` and `@stencil` decorators - Issue #23.
- Replace `np.fft.*` routines used in several submodules with double engine, numpy and pyFFTW - Issue #20.

3.11.4 Modules

avo

- Add possibility to choose different damping factors for each elastic parameter to invert for in *pylops.avo.prestack.PrestackInversion* - Issue #25.

basicoperators

- Create Kronecker operator - Issue #28.
- Deal with edges in FirstDerivative and SecondDerivative operators - Issue #34.

optimization

- Sparse solvers - Issue #44.

signalprocessing

- Compare performance in FTT operator of performing np.swap+np.fft.fft(..., axis=-1) versus np.fft.fft(..., axis=chosen) - Issue #33.
- Add `Wavelet` operator performing the wavelet transform. pywavelets can be used as back-end - Issue #21.
- Fredholm1 operator applying Fredholm integrals of first kind - Issue #31.
- `Fredholm2` operators applying Fredholm integrals of second kind - Issue #31.

utils

Nothing so far

waveeqprocessing

- numpy.matmul as a way to speed up integral computation (i.e., inner for loop) in ‘‘MDC‘ operator‘ - Issue #32.
- `NMO` operator performing NMO modelling - Issue #29.
- WavefieldDecomposition operator performing acoustic wavefield separation by inversion - Issue #30.

3.12 Citing

When using `pylops` in scientific publications, please cite the following paper:

- Ravasi, M., and Vasconcelos I., *PyLops–A Linear-Operator Python Library for large scale optimization*, arXiv preprint, arXiv:1907.12349 (2019).

3.13 Contributors

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Bibliography

[1] http://www.caam.rice.edu/software/ARPACK/

[1] Wapenaar, K. “Reciprocity properties of one-way propagators”, Geophysics, vol. 63, pp. 1795-1798. 1998.

[1] Wapenaar, K., van der Neut, J., Ruigrok, E., Draganov, D., Hunziker, J., Slob, E., Thorbecke, J., and Snieder, R., “Seismic interferometry by crosscorrelation and by multi-dimensional deconvolution: a systematic comparison”, Geophysical Journal International, vol. 185, pp. 1335-1364. 2011.

[1] https://en.wikipedia.org/wiki/Iteratively_reweighted_least_squares

[1] Beck, A., and Teboulle, M., “A Fast Iterative Shrinkage-Thresholding Algorithm for Linear Inverse Problems”, SIAM Journal on Imaging Sciences, vol. 2, pp. 183-202. 2009.

[1] Beck, A., and Teboulle, M., “A Fast Iterative Shrinkage-Thresholding Algorithm for Linear Inverse Problems”, SIAM Journal on Imaging Sciences, vol. 2, pp. 183-202. 2009.

[1] van den Berg E., Friedlander M.P., “Probing the Pareto frontier for basis pursuit solutions”, SIAM J. on Scientific Computing, vol. 31(2), pp. 890-912. 2008.

[1] Goldstein T. and Osher S., “The Split Bregman Method for L1-Regularized Problems”, SIAM J. on Scientific Computing, vol. 2(2), pp. 323-343. 2008.

[1] Wapenaar, K. “Reciprocity properties of one-way propagators”, Geophysics, vol. 63, pp. 1795-1798. 1998.

[1] Wapenaar, K., van der Neut, J., Ruigrok, E., Draganov, D., Hunziker, J., Slob, E., Thorbecke, J., and Snieder, R., “Seismic interferometry by crosscorrelation and by multi-dimensional deconvolution: a systematic comparison”, Geophyscial Journal International, vol. 185, pp. 1335-1364. 2011.

[R1c4066d82e61-1] Wapenaar, K., Thorbecke, J., Van der Neut, J., Broggini, F., Slob, E., and Snieder, R., “Marchenko imaging”, Geophysics, vol. 79, pp. WA39-WA57. 2014.

[R1c4066d82e61-2] van der Neut, J., Vasconcelos, I., and Wapenaar, K. “On Green’s function retrieval by iterative substitution of the coupled Marchenko equations”, Geophysical Journal International, vol. 203, pp. 792-813. 2015.

[1] Dvorkin et al. Seismic Reflections of Rock Properties. Cambridge. 2014.

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