PyLops

unknown

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GETTING STARTED

PyLops is an open-source Python library focused on providing a backend-agnostic, idiomatic, matrix-free library of linear operators and related computations. It is inspired by the iconic MATLAB Spot – A Linear-Operator Toolbox project.

Linear operators and inverse problems are at the core of many of the most used algorithms in signal processing, image processing, and remote sensing. For small-scale problems, matrices can be explicitly computed and manipulated with Python numerical scientific libraries such as NumPy and SciPy.

Large-scale problems often feature matrices that are prohibitive in size—but whose operations can be described by simple functions. PyLops operators exploit this to represent a linear operator not as array of numbers, but by functions which describe matrix-vector products in forward and adjoint modes. Moreover, many iterative methods (e.g. cg, lsqr) are designed to not rely on the elements of the matrix, only these matrix-vector products. PyLops offers such solvers for many different types of problems, in particular least-squares and sparsity-promoting inversions.

Get started by *installing PyLops* and following our quick tour.

CHAPTER

ONE

TERMINOLOGY

A common *terminology* is used within the entire documentation of PyLops. Every linear operator and its application to a model will be referred to as **forward model (or operation)**

$$\mathbf{y} = \mathbf{A}\mathbf{x}$$

while its application to a data is referred to as **adjoint model (or operation)**

$$\mathbf{x} = \mathbf{A}^H\mathbf{y}$$

where $\mathbf{x}$ is called *model* and $\mathbf{y}$ is called *data*. The *operator* $\mathbf{A} : \mathbb{F}^m \to \mathbb{F}^n$ effectively maps a vector of size m in the *model space* to a vector of size n in the *data space*, conversely the *adjoint operator* $\mathbf{A}^H : \mathbb{F}^n \to \mathbb{F}^m$ maps a vector of size n in the *data space* to a vector of size m in the *model space*. As linear operators mimics the effect a matrix on a vector we can also loosely refer to m as the number of *columns* and n as the number of *rows* of the operator.

Ultimately, solving an inverse problems accounts to removing the effect of $\mathbf{A}$ from the data $\mathbf{y}$ to retrieve the model $\mathbf{x}$.

For a more detailed description of the concepts of linear operators, adjoints and inverse problems in general, you can head over to one of Jon Claerbout's books such as Basic Earth Imaging.

CHAPTER

TWO

IMPLEMENTATION

PyLops is build on top of the scipy class `scipy.sparse.linalg.LinearOperator`.

This class allows in fact for the creation of objects (or interfaces) for matrix-vector and matrix-matrix products that can ultimately be used to solve any inverse problem of the form $\mathbf{y} = \mathbf{A}\mathbf{x}$.

As explained in the scipy LinearOperator official documentation, to construct a `scipy.sparse.linalg.LinearOperator`, a user is required to pass appropriate callables to the constructor of this class, or subclass it. More specifically one of the methods `_matvec` and `_matmat` must be implemented for the *forward operator* and one of the methods `_rmatvec` or `_adjoint` may be implemented to apply the *Hermitian adjoint*. The attributes/properties `shape` (pair of integers) and `dtype` (may be None) must also be provided during `__init__` of this class.

Any linear operator developed within the PyLops library follows this philosophy. As explained more in details in *Implementing new operators* section, a linear operator is created by subclassing the `scipy.sparse.linalg.LinearOperator` class and `_matvec` and `_rmatvec` are implemented.

CHAPTER

THREE

HISTORY

PyLops was initially written by Equinor It is a flexible and scalable python library for large-scale optimization with linear operators that can be tailored to our needs, and as contribution to the free software community. Since June 2021, PyLops is a NUMFOCUS Affiliated Project.

3.1 Installation

3.1.1 Dependencies

The PyLops project strives to create a library that is easy to install in any environment and has a very limited number of dependencies. Required dependencies are limited to:

- Python 3.8 or greater
- NumPy
- SciPy

We highly encourage using the Anaconda Python distribution or its standalone package manager Conda. Especially for Intel processors, this ensures a higher performance with no configuration. If you are interested in getting the best code performance, read carefully *Advanced installation*. For learning, however, the standard installation is often good enough.

Some operators have additional, optional "engines" to improve their performance. These often rely on third-party libraries which are added to the list of our optional dependencies. Optional dependencies therefore refer to those dependencies that are not strictly needed nor installed directly as part of a standard installation. For details more details, see *Optional dependencies*.

3.1.2 Step-by-step installation for users

Conda (recommended)

If using `conda`, install our `conda-forge` distribution via:

```
>> conda install --channel conda-forge pylops
```

Using the `conda-forge` distribution is recommended as all the dependencies (both required and optional) will be automatically installed for you.

Pip

If you are using `pip`, and simply type the following command in your terminal to install the PyPI distribution:

```
>> pip install pylops
```

Note that when installing via `pip`, only *required* dependencies are installed.

Docker

If you want to try PyLops but do not have Python in your local machine, you can use our Docker image instead.

After installing Docker in your computer, type the following command in your terminal (note that this will take some time the first time you type it as you will download and install the Docker image):

```
>> docker run -it -v /path/to/local/folder:/home/jupyter/notebook -p 8888:8888 mrava87/
↪pylops:notebook
```

This will give you an address that you can put in your browser and will open a Jupyter notebook environment with PyLops and other basic Python libraries installed. Here, `/path/to/local/folder` is the absolute path of a local folder on your computer where you will create a notebook (or containing notebooks that you want to continue working on). Note that anything you do to the notebook(s) will be saved in your local folder. A larger image with `conda` a distribution is also available:

```
>> docker run -it -v /path/to/local/folder:/home/jupyter/notebook -p 8888:8888 mrava87/
↪pylops:conda_notebook
```

3.1.3 Step-by-step installation for developers

Fork PyLops

Fork the PyLops repository and clone it by executing the following in your terminal:

```
>> git clone https://github.com/YOUR-USERNAME/pylops.git
```

We recommend installing dependencies into a separate environment. For that end, we provide a *Makefile* with useful commands for setting up the environment.

Install dependencies

Conda (recommended)

For a `conda` environment, run

```
>> make dev-install_conda
```

This will create and activate an environment called `pylops`, with all required and optional dependencies.

Pip

If you prefer a `pip` installation, we provide the following command

```
>> make dev-install
```

Note that, differently from the `conda` command, the above **will not** create a virtual environment. Make sure you create and activate your environment previously.

Run tests

To ensure that everything has been setup correctly, run tests:

```
>> make tests
```

Make sure no tests fail, this guarantees that the installation has been successful.

Add remote (optional)

To keep up-to-date on the latest changes while you are developing, you may optionally add the PyLops repository as a *remote*. Run the following command to add the PyLops repo as a remote named *upstream*:

```
>> git remote add upstream https://github.com/PyLops/pylops
```

From then on, you can pull changes (for example, in the dev branch) with:

```
>> git pull upstream dev
```

Install pre-commit hooks

To ensure consistency in the coding style of our developers we rely on pre-commit to perform a series of checks when you are ready to commit and push some changes. This is accomplished by means of git hooks that have been configured in the `.pre-commit-config.yaml` file.

In order to setup such hooks in your local repository, run:

```
>> pre-commit install
```

Once this is set up, when committing changes, `pre-commit` will reject and "fix" your code by running the proper hooks. At this point, the user must check the changes and then stage them before trying to commit again.

Final steps

PyLops does not enforce the use of a linter as a pre-commit hook, but we do highly encourage using one before submitting a Pull Request. A properly configured linter (`flake8`) can be run with:

```
>> make lint
```

In addition, it is highly encouraged to build the docs prior to submitting a Pull Request. Apart from ensuring that docstrings are properly formatted, they can aid in catching bugs during development. Build (or update) the docs with:

```
>> make doc
```

or

```
>> make docupdate
```

3.1.4 Advanced installation

In this section we discuss some important details regarding code performance when using PyLops.

To get the most out of PyLops operators in terms of speed you will need to follow these guidelines as much as possible or ensure that the Python libraries used by PyLops are efficiently installed in your system.

BLAS

PyLops relies on the NumPy and SciPy, and being able to link these to the most performant BLAS library will ensure optimal performance of PyLops when using only *required dependencies*.

We strongly encourage using the Anaconda Python distribution as NumPy and SciPy will, when available, be automatically linked to Intel MKL, the most performant library for basic linear algebra operations to date (see Markus Beuckelmann's benchmarks). The PyPI version installed with `pip`, however, will default to OpenBLAS. For more information, see NumPy's section on BLAS.

To check which BLAS NumPy and SciPy were compiled against, run the following commands in a Python interpreter:

```
import numpy as np
import scipy as sp
print(np.__config__.show())
print(sp.__config__.show())
```

Intel also provides NumPy and SciPy replacement packages in PyPI `intel-numpy` and `intel-scipy`, respectively, which link to Intel MKL. These are an option for an environment without `conda` that needs Intel MKL without requiring manual compilation.

Warning: `intel-numpy` and `intel-scipy` not only link against Intel MKL, but also substitute NumPy and SciPy FFTs for Intel MKL FFT. **MKL FFT is not supported and may break PyLops**.

Multithreading

It is important to ensure that your environment variable which sets threads is correctly assigned to the maximum number of cores you would like to use in your code. Multiprocessing parallelism in NumPy and SciPy can be controlled in different ways depending on where it comes from.

Environment variable	Library
OMP_NUM_THREADS	OpenMP
NUMEXPR_NUM_THREADS	NumExpr
OPENBLAS_NUM_THREADS	OpenBLAS
MKL_NUM_THREADS	Intel MKL
VECLIB_MAXIMUM_THREADS	Apple Accelerate (vecLib)

For example, try setting one processor to be used with (if using OpenBlas)

```
>> export OMP_NUM_THREADS=1
>> export NUMEXPR_NUM_THREADS=1
>> export OPENBLAS_NUM_THREADS=1
```

and run the following code in Python:

```
import os
import numpy as np
from timeit import timeit

size = 1024
A = np.random.random((size, size)),
B = np.random.random((size, size))
print("Time with %s threads: %f s" \
      %(os.environ.get("OMP_NUM_THREADS"),
        timeit(lambda: np.dot(A, B), number=4)))
```

Subsequently set the environment variables to `2` or any higher number of threads available in your hardware (multi-threaded), and run the same code. By looking at both the load on your processors (e.g., using `top`), and at the Python print statement you should see a speed-up in the second case.

Alternatively, you could set the `OMP_NUM_THREADS` variable directly inside your script using `os.environ["OMP_NUM_THREADS"]="2"`, but ensure that this is done *before* loading NumPy.

Note: Always remember to set `OMP_NUM_THREADS` and other relevant variables in your environment when using PyLops

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Optional dependencies

To avoid increasing the number of *required* dependencies, which may lead to conflicts with other libraries that you have in your system, we have decided to build some of the additional features of PyLops in such a way that if an *optional* dependency is not present in your Python environment, a safe fallback to one of the required dependencies will be enforced.

When available in your system, we recommend using the Conda package manager and install all the required and optional dependencies of PyLops at once using the command:

```
>> conda install --channel conda-forge pylops
```

in this case all dependencies will be installed from their Conda distributions.

Alternatively, from version `1.4.0` optional dependencies can also be installed as part of the pip installation via:

```
>> pip install pylops[advanced]
```

Dependencies are however installed from their PyPI wheels. An exception is however represented by CuPy. This library is **not** installed automatically. Users interested to accelerate their computations with the aid of GPUs should install it prior to installing PyLops as described in *Optional Dependencies for GPU*.

Note: If you are a developer, all the optional dependencies below (except GPU) can be installed automatically by cloning the repository and installing PyLops via `make dev-install_conda` (`conda`) or `make dev-install` (`pip`).

In alphabetic order:

Devito

Devito is library used to solve PDEs via the finite-difference method. It is used in PyLops to compute wavefields *pylops.waveeqprocessing.AcousticWave2D*

Install it via `pip` with

```
>> pip install devito
```

FFTW

Three different "engines" are provided by the *pylops.signalprocessing.FFT* operator: `engine="numpy"` (default), `engine="scipy"` and `engine="fftw"`.

The first two engines are part of the required PyLops dependencies. The latter implements the well-known FFTW via the Python wrapper `pyfftw.FFTW`. While this optimized FFT tends to outperform the other two in many cases, it is not included by default. To use this library, install it manually either via `conda`:

```
>> conda install --channel conda-forge pyfftw
```

or via pip:

```
>> pip install pyfftw
```

Note: FFTW is only available for *pylops.signalprocessing.FFT*, not *pylops.signalprocessing.FFT2D* or *pylops.signalprocessing.FFTND*.

Warning: Intel MKL FFT is not supported.

Numba

Although we always strive to write code for forward and adjoint operators that takes advantage of the perks of NumPy and SciPy (e.g., broadcasting, ufunc), in some case we may end up using for loops that may lead to poor performance. In those cases we may decide to implement alternative (optional) back-ends in Numba, a Just-In-Time compiler that translates a subset of Python and NumPy code into fast machine code.

A user can simply switch from the native, always available implementation to the Numba implementation by simply providing the following additional input parameter to the operator `engine="numba"`. This is for example the case in the *pylops.signalprocessing.Radon2D*.

If interested to use Numba backend from `conda`, you will need to manually install it:

```
>> conda install numba
```

It is also advised to install the additional package icc_rt to use optimised transcendental functions as compiler intrinsics.

```
>> conda install --channel numba icc_rt
```

Through `pip` the equivalent would be:

```
>> pip install numba
>> pip install icc_rt
```

However, it is important to note that `icc_rt` will only be identified by Numba if `LD_LIBRARY_PATH` is properly set. If you are using a virtual environment, you can ensure this with:

```
>> export LD_LIBRARY_PATH=/path/to/venv/lib/:$LD_LIBRARY_PATH
```

To ensure that `icc_rt` is being recognized, run

```
>> numba -s | grep SVML
__SVML Information__
SVML State, config.USING_SVML                : True
SVML Library Loaded                          : True
llvmlite Using SVML Patched LLVM             : True
SVML Operational                             : True
```

Numba also offers threading parallelism through a variety of Threading Layers. You may need to set the environment variable `NUMBA_NUM_THREADS` define how many threads to use out of the available ones (`numba -s | grep "CPU Count"`). It can also be checked dynamically with `numba.config.NUMBA_DEFAULT_NUM_THREADS`.

PyWavelets

PyWavelets is used to implement the wavelet operators. Install it via `conda` with:

```
>> conda install pywavelets
```

or via `pip` with

```
>> pip install PyWavelets
```

scikit-fmm

scikit-fmm is a library which implements the fast marching method. It is used in PyLops to compute traveltime tables in the initialization of *pylops.waveeqprocessing.Kirchhoff* when choosing `mode="eikonal"`. As this may not be of interest for many users, this library has not been added to the mandatory requirements of PyLops. With `conda`, install it via

```
>> conda install --channel conda-forge scikit-fmm
```

or with `pip` via

```
>> pip install scikit-fmm
```

SPGL1

SPGL1 is used to solve sparsity-promoting basis pursuit, basis pursuit denoise, and Lasso problems in `pylops.optimization.sparsity.SPGL1` solver.

Install it via `pip` with:

```
>> pip install spgl1
```

Sympy

This library is used to implement the `describe` method, which transforms PyLops operators into their mathematical expression.

Install it via `conda` with:

```
>> conda install sympy
```

or via `pip` with

```
>> pip install sympy
```

Torch

Torch used to allow seamless integration between PyLops and PyTorch operators.

Install it via `conda` with:

```
>> conda install -c pytorch pytorch
```

or via `pip` with

```
>> pip install torch
```

Optional Dependencies for GPU

PyLops will automatically check if the libraries below are installed and, in that case, use them any time the input vector passed to an operator is of compatible type. Users can, however, disable this option. For more details of GPU-accelerated PyLops read *GPU Support*.

CuPy

CuPy is a library used as a drop-in replacement to NumPy for GPU-accelerated computations. Since many different versions of CuPy exist (based on the CUDA drivers of the GPU), users must install CuPy prior to installing PyLops. To do so, follow their installation instructions.

cuSignal

cuSignal is a library is used as a drop-in replacement to SciPy Signal for GPU-accelerated computations. Similar to CuPy, users must install cuSignal prior to installing PyLops. To do so, follow their installation instructions.

3.2 GPU Support

3.2.1 Overview

From `v1.12.0`, PyLops supports computations on GPUs powered by CuPy (`cupy-cudaXX>=8.1.0`) and cuSignal (`cusignal>=0.16.0`). They must be installed *before* PyLops is installed.

Note: Set environment variables `CUPY_PYLOPS=0` and/or `CUSIGNAL_PYLOPS=0` to force PyLops to ignore `cupy` and `cusignal` backends. This can be also used if a previous version of `cupy` or `cusignal` is installed in your system, otherwise you will get an error when importing PyLops.

Apart from a few exceptions, all operators and solvers in PyLops can seamlessly work with `numpy` arrays on CPU as well as with `cupy` arrays on GPU. Users do simply need to consistently create operators and provide data vectors to the solvers, e.g., when using *pylops.MatrixMult* the input matrix must be a `cupy` array if the data provided to a solver is also `cupy` array.

Warning: Some *pylops.LinearOperator* methods are currently on GPU:

- `pylops.LinearOperator.eigs`
- `pylops.LinearOperator.cond`
- `pylops.LinearOperator.tosparse`
- `pylops.LinearOperator.estimate_spectral_norm`

Warning: Some operators are currently not available on GPU:

- *pylops.Spread*
- *pylops.signalprocessing.Radon2D*
- *pylops.signalprocessing.Radon3D*
- *pylops.signalprocessing.DWT*
- *pylops.signalprocessing.DWT2D*
- *pylops.signalprocessing.Seislet*
- `pylops.waveeqprocessing.Demigration`
- *pylops.waveeqprocessing.LSM*

Warning: Some solvers are currently not available on GPU:

- `pylops.optimization.sparsity.SPGL1`

3.2.2 Example

Finally, let's briefly look at an example. First we write a code snippet using `numpy` arrays which PyLops will run on your CPU:

```
ny, nx = 400, 400
G = np.random.normal(0, 1, (ny, nx)).astype(np.float32)
x = np.ones(nx, dtype=np.float32)

Gop = MatrixMult(G, dtype='float32')
y = Gop * x
xest = Gop / y
```

Now we write a code snippet using `cupy` arrays which PyLops will run on your GPU:

```
ny, nx = 400, 400
G = cp.random.normal(0, 1, (ny, nx)).astype(np.float32)
x = cp.ones(nx, dtype=np.float32)

Gop = MatrixMult(G, dtype='float32')
y = Gop * x
xest = Gop / y
```

The code is almost unchanged apart from the fact that we now use `cupy` arrays, PyLops will figure this out!

Note: The CuPy backend is in active development, with many examples not yet in the docs. You can find many other examples from the PyLops Notebooks repository.

3.3 Extensions

PyLops brings to users the power of linear operators in a simple and easy to use programming interface.

While very powerful on its own, this library is further extended to take advantage of more advanced computational resources, either in terms of **multiple-node clusters** or **GPUs**. Moreover, some independent libraries are created to use third party software that cannot be included as dependencies to our main library for licensing issues but may be useful for academic purposes.

Spin-off projects that aim at extending the capabilities of PyLops are:

- PyLops-GPU : PyLops for GPU arrays (incorporated into PyLops).
- PyLops-Distributed: PyLops for distributed systems with many computing nodes.
- PyProximal: Proximal solvers which integrate with PyLops Linear Operators.
- Curvelops: Python wrapper for the Curvelab 2D and 3D digital curvelet transforms.

3.4 Tutorials

3.4.1 01. The LinearOpeator

This first tutorial is aimed at easing the use of the PyLops library for both new users and developers.

Since PyLops heavily relies on the use of the `scipy.sparse.linalg.LinearOperator` class of SciPy, we will start by looking at how to initialize a linear operator as well as different ways to apply the forward and adjoint operations. Finally we will investigate various *special methods*, also called *magic methods* (i.e., methods with the double underscores at the beginning and the end) that have been implemented for such a class and will allow summing, subtractring, chaining, etc. multiple operators in very easy and expressive way.

Let's start by defining a simple operator that applies element-wise multiplication of the model with a vector `d` in forward mode and element-wise multiplication of the data with the same vector `d` in adjoint mode. This operator is present in PyLops under the name of *`pylops.Diagonal`* and its implementation is discussed in more details in the *Implementing new operators* page.

```
import timeit

import matplotlib.pyplot as plt
import numpy as np

import pylops

n = 10
d = np.arange(n) + 1.0
x = np.ones(n)
Dop = pylops.Diagonal(d)
```

First of all we apply the operator in the forward mode. This can be done in four different ways:

- `_matvec`: directly applies the method implemented for forward mode
- `matvec`: performs some checks before and after applying `_matvec`
- `*`: operator used to map the special method `__matmul__` which checks whether the input `x` is a vector or matrix and applies `_matvec` or `_matmul` accordingly.
- `@`: operator used to map the special method `__mul__` which performs like the * opetator

We will time these 4 different executions and see how using `_matvec` (or `matvec`) will result in the faster computation. It is thus advised to use * (or @) in examples when expressivity has priority but prefer `_matvec` (or `matvec`) for efficient implementations.

```
# setup command
cmd_setup = """\
import numpy as np
import pylops
n = 10
d = np.arange(n) + 1.
x = np.ones(n)
Dop = pylops.Diagonal(d)
DopH = Dop.H
"""

# _matvec
```

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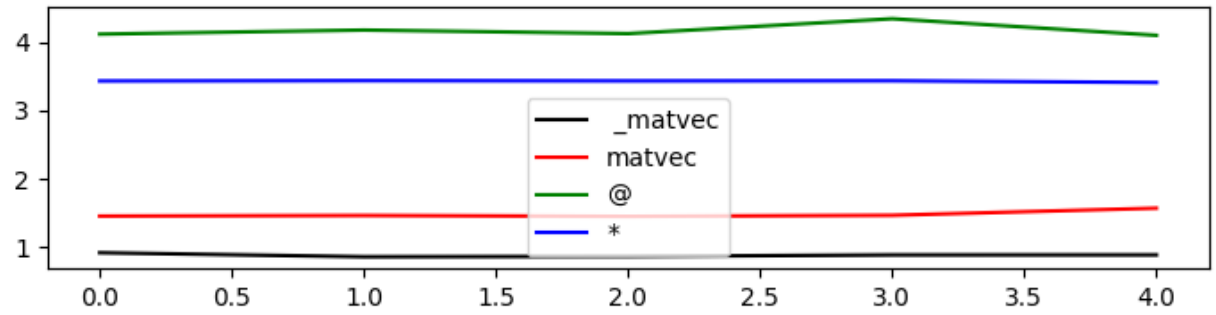
```
cmd1 = "Dop._matvec(x)"

# matvec
cmd2 = "Dop.matvec(x)"

# @
cmd3 = "Dop@x"

# *
cmd4 = "Dop*x"

# timing
t1 = 1.0e3 * np.array(timeit.repeat(cmd1, setup=cmd_setup, number=500, repeat=5))
t2 = 1.0e3 * np.array(timeit.repeat(cmd2, setup=cmd_setup, number=500, repeat=5))
t3 = 1.0e3 * np.array(timeit.repeat(cmd3, setup=cmd_setup, number=500, repeat=5))
t4 = 1.0e3 * np.array(timeit.repeat(cmd4, setup=cmd_setup, number=500, repeat=5))

plt.figure(figsize=(7, 2))
plt.plot(t1, "k", label=" _matvec")
plt.plot(t2, "r", label="matvec")
plt.plot(t3, "g", label="@")
plt.plot(t4, "b", label="*")
plt.axis("tight")
plt.legend()
plt.tight_layout()
```



Similarly we now consider the adjoint mode. This can be done in three different ways:

- `_rmatvec`: directly applies the method implemented for adjoint mode
- `rmatvec`: performs some checks before and after applying `_rmatvec`
- `.H*`: first applies the adjoint `.H` which creates a new *scipy.sparse.linalg._CustomLinearOperator* ` where `_matvec` and `_rmatvec` are swapped and then applies the new `_matvec`.

Once again, after timing these 3 different executions we can see see how using `_rmatvec` (or `rmatvec`) will result in the faster computation while `.H*` is very unefficient and slow. Note that if the adjoint has to be applied multiple times it is at least advised to create the adjoint operator by applying `.H` only once upfront. Not surprisingly, the linear solvers in scipy as well as in PyLops actually use `matvec` and `rmatvec` when dealing with linear operators.

```
# _rmatvec
cmd1 = "Dop._rmatvec(x)"
```

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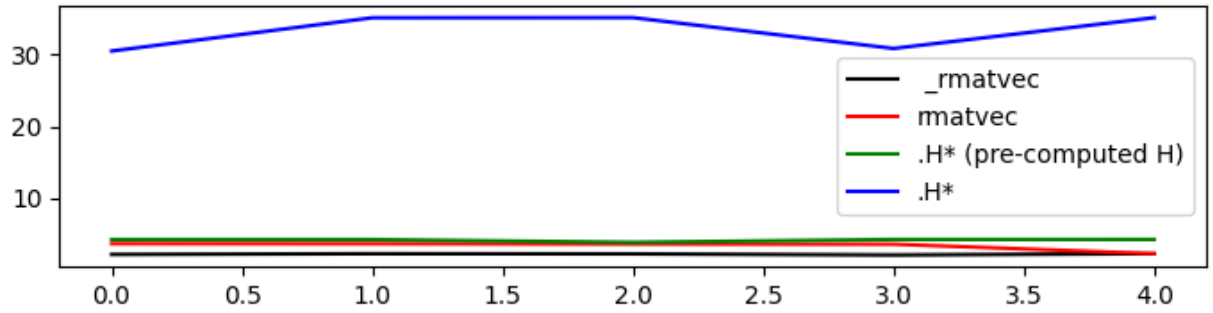
```
# rmatvec
cmd2 = "Dop.rmatvec(x)"

# .H* (pre-computed H)
cmd3 = "DopH*x"

# .H*
cmd4 = "Dop.H*x"

# timing
t1 = 1.0e3 * np.array(timeit.repeat(cmd1, setup=cmd_setup, number=500, repeat=5))
t2 = 1.0e3 * np.array(timeit.repeat(cmd2, setup=cmd_setup, number=500, repeat=5))
t3 = 1.0e3 * np.array(timeit.repeat(cmd3, setup=cmd_setup, number=500, repeat=5))
t4 = 1.0e3 * np.array(timeit.repeat(cmd4, setup=cmd_setup, number=500, repeat=5))

plt.figure(figsize=(7, 2))
plt.plot(t1, "k", label=" _rmatvec")
plt.plot(t2, "r", label="rmatvec")
plt.plot(t3, "g", label=".H* (pre-computed H)")
plt.plot(t4, "b", label=".H*")
plt.axis("tight")
plt.legend()
plt.tight_layout()
```



Just to reiterate once again, it is advised to call `matvec` and `rmatvec` unless PyLops linear operators are used for teaching purposes.

We now go through some other *methods* and *special methods* that are implemented in `scipy.sparse.linalg.LinearOperator` (and *pylops.LinearOperator*):

- `Op1+Op2`: maps the special method `__add__` and performs summation between two operators and returns a *pylops.LinearOperator*
- `-Op`: maps the special method `__neg__` and performs negation of an operators and returns a *pylops.LinearOperator*
- `Op1-Op2`: maps the special method `__sub__` and performs summation between two operators and returns a *pylops.LinearOperator*
- `Op1**N`: maps the special method `__pow__` and performs exponentiation of an operator and returns a *pylops.LinearOperator*
- `Op/y` (and `Op.div(y)`): maps the special method `__truediv__` and performs inversion of an operator

- `Op.eigs()`: estimates the eigenvalues of the operator
- `Op.cond()`: estimates the condition number of the operator
- `Op.conj()`: create complex conjugate operator

```
Dop = pylops.Diagonal(d)

# +
print(Dop + Dop)

# -
print(-Dop)
print(Dop - 0.5 * Dop)

# **
print(Dop**3)

# * and /
y = Dop * x
print(Dop / y)

# eigs
print(Dop.eigs(neigs=3))

# cond
print(Dop.cond())

# conj
print(Dop.conj())
```

```
<10x10 LinearOperator with dtype=float64>
<10x10 LinearOperator with dtype=float64>
<10x10 LinearOperator with dtype=float64>
<10x10 LinearOperator with dtype=float64>
[1. 1. 1. 1. 1. 1. 1. 1. 1. 1.]
[10.+0.j  9.+0.j  8.+0.j]
(10.000000000000003+0j)
<10x10 _ConjLinearOperator with dtype=float64>
```

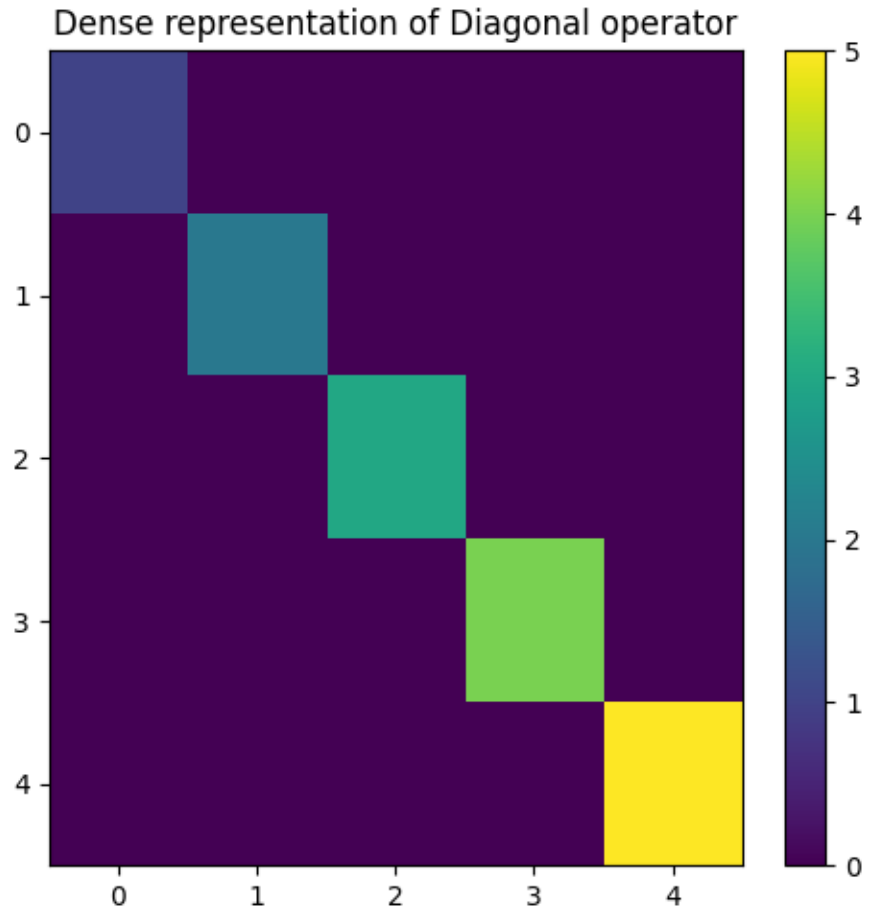
To understand the effect of `conj` we need to look into a problem with an operator in the complex domain. Let's create again our *pylops.Diagonal* operator but this time we populate it with complex numbers. We will see that the action of the operator and its complex conjugate is different even if the model is real.

```
n = 5
d = 1j * (np.arange(n) + 1.0)
x = np.ones(n)
Dop = pylops.Diagonal(d)

print(f"y = Dx = {Dop * x}")
print(f"y = conj(D)x = {Dop.conj() * x}")
```

```
y = Dx = [0.+1.j 0.+2.j 0.+3.j 0.+4.j 0.+5.j]
y = conj(D)x = [0.-1.j 0.-2.j 0.-3.j 0.-4.j 0.-5.j]
```

At this point, the concept of linear operator may sound abstract. The convinience method `pylops.LinearOperator.todense` can be used to create the equivalent dense matrix of any operator. In this case for example we expect to see a diagonal matrix with `d` values along the main diagonal

```
D = Dop.todense()

plt.figure(figsize=(5, 5))
plt.imshow(np.abs(D))
plt.title("Dense representation of Diagonal operator")
plt.axis("tight")
plt.colorbar()
plt.tight_layout()
```



At this point it is worth reiterating that if two linear operators are combined by means of the algebraical operations shown above, the resulting operator is still a *pylops.LinearOperator* operator. This means that we can still apply any of the methods implemented in the original scipy class definition like `*`, as well as those in our class definition like `/`

```
Dop1 = Dop - Dop.conj()

y = Dop1 * x
print(f"x = (Dop - conj(Dop))/y = {Dop1 / y}")
```

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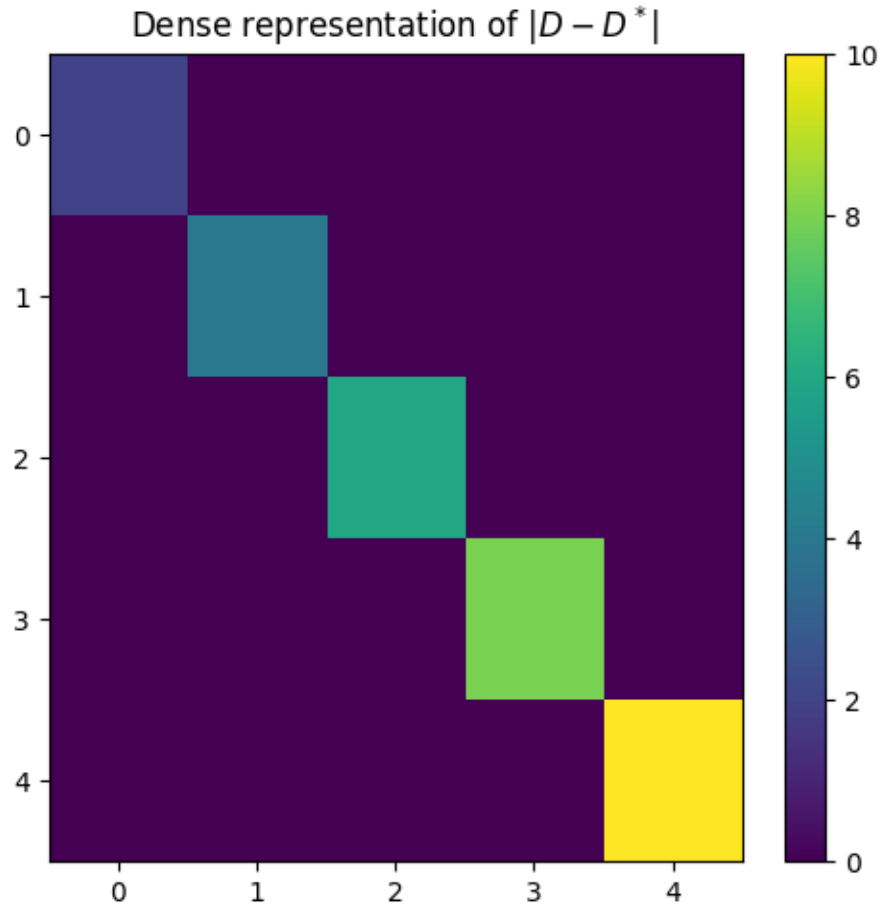
```
D1 = Dop1.todense()

plt.figure(figsize=(5, 5))
plt.imshow(np.abs(D1))
plt.title(r"Dense representation of $|D - D^*|$")
plt.axis("tight")
plt.colorbar()
plt.tight_layout()
```




```
x = (Dop - conj(Dop))/y = [1.+0.j 1.+0.j 1.+0.j 1.+0.j 1.+0.j]
```

Finally, another important feature of PyLops linear operators is that we can always keep track of how many times the forward and adjoint passes have been applied (and reset when needed). This is particularly useful when running a third party solver to see how many evaluations of our operator are performed inside the solver.

```
Dop = pylops.Diagonal(d)

y = Dop.matvec(x)
y = Dop.matvec(x)
y = Dop.rmatvec(y)
```

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```
print(f"Forward evaluations: {Dop.matvec_count}")
print(f"Adjoint evaluations: {Dop.rmatvec_count}")

# Reset
Dop.reset_count()
print(f"Forward evaluations: {Dop.matvec_count}")
print(f"Adjoint evaluations: {Dop.rmatvec_count}")
```

```
Forward evaluations: 2
Adjoint evaluations: 1
Forward evaluations: 0
Adjoint evaluations: 0
```

This first tutorial is completed. You have seen the basic operations that can be performed using `scipy.sparse.linalg.LinearOperator` and our overload of such a class *pylops.LinearOperator* and you should be able to get started combining various PyLops operators and solving your own inverse problems.

Total running time of the script: (0 minutes 1.049 seconds)

3.4.2 02. The Dot-Test

One of the most important aspect of writing a *Linear operator* is to be able to verify that the code implemented in *forward mode* and the code implemented in *adjoint mode* are effectively adjoint to each other. If this is the case, your Linear operator will successfully pass the so-called **dot-test**. Refer to the *Notes* section of *pylops.utils.dottest*) for a more detailed description.

In this example, I will show you how to use the dot-test for a variety of operator when model and data are either real or complex numbers.

```
import matplotlib.gridspec as pltgs
import matplotlib.pyplot as plt

# pylint: disable=C0103
import numpy as np

import pylops
from pylops.utils import dottest

plt.close("all")
```

Let's start with something very simple. We will make a *pylops.MatrixMult* operator and verify that its implementation passes the dot-test. For this time, we will do this step-by-step, replicating what happens in the *pylops.utils.dottest* routine.

```
N, M = 5, 3
Mat = np.arange(N * M).reshape(N, M)
Op = pylops.MatrixMult(Mat)

v = np.random.randn(N)
u = np.random.randn(M)

# Op * u
```

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```
y = Op.matvec(u)
# Op'* v
x = Op.rmatvec(v)

yy = np.dot(y, v)  # (Op  * u)' * v
xx = np.dot(u, x)  # u' * (Op' * v)

print(f"Dot-test {np.abs((yy - xx) / ((yy + xx + 1e-15) / 2)):.2e}")
```

```
Dot-test 1.37e-16
```

And here is a visual intepretation of what a dot-test is

```
gs = pltgs.GridSpec(1, 9)
fig = plt.figure(figsize=(7, 3))
ax = plt.subplot(gs[0, 0:2])
ax.imshow(Op.A, cmap="rainbow")
ax.set_title(r"$(Op*$", size=20, fontweight="bold")
ax.set_xticks(np.arange(M - 1) + 0.5)
ax.set_yticks(np.arange(N - 1) + 0.5)
ax.grid(linewidth=3, color="white")
ax.xaxis.set_ticklabels([])
ax.yaxis.set_ticklabels([])
ax.axis("tight")
ax = plt.subplot(gs[0, 2])
ax.imshow(u[:, np.newaxis], cmap="rainbow")
ax.set_title(r"$u)^T$", size=20, fontweight="bold")
ax.set_xticks([])
ax.set_yticks(np.arange(M - 1) + 0.5)
ax.grid(linewidth=3, color="white")
ax.xaxis.set_ticklabels([])
ax.yaxis.set_ticklabels([])
ax.axis("tight")
ax = plt.subplot(gs[0, 3])
ax.imshow(v[:, np.newaxis], cmap="rainbow")
ax.set_title(r"$v$", size=20, fontweight="bold")
ax.set_xticks([])
ax.set_yticks(np.arange(N - 1) + 0.5)
ax.grid(linewidth=3, color="white")
ax.xaxis.set_ticklabels([])
ax.yaxis.set_ticklabels([])
ax = plt.subplot(gs[0, 4])
ax.text(
    0.35,
    0.5,
    "=",
    horizontalalignment="center",
    verticalalignment="center",
    size=40,
    fontweight="bold",
)
ax.axis("off")
```

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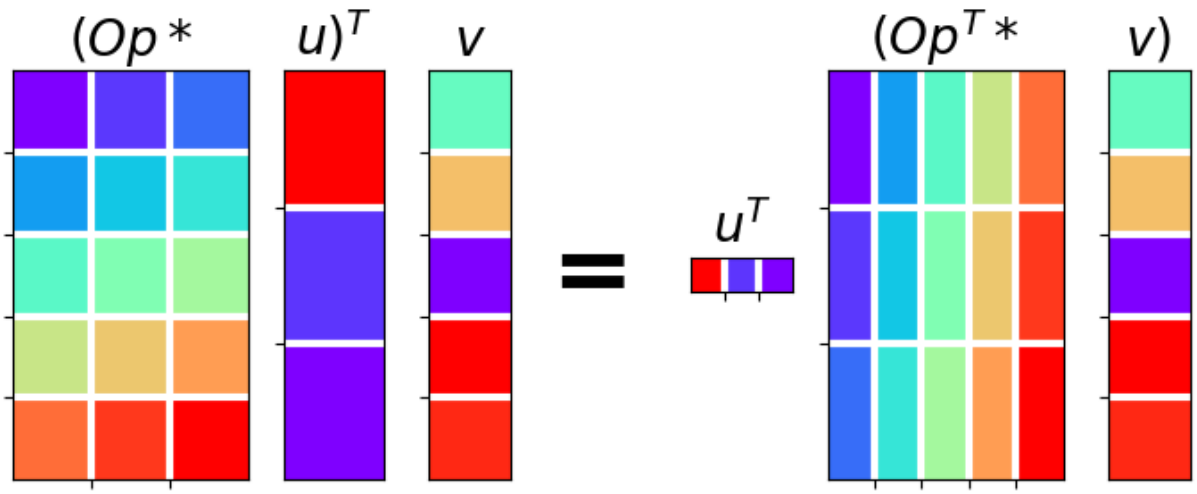
```
ax = plt.subplot(gs[0, 5])
ax.imshow(u[:, np.newaxis].T, cmap="rainbow")
ax.set_title(r"$u^T$", size=20, fontweight="bold")
ax.set_xticks(np.arange(M - 1) + 0.5)
ax.set_yticks([])
ax.grid(linewidth=3, color="white")
ax.xaxis.set_ticklabels([])
ax.yaxis.set_ticklabels([])
ax = plt.subplot(gs[0, 6:8])
ax.imshow(Op.A.T, cmap="rainbow")
ax.set_title(r"$(Op^T*$", size=20, fontweight="bold")
ax.set_xticks(np.arange(N - 1) + 0.5)
ax.set_yticks(np.arange(M - 1) + 0.5)
ax.grid(linewidth=3, color="white")
ax.xaxis.set_ticklabels([])
ax.yaxis.set_ticklabels([])
ax.axis("tight")
ax = plt.subplot(gs[0, 8])
ax.imshow(v[:, np.newaxis], cmap="rainbow")
ax.set_title(r"$v)$", size=20, fontweight="bold")
ax.set_xticks([])
ax.set_yticks(np.arange(N - 1) + 0.5)
ax.grid(linewidth=3, color="white")
ax.xaxis.set_ticklabels([])
ax.yaxis.set_ticklabels([])
plt.tight_layout()
```



From now on, we can simply use the *pylops.utils.dottest* implementation of the dot-test and pass the operator we would like to validate, its size in the model and data spaces and optionally the tolerance we will be accepting for the dot-test to be considered succesfull. Finally we need to specify if our data or/and model vectors contain complex numbers using the `complexflag` parameter. While the dot-test will return `True` when succesfull and `False` otherwise, we can also ask to print its outcome putting the `verb` parameters to `True`.

```
N = 10
d = np.arange(N)
```

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```
Dop = pylops.Diagonal(d)

_ = dottest(Dop, N, N, rtol=1e-6, complexflag=0, verb=True)
```

```
Dot test passed, v^H(Opu)=-6.91965436475742 - u^H(Op^Hv)=-6.91965436475742
```

We move now to a more complicated operator, the *pylops.signalprocessing.FFT* operator. We use once again the *pylops.utils.dottest* to verify its implementation and since we are dealing with a transform that can be applied to both real and complex array, we try different combinations using the `complexflag` input.

```
dt = 0.005
nt = 100
nfft = 2**10

FFTop = pylops.signalprocessing.FFT(
    dims=(nt,), nfft=nfft, sampling=dt, dtype=np.complex128
)
dottest(FFTop, nfft, nt, complexflag=2, verb=True)
_ = dottest(FFTop, nfft, nt, complexflag=3, verb=True)
```

```
Dot test passed, v^H(Opu)=(-17.13910350422714+3.8288339959112507j) - u^H(Op^Hv)=(-17.
↪139103504227144+3.8288339959112503j)
Dot test passed, v^H(Opu)=(11.42080013111478-3.688151329246325j) - u^H(Op^Hv)=(11.
↪420800131114781-3.6881513292463266j)
```

Total running time of the script: (0 minutes 0.266 seconds)

3.4.3 03. Solvers

This tutorial will guide you through the `pylops.optimization` module and show how to use various solvers that are included in the PyLops library.

The main idea here is to provide the user of PyLops with very high-level functionalities to quickly and easily set up and solve complex systems of linear equations as well as include regularization and/or preconditioning terms (all of those constructed by means of PyLops linear operators).

To make this tutorial more interesting, we will present a real life problem and show how the choice of the solver and regularization/preconditioning terms is vital in many circumstances to successfully retrieve an estimate of the model. The problem that we are going to consider is generally referred to as the *data reconstruction* problem and aims at reconstructing a regularly sampled signal of size M from N randomly selected samples:

$$\mathbf{y} = \mathbf{R}\mathbf{x}$$

where the restriction operator $\mathbf{R}$ that selects the M elements from $\mathbf{x}$ at random locations is implemented using *pylops.Restriction*, and

$$\mathbf{y} = [y_1, y_2, \ldots, y_N]^T, \qquad \mathbf{x} = [x_1, x_2, \ldots, x_M]^T,$$

with $M \gg N$.

```
import matplotlib.pyplot as plt
```

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```
# pylint: disable=C0103
import numpy as np

import pylops

plt.close("all")
np.random.seed(10)
```

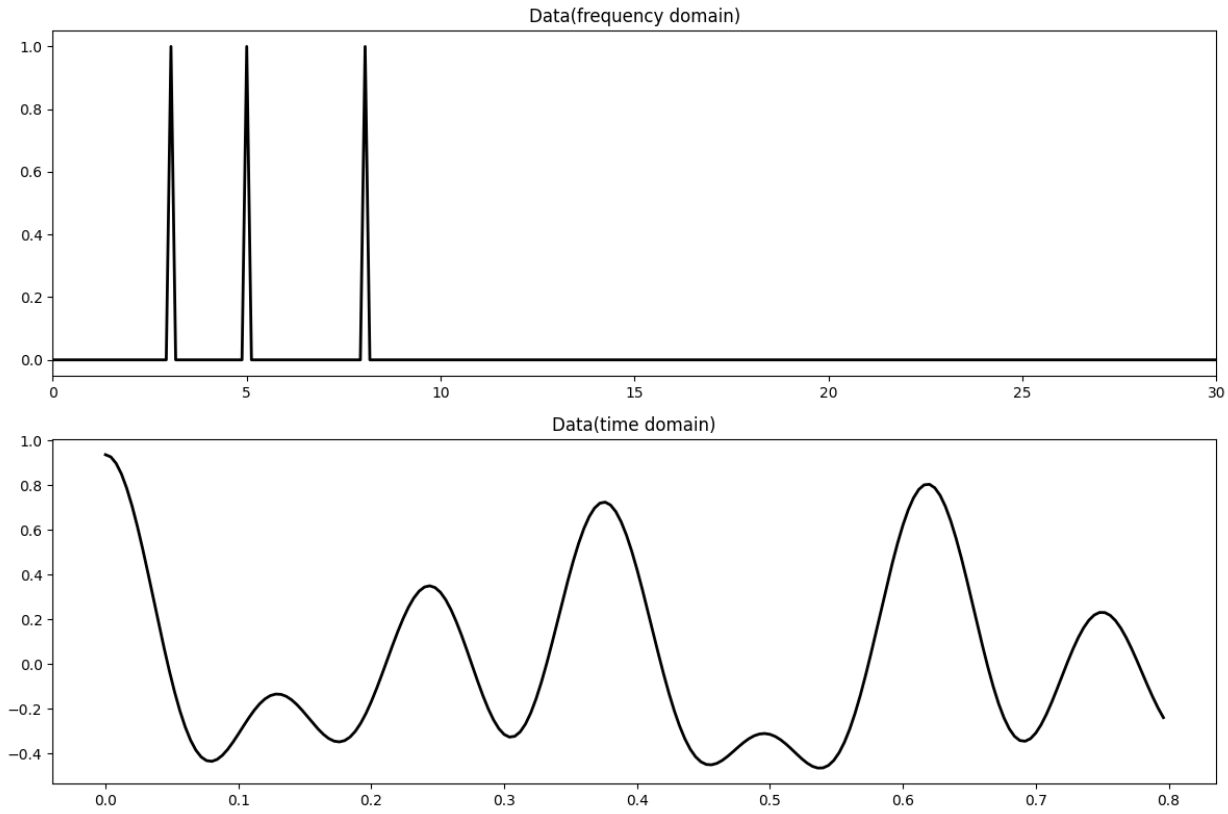
Let's first create the data in the frequency domain. The data is composed by the superposition of 3 sinusoids with different frequencies.

```
# Signal creation in frequency domain
ifreqs = [41, 25, 66]
amps = [1.0, 1.0, 1.0]
N = 200
nfft = 2**11
dt = 0.004
t = np.arange(N) * dt
f = np.fft.rfftfreq(nfft, dt)

FFTop = 10 * pylops.signalprocessing.FFT(N, nfft=nfft, real=True)

X = np.zeros(nfft // 2 + 1, dtype="complex128")
X[ifreqs] = amps
x = FFTop.H * X

fig, axs = plt.subplots(2, 1, figsize=(12, 8))
axs[0].plot(f, np.abs(X), "k", lw=2)
axs[0].set_xlim(0, 30)
axs[0].set_title("Data(frequency domain)")
axs[1].plot(t, x, "k", lw=2)
axs[1].set_title("Data(time domain)")
axs[1].axis("tight")
plt.tight_layout()
```



We now define the locations at which the signal will be sampled.

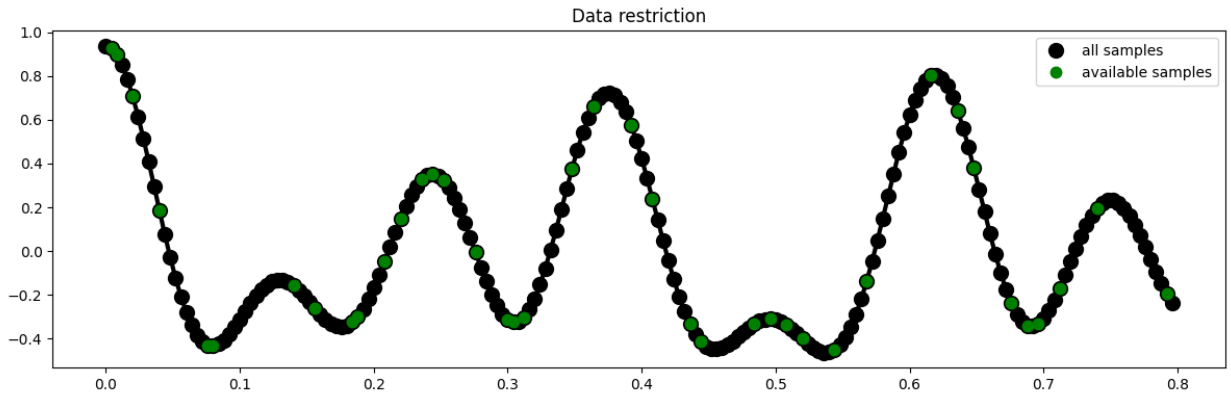
```
# subsampling locations
perc_subsampling = 0.2
Nsub = int(np.round(N * perc_subsampling))

iava = np.sort(np.random.permutation(np.arange(N))[:Nsub])

# Create restriction operator
Rop = pylops.Restriction(N, iava, dtype="float64")

y = Rop * x
ymask = Rop.mask(x)

# Visualize data
fig = plt.figure(figsize=(12, 4))
plt.plot(t, x, "k", lw=3)
plt.plot(t, x, ".k", ms=20, label="all samples")
plt.plot(t, ymask, ".g", ms=15, label="available samples")
plt.legend()
plt.title("Data restriction")
plt.tight_layout()
```



To start let's consider the simplest *'solver'*, i.e., *least-square inversion without regularization*. We aim here to minimize the following cost function:

$$J = \|\mathbf{y} - \mathbf{R}\mathbf{x}\|_2^2$$

Depending on the choice of the operator $\mathbf{R}$, such problem can be solved using explicit matrix solvers as well as iterative solvers. In this case we will be using the latter approach (more specifically the scipy implementation of the *LSQR* solver - i.e., `scipy.sparse.linalg.lsqr`) as we do not want to explicitly create and invert a matrix. In most cases this will be the only viable approach as most of the large-scale optimization problems that we are interested to solve using PyLops do not lend naturally to the creation and inversion of explicit matrices.

This first solver can be very easily implemented using the `/` for PyLops operators, which will automatically call the `scipy.sparse.linalg.lsqr` with some default parameters.

```
xinv = Rop / y
```

We can also use *`pylops.optimization.leastsquares.regularized_inversion`* (without regularization term for now) and customize our solvers using `kwargs`.

```
xinv = pylops.optimization.leastsquares.regularized_inversion(
    Rop, y, [], **dict(damp=0, iter_lim=10, show=True)
)[0]
```

```
RegularizedInversion
-----------------------------------------------------------------
The Operator Op has 40 rows and 200 cols
Regs=[]
epsRs=[]
-----------------------------------------------------------------

LSQR            Least-squares solution of  Ax = b
The matrix A has 40 rows and 200 columns
damp = 0.00000000000000e+00   calc_var =        0
atol = 1.00e-06                 conlim = 1.00e+08
btol = 1.00e-06               iter_lim =       10

   Itn      x[0]       r1norm     r2norm   Compatible    LS      Norm A   Cond A
     0  0.00000e+00   2.658e+00  2.658e+00    1.0e+00  3.8e-01
```

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```
     1  0.00000e+00   0.000e+00  0.000e+00    0.0e+00  0.0e+00   0.0e+00  0.0e+00

LSQR finished
Ax - b is small enough, given atol, btol

istop =       1   r1norm = 0.0e+00   anorm = 0.0e+00   arnorm = 0.0e+00
itn   =       1   r2norm = 0.0e+00   acond = 0.0e+00   xnorm  = 2.7e+00
```

Finally we can select a different starting guess from the null vector

```
xinv_fromx0 = pylops.optimization.leastsquares.regularized_inversion(
    Rop, y, [], x0=np.ones(N), **dict(damp=0, iter_lim=10, show=True)
)[0]
```

```
RegularizedInversion
-----------------------------------------------------------------
The Operator Op has 40 rows and 200 cols
Regs=[]
epsRs=[]
-----------------------------------------------------------------

LSQR            Least-squares solution of  Ax = b
The matrix A has 40 rows and 200 columns
damp = 0.00000000000000e+00   calc_var =        0
atol = 1.00e-06                 conlim = 1.00e+08
btol = 1.00e-06               iter_lim =       10

   Itn      x[0]       r1norm     r2norm   Compatible    LS      Norm A   Cond A
     0  0.00000e+00   6.737e+00  6.737e+00    1.0e+00  1.5e-01
     1  0.00000e+00   0.000e+00  0.000e+00    0.0e+00  0.0e+00   0.0e+00  0.0e+00

LSQR finished
Ax - b is small enough, given atol, btol

istop =       1   r1norm = 0.0e+00   anorm = 0.0e+00   arnorm = 0.0e+00
itn   =       1   r2norm = 0.0e+00   acond = 0.0e+00   xnorm  = 6.7e+00
```

The cost function above can be also expanded in terms of its *normal equations*

$$\mathbf{x}_{ne} = (\mathbf{R}^T\mathbf{R})^{-1}\mathbf{R}^T\mathbf{y}$$

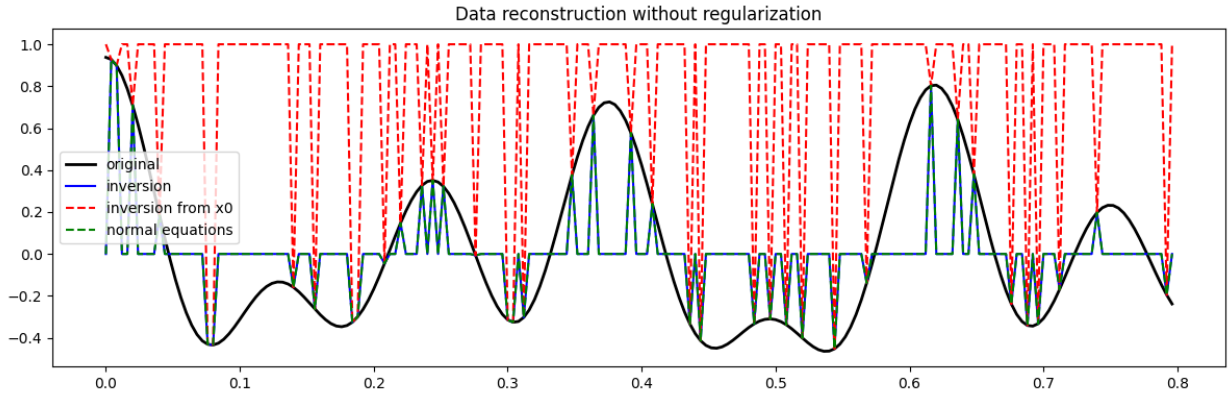
The method *pylops.optimization.leastsquares.normal_equations_inversion* implements such system of equations explicitly and solves them using an iterative scheme suitable for square matrices (i.e., $M = N$).

While this approach may seem not very useful, we will soon see how regularization terms could be easily added to the normal equations using this method.

```
xne = pylops.optimization.leastsquares.normal_equations_inversion(Rop, y, [])[0]
```

Let's now visualize the different inversion results

```
fig = plt.figure(figsize=(12, 4))
plt.plot(t, x, "k", lw=2, label="original")
plt.plot(t, xinv, "b", ms=10, label="inversion")
plt.plot(t, xinv_fromx0, "--r", ms=10, label="inversion from x0")
plt.plot(t, xne, "--g", ms=10, label="normal equations")
plt.legend()
plt.title("Data reconstruction without regularization")
plt.tight_layout()
```



Regularization

You may have noticed that none of the inversion has been successfull in recovering the original signal. This is a clear indication that the problem we are trying to solve is highly ill-posed and requires some prior knowledge from the user.

We will now see how to add prior information to the inverse process in the form of regularization (or preconditioning). This can be done in two different ways

- regularization via *pylops.optimization.leastsquares.normal_equations_inversion* or *pylops.optimization.leastsquares.regularized_inversion*)
- preconditioning via *pylops.optimization.leastsquares.preconditioned_inversion*

Let's start by regularizing the normal equations using a second derivative operator

$$\mathbf{x} = (\mathbf{R^T R} + \epsilon_\nabla^2 \nabla^T \nabla)^{-1} \mathbf{R^T y}$$

```
# Create regularization operator
D2op = pylops.SecondDerivative(N, dtype="float64")

# Regularized inversion
epsR = np.sqrt(0.1)
epsI = np.sqrt(1e-4)

xne = pylops.optimization.leastsquares.normal_equations_inversion(
    Rop, y, [D2op], epsI=epsI, epsRs=[epsR], **dict(maxiter=50)
)[0]
```

Note that in case we have access to a fast implementation for the chain of forward and adjoint for the regularization operator (i.e., $\nabla^T\nabla$), we can modify our call to *pylops.optimization.leastsquares.normal_equations_inversion* as follows:

```
ND2op = pylops.MatrixMult((D2op.H * D2op).tosparse())  # mimic fast D^T D

xne1 = pylops.optimization.leastsquares.normal_equations_inversion(
    Rop, y, [], NRegs=[ND2op], epsI=epsI, epsNRs=[epsR], **dict(maxiter=50)
)[0]
```

We can do the same while using *pylops.optimization.leastsquares.regularized_inversion* which solves the following augmented problem

$$\begin{bmatrix} \mathbf{R} \\ \epsilon_\nabla \nabla \end{bmatrix} \mathbf{x} = \begin{bmatrix} \mathbf{y} \\ 0 \end{bmatrix}$$

```
xreg = pylops.optimization.leastsquares.regularized_inversion(
    Rop,
    y,
    [D2op],
    epsRs=[np.sqrt(0.1)],
    **dict(damp=np.sqrt(1e-4), iter_lim=50, show=0)
)[0]
```

We can also write a preconditioned problem, whose cost function is

$$J = \|\mathbf{y} - \mathbf{R}\mathbf{P}\mathbf{p}\|_2^2$$

where $\mathbf{P}$ is the precondioned operator, $\mathbf{p}$ is the projected model in the preconditioned space, and $\mathbf{x} = \mathbf{P}\mathbf{p}$ is the model in the original model space we want to solve for. Note that a preconditioned problem converges much faster to its solution than its corresponding regularized problem. This can be done using the routine *pylops.optimization.leastsquares.preconditioned_inversion*.

```
# Create regularization operator
Sop = pylops.Smoothing1D(nsmooth=11, dims=[N], dtype="float64")

# Invert for interpolated signal
xprec = pylops.optimization.leastsquares.preconditioned_inversion(
    Rop, y, Sop, **dict(damp=np.sqrt(1e-9), iter_lim=20, show=0)
)[0]
```

Let's finally visualize these solutions

```
# sphinx_gallery_thumbnail_number=4
fig = plt.figure(figsize=(12, 4))
plt.plot(t[iava], y, ".k", ms=20, label="available samples")
plt.plot(t, x, "k", lw=3, label="original")
plt.plot(t, xne, "b", lw=3, label="normal equations")
plt.plot(t, xne1, "--c", lw=3, label="normal equations (with direct D^T D)")
plt.plot(t, xreg, "-.r", lw=3, label="regularized")
plt.plot(t, xprec, "--g", lw=3, label="preconditioned equations")
```

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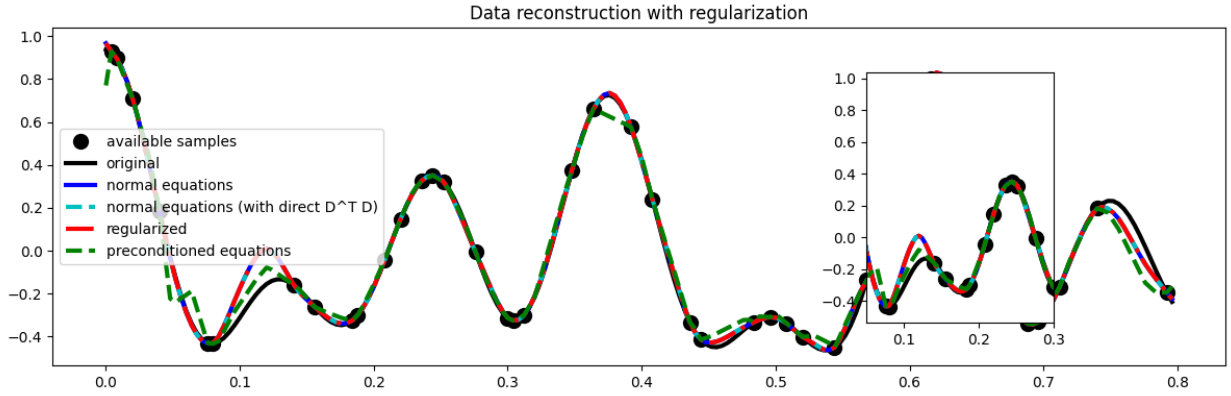
```
plt.legend()
plt.title("Data reconstruction with regularization")

subax = fig.add_axes([0.7, 0.2, 0.15, 0.6])
subax.plot(t[iava], y, ".k", ms=20)
subax.plot(t, x, "k", lw=3)
subax.plot(t, xne, "b", lw=3)
subax.plot(t, xne1, "--c", lw=3)
subax.plot(t, xreg, "-.r", lw=3)
subax.plot(t, xprec, "--g", lw=3)
subax.set_xlim(0.05, 0.3)
plt.tight_layout()
```



Much better estimates! We have seen here how regularization and/or preconditioning can be vital to succesfully solve some ill-posed inverse problems.

We have however so far only considered solvers that can include additional norm-2 regularization terms. A very active area of research is that of *sparsity-promoting* solvers (also sometimes referred to as *compressive sensing*): the regularization term added to the cost function to minimize has norm-p ($p \leq 1$) and the problem is generally recasted by considering the model to be sparse in some domain. We can follow this philosophy as our signal to invert was actually created as superposition of 3 sinusoids (i.e., three spikes in the Fourier domain). Our new cost function is:

$$J_1 = \|\mathbf{y} - \mathbf{RFp}\|_2^2 + \epsilon\|\mathbf{p}\|_1$$

where $\mathbf{F}$ is the FFT operator. We will thus use the *pylops.optimization.sparsity.ista* and *pylops.optimization.sparsity.fista* solvers to estimate our input signal.

```
pista, niteri, costi = pylops.optimization.sparsity.ista(
    Rop * FFTop.H,
    y,
    niter=1000,
    eps=0.1,
    tol=1e-7,
)
xista = FFTop.H * pista

pfista, niterf, costf = pylops.optimization.sparsity.fista(
```

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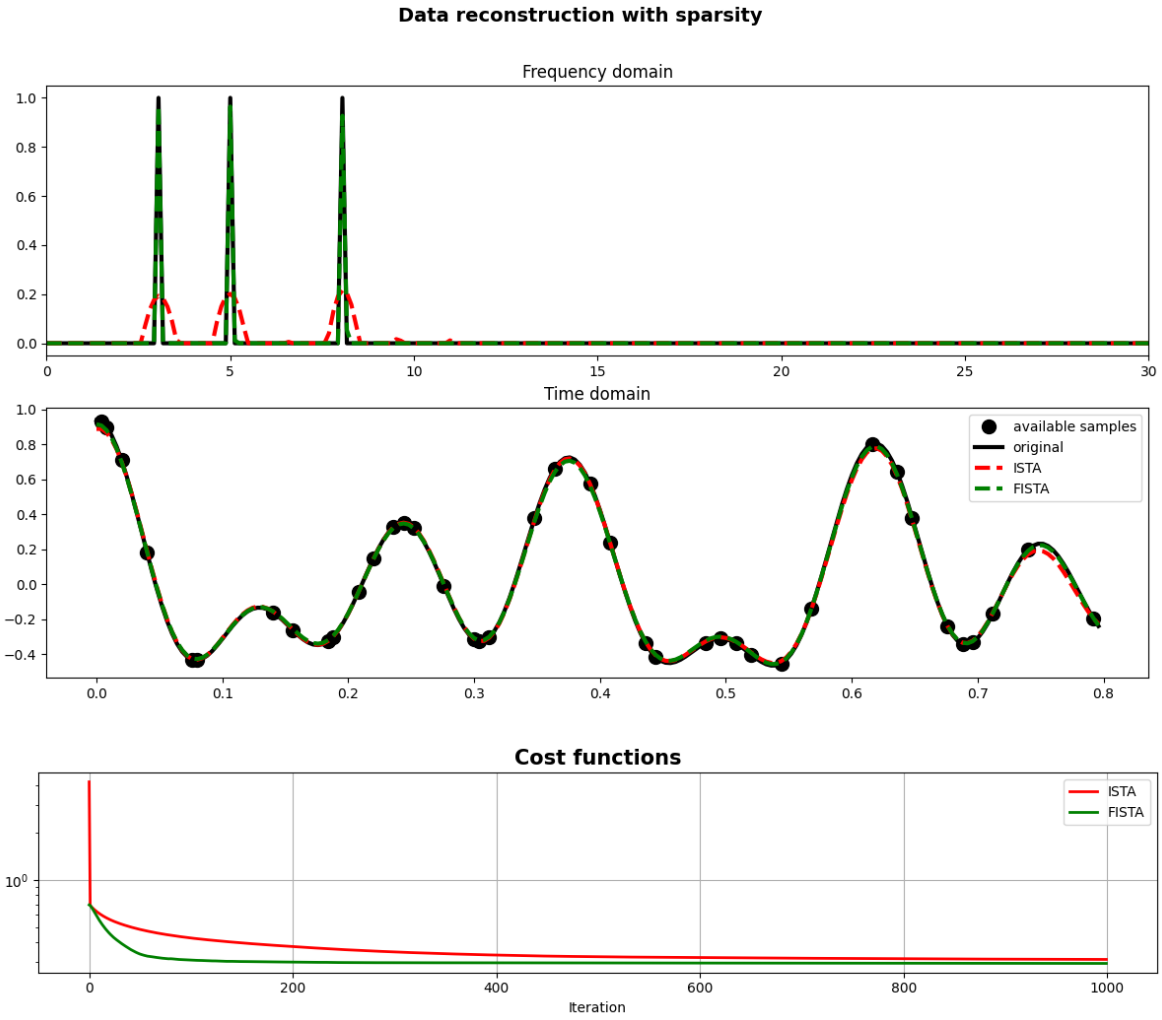
```
    Rop * FFTop.H,
    y,
    niter=1000,
    eps=0.1,
    tol=1e-7,
)
xfista = FFTop.H * pfista

fig, axs = plt.subplots(2, 1, figsize=(12, 8))
fig.suptitle("Data reconstruction with sparsity", fontsize=14, fontweight="bold", y=0.9)
axs[0].plot(f, np.abs(X), "k", lw=3)
axs[0].plot(f, np.abs(pista), "--r", lw=3)
axs[0].plot(f, np.abs(pfista), "--g", lw=3)
axs[0].set_xlim(0, 30)
axs[0].set_title("Frequency domain")
axs[1].plot(t[iava], y, ".k", ms=20, label="available samples")
axs[1].plot(t, x, "k", lw=3, label="original")
axs[1].plot(t, xista, "--r", lw=3, label="ISTA")
axs[1].plot(t, xfista, "--g", lw=3, label="FISTA")
axs[1].set_title("Time domain")
axs[1].axis("tight")
axs[1].legend()
plt.tight_layout()
plt.subplots_adjust(top=0.8)

fig, ax = plt.subplots(1, 1, figsize=(12, 3))
ax.semilogy(costi, "r", lw=2, label="ISTA")
ax.semilogy(costf, "g", lw=2, label="FISTA")
ax.set_title("Cost functions", size=15, fontweight="bold")
ax.set_xlabel("Iteration")
ax.legend()
ax.grid(True)
plt.tight_layout()
```



As you can see, changing parametrization of the model and imposing sparsity in the Fourier domain has given an extra improvement to our ability of recovering the underlying densely sampled input signal. Moreover, FISTA converges much faster than ISTA as expected and should be preferred when using sparse solvers.

Finally we consider a slightly different cost function (note that in this case we try to solve a constrained problem):

$$J_1 = \|\mathbf{p}\|_1 \quad \text{subject to} \quad \|\mathbf{y} - \mathbf{RFp}\|$$

A very popular solver to solve such kind of cost function is called *spgl1* and can be accessed via *pylops.optimization.sparsity.spgl1*.

```
xspgl1, pspgl1, info = pylops.optimization.sparsity.spgl1(
    Rop, y, SOp=FFTop, tau=3, iter_lim=200
)

fig, axs = plt.subplots(2, 1, figsize=(12, 8))
fig.suptitle("Data reconstruction with SPGL1", fontsize=14, fontweight="bold", y=0.9)
```

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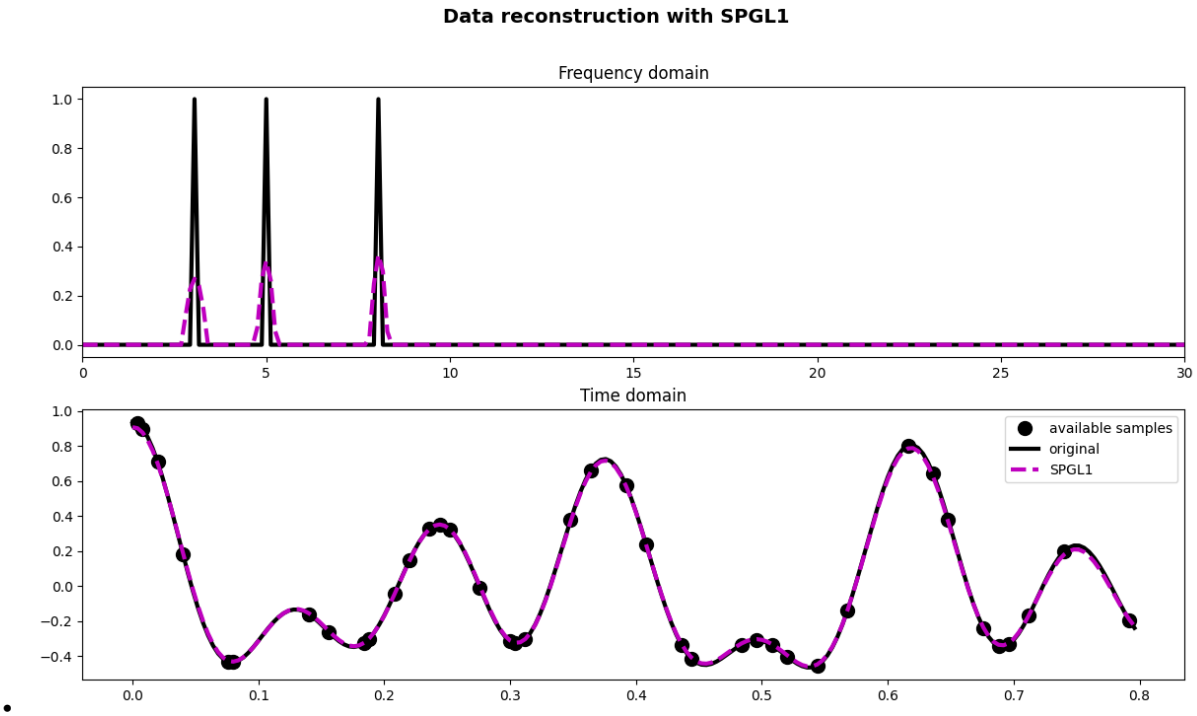
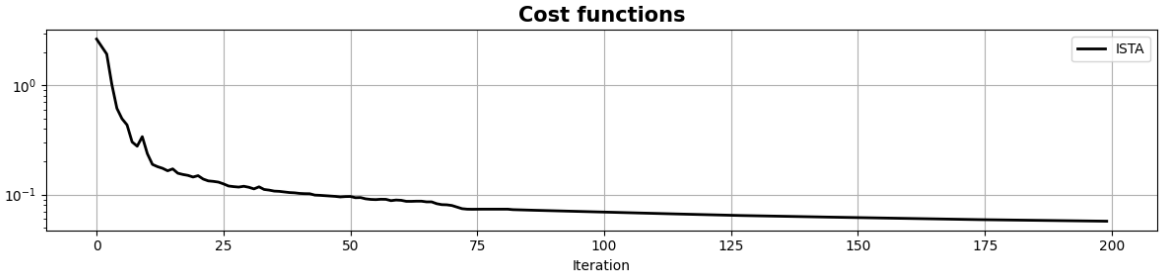
```
axs[0].plot(f, np.abs(X), "k", lw=3)
axs[0].plot(f, np.abs(pspgl1), "--m", lw=3)
axs[0].set_xlim(0, 30)
axs[0].set_title("Frequency domain")
axs[1].plot(t[iava], y, ".k", ms=20, label="available samples")
axs[1].plot(t, x, "k", lw=3, label="original")
axs[1].plot(t, xspgl1, "--m", lw=3, label="SPGL1")
axs[1].set_title("Time domain")
axs[1].axis("tight")
axs[1].legend()
plt.tight_layout()
plt.subplots_adjust(top=0.8)

fig, ax = plt.subplots(1, 1, figsize=(12, 3))
ax.semilogy(info["rnorm2"], "k", lw=2, label="ISTA")
ax.set_title("Cost functions", size=15, fontweight="bold")
ax.set_xlabel("Iteration")
ax.legend()
ax.grid(True)
plt.tight_layout()
```





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Total running time of the script: (0 minutes 3.996 seconds)

3.4.4 03. Solvers (Advanced)

This is a follow up tutorial to the *03. Solvers* tutorial. The same example will be considered, however we will showcase how to use the class-based version of our solvers (introduced in PyLops v2).

First of all, when shall you use class-based solvers over function-based ones? The answer is simple, every time you feel you would have like to have more flexibility when using one PyLops function-based solvers.

In fact, a function-based solver in PyLops v2 is nothing more than a thin wrapper over its class-based equivalent, which generally performs the following steps:

- solver initialization
- `setup`
- `run` (by calling multiple times `step`)
- `finalize`

The nice thing about class-based solvers is that i) a user can manually orchestrate these steps and do anything in between them; ii) a user can create a class-based *pylops.optimization.callback.Callbacks* object and define a set of callbacks that will be run pre and post setup, step and run. One example of how such callbacks can be handy to track evolving variables in the solver can be found in *Linear Regression*.

In the following we will leverage the very same mechanism to keep track of a number of metrics using the predefined *pylops.optimization.callback.MetricsCallback* callback. Finally we show how to create a customized callback that can track the percentage change of the solution and residual. This is of course just an example, we expect users will find different use cases based on the problem at hand.

```
import matplotlib.pyplot as plt

# pylint: disable=C0103
import numpy as np

import pylops

plt.close("all")
np.random.seed(10)
```

Let's first create the data in the frequency domain. The data is composed by the superposition of 3 sinusoids with different frequencies.

```
# Signal creation in frequency domain
ifreqs = [41, 25, 66]
amps = [1.0, 1.0, 1.0]
```

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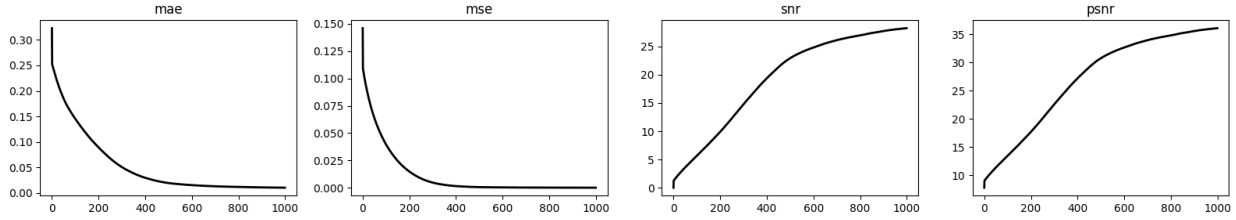
```
N = 200
nfft = 2**11
dt = 0.004
t = np.arange(N) * dt
f = np.fft.rfftfreq(nfft, dt)

FFTop = 10 * pylops.signalprocessing.FFT(N, nfft=nfft, real=True)

X = np.zeros(nfft // 2 + 1, dtype="complex128")
X[ifreqs] = amps
x = FFTop.H * X
```

We now define the locations at which the signal will be sampled.

```
# subsampling locations
perc_subsampling = 0.2
Nsub = int(np.round(N * perc_subsampling))

iava = np.sort(np.random.permutation(np.arange(N))[:Nsub])

# Create restriction operator
Rop = pylops.Restriction(N, iava, dtype="float64")

y = Rop * x
ymask = Rop.mask(x)
```

Let's now solve the interpolation problem using the `pylops.optimization.sparsity.ISTA` class-based solver.

```
cb = pylops.optimization.callback.MetricsCallback(x, FFTop.H)

istasolve = pylops.optimization.sparsity.ISTA(
    Rop * FFTop.H,
    callbacks=[
        cb,
    ],
)
pista, niteri, costi = istasolve.solve(y, niter=1000, eps=0.1, tol=1e-7)
xista = FFTop.H * pista

fig, axs = plt.subplots(1, 4, figsize=(16, 3))
for i, metric in enumerate(["mae", "mse", "snr", "psnr"]):
    axs[i].plot(cb.metrics[metric], "k", lw=2)
    axs[i].set_title(metric)
plt.tight_layout()
```



Finally, we show how we can also define customized callbacks. What we are really interested in here is to store the first residual norm once the setup of the solver is over, and repeat the same after each step (using the previous estimate to compute the percentage change). And, we do the same for the solution norm.

```
class CallbackISTA(pylops.optimization.callback.Callbacks):
    def __init__(self):
        self.res_perc = []
        self.x_perc = []

    def on_setup_end(self, solver, x):
        self.x = x
        if x is not None:
            self.rec = solver.Op @ x - solver.y
        else:
            self.rec = None

    def on_step_end(self, solver, x):
        self.xold = self.x
        self.x = x
        self.recold = self.rec
        self.rec = solver.Op @ x - solver.y
        if self.xold is not None:
            self.x_perc.append(
                100 * np.linalg.norm(self.x - self.xold) / np.linalg.norm(self.xold)
            )
            self.res_perc.append(
                100
                * np.linalg.norm(self.rec - self.recold)
                / np.linalg.norm(self.recold)
            )

    def on_run_end(self, solver, x):
        # remove first percentage
        self.x_perc = np.array(self.x_perc[1:])
        self.res_perc = np.array(self.res_perc[1:])


cb = CallbackISTA()
istasolve = pylops.optimization.sparsity.ISTA(
    Rop * FFTop.H,
    callbacks=[
        cb,
    ],
)
pista, niteri, costi = istasolve.solve(y, niter=1000, eps=0.1, tol=1e-7)
xista = FFTop.H * pista

cbf = CallbackISTA()
fistasolve = pylops.optimization.sparsity.FISTA(
    Rop * FFTop.H,
    callbacks=[
        cbf,
    ],
```

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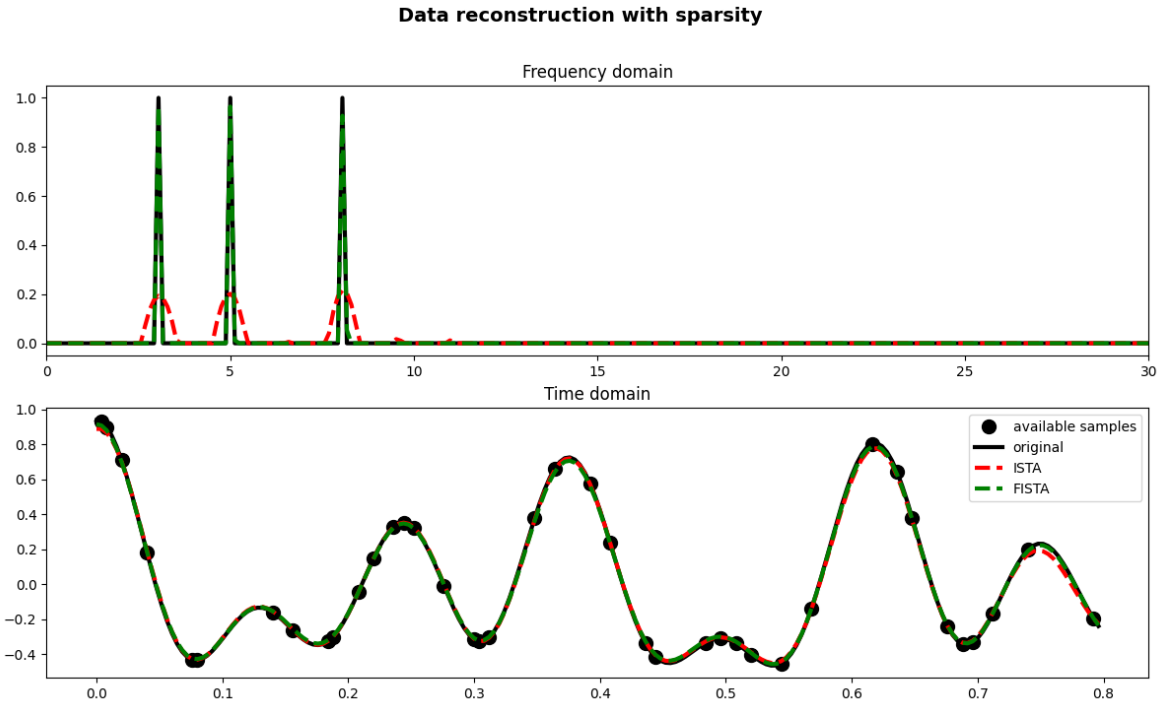
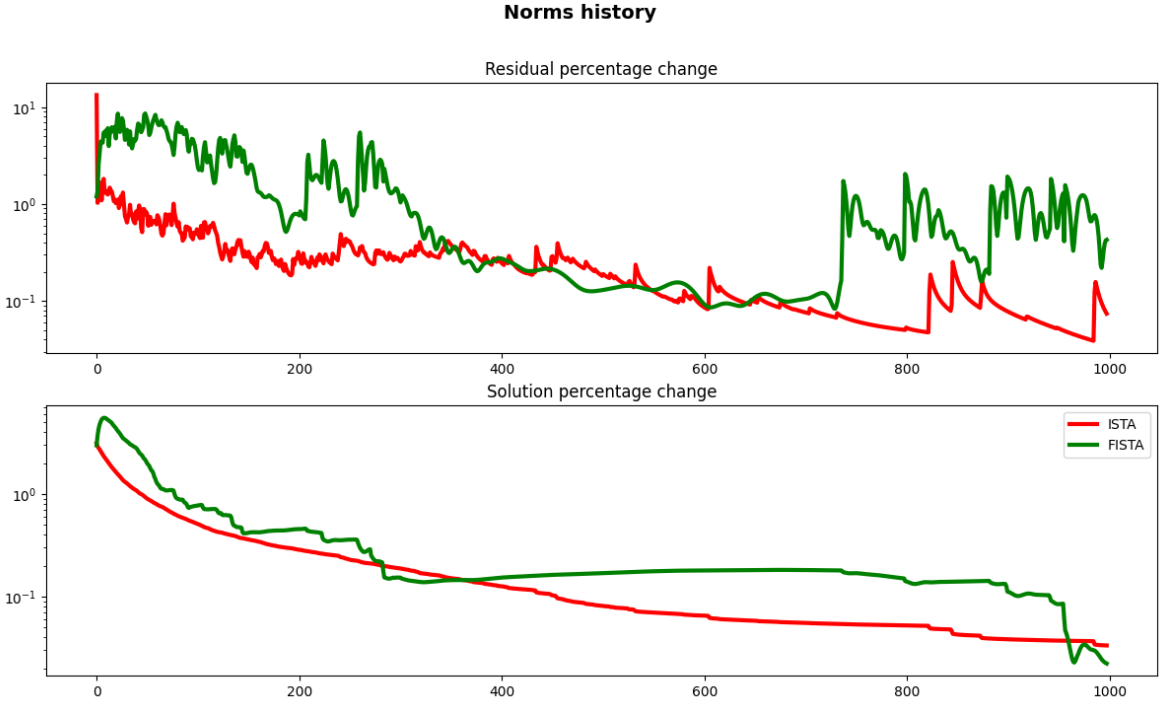
```
)
pfista, niterf, costf = fistasolve.solve(y, niter=1000, eps=0.1, tol=1e-7)
xfista = FFTop.H * pfista

fig, axs = plt.subplots(2, 1, figsize=(12, 8))
fig.suptitle("Data reconstruction with sparsity", fontsize=14, fontweight="bold", y=0.9)
axs[0].plot(f, np.abs(X), "k", lw=3)
axs[0].plot(f, np.abs(pista), "--r", lw=3)
axs[0].plot(f, np.abs(pfista), "--g", lw=3)
axs[0].set_xlim(0, 30)
axs[0].set_title("Frequency domain")
axs[1].plot(t[iava], y, ".k", ms=20, label="available samples")
axs[1].plot(t, x, "k", lw=3, label="original")
axs[1].plot(t, xista, "--r", lw=3, label="ISTA")
axs[1].plot(t, xfista, "--g", lw=3, label="FISTA")
axs[1].set_title("Time domain")
axs[1].axis("tight")
axs[1].legend()
plt.tight_layout()
plt.subplots_adjust(top=0.8)

fig, axs = plt.subplots(2, 1, figsize=(12, 8))
fig.suptitle("Norms history", fontsize=14, fontweight="bold", y=0.9)
axs[0].semilogy(cb.res_perc, "r", lw=3)
axs[0].semilogy(cbf.res_perc, "g", lw=3)
axs[0].set_title("Residual percentage change")
axs[1].semilogy(cb.x_perc, "r", lw=3, label="ISTA")
axs[1].semilogy(cbf.x_perc, "g", lw=3, label="FISTA")
axs[1].set_title("Solution percentage change")
axs[1].legend()
plt.tight_layout()
plt.subplots_adjust(top=0.8)
```





Total running time of the script: (0 minutes 3.739 seconds)

3.4.5 04. Bayesian Inversion

This tutorial focuses on Bayesian inversion, a special type of inverse problem that aims at incorporating prior information in terms of model and data probabilities in the inversion process.

In this case we will be dealing with the same problem that we discussed in *03. Solvers*, but instead of defining ad-hoc regularization or preconditioning terms we parametrize and model our input signal in the frequency domain in a probabilistic fashion: the central frequency, amplitude and phase of the three sinusoids have gaussian distributions as follows:

$$X(f) = \sum_{i=1}^{3} a_i e^{j\phi_i} \delta(f - f_i)$$

where $f_i \sim N(f_{0,i}, \sigma_{f,i})$, $a_i \sim N(a_{0,i}, \sigma_{a,i})$, and $\phi_i \sim N(\phi_{0,i}, \sigma_{\phi,i})$.

Based on the above definition, we construct some prior models in the frequency domain, convert each of them to the time domain and use such an ensemble to estimate the prior mean $\mu_{\mathbf{x}}$ and model covariance $\mathbf{C}_{\mathbf{x}}$.

We then create our data by sampling the true signal at certain locations and solve the resconstruction problem within a Bayesian framework. Since we are assuming gaussianity in our priors, the equation to obtain the posterion mean can be derived analytically:

$$\mathbf{x} = \mathbf{x_0} + \mathbf{C}_x\mathbf{R}^T(\mathbf{R}\mathbf{C}_x\mathbf{R}^T + \mathbf{C}_y)^{-1}(\mathbf{y} - \mathbf{R}\mathbf{x_0})$$

```
import matplotlib.pyplot as plt

# sphinx_gallery_thumbnail_number = 2
import numpy as np
from scipy.sparse.linalg import lsqr

import pylops

plt.close("all")
np.random.seed(10)
```

Let's start by creating our true model and prior realizations

```
def prior_realization(f0, a0, phi0, sigmaf, sigmaa, sigmaphi, dt, nt, nfft):
    """Create realization from prior mean and std for amplitude, frequency and
    phase
    """
    f = np.fft.rfftfreq(nfft, dt)
    df = f[1] - f[0]
    ifreqs = [int(np.random.normal(f, sigma) / df) for f, sigma in zip(f0, sigmaf)]
    amps = [np.random.normal(a, sigma) for a, sigma in zip(a0, sigmaa)]
    phis = [np.random.normal(phi, sigma) for phi, sigma in zip(phi0, sigmaphi)]

    # input signal in frequency domain
    X = np.zeros(nfft // 2 + 1, dtype="complex128")
    X[ifreqs] = (
        np.array(amps).squeeze() * np.exp(1j * np.deg2rad(np.array(phis))).squeeze()
    )

    # input signal in time domain
```

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```
    FFTop = pylops.signalprocessing.FFT(nt, nfft=nfft, real=True)
    x = FFTop.H * X
    return x


# Priors
nreals = 100
f0 = [5, 3, 8]
sigmaf = [0.5, 1.0, 0.6]
a0 = [1.0, 1.0, 1.0]
sigmaa = [0.1, 0.5, 0.6]
phi0 = [-90.0, 0.0, 0.0]
sigmaphi = [0.1, 0.2, 0.4]
sigmad = 1e-2

# Prior models
nt = 200
nfft = 2**11
dt = 0.004
t = np.arange(nt) * dt
xs = np.array(
    [
        prior_realization(f0, a0, phi0, sigmaf, sigmaa, sigmaphi, dt, nt, nfft)
        for _ in range(nreals)
    ]
)

# True model (taken as one possible realization)
x = prior_realization(f0, a0, phi0, [0, 0, 0], [0, 0, 0], [0, 0, 0], dt, nt, nfft)
```

We have now a set of prior models in time domain. We can easily use sample statistics to estimate the prior mean and covariance. For the covariance, we perform a second step where we average values around the main diagonal for each row and find a smooth, compact filter that we use to define a convolution linear operator that mimics the action of the covariance matrix on a vector

```
x0 = np.average(xs, axis=0)
Cm = ((xs - x0).T @ (xs - x0)) / nreals

N = 30  # lenght of decorrelation
diags = np.array([Cm[i, i - N : i + N + 1] for i in range(N, nt - N)])
diag_ave = np.average(diags, axis=0)
# add a taper at the end to avoid edge effects
diag_ave *= np.hamming(2 * N + 1)

fig, ax = plt.subplots(1, 1, figsize=(12, 4))
ax.plot(t, xs.T, "r", lw=1)
ax.plot(t, x0, "g", lw=4)
ax.plot(t, x, "k", lw=4)
ax.set_title("Prior realizations and mean")
ax.set_xlim(0, 0.8)

fig, (ax1, ax2) = plt.subplots(1, 2, figsize=(10, 4))
```

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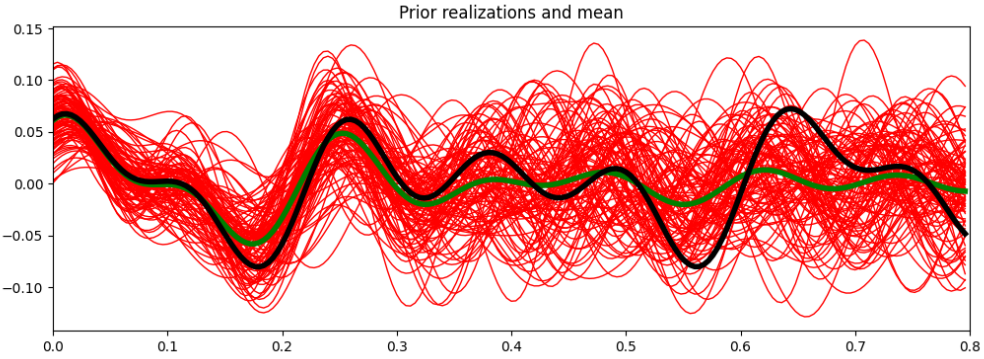
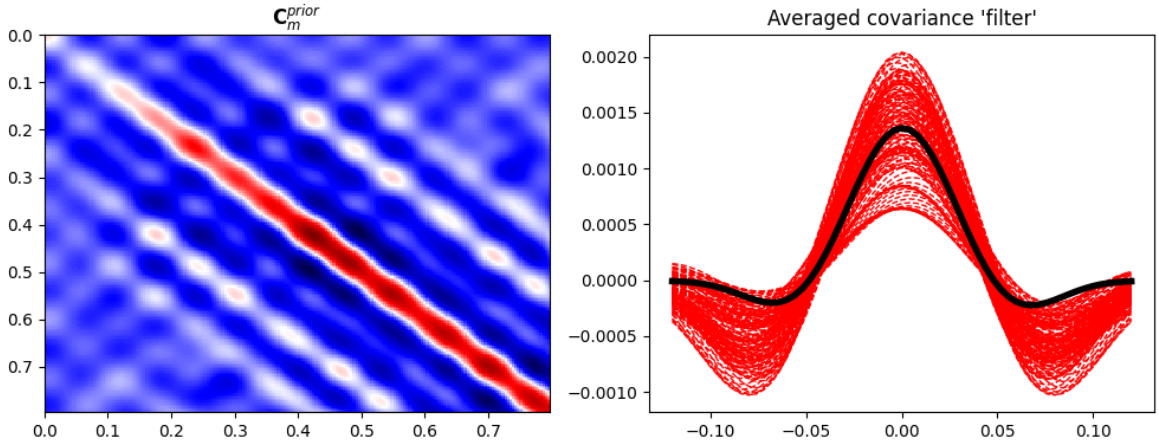
```
im = ax1.imshow(
    Cm, interpolation="nearest", cmap="seismic", extent=(t[0], t[-1], t[-1], t[0])
)
ax1.set_title(r"$\mathbf{C}_m^{prior}$")
ax1.axis("tight")
ax2.plot(np.arange(-N, N + 1) * dt, diags.T, "--r", lw=1)
ax2.plot(np.arange(-N, N + 1) * dt, diag_ave, "k", lw=4)
ax2.set_title("Averaged covariance 'filter'")
plt.tight_layout()
```



-



-

Let's define now the sampling operator as well as create our covariance matrices in terms of linear operators. This may not be strictly necessary here but shows how even Bayesian-type of inversion can very easily scale to large model and data spaces.

```
# Sampling operator
perc_subsampling = 0.2
ntsub = int(np.round(nt * perc_subsampling))
iava = np.sort(np.random.permutation(np.arange(nt))[:ntsub])
iava[-1] = nt - 1  # assume we have the last sample to avoid instability
Rop = pylops.Restriction(nt, iava, dtype="float64")

# Covariance operators
Cm_op = pylops.signalprocessing.Convolve1D(nt, diag_ave, offset=N)
Cd_op = sigmad**2 * pylops.Identity(ntsub)
```

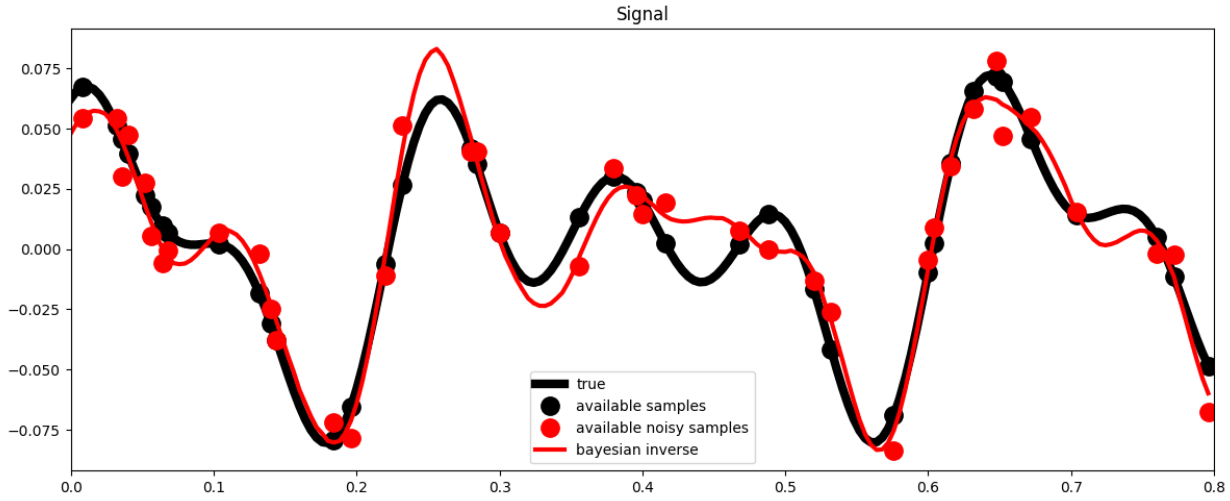
We model now our data and add noise that respects our prior definition

```
n = np.random.normal(0, sigmad, nt)
y = Rop * x
yn = Rop * (x + n)
ymask = Rop.mask(x)
ynmask = Rop.mask(x + n)
```

First we apply the Bayesian inversion equation

```
xbayes = x0 + Cm_op * Rop.H * (
    lsqr(Rop * Cm_op * Rop.H + Cd_op, yn - Rop * x0, iter_lim=400)[0]
)

# Visualize
fig, ax = plt.subplots(1, 1, figsize=(12, 5))
ax.plot(t, x, "k", lw=6, label="true")
ax.plot(t, ymask, ".k", ms=25, label="available samples")
ax.plot(t, ynmask, ".r", ms=25, label="available noisy samples")
ax.plot(t, xbayes, "r", lw=3, label="bayesian inverse")
ax.legend()
ax.set_title("Signal")
ax.set_xlim(0, 0.8)
plt.tight_layout()
```



So far we have been able to estimate our posterion mean. What about its uncertainties (i.e., posterion covariance)?

In real-life applications it is very difficult (if not impossible) to directly compute the posterior covariance matrix. It is much more useful to create a set of models that sample the posterion probability. We can do that by solving our problem several times using different prior realizations as starting guesses:

```
xpost = [
    x0
    + Cm_op
    * Rop.H
    * (lsqr(Rop * Cm_op * Rop.H + Cd_op, yn - Rop * x0, iter_lim=400)[0])
    for x0 in xs[:30]
```

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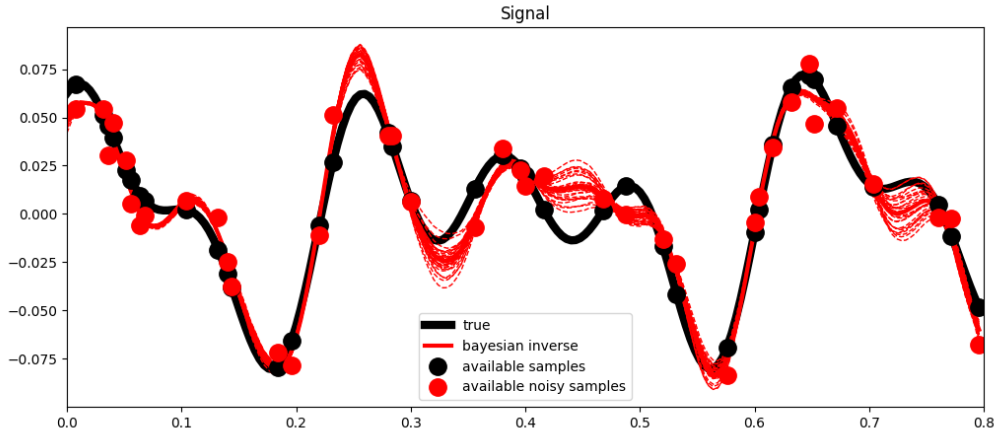
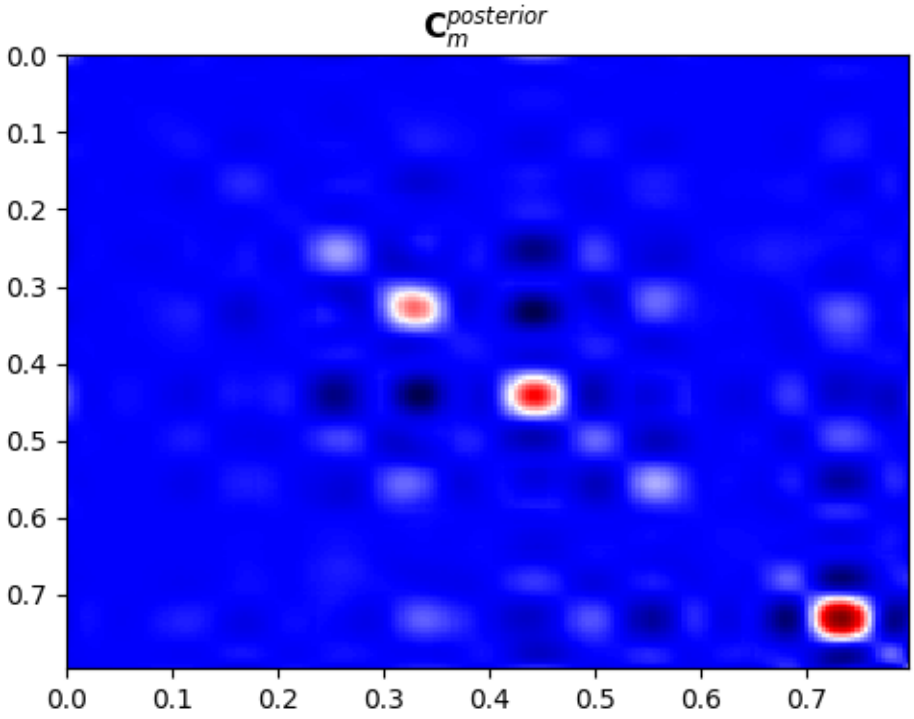
```
]
xpost = np.array(xpost)

x0post = np.average(xpost, axis=0)
Cm_post = ((xpost - x0post).T @ (xpost - x0post)) / nreals

# Visualize
fig, ax = plt.subplots(1, 1, figsize=(12, 5))
ax.plot(t, x, "k", lw=6, label="true")
ax.plot(t, xpost.T, "--r", lw=1)
ax.plot(t, x0post, "r", lw=3, label="bayesian inverse")
ax.plot(t, ymask, ".k", ms=25, label="available samples")
ax.plot(t, ynmask, ".r", ms=25, label="available noisy samples")
ax.legend()
ax.set_title("Signal")
ax.set_xlim(0, 0.8)

fig, ax = plt.subplots(1, 1, figsize=(5, 4))
im = ax.imshow(
    Cm_post, interpolation="nearest", cmap="seismic", extent=(t[0], t[-1], t[-1], t[0])
)
ax.set_title(r"$\mathbf{C}_m^{posterior}$")
ax.axis("tight")
plt.tight_layout()
```





Note that here we have been able to compute a sample posterior covariance from its estimated samples. By displaying it we can see how both the overall variances and the correlation between different parameters have become narrower compared to their prior counterparts.

Total running time of the script: (0 minutes 1.659 seconds)

3.4.6 05. Image deblurring

Deblurring is the process of removing blurring effects from images, caused for example by defocus aberration or motion blur.

In forward mode, such blurring effect is typically modelled as a 2-dimensional convolution between the so-called *point spread function* and a target sharp input image, where the sharp input image (which has to be recovered) is unknown and the point-spread function can be either known or unknown.

In this tutorial, an example of 2d blurring and deblurring will be shown using the *pylops.signalprocessing.Convolve2D* operator assuming knowledge of the point-spread function.

```
import matplotlib.pyplot as plt
import numpy as np

import pylops
```

Let's start by importing a 2d image and defining the blurring operator

```
im = np.load("../testdata/python.npy")[::5, ::5, 0]

Nz, Nx = im.shape
```

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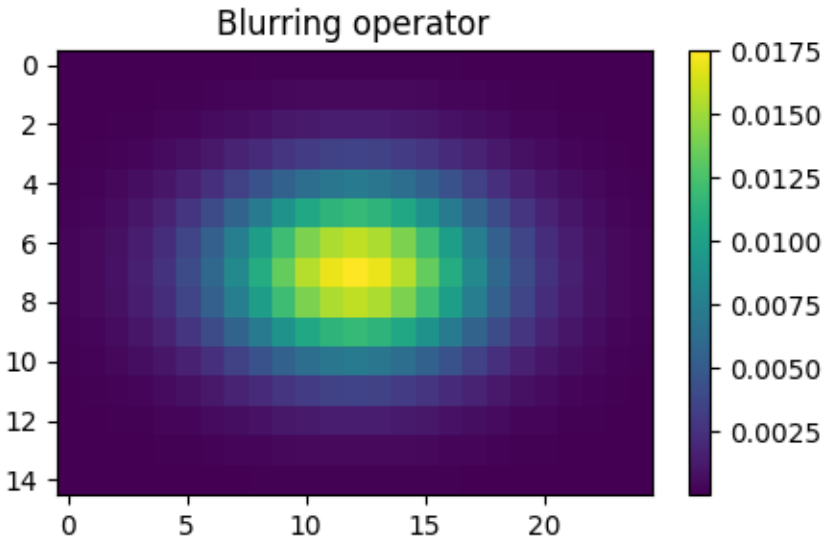
```
# Blurring guassian operator
nh = [15, 25]
hz = np.exp(-0.1 * np.linspace(-(nh[0] // 2), nh[0] // 2, nh[0]) ** 2)
hx = np.exp(-0.03 * np.linspace(-(nh[1] // 2), nh[1] // 2, nh[1]) ** 2)
hz /= np.trapz(hz)  # normalize the integral to 1
hx /= np.trapz(hx)  # normalize the integral to 1
h = hz[:, np.newaxis] * hx[np.newaxis, :]

fig, ax = plt.subplots(1, 1, figsize=(5, 3))
him = ax.imshow(h)
ax.set_title("Blurring operator")
fig.colorbar(him, ax=ax)
ax.axis("tight")

Cop = pylops.signalprocessing.Convolve2D(
    (Nz, Nx), h=h, offset=(nh[0] // 2, nh[1] // 2), dtype="float32"
)
```



We first apply the blurring operator to the sharp image. We then try to recover the sharp input image by inverting the convolution operator from the blurred image. Note that when we perform inversion without any regularization, the deblurred image will show some ringing due to the instabilities of the inverse process. Using a L1 solver with a DWT preconditioner or TV regularization allows to recover sharper contrasts.

```
imblur = Cop * im

imdeblur = pylops.optimization.leastsquares.normal_equations_inversion(
    Cop, imblur.ravel(), None, maxiter=50  # solvers need 1D arrays
)[0]
imdeblur = imdeblur.reshape(Cop.dims)

Wop = pylops.signalprocessing.DWT2D((Nz, Nx), wavelet="haar", level=3)
Dop = [
    pylops.FirstDerivative((Nz, Nx), axis=0, edge=False),
```

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```
    pylops.FirstDerivative((Nz, Nx), axis=1, edge=False),
]
DWop = Dop + [Wop]

imdeblurfista = pylops.optimization.sparsity.fista(
    Cop * Wop.H, imblur.ravel(), eps=1e-1, niter=100
)[0]
imdeblurfista = imdeblurfista.reshape((Cop * Wop.H).dims)
imdeblurfista = Wop.H * imdeblurfista

imdeblurtv = pylops.optimization.sparsity.splitbregman(
    Cop,
    imblur.ravel(),
    Dop,
    niter_outer=10,
    niter_inner=5,
    mu=1.5,
    epsRL1s=[2e0, 2e0],
    tol=1e-4,
    tau=1.0,
    show=False,
    **dict(iter_lim=5, damp=1e-4)
)[0]
imdeblurtv = imdeblurtv.reshape(Cop.dims)

imdeblurtv1 = pylops.optimization.sparsity.splitbregman(
    Cop,
    imblur.ravel(),
    DWop,
    niter_outer=10,
    niter_inner=5,
    mu=1.5,
    epsRL1s=[1e0, 1e0, 1e0],
    tol=1e-4,
    tau=1.0,
    show=False,
    **dict(iter_lim=5, damp=1e-4)
)[0]
imdeblurtv1 = imdeblurtv1.reshape(Cop.dims)
```

Finally we visualize the original, blurred, and recovered images.

```
# sphinx_gallery_thumbnail_number = 2
fig = plt.figure(figsize=(12, 6))
fig.suptitle("Deblurring", fontsize=14, fontweight="bold", y=0.95)
ax1 = plt.subplot2grid((2, 5), (0, 0))
ax2 = plt.subplot2grid((2, 5), (0, 1))
ax3 = plt.subplot2grid((2, 5), (0, 2))
ax4 = plt.subplot2grid((2, 5), (1, 0))
ax5 = plt.subplot2grid((2, 5), (1, 1))
ax6 = plt.subplot2grid((2, 5), (1, 2))
ax7 = plt.subplot2grid((2, 5), (0, 3), colspan=2)
```

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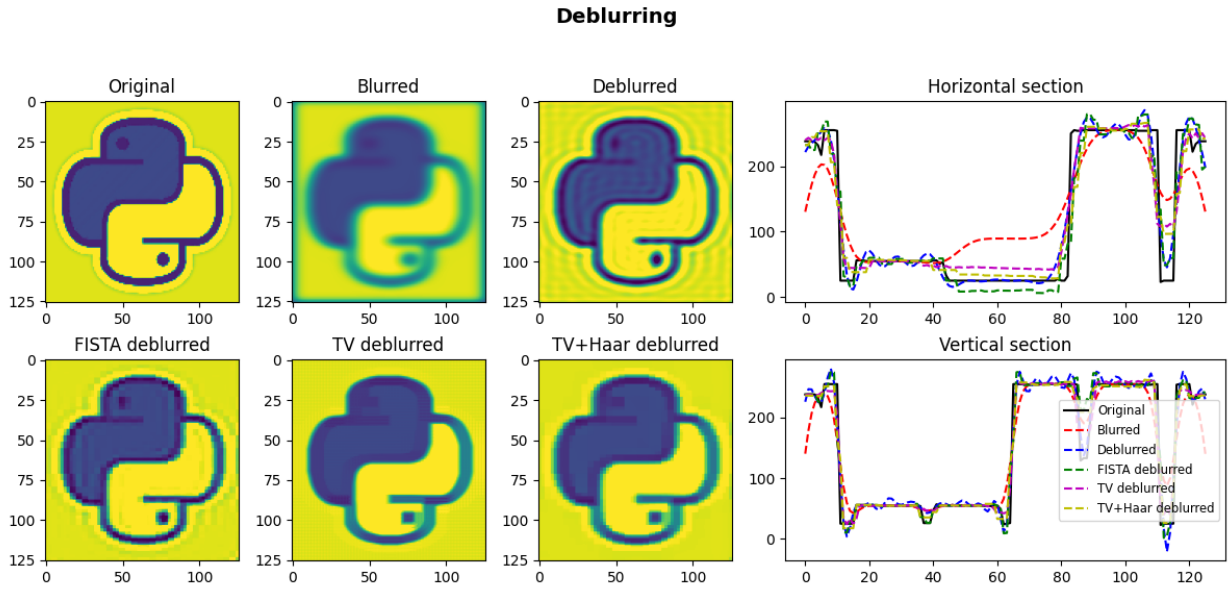
```
ax8 = plt.subplot2grid((2, 5), (1, 3), colspan=2)
ax1.imshow(im, cmap="viridis", vmin=0, vmax=250)
ax1.axis("tight")
ax1.set_title("Original")
ax2.imshow(imblur, cmap="viridis", vmin=0, vmax=250)
ax2.axis("tight")
ax2.set_title("Blurred")
ax3.imshow(imdeblur, cmap="viridis", vmin=0, vmax=250)
ax3.axis("tight")
ax3.set_title("Deblurred")
ax4.imshow(imdeblurfista, cmap="viridis", vmin=0, vmax=250)
ax4.axis("tight")
ax4.set_title("FISTA deblurred")
ax5.imshow(imdeblurtv, cmap="viridis", vmin=0, vmax=250)
ax5.axis("tight")
ax5.set_title("TV deblurred")
ax6.imshow(imdeblurtv1, cmap="viridis", vmin=0, vmax=250)
ax6.axis("tight")
ax6.set_title("TV+Haar deblurred")
ax7.plot(im[Nz // 2], "k")
ax7.plot(imblur[Nz // 2], "--r")
ax7.plot(imdeblur[Nz // 2], "--b")
ax7.plot(imdeblurfista[Nz // 2], "--g")
ax7.plot(imdeblurtv[Nz // 2], "--m")
ax7.plot(imdeblurtv1[Nz // 2], "--y")
ax7.axis("tight")
ax7.set_title("Horizontal section")
ax8.plot(im[:, Nx // 2], "k", label="Original")
ax8.plot(imblur[:, Nx // 2], "--r", label="Blurred")
ax8.plot(imdeblur[:, Nx // 2], "--b", label="Deblurred")
ax8.plot(imdeblurfista[:, Nx // 2], "--g", label="FISTA deblurred")
ax8.plot(imdeblurtv[:, Nx // 2], "--m", label="TV deblurred")
ax8.plot(imdeblurtv1[:, Nx // 2], "--y", label="TV+Haar deblurred")
ax8.axis("tight")
ax8.set_title("Vertical section")
ax8.legend(loc=5, fontsize="small")
plt.tight_layout()
plt.subplots_adjust(top=0.8)
```



Total running time of the script: (0 minutes 6.555 seconds)

3.4.7 06. 2D Interpolation

In the mathematical field of numerical analysis, interpolation is the problem of constructing new data points within the range of a discrete set of known data points. In signal and image processing, the data may be recorded at irregular locations and it is often required to *regularize* the data into a regular grid.

In this tutorial, an example of 2d interpolation of an image is carried out using a combination of PyLops operators (*pylops.Restriction* and *pylops.Laplacian*) and the `pylops.optimization` module.

Mathematically speaking, if we want to interpolate a signal using the theory of inverse problems, we can define the following forward problem:

$$\mathbf{y} = \mathbf{R}\mathbf{x}$$

where the restriction operator $\mathbf{R}$ selects M elements from the regularly sampled signal $\mathbf{x}$ at random locations. The input and output signals are:

$$\mathbf{y} = [y_1, y_2, \ldots, y_N]^T, \qquad \mathbf{x} = [x_1, x_2, \ldots, x_M]^T,$$

with $M \gg N$.

```
import matplotlib.pyplot as plt
import numpy as np

import pylops

plt.close("all")
np.random.seed(0)
```

To start we import a 2d image and define our restriction operator to irregularly and randomly sample the image for 30% of the entire grid

```
im = np.load("../testdata/python.npy")[:, :, 0]

Nz, Nx = im.shape
N = Nz * Nx

# Subsample signal
perc_subsampling = 0.2

Nsub2d = int(np.round(N * perc_subsampling))
iava = np.sort(np.random.permutation(np.arange(N))[:Nsub2d])

# Create operators and data
Rop = pylops.Restriction(N, iava, dtype="float64")
D2op = pylops.Laplacian((Nz, Nx), weights=(1, 1), dtype="float64")

x = im.ravel()
y = Rop * x
y1 = Rop.mask(x)
```

We will now use two different routines from our optimization toolbox to estimate our original image in the regular grid.

```
xcg_reg_lop = pylops.optimization.leastsquares.normal_equations_inversion(
    Rop, y, [D2op], epsRs=[np.sqrt(0.1)], **dict(maxiter=200)
)[0]

# Invert for interpolated signal, lsqrt
xlsqr_reg_lop = pylops.optimization.leastsquares.regularized_inversion(
    Rop,
    y,
    [D2op],
    epsRs=[np.sqrt(0.1)],
    **dict(damp=0, iter_lim=200, show=0),
)[0]

# Reshape estimated images
im_sampled = y1.reshape((Nz, Nx))
im_rec_lap_cg = xcg_reg_lop.reshape((Nz, Nx))
im_rec_lap_lsqr = xlsqr_reg_lop.reshape((Nz, Nx))
```

Finally we visualize the original image, the reconstructed images and their error

```
fig, axs = plt.subplots(1, 4, figsize=(12, 4))
fig.suptitle("Data reconstruction - normal eqs", fontsize=14, fontweight="bold", y=0.95)
axs[0].imshow(im, cmap="viridis", vmin=0, vmax=250)
axs[0].axis("tight")
axs[0].set_title("Original")
axs[1].imshow(im_sampled.data, cmap="viridis", vmin=0, vmax=250)
axs[1].axis("tight")
axs[1].set_title("Sampled")
axs[2].imshow(im_rec_lap_cg, cmap="viridis", vmin=0, vmax=250)
axs[2].axis("tight")
axs[2].set_title("2D Regularization")
axs[3].imshow(im - im_rec_lap_cg, cmap="gray", vmin=-80, vmax=80)
```

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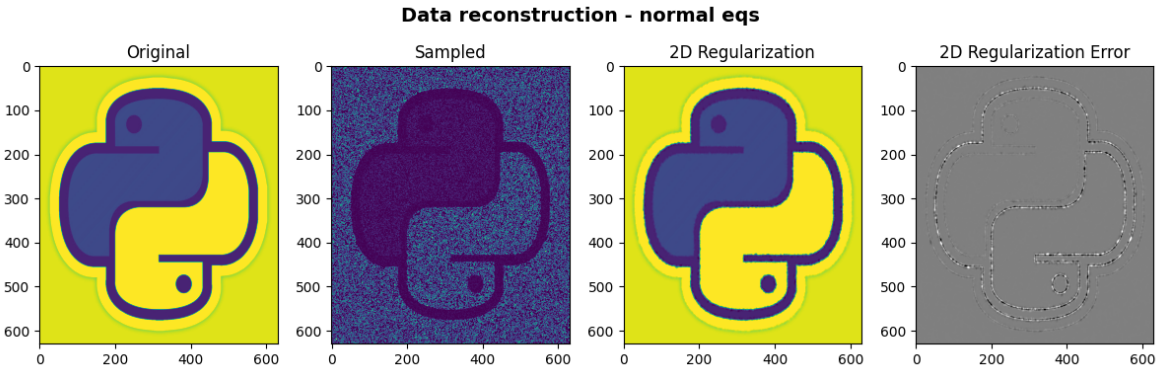
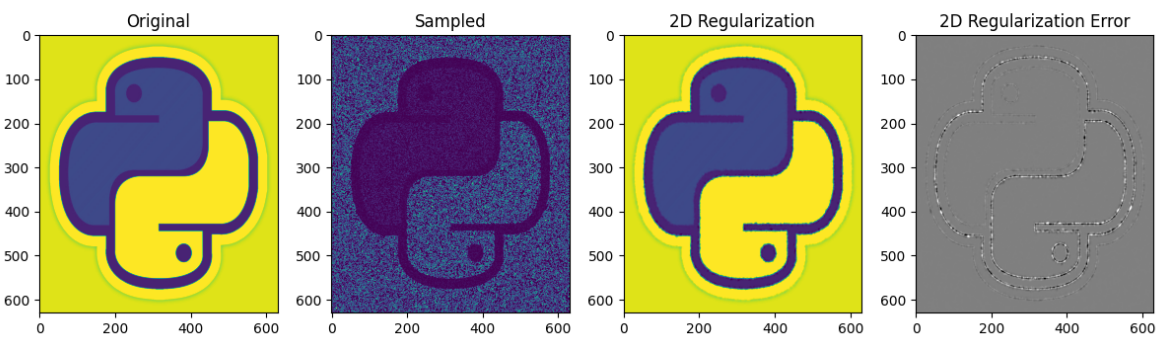
```
axs[3].axis("tight")
axs[3].set_title("2D Regularization Error")
plt.tight_layout()
plt.subplots_adjust(top=0.8)

fig, axs = plt.subplots(1, 4, figsize=(12, 4))
fig.suptitle(
    "Data reconstruction - regularized eqs", fontsize=14, fontweight="bold", y=0.95
)
axs[0].imshow(im, cmap="viridis", vmin=0, vmax=250)
axs[0].axis("tight")
axs[0].set_title("Original")
axs[1].imshow(im_sampled.data, cmap="viridis", vmin=0, vmax=250)
axs[1].axis("tight")
axs[1].set_title("Sampled")
axs[2].imshow(im_rec_lap_lsqr, cmap="viridis", vmin=0, vmax=250)
axs[2].axis("tight")
axs[2].set_title("2D Regularization")
axs[3].imshow(im - im_rec_lap_lsqr, cmap="gray", vmin=-80, vmax=80)
axs[3].axis("tight")
axs[3].set_title("2D Regularization Error")
plt.tight_layout()
plt.subplots_adjust(top=0.8)
```

- 

- 

Total running time of the script: (0 minutes 17.005 seconds)

3.4.8 07. Post-stack inversion

Estimating subsurface properties from band-limited seismic data represents an important task for geophysical subsurface characterization.

In this tutorial, the *pylops.avo.poststack.PoststackLinearModelling* operator is used for modelling of both 1d and 2d synthetic post-stack seismic data from a profile or 2d model of the subsurface acoustic impedence.

$$d(t, \theta = 0) = \frac{1}{2} w(t) * \frac{\mathrm{d} \ln \mathrm{AI}(t)}{\mathrm{d}t}$$

where $\mathrm{AI}(t)$ is the acoustic impedance profile and $w(t)$ is the time domain seismic wavelet. In compact form:

$$\mathbf{d} = \mathbf{WDai}$$

where $\mathbf{W}$ is a convolution operator, $\mathbf{D}$ is a first derivative operator, and $\mathbf{ai}$ is the input model. Subsequently the acoustic impedance model is estimated via the *pylops.avo.poststack.PoststackInversion* module. A two-steps inversion strategy is finally presented to deal with the case of noisy data.

```
import matplotlib.pyplot as plt

# sphinx_gallery_thumbnail_number = 4
import numpy as np
from scipy.signal import filtfilt

import pylops
from pylops.utils.wavelets import ricker

plt.close("all")
np.random.seed(10)
```

Let's start with a 1d example. A synthetic profile of acoustic impedance is created and data is modelled using both the dense and linear operator version of *pylops.avo.poststack.PoststackLinearModelling* operator.

```
# model
nt0 = 301
dt0 = 0.004
t0 = np.arange(nt0) * dt0
vp = 1200 + np.arange(nt0) + filtfilt(np.ones(5) / 5.0, 1, np.random.normal(0, 80, nt0))
rho = 1000 + vp + filtfilt(np.ones(5) / 5.0, 1, np.random.normal(0, 30, nt0))
vp[131:] += 500
rho[131:] += 100
m = np.log(vp * rho)

# smooth model
nsmooth = 100
mback = filtfilt(np.ones(nsmooth) / float(nsmooth), 1, m)

# wavelet
ntwav = 41
wav, twav, wavc = ricker(t0[: ntwav // 2 + 1], 20)

# dense operator
PPop_dense = pylops.avo.poststack.PoststackLinearModelling(
    wav / 2, nt0=nt0, explicit=True
```

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```
)

# lop operator
PPop = pylops.avo.poststack.PoststackLinearModelling(wav / 2, nt0=nt0)

# data
d_dense = PPop_dense * m.ravel()
d = PPop * m

# add noise
dn_dense = d_dense + np.random.normal(0, 2e-2, d_dense.shape)
```

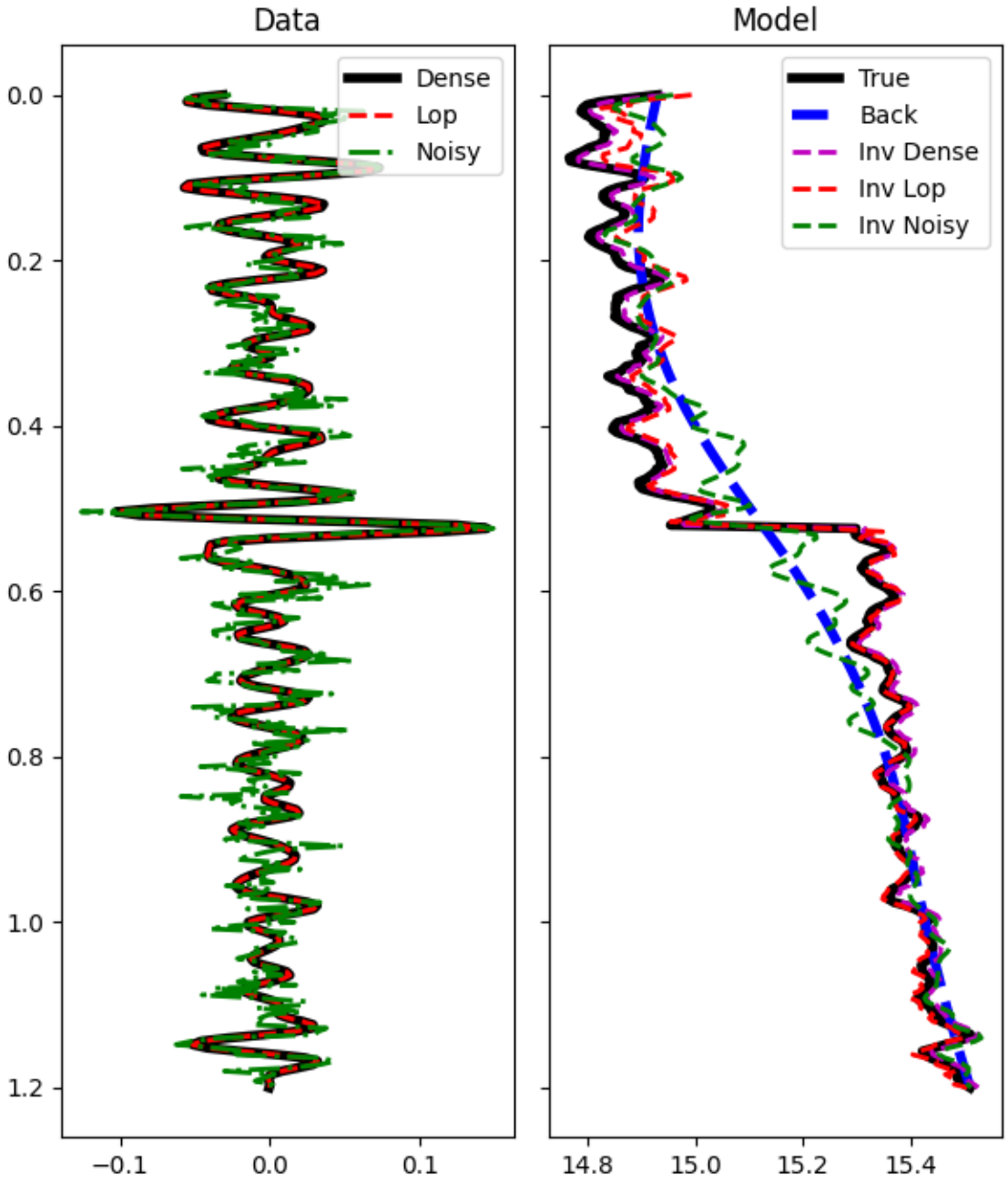
We can now estimate the acoustic profile from band-limited data using either the dense operator or linear operator.

```
# solve dense
minv_dense = pylops.avo.poststack.PoststackInversion(
    d, wav / 2, m0=mback, explicit=True, simultaneous=False
)[0]

# solve lop
minv = pylops.avo.poststack.PoststackInversion(
    d_dense,
    wav / 2,
    m0=mback,
    explicit=False,
    simultaneous=False,
    **dict(iter_lim=2000)
)[0]

# solve noisy
mn = pylops.avo.poststack.PoststackInversion(
    dn_dense, wav / 2, m0=mback, explicit=True, epsR=1e0, **dict(damp=1e-1)
)[0]

fig, axs = plt.subplots(1, 2, figsize=(6, 7), sharey=True)
axs[0].plot(d_dense, t0, "k", lw=4, label="Dense")
axs[0].plot(d, t0, "--r", lw=2, label="Lop")
axs[0].plot(dn_dense, t0, "-.g", lw=2, label="Noisy")
axs[0].set_title("Data")
axs[0].invert_yaxis()
axs[0].axis("tight")
axs[0].legend(loc=1)
axs[1].plot(m, t0, "k", lw=4, label="True")
axs[1].plot(mback, t0, "--b", lw=4, label="Back")
axs[1].plot(minv_dense, t0, "--m", lw=2, label="Inv Dense")
axs[1].plot(minv, t0, "--r", lw=2, label="Inv Lop")
axs[1].plot(mn, t0, "--g", lw=2, label="Inv Noisy")
axs[1].set_title("Model")
axs[1].axis("tight")
axs[1].legend(loc=1)
plt.tight_layout()
```



We see how inverting a dense matrix is in this case faster than solving for the linear operator (a good estimate of the model is in fact obtained only after 2000 iterations of lsqr). Nevertheless, having a linear operator is useful when we deal with larger dimensions (2d or 3d) and we want to couple our modelling operator with different types of spatial regularizations or preconditioning.

Before we move onto a 2d example, let's consider the case of non-stationary wavelet and see how we can easily use the same routines in this case

```
# wavelet
ntwav = 41
f0s = np.flip(np.arange(nt0) * 0.05 + 3)
```

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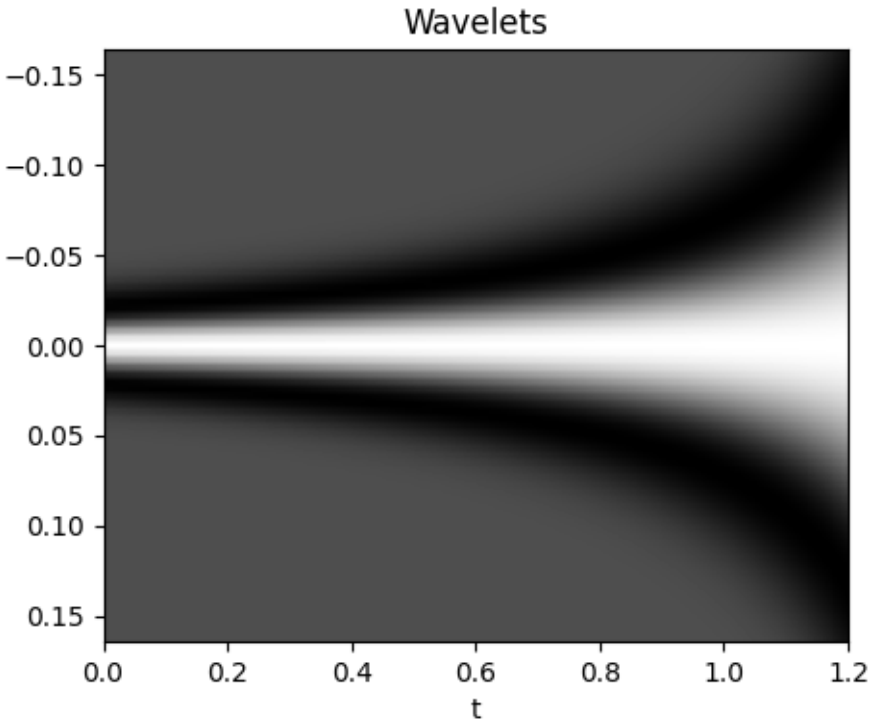
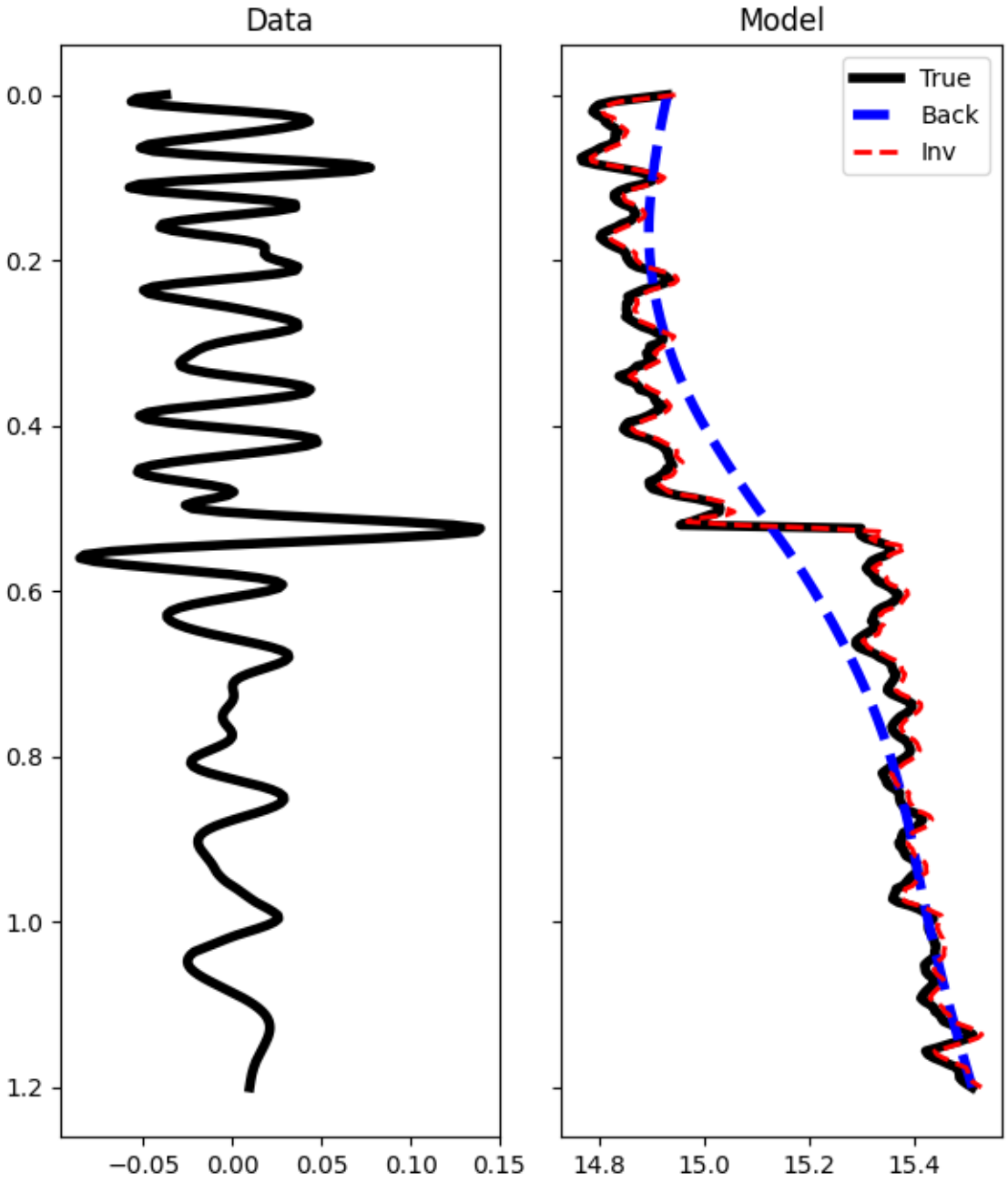
```
wavs = np.array([ricker(t0[:ntwav], f0)[0] for f0 in f0s])
wavc = np.argmax(wavs[0])

plt.figure(figsize=(5, 4))
plt.imshow(wavs.T, cmap="gray", extent=(t0[0], t0[-1], t0[ntwav], -t0[ntwav]))
plt.xlabel("t")
plt.title("Wavelets")
plt.axis("tight")

# operator
PPop = pylops.avo.poststack.PoststackLinearModelling(wavs / 2, nt0=nt0, explicit=True)

# data
d = PPop * m

# solve
minv = pylops.avo.poststack.PoststackInversion(
    d, wavs / 2, m0=mback, explicit=True, **dict(cond=1e-10)
)[0]

fig, axs = plt.subplots(1, 2, figsize=(6, 7), sharey=True)
axs[0].plot(d, t0, "k", lw=4)
axs[0].set_title("Data")
axs[0].invert_yaxis()
axs[0].axis("tight")
axs[1].plot(m, t0, "k", lw=4, label="True")
axs[1].plot(mback, t0, "--b", lw=4, label="Back")
axs[1].plot(minv, t0, "--r", lw=2, label="Inv")
axs[1].set_title("Model")
axs[1].axis("tight")
axs[1].legend(loc=1)
plt.tight_layout()
```


Wavelets
−0.15
−0.10
−0.05
0.00
0.05
0.10
0.15
0.0
0.2
0.4
0.6
0.8
1.0
1.2
t



We move now to a 2d example. First of all the model is loaded and data generated.

```
# model
inputfile = "../testdata/avo/poststack_model.npz"

model = np.load(inputfile)
m = np.log(model["model"][:, ::3])
x, z = model["x"][::3] / 1000.0, model["z"] / 1000.0
nx, nz = len(x), len(z)
```

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```
# smooth model
nsmoothz, nsmoothx = 60, 50
mback = filtfilt(np.ones(nsmoothz) / float(nsmoothz), 1, m, axis=0)
mback = filtfilt(np.ones(nsmoothx) / float(nsmoothx), 1, mback, axis=1)

# dense operator
PPop_dense = pylops.avo.poststack.PoststackLinearModelling(
    wav / 2, nt0=nz, spatdims=nx, explicit=True
)

# lop operator
PPop = pylops.avo.poststack.PoststackLinearModelling(wav / 2, nt0=nz, spatdims=nx)

# data
d = (PPop_dense * m.ravel()).reshape(nz, nx)
n = np.random.normal(0, 1e-1, d.shape)
dn = d + n
```

Finally we perform 4 different inversions:

- trace-by-trace inversion with explicit solver and dense operator with noise-free data
- trace-by-trace inversion with explicit solver and dense operator with noisy data
- multi-trace regularized inversion with iterative solver and linear operator using the result of trace-by-trace inversion as starting guess

$$J = ||\Delta\mathbf{d} - \mathbf{W}\Delta\mathbf{ai}||_2 + \epsilon_\nabla^2 ||\nabla\mathbf{ai}||_2$$

where $\Delta\mathbf{d} = \mathbf{d} - \mathbf{WAI_0}$ is the residual data

- multi-trace blocky inversion with iterative solver and linear operator

```
# dense inversion with noise-free data
minv_dense = pylops.avo.poststack.PoststackInversion(
    d, wav / 2, m0=mback, explicit=True, simultaneous=False
)[0]

# dense inversion with noisy data
minv_dense_noisy = pylops.avo.poststack.PoststackInversion(
    dn, wav / 2, m0=mback, explicit=True, epsI=4e-2, simultaneous=False
)[0]

# spatially regularized lop inversion with noisy data
minv_lop_reg = pylops.avo.poststack.PoststackInversion(
    dn,
    wav / 2,
    m0=minv_dense_noisy,
    explicit=False,
    epsR=5e1,
    **dict(damp=np.sqrt(1e-4), iter_lim=80)
)[0]

# blockiness promoting inversion with noisy data
```

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```
minv_lop_blocky = pylops.avo.poststack.PoststackInversion(
    dn,
    wav / 2,
    m0=mback,
    explicit=False,
    epsR=[0.4],
    epsRL1=[0.1],
    **dict(mu=0.1, niter_outer=5, niter_inner=10, iter_lim=5, damp=1e-3)
)[0]

fig, axs = plt.subplots(2, 4, figsize=(15, 9))
axs[0][0].imshow(d, cmap="gray", extent=(x[0], x[-1], z[-1], z[0]), vmin=-0.4, vmax=0.4)
axs[0][0].set_title("Data")
axs[0][0].axis("tight")
axs[0][1].imshow(
    dn, cmap="gray", extent=(x[0], x[-1], z[-1], z[0]), vmin=-0.4, vmax=0.4
)
axs[0][1].set_title("Noisy Data")
axs[0][1].axis("tight")
axs[0][2].imshow(
    m,
    cmap="gist_rainbow",
    extent=(x[0], x[-1], z[-1], z[0]),
    vmin=m.min(),
    vmax=m.max(),
)
axs[0][2].set_title("Model")
axs[0][2].axis("tight")
axs[0][3].imshow(
    mback,
    cmap="gist_rainbow",
    extent=(x[0], x[-1], z[-1], z[0]),
    vmin=m.min(),
    vmax=m.max(),
)
axs[0][3].set_title("Smooth Model")
axs[0][3].axis("tight")
axs[1][0].imshow(
    minv_dense,
    cmap="gist_rainbow",
    extent=(x[0], x[-1], z[-1], z[0]),
    vmin=m.min(),
    vmax=m.max(),
)
axs[1][0].set_title("Noise-free Inversion")
axs[1][0].axis("tight")
axs[1][1].imshow(
    minv_dense_noisy,
    cmap="gist_rainbow",
    extent=(x[0], x[-1], z[-1], z[0]),
    vmin=m.min(),
    vmax=m.max(),
```

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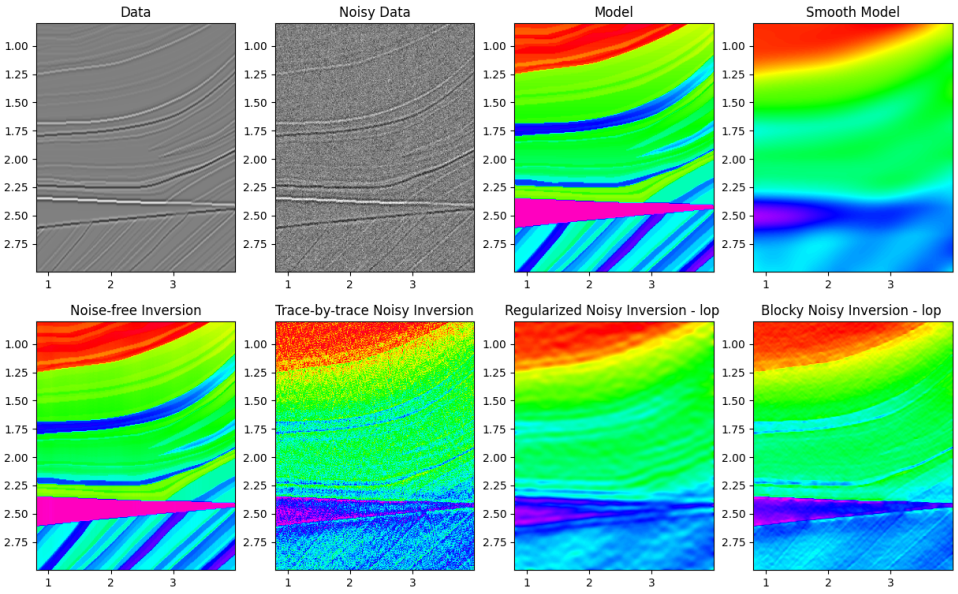
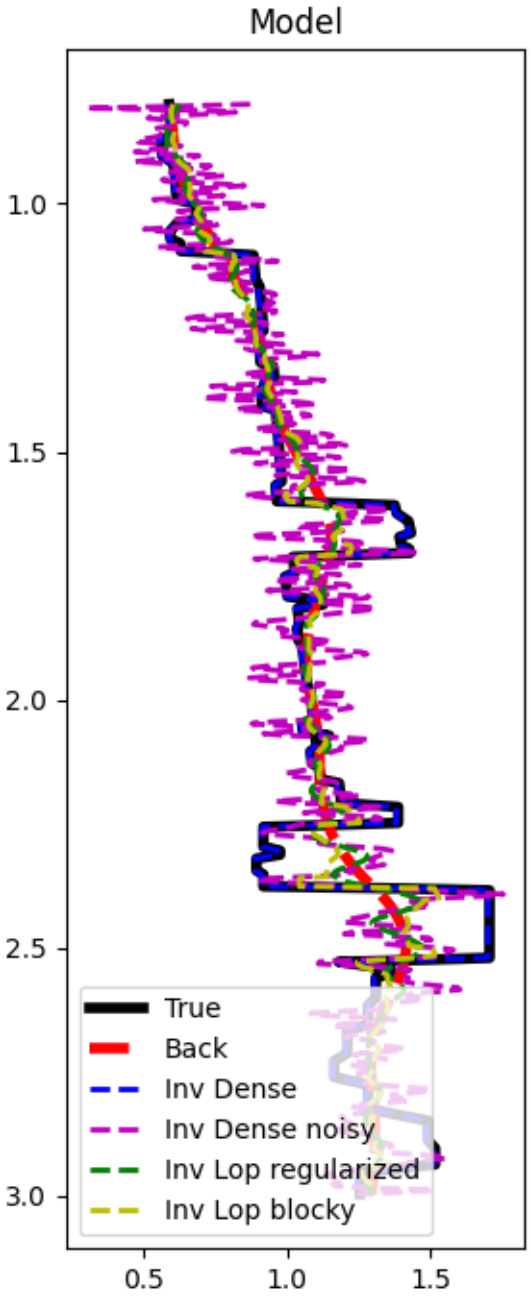
```
)
axs[1][1].set_title("Trace-by-trace Noisy Inversion")
axs[1][1].axis("tight")
axs[1][2].imshow(
    minv_lop_reg,
    cmap="gist_rainbow",
    extent=(x[0], x[-1], z[-1], z[0]),
    vmin=m.min(),
    vmax=m.max(),
)
axs[1][2].set_title("Regularized Noisy Inversion - lop ")
axs[1][2].axis("tight")
axs[1][3].imshow(
    minv_lop_blocky,
    cmap="gist_rainbow",
    extent=(x[0], x[-1], z[-1], z[0]),
    vmin=m.min(),
    vmax=m.max(),
)
axs[1][3].set_title("Blocky Noisy Inversion - lop ")
axs[1][3].axis("tight")

fig, ax = plt.subplots(1, 1, figsize=(3, 7))
ax.plot(m[:, nx // 2], z, "k", lw=4, label="True")
ax.plot(mback[:, nx // 2], z, "--r", lw=4, label="Back")
ax.plot(minv_dense[:, nx // 2], z, "--b", lw=2, label="Inv Dense")
ax.plot(minv_dense_noisy[:, nx // 2], z, "--m", lw=2, label="Inv Dense noisy")
ax.plot(minv_lop_reg[:, nx // 2], z, "--g", lw=2, label="Inv Lop regularized")
ax.plot(minv_lop_blocky[:, nx // 2], z, "--y", lw=2, label="Inv Lop blocky")
ax.set_title("Model")
ax.invert_yaxis()
ax.axis("tight")
ax.legend()
plt.tight_layout()
```


Data
Noisy Data
Model
Smooth Model
Noise-free Inversion
Trace-by-trace Noisy Inversion
Regularized Noisy Inversion - lop
Blocky Noisy Inversion - lop



That's almost it. If you wonder how this can be applied to real data, head over to the following notebook where the open-source segyio library is used alongside pylops to create an end-to-end open-source seismic inversion workflow with SEG-Y input data.

Total running time of the script: (0 minutes 12.010 seconds)

3.4.9 08. Pre-stack (AVO) inversion

Pre-stack inversion represents one step beyond post-stack inversion in that not only the profile of acoustic impedance can be inferred from seismic data, rather a set of elastic parameters is estimated from pre-stack data (i.e., angle gathers) using the information contained in the so-called AVO (amplitude versus offset) response. Such elastic parameters represent vital information for more sophisticated geophysical subsurface characterization than it would be possible to achieve working with post-stack seismic data.

In this tutorial, the *pylops.avo.prestack.PrestackLinearModelling* operator is used for modelling of both 1d and 2d synthetic pre-stack seismic data using 1d profiles or 2d models of different subsurface elastic parameters (P-wave velocity, S-wave velocity, and density) as input.

$$d(t,\theta) = w(t) * \sum_{i=1}^{N} G_i(t,\theta) \frac{\mathrm{d} \ln m_i(t)}{\mathrm{d}t}$$

where $\mathbf{m}(t) = [V_P(t), V_S(t), \rho(t)]$ is a vector containing three elastic parameters at time t, $G_i(t,\theta)$ are the coefficients of the AVO parametrization used to model pre-stack data and $w(t)$ is the time domain seismic wavelet. In compact form:

$$\mathbf{d} = \mathbf{WGDm}$$

where $\mathbf{W}$ is a convolution operator, $\mathbf{G}$ is the AVO modelling operator, $\mathbf{D}$ is a block-diagonal derivative operator, and $\mathbf{m}$ is the input model. Subsequently the elastic parameters are estimated via the *pylops.avo.prestack.PrestackInversion* module. Once again, a two-steps inversion strategy can also be used to deal with the case of noisy data.

```
import matplotlib.pyplot as plt
import numpy as np
from scipy.signal import filtfilt

import pylops
from pylops.utils.wavelets import ricker

plt.close("all")
np.random.seed(0)
```

Let's start with a 1d example. A synthetic profile of acoustic impedance is created and data is modelled using both the dense and linear operator version of *pylops.avo.prestack.PrestackLinearModelling* operator

```
# sphinx_gallery_thumbnail_number = 5

# model
nt0 = 301
dt0 = 0.004

t0 = np.arange(nt0) * dt0
vp = 1200 + np.arange(nt0) + filtfilt(np.ones(5) / 5.0, 1, np.random.normal(0, 80, nt0))
vs = 600 + vp / 2 + filtfilt(np.ones(5) / 5.0, 1, np.random.normal(0, 20, nt0))
rho = 1000 + vp + filtfilt(np.ones(5) / 5.0, 1, np.random.normal(0, 30, nt0))
vp[131:] += 500
vs[131:] += 200
rho[131:] += 100
vsvp = 0.5
m = np.stack((np.log(vp), np.log(vs), np.log(rho)), axis=1)
```

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```
# background model
nsmooth = 50
mback = filtfilt(np.ones(nsmooth) / float(nsmooth), 1, m, axis=0)

# angles
ntheta = 21
thetamin, thetamax = 0, 40
theta = np.linspace(thetamin, thetamax, ntheta)

# wavelet
ntwav = 41
wav = ricker(t0[: ntwav // 2 + 1], 15)[0]

# lop
PPop = pylops.avo.prestack.PrestackLinearModelling(
    wav, theta, vsvp=vsvp, nt0=nt0, linearization="akirich"
)

# dense
PPop_dense = pylops.avo.prestack.PrestackLinearModelling(
    wav, theta, vsvp=vsvp, nt0=nt0, linearization="akirich", explicit=True
)

# data lop
dPP = PPop * m.ravel()
dPP = dPP.reshape(nt0, ntheta)

# data dense
dPP_dense = PPop_dense * m.T.ravel()
dPP_dense = dPP_dense.reshape(ntheta, nt0).T

# noisy data
dPPn_dense = dPP_dense + np.random.normal(0, 1e-2, dPP_dense.shape)
```

We can now invert our data and retrieve elastic profiles for both noise-free and noisy data using *pylops.avo.prestack.PrestackInversion*.

```
# dense
minv_dense, dPP_dense_res = pylops.avo.prestack.PrestackInversion(
    dPP_dense,
    theta,
    wav,
    m0=mback,
    linearization="akirich",
    explicit=True,
    returnres=True,
    **dict(cond=1e-10)
)

# lop
minv, dPP_res = pylops.avo.prestack.PrestackInversion(
```

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```
    dPP,
    theta,
    wav,
    m0=mback,
    linearization="akirich",
    explicit=False,
    returnres=True,
    **dict(damp=1e-10, iter_lim=2000)
)

# dense noisy
minv_dense_noise, dPPn_dense_res = pylops.avo.prestack.PrestackInversion(
    dPPn_dense,
    theta,
    wav,
    m0=mback,
    linearization="akirich",
    explicit=True,
    returnres=True,
    **dict(cond=1e-1)
)

# lop noisy (with vertical smoothing)
minv_noise, dPPn_res = pylops.avo.prestack.PrestackInversion(
    dPPn_dense,
    theta,
    wav,
    m0=mback,
    linearization="akirich",
    explicit=False,
    epsR=5e-1,
    returnres=True,
    **dict(damp=1e-1, iter_lim=100)
)
```

The data, inverted models and residuals are now displayed.

```
# data and model
fig, (axd, axdn, axvp, axvs, axrho) = plt.subplots(1, 5, figsize=(8, 5), sharey=True)
axd.imshow(
    dPP_dense,
    cmap="gray",
    extent=(theta[0], theta[-1], t0[-1], t0[0]),
    vmin=-np.abs(dPP_dense).max(),
    vmax=np.abs(dPP_dense).max(),
)
axd.set_title("Data")
axd.axis("tight")
axdn.imshow(
    dPPn_dense,
    cmap="gray",
    extent=(theta[0], theta[-1], t0[-1], t0[0]),
```

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```
    vmin=-np.abs(dPP_dense).max(),
    vmax=np.abs(dPP_dense).max(),
)
axdn.set_title("Noisy Data")
axdn.axis("tight")
axvp.plot(vp, t0, "k", lw=4, label="True")
axvp.plot(np.exp(mback[:, 0]), t0, "--r", lw=4, label="Back")
axvp.plot(np.exp(minv_dense[:, 0]), t0, "--b", lw=2, label="Inv Dense")
axvp.plot(np.exp(minv[:, 0]), t0, "--m", lw=2, label="Inv Lop")
axvp.plot(np.exp(minv_dense_noise[:, 0]), t0, "--c", lw=2, label="Noisy Dense")
axvp.plot(np.exp(minv_noise[:, 0]), t0, "--g", lw=2, label="Noisy Lop")
axvp.set_title(r"$V_P$")
axvs.plot(vs, t0, "k", lw=4, label="True")
axvs.plot(np.exp(mback[:, 1]), t0, "--r", lw=4, label="Back")
axvs.plot(np.exp(minv_dense[:, 1]), t0, "--b", lw=2, label="Inv Dense")
axvs.plot(np.exp(minv[:, 1]), t0, "--m", lw=2, label="Inv Lop")
axvs.plot(np.exp(minv_dense_noise[:, 1]), t0, "--c", lw=2, label="Noisy Dense")
axvs.plot(np.exp(minv_noise[:, 1]), t0, "--g", lw=2, label="Noisy Lop")
axvs.set_title(r"$V_S$")
axrho.plot(rho, t0, "k", lw=4, label="True")
axrho.plot(np.exp(mback[:, 2]), t0, "--r", lw=4, label="Back")
axrho.plot(np.exp(minv_dense[:, 2]), t0, "--b", lw=2, label="Inv Dense")
axrho.plot(np.exp(minv[:, 2]), t0, "--m", lw=2, label="Inv Lop")
axrho.plot(np.exp(minv_dense_noise[:, 2]), t0, "--c", lw=2, label="Noisy Dense")
axrho.plot(np.exp(minv_noise[:, 2]), t0, "--g", lw=2, label="Noisy Lop")
axrho.set_title(r"$\rho$")
axrho.legend(loc="center left", bbox_to_anchor=(1, 0.5))
axd.axis("tight")
plt.tight_layout()

# residuals
fig, axs = plt.subplots(1, 4, figsize=(8, 5), sharey=True)
fig.suptitle("Residuals", fontsize=14, fontweight="bold", y=0.95)
im = axs[0].imshow(
    dPP_dense_res,
    cmap="gray",
    extent=(theta[0], theta[-1], t0[-1], t0[0]),
    vmin=-0.1,
    vmax=0.1,
)
axs[0].set_title("Dense")
axs[0].set_xlabel(r"$\theta$")
axs[0].set_ylabel("t[s]")
axs[0].axis("tight")
axs[1].imshow(
    dPP_res,
    cmap="gray",
    extent=(theta[0], theta[-1], t0[-1], t0[0]),
    vmin=-0.1,
    vmax=0.1,
)
axs[1].set_title("Lop")
```

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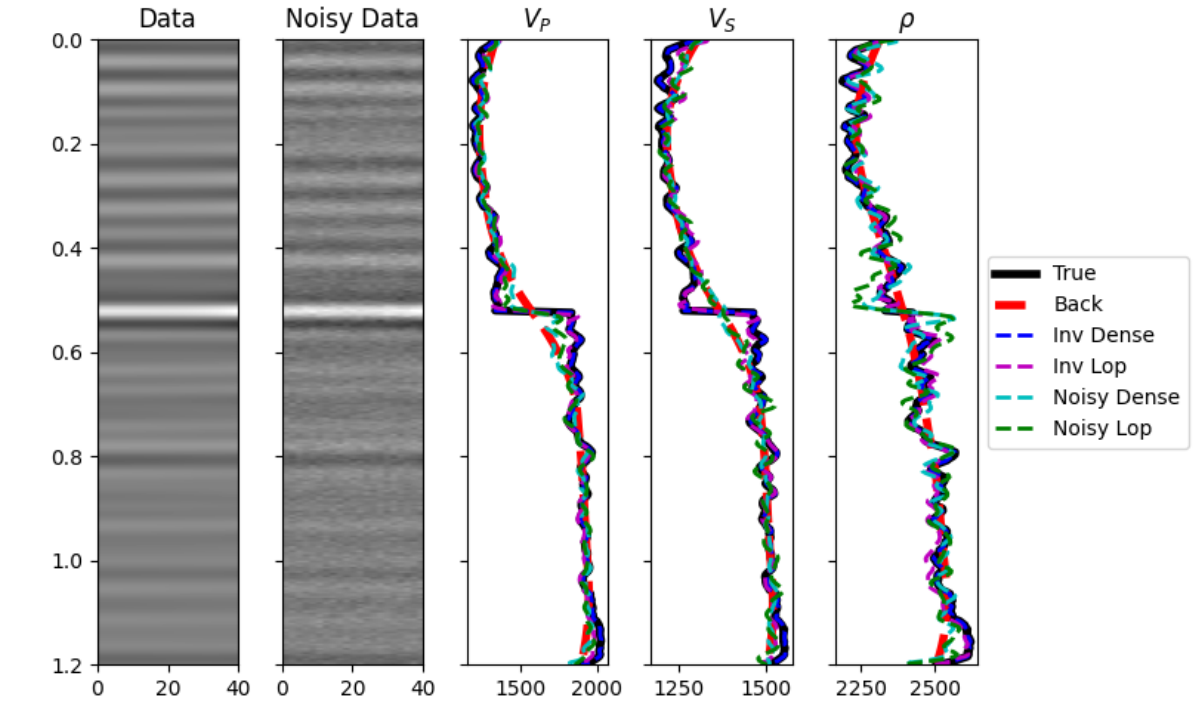
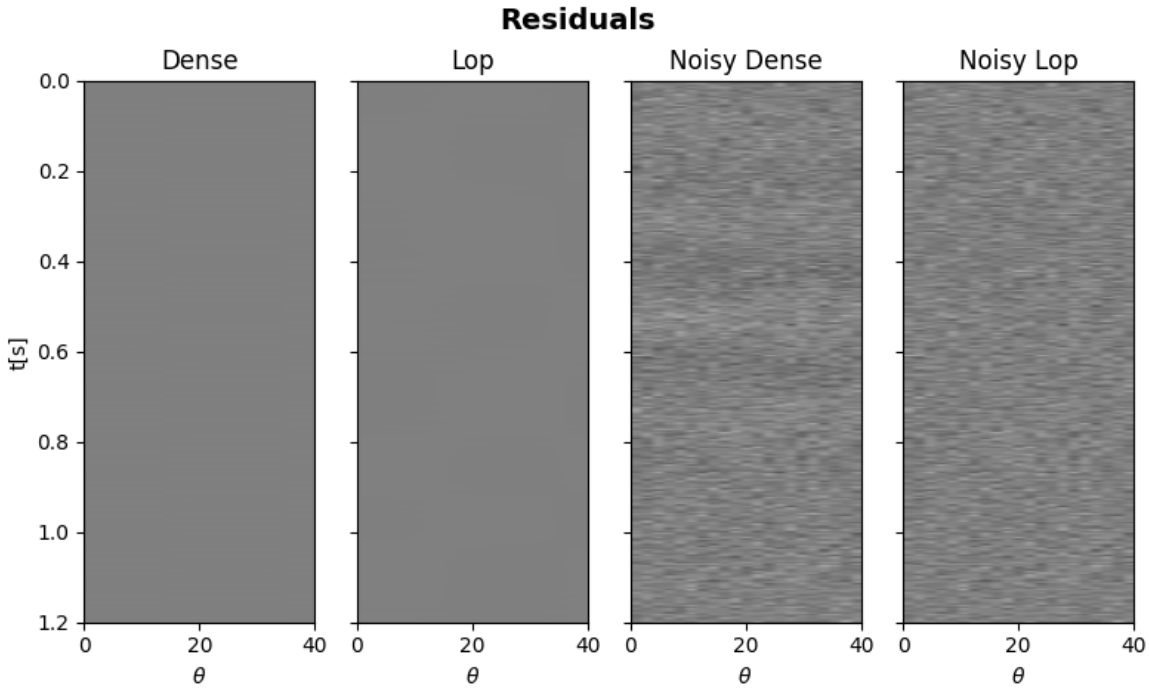
```
axs[1].set_xlabel(r"$\theta$")
axs[1].axis("tight")
axs[2].imshow(
    dPPn_dense_res,
    cmap="gray",
    extent=(theta[0], theta[-1], t0[-1], t0[0]),
    vmin=-0.1,
    vmax=0.1,
)
axs[2].set_title("Noisy Dense")
axs[2].set_xlabel(r"$\theta$")
axs[2].axis("tight")
axs[3].imshow(
    dPPn_res,
    cmap="gray",
    extent=(theta[0], theta[-1], t0[-1], t0[0]),
    vmin=-0.1,
    vmax=0.1,
)
axs[3].set_title("Noisy Lop")
axs[3].set_xlabel(r"$\theta$")
axs[3].axis("tight")
plt.tight_layout()
plt.subplots_adjust(top=0.85)
```





Finally before moving to the 2d example, we consider the case when both PP and PS data are available. A joint PP-PS inversion can be easily solved as follows.

```
PSop = pylops.avo.prestack.PrestackLinearModelling(
    2 * wav, theta, vsvp=vsvp, nt0=nt0, linearization="ps"
)
PPPSop = pylops.VStack((PPop, PSop))

# data
dPPPS = PPPSop * m.ravel()
dPPPS = dPPPS.reshape(2, nt0, ntheta)

dPPPSn = dPPPS + np.random.normal(0, 1e-2, dPPPS.shape)

# Invert
minvPPSP, dPPPS_res = pylops.avo.prestack.PrestackInversion(
    dPPPS,
    theta,
    [wav, 2 * wav],
    m0=mback,
    linearization=["fatti", "ps"],
    epsR=5e-1,
    returnres=True,
    **dict(damp=1e-1, iter_lim=100)
)

# Data and model
fig, (axd, axdn, axvp, axvs, axrho) = plt.subplots(1, 5, figsize=(8, 5), sharey=True)
axd.imshow(
```

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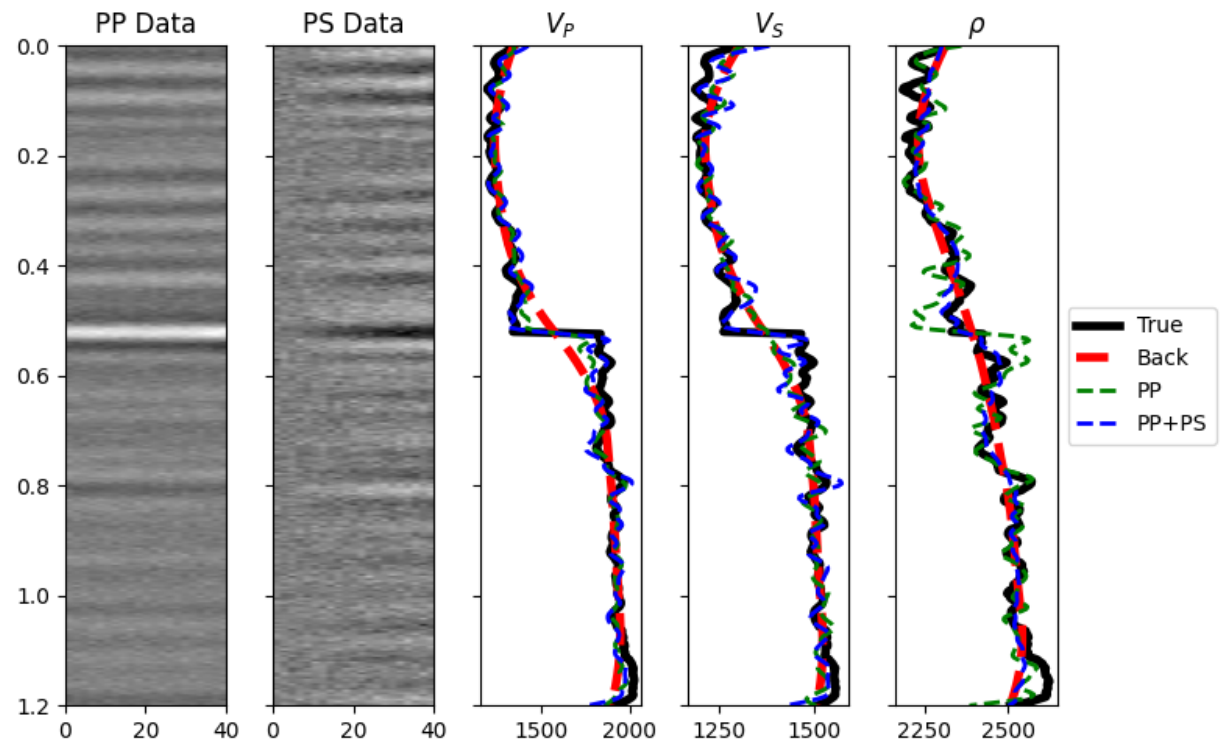
```
    dPPPSn[0],
    cmap="gray",
    extent=(theta[0], theta[-1], t0[-1], t0[0]),
    vmin=-np.abs(dPPPSn[0]).max(),
    vmax=np.abs(dPPPSn[0]).max(),
)
axd.set_title("PP Data")
axd.axis("tight")
axdn.imshow(
    dPPPSn[1],
    cmap="gray",
    extent=(theta[0], theta[-1], t0[-1], t0[0]),
    vmin=-np.abs(dPPPSn[1]).max(),
    vmax=np.abs(dPPPSn[1]).max(),
)
axdn.set_title("PS Data")
axdn.axis("tight")
axvp.plot(vp, t0, "k", lw=4, label="True")
axvp.plot(np.exp(mback[:, 0]), t0, "--r", lw=4, label="Back")
axvp.plot(np.exp(minv_noise[:, 0]), t0, "--g", lw=2, label="PP")
axvp.plot(np.exp(minvPPSP[:, 0]), t0, "--b", lw=2, label="PP+PS")
axvp.set_title(r"$V_P$")
axvs.plot(vs, t0, "k", lw=4, label="True")
axvs.plot(np.exp(mback[:, 1]), t0, "--r", lw=4, label="Back")
axvs.plot(np.exp(minv_noise[:, 1]), t0, "--g", lw=2, label="PP")
axvs.plot(np.exp(minvPPSP[:, 1]), t0, "--b", lw=2, label="PP+PS")
axvs.set_title(r"$V_S$")
axrho.plot(rho, t0, "k", lw=4, label="True")
axrho.plot(np.exp(mback[:, 2]), t0, "--r", lw=4, label="Back")
axrho.plot(np.exp(minv_noise[:, 2]), t0, "--g", lw=2, label="PP")
axrho.plot(np.exp(minvPPSP[:, 2]), t0, "--b", lw=2, label="PP+PS")
axrho.set_title(r"$\rho$")
axrho.legend(loc="center left", bbox_to_anchor=(1, 0.5))
axd.axis("tight")
plt.tight_layout()
```



We move now to a 2d example. First of all the model is loaded and data generated.

```
# model
inputfile = "../testdata/avo/poststack_model.npz"

model = np.load(inputfile)
x, z = model["x"][::6] / 1000.0, model["z"][:300] / 1000.0
nx, nz = len(x), len(z)
m = 1000 * model["model"][:300, ::6]

mvp = m.copy()
mvs = m / 2
mrho = m / 3 + 300
m = np.log(np.stack((mvp, mvs, mrho), axis=1))

# smooth model
nsmoothz, nsmoothx = 30, 25
mback = filtfilt(np.ones(nsmoothz) / float(nsmoothz), 1, m, axis=0)
mback = filtfilt(np.ones(nsmoothx) / float(nsmoothx), 1, mback, axis=2)

# dense operator
PPop_dense = pylops.avo.prestack.PrestackLinearModelling(
    wav,
    theta,
    vsvp=vsvp,
    nt0=nz,
    spatdims=(nx,),
    linearization="akirich",
```

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```
    explicit=True,
)

# lop operator
PPop = pylops.avo.prestack.PrestackLinearModelling(
    wav, theta, vsvp=vsvp, nt0=nz, spatdims=(nx,), linearization="akirich"
)

# data
dPP = PPop_dense * m.swapaxes(0, 1).ravel()
dPP = dPP.reshape(ntheta, nz, nx).swapaxes(0, 1)
dPPn = dPP + np.random.normal(0, 5e-2, dPP.shape)
```

Finally we perform the same 4 different inversions as in the post-stack tutorial (see *07. Post-stack inversion* for more details).

```
# dense inversion with noise-free data
minv_dense = pylops.avo.prestack.PrestackInversion(
    dPP, theta, wav, m0=mback, explicit=True, simultaneous=False
)

# dense inversion with noisy data
minv_dense_noisy = pylops.avo.prestack.PrestackInversion(
    dPPn, theta, wav, m0=mback, explicit=True, epsI=4e-2, simultaneous=False
)

# spatially regularized lop inversion with noisy data
minv_lop_reg = pylops.avo.prestack.PrestackInversion(
    dPPn,
    theta,
    wav,
    m0=minv_dense_noisy,
    explicit=False,
    epsR=1e1,
    **dict(damp=np.sqrt(1e-4), iter_lim=20)
)

# blockiness promoting inversion with noisy data
minv_blocky = pylops.avo.prestack.PrestackInversion(
    dPPn,
    theta,
    wav,
    m0=mback,
    explicit=False,
    epsR=0.4,
    epsRL1=0.1,
    **dict(mu=0.1, niter_outer=3, niter_inner=3, iter_lim=5, damp=1e-3)
)
```

Let's now visualize the inverted elastic parameters for the different scenarios

```
def plotmodel(
```

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```
    axs,
    m,
    x,
    z,
    vmin,
    vmax,
    params=("VP", "VS", "Rho"),
    cmap="gist_rainbow",
    title=None,
):
    """Quick visualization of model"""
    for ip, param in enumerate(params):
        axs[ip].imshow(
            m[:, ip], extent=(x[0], x[-1], z[-1], z[0]), vmin=vmin, vmax=vmax, cmap=cmap
        )
        axs[ip].set_title("%s - %s" % (param, title))
        axs[ip].axis("tight")
    plt.setp(axs[1].get_yticklabels(), visible=False)
    plt.setp(axs[2].get_yticklabels(), visible=False)


# data
fig = plt.figure(figsize=(8, 9))
ax1 = plt.subplot2grid((2, 3), (0, 0), colspan=3)
ax2 = plt.subplot2grid((2, 3), (1, 0))
ax3 = plt.subplot2grid((2, 3), (1, 1), sharey=ax2)
ax4 = plt.subplot2grid((2, 3), (1, 2), sharey=ax2)
ax1.imshow(
    dPP[:, 0], cmap="gray", extent=(x[0], x[-1], z[-1], z[0]), vmin=-0.4, vmax=0.4
)
ax1.vlines(
    [x[nx // 5], x[nx // 2], x[4 * nx // 5]],
    ymin=z[0],
    ymax=z[-1],
    colors="w",
    linestyles="--",
)
ax1.set_xlabel("x [km]")
ax1.set_ylabel("z [km]")
ax1.set_title(r"Stack ($\theta$=0)")
ax1.axis("tight")
ax2.imshow(
    dPP[:, :, nx // 5],
    cmap="gray",
    extent=(theta[0], theta[-1], z[-1], z[0]),
    vmin=-0.4,
    vmax=0.4,
)
ax2.set_xlabel(r"$\theta$")
ax2.set_ylabel("z [km]")
ax2.set_title(r"Gather (x=%.2f)" % x[nx // 5])
ax2.axis("tight")
```

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```
ax3.imshow(
    dPP[:, :, nx // 2],
    cmap="gray",
    extent=(theta[0], theta[-1], z[-1], z[0]),
    vmin=-0.4,
    vmax=0.4,
)
ax3.set_xlabel(r"$\theta$")
ax3.set_title(r"Gather (x=%.2f)" % x[nx // 2])
ax3.axis("tight")
ax4.imshow(
    dPP[:, :, 4 * nx // 5],
    cmap="gray",
    extent=(theta[0], theta[-1], z[-1], z[0]),
    vmin=-0.4,
    vmax=0.4,
)
ax4.set_xlabel(r"$\theta$")
ax4.set_title(r"Gather (x=%.2f)" % x[4 * nx // 5])
ax4.axis("tight")
plt.setp(ax3.get_yticklabels(), visible=False)
plt.setp(ax4.get_yticklabels(), visible=False)

# noisy data
fig = plt.figure(figsize=(8, 9))
ax1 = plt.subplot2grid((2, 3), (0, 0), colspan=3)
ax2 = plt.subplot2grid((2, 3), (1, 0))
ax3 = plt.subplot2grid((2, 3), (1, 1), sharey=ax2)
ax4 = plt.subplot2grid((2, 3), (1, 2), sharey=ax2)
ax1.imshow(
    dPPn[:, 0], cmap="gray", extent=(x[0], x[-1], z[-1], z[0]), vmin=-0.4, vmax=0.4
)
ax1.vlines(
    [x[nx // 5], x[nx // 2], x[4 * nx // 5]],
    ymin=z[0],
    ymax=z[-1],
    colors="w",
    linestyles="--",
)
ax1.set_xlabel("x [km]")
ax1.set_ylabel("z [km]")
ax1.set_title(r"Noisy Stack ($\theta$=0)")
ax1.axis("tight")
ax2.imshow(
    dPPn[:, :, nx // 5],
    cmap="gray",
    extent=(theta[0], theta[-1], z[-1], z[0]),
    vmin=-0.4,
    vmax=0.4,
)
ax2.set_xlabel(r"$\theta$")
ax2.set_ylabel("z [km]")
```

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```
ax2.set_title(r"Gather (x=%.2f)" % x[nx // 5])
ax2.axis("tight")
ax3.imshow(
    dPPn[:, :, nx // 2],
    cmap="gray",
    extent=(theta[0], theta[-1], z[-1], z[0]),
    vmin=-0.4,
    vmax=0.4,
)
ax3.set_title(r"Gather (x=%.2f)" % x[nx // 2])
ax3.set_xlabel(r"$\theta$")
ax3.axis("tight")
ax4.imshow(
    dPPn[:, :, 4 * nx // 5],
    cmap="gray",
    extent=(theta[0], theta[-1], z[-1], z[0]),
    vmin=-0.4,
    vmax=0.4,
)
ax4.set_xlabel(r"$\theta$")
ax4.set_title(r"Gather (x=%.2f)" % x[4 * nx // 5])
ax4.axis("tight")
plt.setp(ax3.get_yticklabels(), visible=False)
plt.setp(ax4.get_yticklabels(), visible=False)

# inverted models
fig, axs = plt.subplots(6, 3, figsize=(8, 19))
fig.suptitle("Model", fontsize=12, fontweight="bold", y=0.95)
plotmodel(axs[0], m, x, z, m.min(), m.max(), title="True")
plotmodel(axs[1], mback, x, z, m.min(), m.max(), title="Back")
plotmodel(axs[2], minv_dense, x, z, m.min(), m.max(), title="Dense")
plotmodel(axs[3], minv_dense_noisy, x, z, m.min(), m.max(), title="Dense noisy")
plotmodel(axs[4], minv_lop_reg, x, z, m.min(), m.max(), title="Lop regularized")
plotmodel(axs[5], minv_blocky, x, z, m.min(), m.max(), title="Lop blocky")
plt.tight_layout()
plt.subplots_adjust(top=0.92)

fig, axs = plt.subplots(1, 3, figsize=(8, 7))
for ip, param in enumerate(["VP", "VS", "Rho"]):
    axs[ip].plot(m[:, ip, nx // 2], z, "k", lw=4, label="True")
    axs[ip].plot(mback[:, ip, nx // 2], z, "--r", lw=4, label="Back")
    axs[ip].plot(minv_dense[:, ip, nx // 2], z, "--b", lw=2, label="Inv Dense")
    axs[ip].plot(
        minv_dense_noisy[:, ip, nx // 2], z, "--m", lw=2, label="Inv Dense noisy"
    )
    axs[ip].plot(
        minv_lop_reg[:, ip, nx // 2], z, "--g", lw=2, label="Inv Lop regularized"
    )
    axs[ip].plot(minv_blocky[:, ip, nx // 2], z, "--y", lw=2, label="Inv Lop blocky")
    axs[ip].set_title(param)
    axs[ip].invert_yaxis()
axs[2].legend(loc=8, fontsize="small")
```

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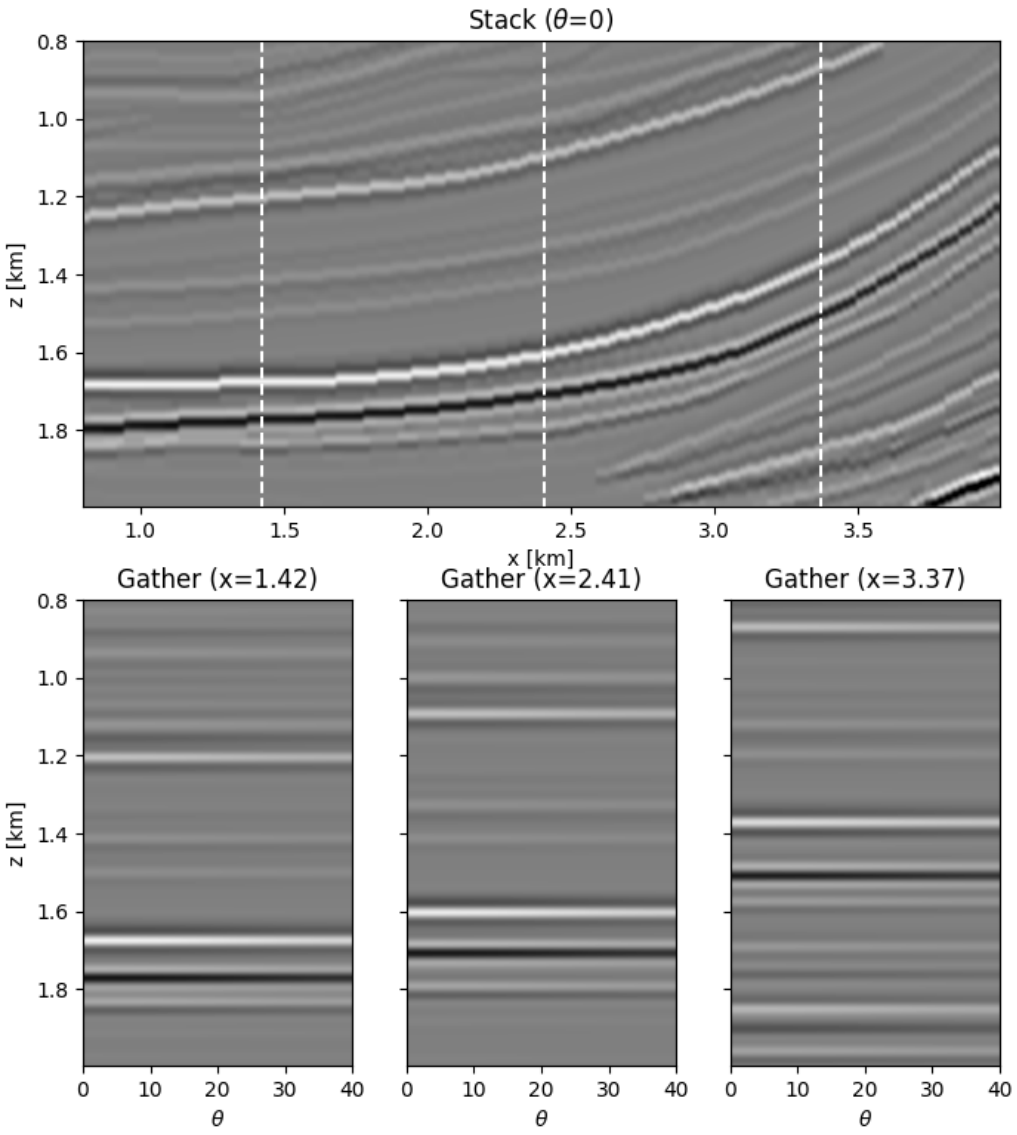
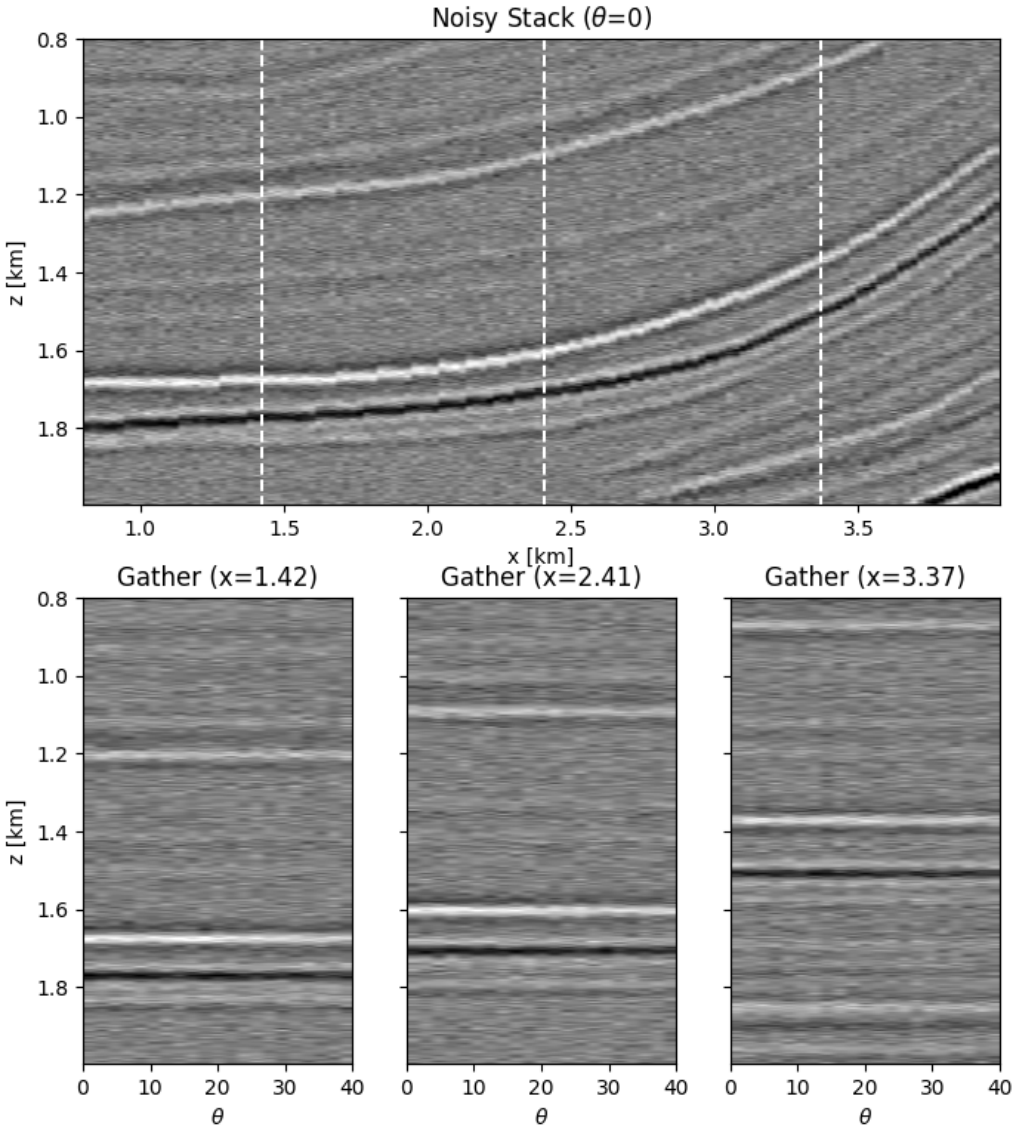
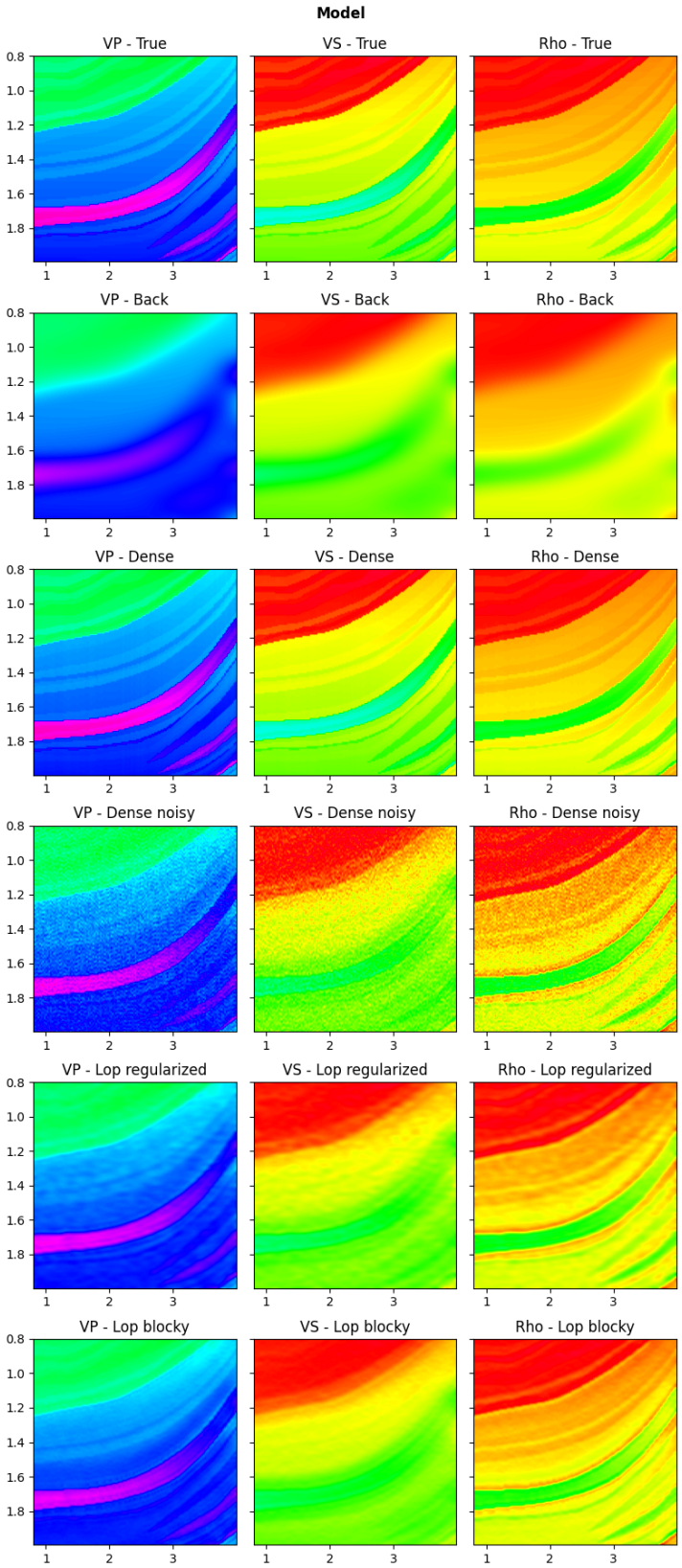
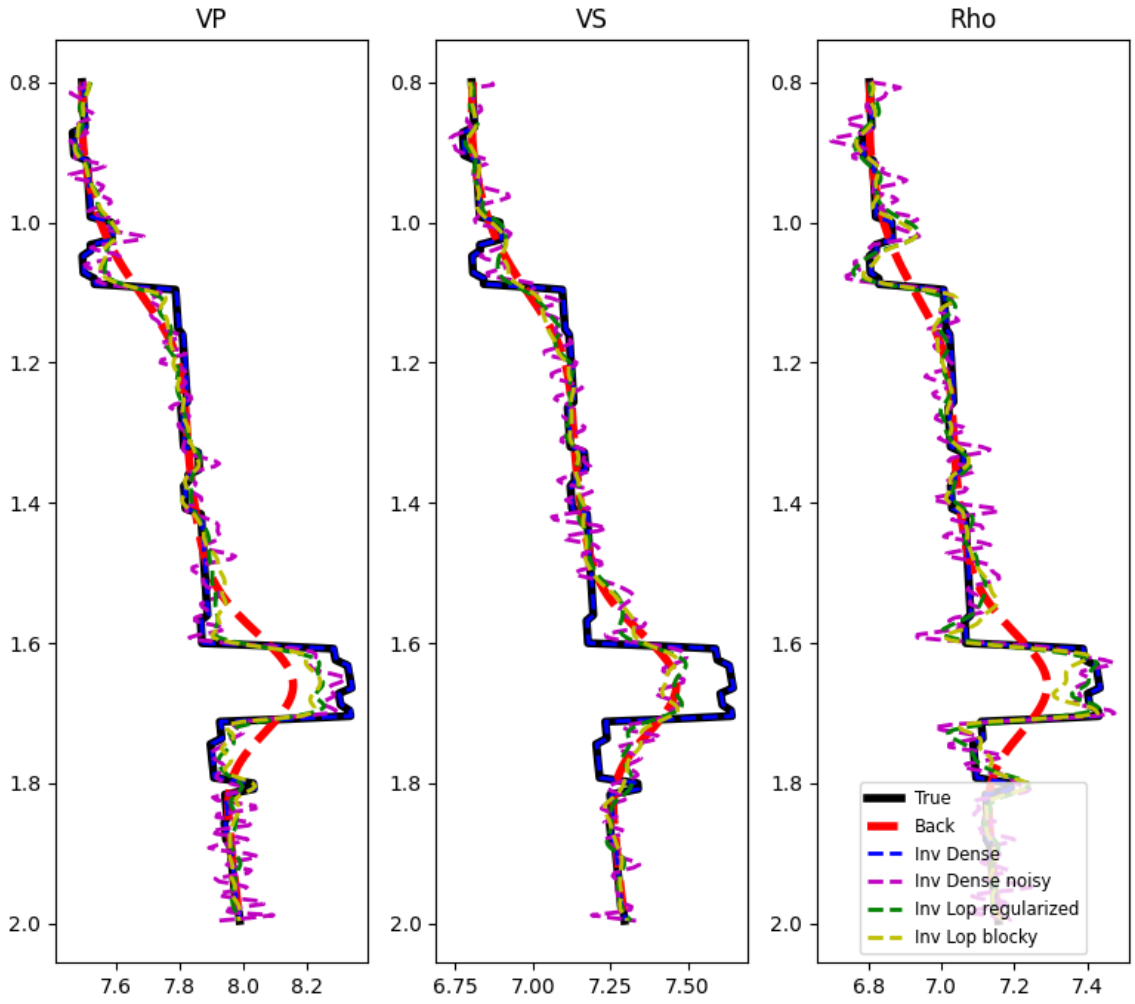
```
plt.tight_layout()
```




Noisy Stack (θ=0)
z [km]
x [km]
Gather (x=1.42)
Gather (x=2.41)
Gather (x=3.37)
θ

-


Model
VP - True
VS - True
Rho - True
VP - Back
VS - Back
Rho - Back
VP - Dense
VS - Dense
Rho - Dense
VP - Dense noisy
VS - Dense noisy
Rho - Dense noisy
VP - Lop regularized
VS - Lop regularized
Rho - Lop regularized
VP - Lop blocky
VS - Lop blocky
Rho - Lop blocky

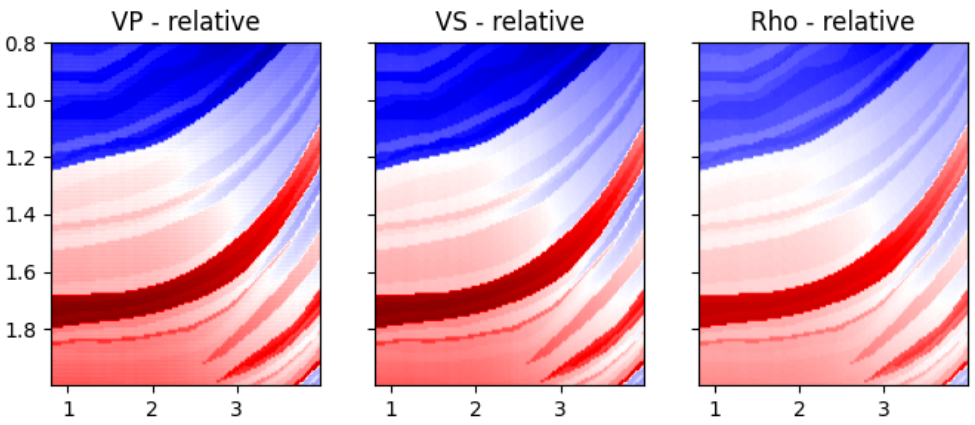


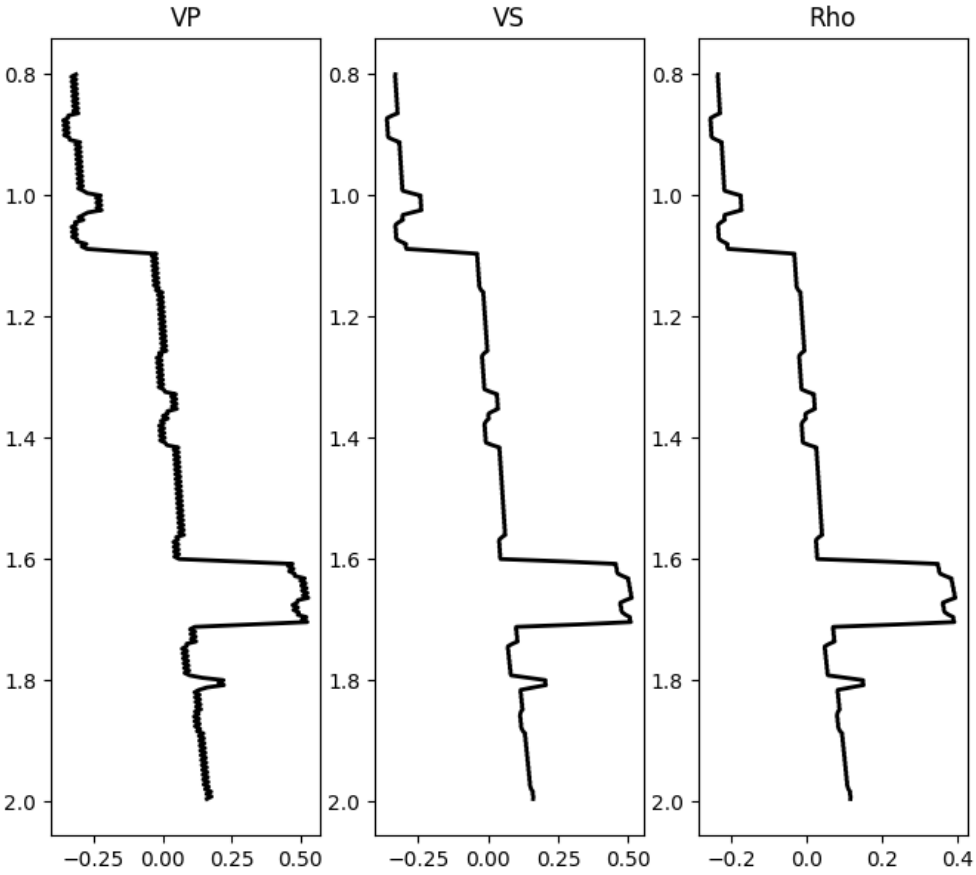
While the background model `m0` has been provided in all the examples so far, it is worth showing that the module *pylops.avo.prestack.PrestackInversion* can also produce so-called relative elastic parameters (i.e., variations from an average medium property) when the background model `m0` is not available.

```
dminv = pylops.avo.prestack.PrestackInversion(
    dPP, theta, wav, m0=None, explicit=True, simultaneous=False
)

fig, axs = plt.subplots(1, 3, figsize=(8, 3))
plotmodel(axs, dminv, x, z, -dminv.max(), dminv.max(), cmap="seismic", title="relative")

fig, axs = plt.subplots(1, 3, figsize=(8, 7))
for ip, param in enumerate(["VP", "VS", "Rho"]):
    axs[ip].plot(dminv[:, ip, nx // 2], z, "k", lw=2)
    axs[ip].set_title(param)
    axs[ip].invert_yaxis()
```





Total running time of the script: (0 minutes 31.440 seconds)

3.4.10 09. Multi-Dimensional Deconvolution

This example shows how to set-up and run the *pylops.waveeqprocessing.MDD* inversion using synthetic data.

```
import warnings

import matplotlib.pyplot as plt
import numpy as np

import pylops
from pylops.utils.seismicevents import hyperbolic2d, makeaxis
from pylops.utils.tapers import taper3d
from pylops.utils.wavelets import ricker

warnings.filterwarnings("ignore")
plt.close("all")

# sphinx_gallery_thumbnail_number = 5
```

Let's start by creating a set of hyperbolic events to be used as our MDC kernel

```
# Input parameters
par = {
    "ox": -150,
    "dx": 10,
    "nx": 31,
    "oy": -250,
    "dy": 10,
    "ny": 51,
    "ot": 0,
    "dt": 0.004,
    "nt": 300,
    "f0": 20,
    "nfmax": 200,
}

t0_m = [0.2]
vrms_m = [700.0]
amp_m = [1.0]

t0_G = [0.2, 0.5, 0.7]
vrms_G = [800.0, 1200.0, 1500.0]
amp_G = [1.0, 0.6, 0.5]

# Taper
tap = taper3d(par["nt"], [par["ny"], par["nx"]], (5, 5), tapertype="hanning")

# Create axis
t, t2, x, y = makeaxis(par)

# Create wavelet
wav = ricker(t[:41], f0=par["f0"])[0]

# Generate model
```

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```
m, mwav = hyperbolic2d(x, t, t0_m, vrms_m, amp_m, wav)

# Generate operator
G, Gwav = np.zeros((par["ny"], par["nx"], par["nt"])), np.zeros(
    (par["ny"], par["nx"], par["nt"])
)
for iy, y0 in enumerate(y):
    G[iy], Gwav[iy] = hyperbolic2d(x - y0, t, t0_G, vrms_G, amp_G, wav)
G, Gwav = G * tap, Gwav * tap

# Add negative part to data and model
m = np.concatenate((np.zeros((par["nx"], par["nt"] - 1)), m), axis=-1)
mwav = np.concatenate((np.zeros((par["nx"], par["nt"] - 1)), mwav), axis=-1)
Gwav2 = np.concatenate((np.zeros((par["ny"], par["nx"], par["nt"] - 1)), Gwav), axis=-1)

# Define MDC linear operator
Gwav_fft = np.fft.rfft(Gwav2, 2 * par["nt"] - 1, axis=-1)
Gwav_fft = (Gwav_fft[..., : par["nfmax"]]).transpose(2, 0, 1)

MDCop = pylops.waveeqprocessing.MDC(
    Gwav_fft,
    nt=2 * par["nt"] - 1,
    nv=1,
    dt=0.004,
    dr=1.0,
)

# Create data
d = MDCop * m.T.ravel()
d = d.reshape(2 * par["nt"] - 1, par["ny"]).T
```

Let's display what we have so far: operator, input model, and data

```
fig, axs = plt.subplots(1, 2, figsize=(8, 6))
axs[0].imshow(
    Gwav2[int(par["ny"] / 2)].T,
    aspect="auto",
    interpolation="nearest",
    cmap="gray",
    vmin=-np.abs(Gwav2.max()),
    vmax=np.abs(Gwav2.max()),
    extent=(x.min(), x.max(), t2.max(), t2.min()),
)
axs[0].set_title("G - inline view", fontsize=15)
axs[0].set_xlabel(r"$x_R$")
axs[1].set_ylabel(r"$t$")
axs[1].imshow(
    Gwav2[:, int(par["nx"] / 2)].T,
    aspect="auto",
    interpolation="nearest",
    cmap="gray",
    vmin=-np.abs(Gwav2.max()),
```

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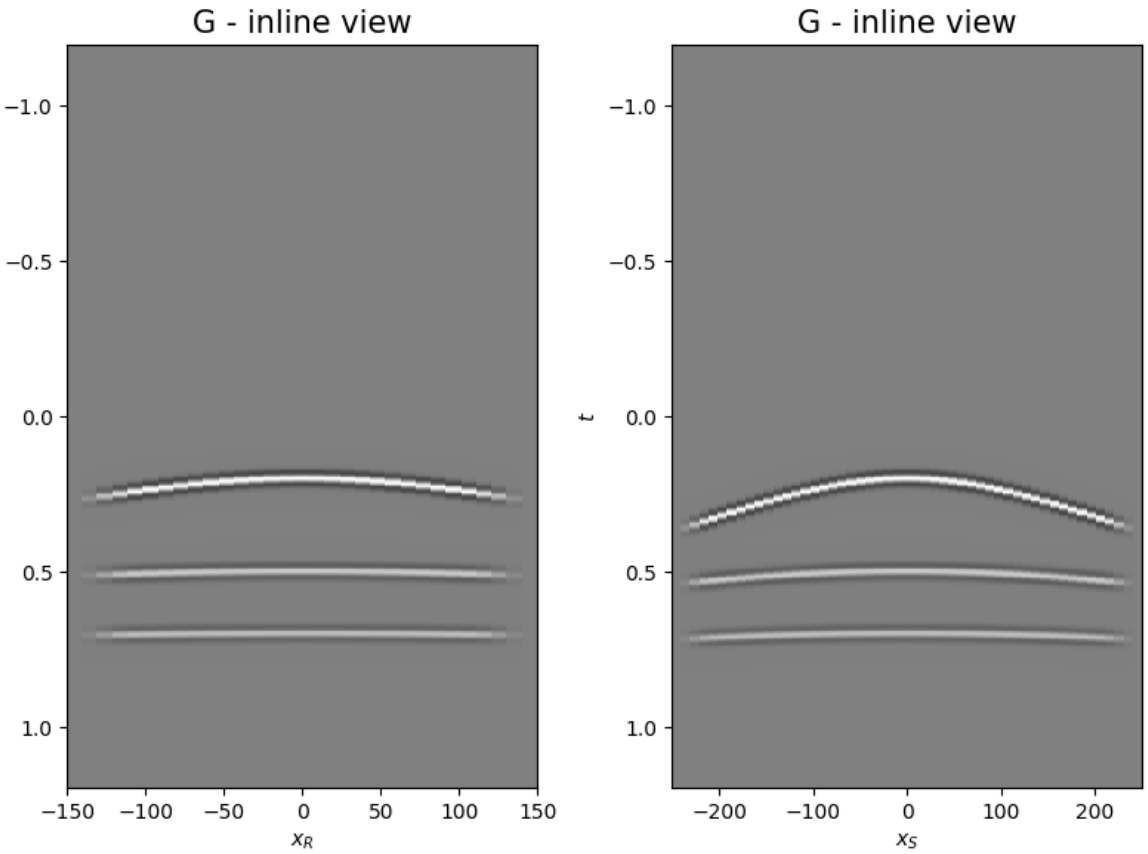
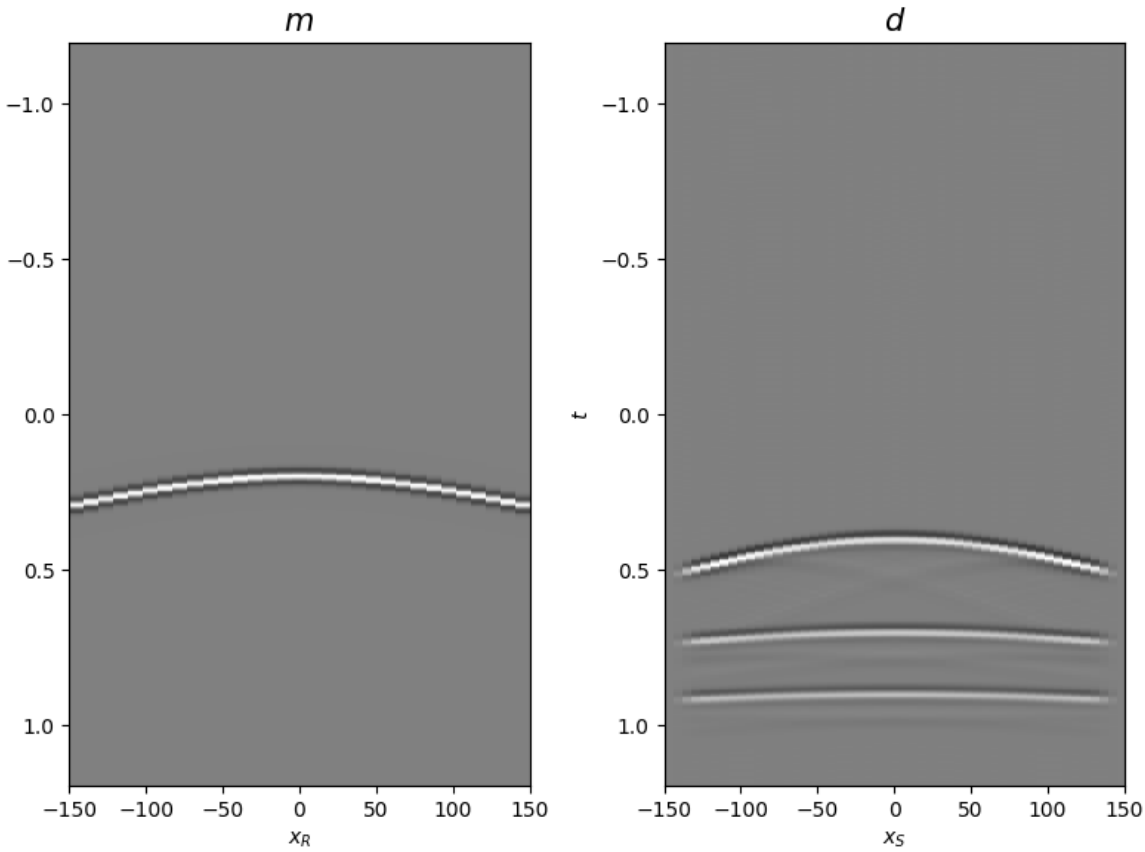
```
    vmax=np.abs(Gwav2.max()),
    extent=(y.min(), y.max(), t2.max(), t2.min()),
)
axs[1].set_title("G - inline view", fontsize=15)
axs[1].set_xlabel(r"$x_S$")
axs[1].set_ylabel(r"$t$")
fig.tight_layout()

fig, axs = plt.subplots(1, 2, figsize=(8, 6))
axs[0].imshow(
    mwav.T,
    aspect="auto",
    interpolation="nearest",
    cmap="gray",
    vmin=-np.abs(mwav.max()),
    vmax=np.abs(mwav.max()),
    extent=(x.min(), x.max(), t2.max(), t2.min()),
)
axs[0].set_title(r"$m$", fontsize=15)
axs[0].set_xlabel(r"$x_R$")
axs[1].set_ylabel(r"$t$")
axs[1].imshow(
    d.T,
    aspect="auto",
    interpolation="nearest",
    cmap="gray",
    vmin=-np.abs(d.max()),
    vmax=np.abs(d.max()),
    extent=(x.min(), x.max(), t2.max(), t2.min()),
)
axs[1].set_title(r"$d$", fontsize=15)
axs[1].set_xlabel(r"$x_S$")
axs[1].set_ylabel(r"$t$")
fig.tight_layout()
```


G - inline view
G - inline view
−1.0
−0.5
0.0
0.5
1.0
t
−150
−100
−50
0
50
100
150
x_R
−200
−100
0
100
200
x_S



We are now ready to feed our operator to *pylops.waveeqprocessing.MDD* and invert back for our input model

```
minv, madj, psfinv, psfadj = pylops.waveeqprocessing.MDD(
    Gwav,
    d[:, par["nt"] - 1 :],
    dt=par["dt"],
    dr=par["dx"],
    nfmax=par["nfmax"],
    wav=wav,
    twosided=True,
    add_negative=True,
    adjoint=True,
    psf=True,
    dottest=False,
    **dict(damp=1e-4, iter_lim=20, show=0)
)

fig = plt.figure(figsize=(8, 6))
ax1 = plt.subplot2grid((1, 5), (0, 0), colspan=2)
ax2 = plt.subplot2grid((1, 5), (0, 2), colspan=2)
ax3 = plt.subplot2grid((1, 5), (0, 4))
ax1.imshow(
    madj.T,
    aspect="auto",
    interpolation="nearest",
```

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```
    cmap="gray",
    vmin=-np.abs(madj.max()),
    vmax=np.abs(madj.max()),
    extent=(x.min(), x.max(), t2.max(), t2.min()),
)
ax1.set_title("Adjoint m", fontsize=15)
ax1.set_xlabel(r"$x_V$")
axs[1].set_ylabel(r"$t$")
ax2.imshow(
    minv.T,
    aspect="auto",
    interpolation="nearest",
    cmap="gray",
    vmin=-np.abs(minv.max()),
    vmax=np.abs(minv.max()),
    extent=(x.min(), x.max(), t2.max(), t2.min()),
)
ax2.set_title("Inverted m", fontsize=15)
ax2.set_xlabel(r"$x_V$")
axs[1].set_ylabel(r"$t$")
ax3.plot(
    madj[int(par["nx"] / 2)] / np.abs(madj[int(par["nx"] / 2)]).max(), t2, "r", lw=5
)
ax3.plot(
    minv[int(par["nx"] / 2)] / np.abs(minv[int(par["nx"] / 2)]).max(), t2, "k", lw=3
)
ax3.set_ylim([t2[-1], t2[0]])
fig.tight_layout()

fig, axs = plt.subplots(1, 2, figsize=(8, 6))
axs[0].imshow(
    psfinv[int(par["nx"] / 2)].T,
    aspect="auto",
    interpolation="nearest",
    vmin=-np.abs(psfinv.max()),
    vmax=np.abs(psfinv.max()),
    cmap="gray",
    extent=(x.min(), x.max(), t2.max(), t2.min()),
)
axs[0].set_title("Inverted psf - inline view", fontsize=15)
axs[0].set_xlabel(r"$x_V$")
axs[1].set_ylabel(r"$t$")
axs[1].imshow(
    psfinv[:, int(par["nx"] / 2)].T,
    aspect="auto",
    interpolation="nearest",
    vmin=-np.abs(psfinv.max()),
    vmax=np.abs(psfinv.max()),
    cmap="gray",
    extent=(y.min(), y.max(), t2.max(), t2.min()),
)
axs[1].set_title("Inverted psf - xline view", fontsize=15)
```

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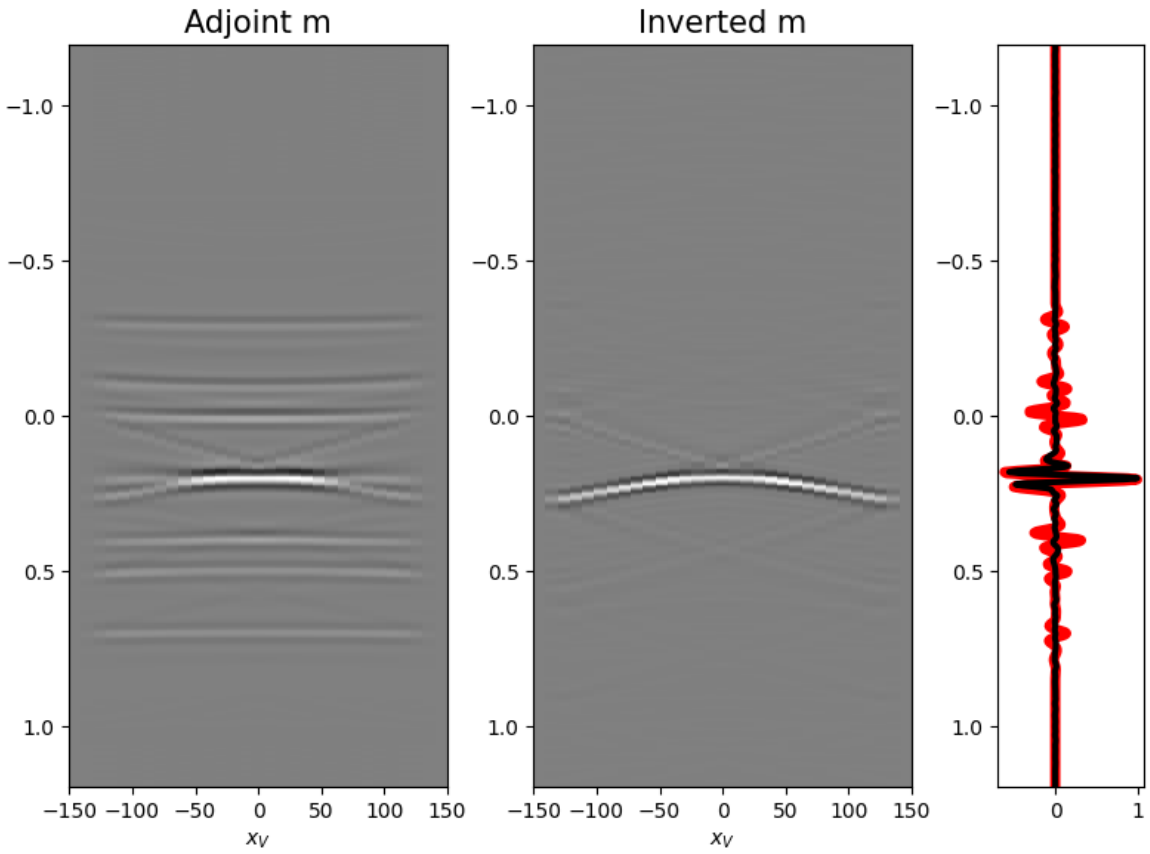
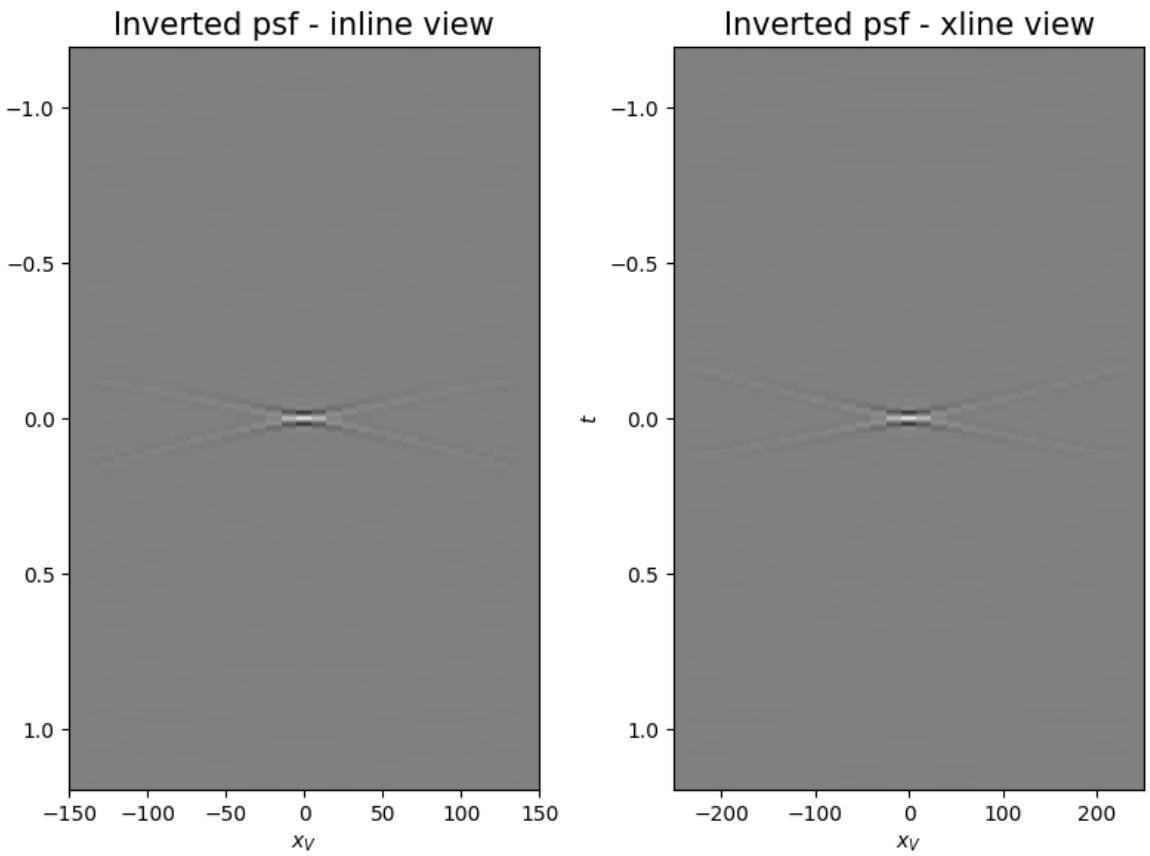
```
axs[1].set_xlabel(r"$x_V$")
axs[1].set_ylabel(r"$t$")
fig.tight_layout()
```





We repeat the same procedure but this time we will add a preconditioning by means of `causality_precond` parameter, which enforces the inverted model to be zero in the negative part of the time axis (as expected by theory). This preconditioning will have the effect of speeding up the convergence of the iterative solver and thus reduce the computation time of the deconvolution

```
minvprec = pylops.waveeqprocessing.MDD(
    Gwav,
    d[:, par["nt"] - 1 :],
    dt=par["dt"],
    dr=par["dx"],
    nfmax=par["nfmax"],
    wav=wav,
    twosided=True,
    add_negative=True,
    adjoint=False,
    psf=False,
    causality_precond=True,
    dottest=False,
    **dict(damp=1e-4, iter_lim=50, show=0)
)

fig = plt.figure(figsize=(8, 6))
ax1 = plt.subplot2grid((1, 5), (0, 0), colspan=2)
ax2 = plt.subplot2grid((1, 5), (0, 2), colspan=2)
ax3 = plt.subplot2grid((1, 5), (0, 4))
```

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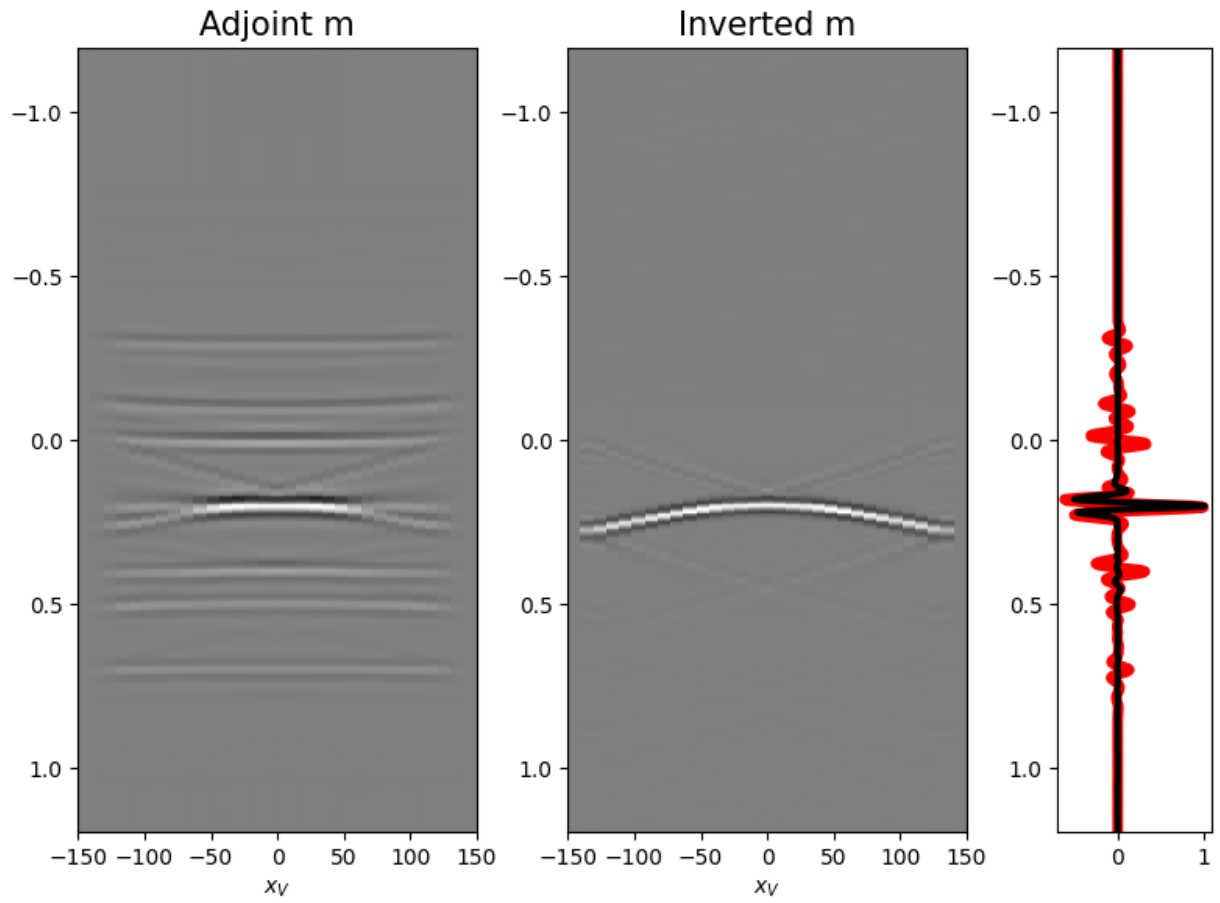
```
ax1.imshow(
    madj.T,
    aspect="auto",
    interpolation="nearest",
    cmap="gray",
    vmin=-np.abs(madj.max()),
    vmax=np.abs(madj.max()),
    extent=(x.min(), x.max(), t2.max(), t2.min()),
)
ax1.set_title("Adjoint m", fontsize=15)
ax1.set_xlabel(r"$x_V$")
axs[1].set_ylabel(r"$t$")
ax2.imshow(
    minvprec.T,
    aspect="auto",
    interpolation="nearest",
    cmap="gray",
    vmin=-np.abs(minvprec.max()),
    vmax=np.abs(minvprec.max()),
    extent=(x.min(), x.max(), t2.max(), t2.min()),
)
ax2.set_title("Inverted m", fontsize=15)
ax2.set_xlabel(r"$x_V$")
axs[1].set_ylabel(r"$t$")
ax3.plot(
    madj[int(par["nx"] / 2)] / np.abs(madj[int(par["nx"] / 2)]).max(), t2, "r", lw=5
)
ax3.plot(
    minvprec[int(par["nx"] / 2)] / np.abs(minv[int(par["nx"] / 2)]).max(), t2, "k", lw=3
)
ax3.set_ylim([t2[-1], t2[0]])
fig.tight_layout()
```



Total running time of the script: (0 minutes 11.100 seconds)

3.4.11 10. Marchenko redatuming by inversion

This example shows how to set-up and run the *pylops.waveeqprocessing.Marchenko* inversion using synthetic data.

```
# sphinx_gallery_thumbnail_number = 5
# pylint: disable=C0103
import warnings

import matplotlib.pyplot as plt
import numpy as np
from scipy.signal import convolve

from pylops.waveeqprocessing import Marchenko

warnings.filterwarnings("ignore")
plt.close("all")
```

Let's start by defining some input parameters and loading the test data

```
# Input parameters
inputfile = "../testdata/marchenko/input.npz"

vel = 2400.0  # velocity
toff = 0.045  # direct arrival time shift
nsmooth = 10  # time window smoothing
nfmax = 1000  # max frequency for MDC (#samples)
niter = 10  # iterations

inputdata = np.load(inputfile)

# Receivers
r = inputdata["r"]
nr = r.shape[1]
dr = r[0, 1] - r[0, 0]

# Sources
s = inputdata["s"]
ns = s.shape[1]
ds = s[0, 1] - s[0, 0]

# Virtual points
vs = inputdata["vs"]

# Density model
rho = inputdata["rho"]
z, x = inputdata["z"], inputdata["x"]

# Reflection data (R[s, r, t]) and subsurface fields
R = inputdata["R"][:, :, :-100]
R = np.swapaxes(R, 0, 1)  # just because of how the data was saved

Gsub = inputdata["Gsub"][:-100]
G0sub = inputdata["G0sub"][:-100]
wav = inputdata["wav"]
wav_c = np.argmax(wav)

t = inputdata["t"][:-100]
ot, dt, nt = t[0], t[1] - t[0], len(t)

Gsub = np.apply_along_axis(convolve, 0, Gsub, wav, mode="full")
Gsub = Gsub[wav_c:][:nt]
G0sub = np.apply_along_axis(convolve, 0, G0sub, wav, mode="full")
G0sub = G0sub[wav_c:][:nt]

plt.figure(figsize=(10, 5))
plt.imshow(rho, cmap="gray", extent=(x[0], x[-1], z[-1], z[0]))
plt.scatter(s[0, 5::10], s[1, 5::10], marker="*", s=150, c="r", edgecolors="k")
plt.scatter(r[0, ::10], r[1, ::10], marker="v", s=150, c="b", edgecolors="k")
plt.scatter(vs[0], vs[1], marker=".", s=250, c="m", edgecolors="k")
plt.axis("tight")
plt.xlabel("x [m]")
plt.ylabel("y [m]")
```

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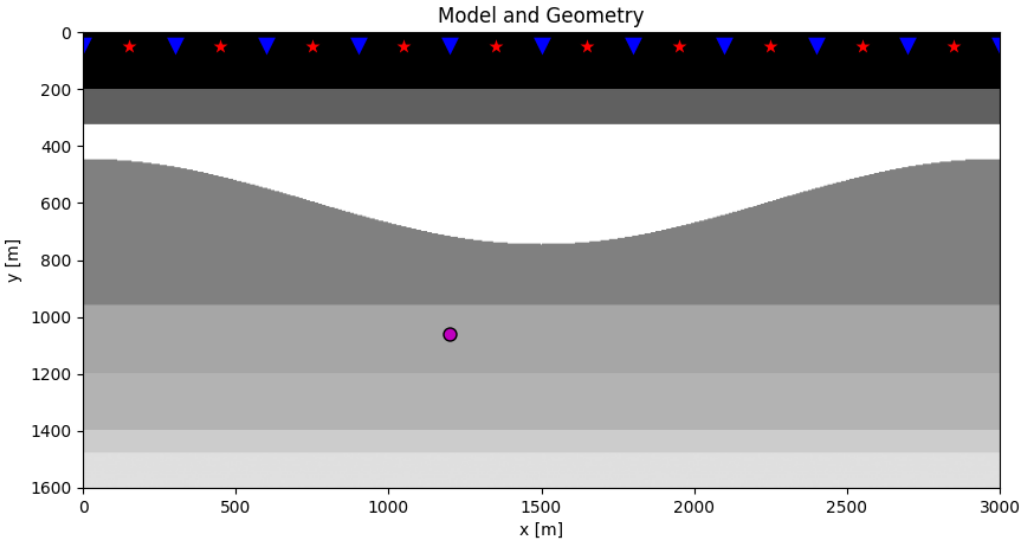
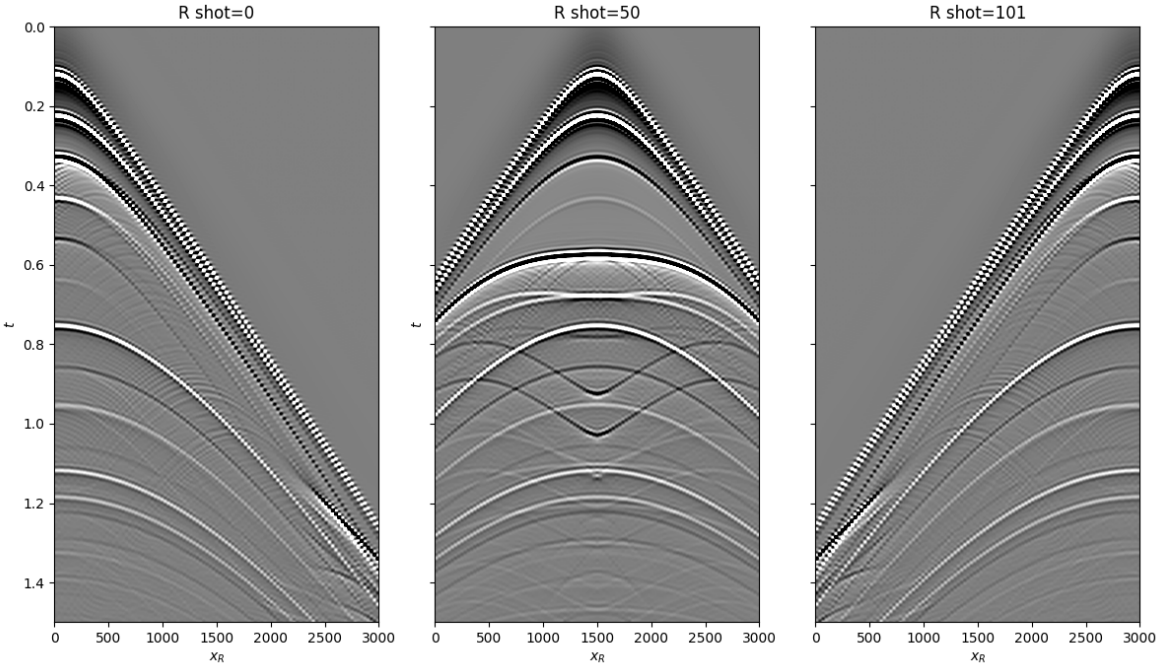
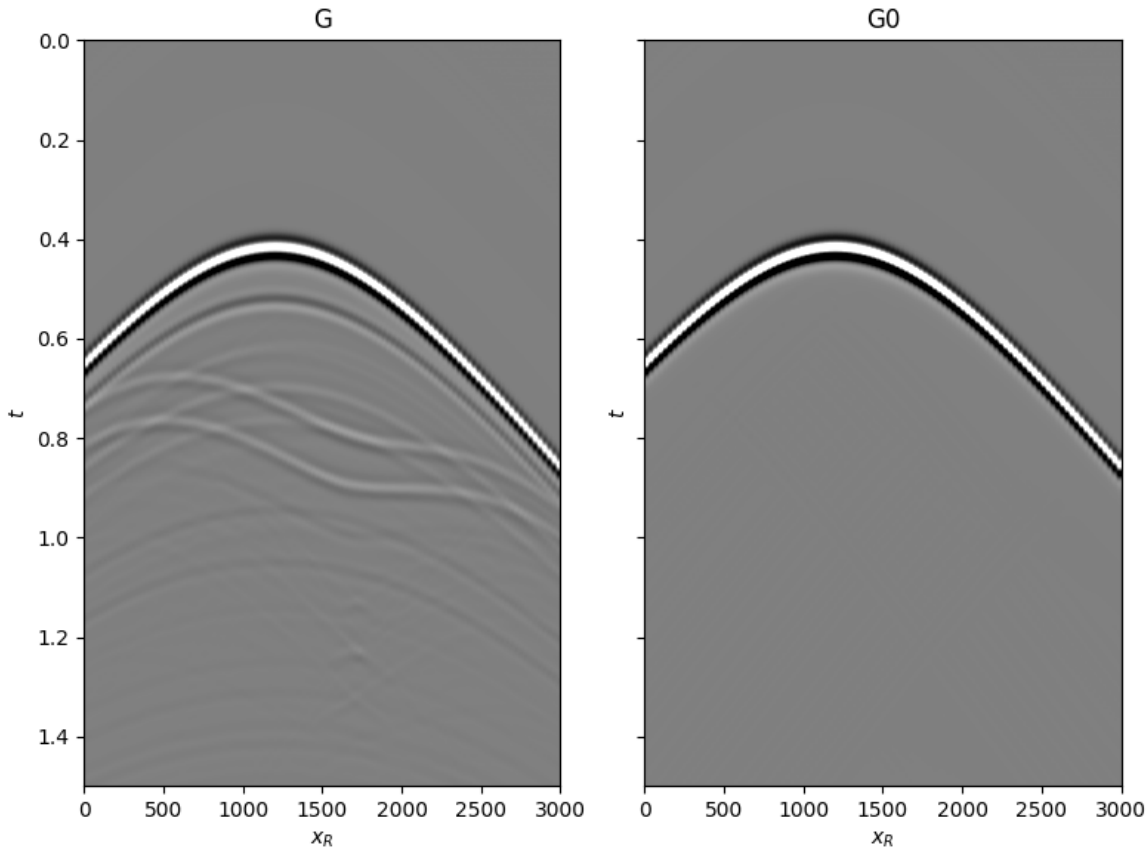
```
plt.title("Model and Geometry")
plt.xlim(x[0], x[-1])

fig, axs = plt.subplots(1, 3, sharey=True, figsize=(12, 7))
axs[0].imshow(
    R[0].T, cmap="gray", vmin=-1e-2, vmax=1e-2, extent=(r[0, 0], r[0, -1], t[-1], t[0])
)
axs[0].set_title("R shot=0")
axs[0].set_xlabel(r"$x_R$")
axs[0].set_ylabel(r"$t$")
axs[0].axis("tight")
axs[0].set_ylim(1.5, 0)
axs[1].imshow(
    R[ns // 2].T,
    cmap="gray",
    vmin=-1e-2,
    vmax=1e-2,
    extent=(r[0, 0], r[0, -1], t[-1], t[0]),
)
axs[1].set_title("R shot=%d" % (ns // 2))
axs[1].set_xlabel(r"$x_R$")
axs[1].set_ylabel(r"$t$")
axs[1].axis("tight")
axs[1].set_ylim(1.5, 0)
axs[2].imshow(
    R[-1].T, cmap="gray", vmin=-1e-2, vmax=1e-2, extent=(r[0, 0], r[0, -1], t[-1], t[0])
)
axs[2].set_title("R shot=%d" % ns)
axs[2].set_xlabel(r"$x_R$")
axs[2].axis("tight")
axs[2].set_ylim(1.5, 0)
fig.tight_layout()

fig, axs = plt.subplots(1, 2, sharey=True, figsize=(8, 6))
axs[0].imshow(
    Gsub, cmap="gray", vmin=-1e6, vmax=1e6, extent=(r[0, 0], r[0, -1], t[-1], t[0])
)
axs[0].set_title("G")
axs[0].set_xlabel(r"$x_R$")
axs[0].set_ylabel(r"$t$")
axs[0].axis("tight")
axs[0].set_ylim(1.5, 0)
axs[1].imshow(
    G0sub, cmap="gray", vmin=-1e6, vmax=1e6, extent=(r[0, 0], r[0, -1], t[-1], t[0])
)
axs[1].set_title("G0")
axs[1].set_xlabel(r"$x_R$")
axs[1].set_ylabel(r"$t$")
axs[1].axis("tight")
axs[1].set_ylim(1.5, 0)
fig.tight_layout()
```


Model and Geometry
y [m]
x [m]
0
200
400
600
800
1000
1200
1400
1600
0
500
1000
1500
2000
2500
3000


R shot=0
R shot=50
R shot=101
t
x_R
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
0
500
1000
1500
2000
2500
3000



Let's now create an object of the *pylops.waveeqprocessing.Marchenko* class and apply redatuming for a single subsurface point `vs`.

```
# direct arrival window
trav = np.sqrt((vs[0] - r[0]) ** 2 + (vs[1] - r[1]) ** 2) / vel

MarchenkoWM = Marchenko(
    R, dt=dt, dr=dr, nfmax=nfmax, wav=wav, toff=toff, nsmooth=nsmooth
)

(
    f1_inv_minus,
    f1_inv_plus,
    p0_minus,
    g_inv_minus,
    g_inv_plus,
) = MarchenkoWM.apply_onepoint(
    trav,
    G0=G0sub.T,
    rtm=True,
    greens=True,
    dottest=True,
    **dict(iter_lim=niter, show=True)
)
g_inv_tot = g_inv_minus + g_inv_plus
```

```
Dot test passed, v^H(Opu)=405.16509639614344 - u^H(Op^Hv)=405.1650963961446
Dot test passed, v^H(Opu)=172.06507561051328 - u^H(Op^Hv)=172.06507561051316

LSQR            Least-squares solution of  Ax = b
The matrix A has 282598 rows and 282598 columns
damp = 0.00000000000000e+00   calc_var =        0
atol = 1.00e-06                 conlim = 1.00e+08
btol = 1.00e-06               iter_lim =       10

   Itn      x[0]       r1norm     r2norm   Compatible    LS      Norm A   Cond A
     0  0.00000e+00   3.134e+07  3.134e+07    1.0e+00  3.3e-08
     1  0.00000e+00   1.374e+07  1.374e+07    4.4e-01  9.3e-01   1.1e+00  1.0e+00
     2  0.00000e+00   7.770e+06  7.770e+06    2.5e-01  3.9e-01   1.8e+00  2.2e+00
     3  0.00000e+00   5.750e+06  5.750e+06    1.8e-01  3.3e-01   2.1e+00  3.4e+00
     4  0.00000e+00   3.930e+06  3.930e+06    1.3e-01  3.4e-01   2.5e+00  5.1e+00
     5  0.00000e+00   3.042e+06  3.042e+06    9.7e-02  2.6e-01   2.9e+00  6.8e+00
     6  0.00000e+00   2.423e+06  2.423e+06    7.7e-02  2.2e-01   3.3e+00  8.6e+00
     7  0.00000e+00   1.675e+06  1.675e+06    5.3e-02  2.5e-01   3.6e+00  1.1e+01
     8  0.00000e+00   1.248e+06  1.248e+06    4.0e-02  2.0e-01   3.9e+00  1.3e+01
     9  0.00000e+00   1.004e+06  1.004e+06    3.2e-02  1.5e-01   4.2e+00  1.4e+01
    10  0.00000e+00   7.762e+05  7.762e+05    2.5e-02  1.8e-01   4.4e+00  1.6e+01

LSQR finished
The iteration limit has been reached

istop =       7   r1norm = 7.8e+05   anorm = 4.4e+00   arnorm = 6.1e+05
itn   =      10   r2norm = 7.8e+05   acond = 1.6e+01   xnorm  = 3.6e+07
```

We can now compare the result of Marchenko redatuming via LSQR with standard redatuming

```
fig, axs = plt.subplots(1, 3, sharey=True, figsize=(12, 7))
axs[0].imshow(
    p0_minus.T,
    cmap="gray",
    vmin=-5e5,
    vmax=5e5,
    extent=(r[0, 0], r[0, -1], t[-1], -t[-1]),
)
axs[0].set_title(r"$p_0^-$")
axs[0].set_xlabel(r"$x_R$")
axs[0].set_ylabel(r"$t$")
axs[0].axis("tight")
axs[0].set_ylim(1.2, 0)
axs[1].imshow(
    g_inv_minus.T,
    cmap="gray",
    vmin=-5e5,
    vmax=5e5,
    extent=(r[0, 0], r[0, -1], t[-1], -t[-1]),
)
axs[1].set_title(r"$g^-$")
axs[1].set_xlabel(r"$x_R$")
axs[1].set_ylabel(r"$t$")
```

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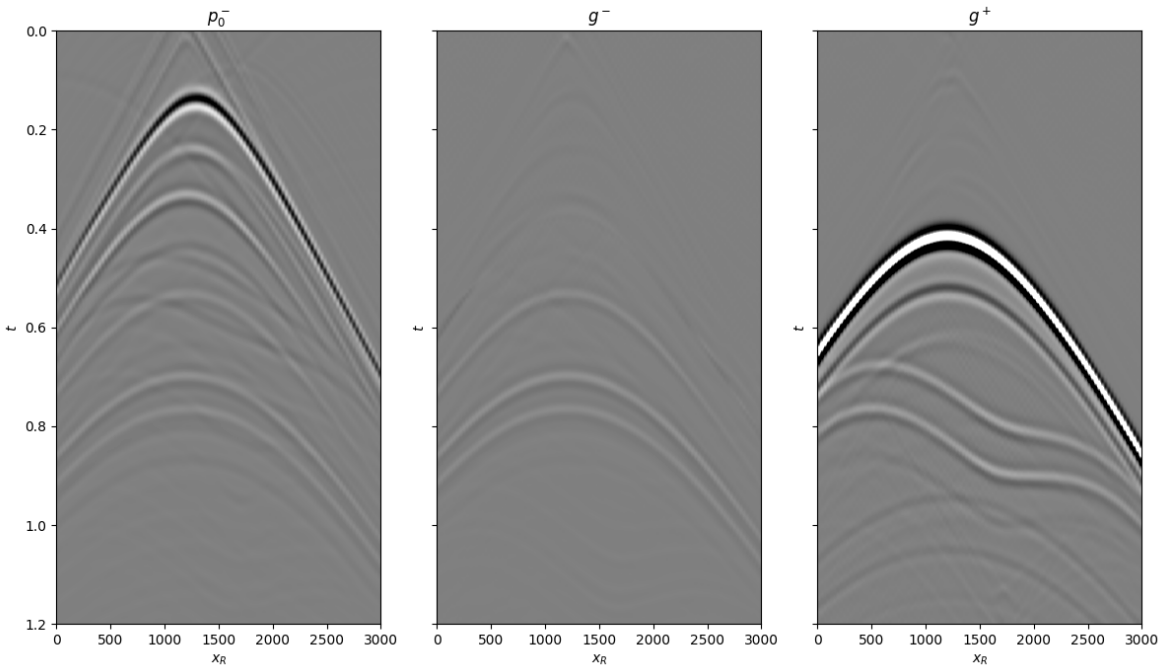
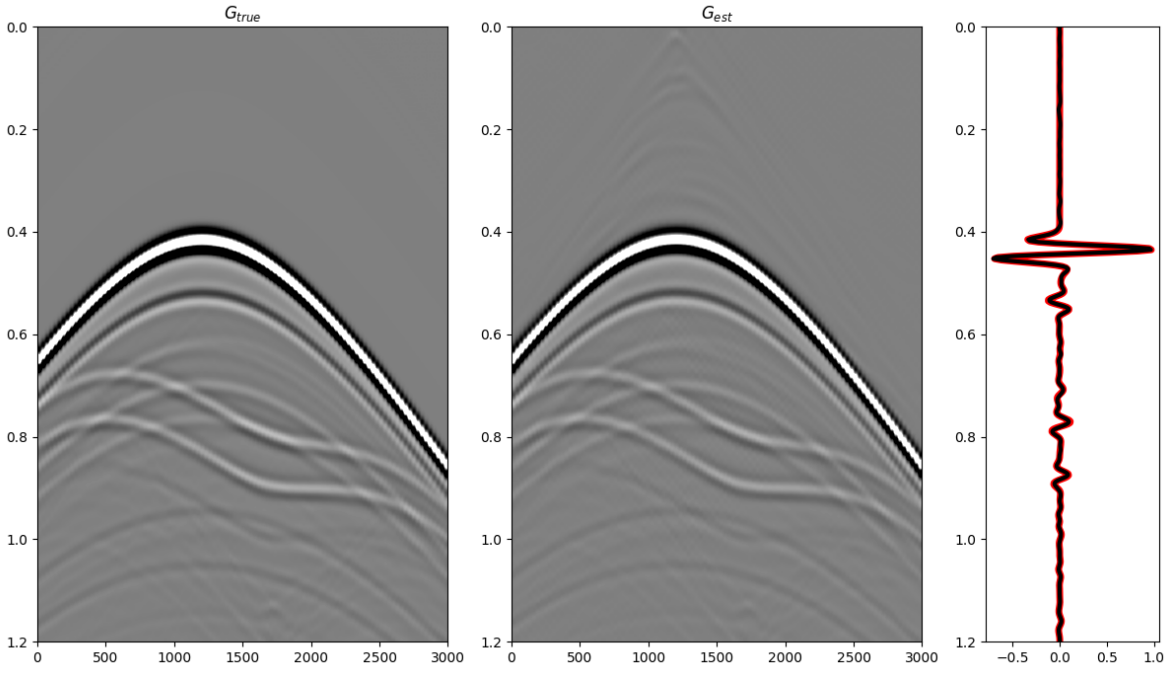
```
axs[1].axis("tight")
axs[1].set_ylim(1.2, 0)
axs[2].imshow(
    g_inv_plus.T,
    cmap="gray",
    vmin=-5e5,
    vmax=5e5,
    extent=(r[0, 0], r[0, -1], t[-1], -t[-1]),
)
axs[2].set_title(r"$g^+$")
axs[2].set_xlabel(r"$x_R$")
axs[2].set_ylabel(r"$t$")
axs[2].axis("tight")
axs[2].set_ylim(1.2, 0)
fig.tight_layout()

fig = plt.figure(figsize=(12, 7))
ax1 = plt.subplot2grid((1, 5), (0, 0), colspan=2)
ax2 = plt.subplot2grid((1, 5), (0, 2), colspan=2)
ax3 = plt.subplot2grid((1, 5), (0, 4))
ax1.imshow(
    Gsub, cmap="gray", vmin=-5e5, vmax=5e5, extent=(r[0, 0], r[0, -1], t[-1], t[0])
)
ax1.set_title(r"$G_{true}$")
axs[0].set_xlabel(r"$x_R$")
axs[0].set_ylabel(r"$t$")
ax1.axis("tight")
ax1.set_ylim(1.2, 0)
ax2.imshow(
    g_inv_tot.T,
    cmap="gray",
    vmin=-5e5,
    vmax=5e5,
    extent=(r[0, 0], r[0, -1], t[-1], -t[-1]),
)
ax2.set_title(r"$G_{est}$")
axs[1].set_xlabel(r"$x_R$")
axs[1].set_ylabel(r"$t$")
ax2.axis("tight")
ax2.set_ylim(1.2, 0)
ax3.plot(Gsub[:, nr // 2] / Gsub.max(), t, "r", lw=5)
ax3.plot(g_inv_tot[nr // 2, nt - 1 :] / g_inv_tot.max(), t, "k", lw=3)
ax3.set_ylim(1.2, 0)
fig.tight_layout()
```





Note that Marchenko redatuming can also be applied simultaneously to multiple subsurface points. Use `pylops.waveeqprocessing.Marchenko.apply_multiplepoints` instead of `pylops.waveeqprocessing.Marchenko.apply_onepoint`.

Total running time of the script: (0 minutes 7.702 seconds)

3.4.12 11. Radon filtering

In this example we will be taking advantage of the *pylops.signalprocessing.Radon2D* operator to perform filtering of unwanted events from a seismic data. For those of you not familiar with seismic data, let's imagine that we have a data composed of a certain number of flat events and a parabolic event , we are after a transform that allows us to separate such an event from the others and filter it out. Those of you with a geophysics background may immediately realize this is the case of seismic angle (or offset) gathers after migration and those events with parabolic moveout are generally residual multiples that we would like to suppress prior to performing further analysis of our data.

The Radon transform is actually a very good transform to perform such a separation. We can thus devise a simple workflow that takes our data as input, applies a Radon transform, filters some of the events out and goes back to the original domain.

```
import matplotlib.pyplot as plt
import numpy as np

import pylops
from pylops.utils.wavelets import ricker

plt.close("all")
np.random.seed(0)
```

Let's first create a data composed on 3 linear events and a parabolic event.

```
par = {"ox": 0, "dx": 2, "nx": 121, "ot": 0, "dt": 0.004, "nt": 100, "f0": 30}

# linear events
v = 1500  # m/s
t0 = [0.1, 0.2, 0.3]  # s
theta = [0, 0, 0]
amp = [1.0, -2, 0.5]

# parabolic event
tp0 = [0.13]  # s
px = [0]  # s/m
pxx = [5e-7]  # s^2/m^2
ampp = [0.7]

# create axis
taxis, taxis2, xaxis, yaxis = pylops.utils.seismicevents.makeaxis(par)

# create wavelet
wav = ricker(taxis[:41], f0=par["f0"])[0]

# generate model
y = (
    pylops.utils.seismicevents.linear2d(xaxis, taxis, v, t0, theta, amp, wav)[1]
    + pylops.utils.seismicevents.parabolic2d(xaxis, taxis, tp0, px, pxx, ampp, wav)[1]
)
```

We can now create the *pylops.signalprocessing.Radon2D* operator. We also apply its adjoint to the data to obtain a representation of those 3 linear events overlapping to a parabolic event in the Radon domain. Similarly, we feed the operator to a sparse solver like `pylops.optimization.sparsity.FISTA` to obtain a sparse represention of the data in the Radon domain. At this point we try to filter out the unwanted event. We can see how this is much easier for the sparse transform as each event has a much more compact representation in the Radon domain than for the adjoint

transform.

```
# radon operator
npx = 61
pxmax = 5e-4  # s/m
px = np.linspace(-pxmax, pxmax, npx)

Rop = pylops.signalprocessing.Radon2D(
    taxis, xaxis, px, kind="linear", interp="nearest", centeredh=False, dtype="float64"
)

# adjoint Radon transform
xadj = Rop.H * y

# sparse Radon transform
xinv, niter, cost = pylops.optimization.sparsity.fista(
    Rop, y.ravel(), niter=15, eps=1e1
)
xinv = xinv.reshape(Rop.dims)

# filtering
xfilt = np.zeros_like(xadj)
xfilt[npx // 2 - 3 : npx // 2 + 4] = xadj[npx // 2 - 3 : npx // 2 + 4]

yfilt = Rop * xfilt

# filtering on sparse transform
xinvfilt = np.zeros_like(xinv)
xinvfilt[npx // 2 - 3 : npx // 2 + 4] = xinv[npx // 2 - 3 : npx // 2 + 4]

yinvfilt = Rop * xinvfilt
```

Finally we visualize our results.

```
pclip = 0.7
fig, axs = plt.subplots(1, 5, sharey=True, figsize=(12, 5))
axs[0].imshow(
    y.T,
    cmap="gray",
    vmin=-pclip * np.abs(y).max(),
    vmax=pclip * np.abs(y).max(),
    extent=(xaxis[0], xaxis[-1], taxis[-1], taxis[0]),
)
axs[0].set(xlabel="$x$ [m]", ylabel="$t$ [s]", title="Data")
axs[0].axis("tight")
axs[1].imshow(
    xadj.T,
    cmap="gray",
    vmin=-pclip * np.abs(xadj).max(),
    vmax=pclip * np.abs(xadj).max(),
    extent=(px[0], px[-1], taxis[-1], taxis[0]),
)
axs[1].axvline(px[npx // 2 - 3], color="r", linestyle="--")
axs[1].axvline(px[npx // 2 + 3], color="r", linestyle="--")
```

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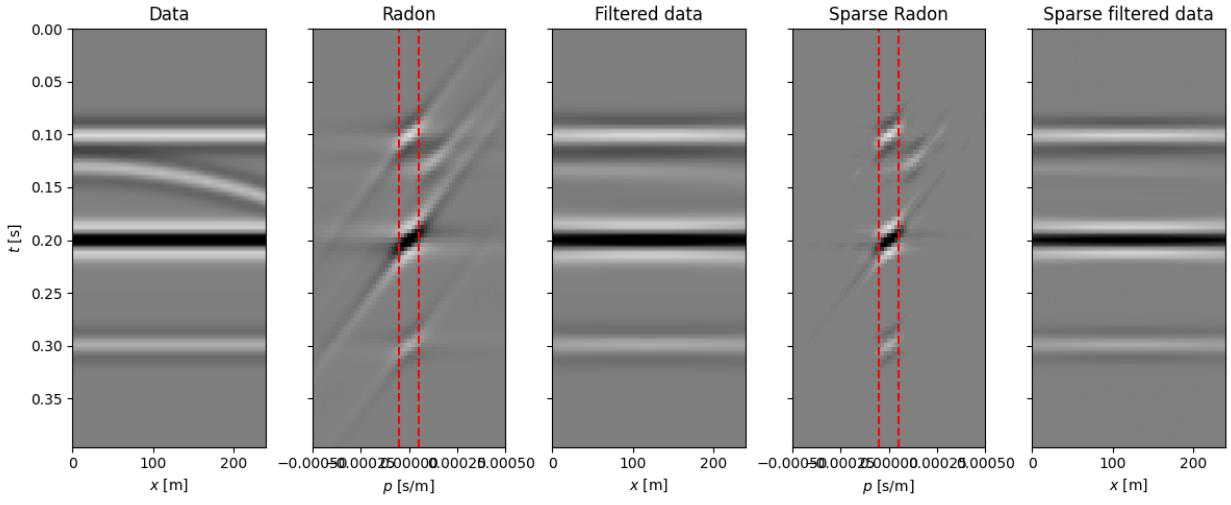
```
axs[1].set(xlabel="$p$ [s/m]", title="Radon")
axs[1].axis("tight")
axs[2].imshow(
    yfilt.T,
    cmap="gray",
    vmin=-pclip * np.abs(yfilt).max(),
    vmax=pclip * np.abs(yfilt).max(),
    extent=(xaxis[0], xaxis[-1], taxis[-1], taxis[0]),
)
axs[2].set(xlabel="$x$ [m]", title="Filtered data")
axs[2].axis("tight")
axs[3].imshow(
    xinv.T,
    cmap="gray",
    vmin=-pclip * np.abs(xinv).max(),
    vmax=pclip * np.abs(xinv).max(),
    extent=(px[0], px[-1], taxis[-1], taxis[0]),
)
axs[3].axvline(px[npx // 2 - 3], color="r", linestyle="--")
axs[3].axvline(px[npx // 2 + 3], color="r", linestyle="--")
axs[3].set(xlabel="$p$ [s/m]", title="Sparse Radon")
axs[3].axis("tight")
axs[4].imshow(
    yinvfilt.T,
    cmap="gray",
    vmin=-pclip * np.abs(y).max(),
    vmax=pclip * np.abs(y).max(),
    extent=(xaxis[0], xaxis[-1], taxis[-1], taxis[0]),
)

axs[4].set(xlabel="$x$ [m]", title="Sparse filtered data")
axs[4].axis("tight")
plt.tight_layout()
```



As expected, the Radon domain is a suitable domain for this type of filtering and the sparse transform improves the ability to filter out parabolic events with small curvature.

On the other hand, it is important to note that we have not been able to correctly preserve the amplitudes of each event. This is because the sparse Radon transform can only identify a sparsest response that explain the data within a certain threshold. For this reason a more suitable approach for preserving amplitudes could be to apply a parabolic Raodn transform with the aim of reconstructing only the unwanted event and apply an adaptive subtraction between the input data and the reconstructed unwanted event.

Total running time of the script: (0 minutes 26.034 seconds)

3.4.13 12. Seismic regularization

The problem of *seismic data regularization* (or interpolation) is a very simple one to write, yet ill-posed and very hard to solve.

The forward modelling operator is a simple *pylops.Restriction* operator which is applied along the spatial direction(s).

$$\mathbf{y} = \mathbf{R}\mathbf{x}$$

Here $\mathbf{y} = [\mathbf{y}_{R_1}^T, \mathbf{y}_{R_2}^T, \ldots, \mathbf{y}_{R_N^T}]^T$ where each vector $\mathbf{y}_{R_i}$ contains all time samples recorded in the seismic data at the specific receiver R_i. Similarly, $\mathbf{x} = [\mathbf{x}_{r_1}^T, \mathbf{x}_{r_2}^T, \ldots, \mathbf{x}_{r_M}^T]$, contains all traces at the regularly and finely sampled receiver locations r_i.

By inverting such an equation we can create a regularized data with densely and regularly spatial direction(s).

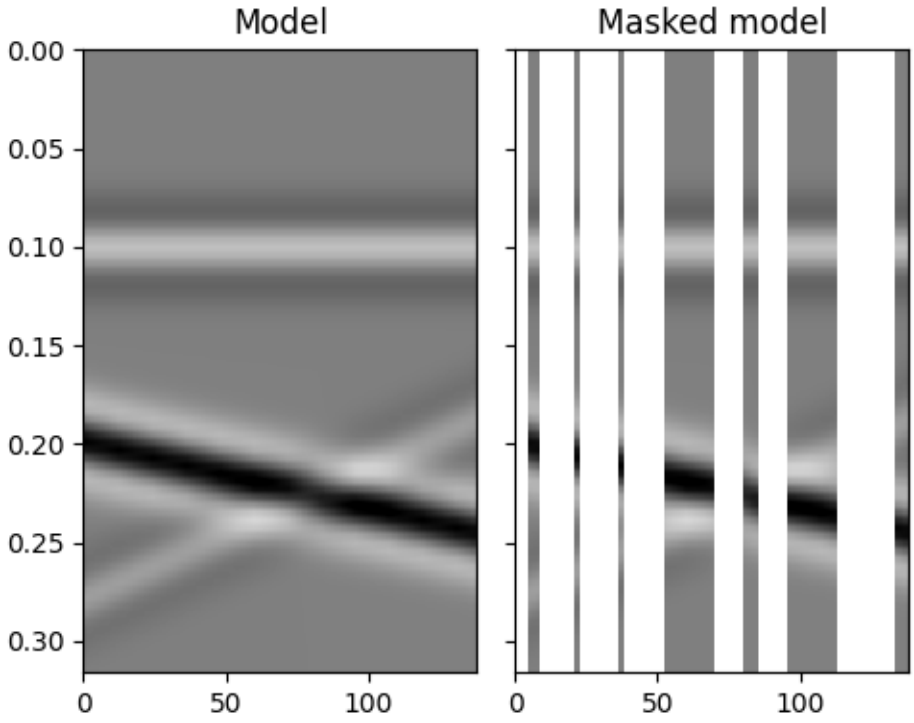
```
import matplotlib.pyplot as plt
import numpy as np
from scipy.signal import convolve

import pylops
from pylops.utils.seismicevents import linear2d, makeaxis
from pylops.utils.wavelets import ricker

np.random.seed(0)
plt.close("all")
```

Let's start by creating a very simple 2d data composed of 3 linear events input parameters

```
par = {"ox": 0, "dx": 2, "nx": 70, "ot": 0, "dt": 0.004, "nt": 80, "f0": 20}

v = 1500
t0_m = [0.1, 0.2, 0.28]
theta_m = [0, 30, -80]
phi_m = [0]
amp_m = [1.0, -2, 0.5]

# axis
taxis, t2, xaxis, y = makeaxis(par)

# wavelet
wav = ricker(taxis[:41], f0=par["f0"])[0]

# model
_, x = linear2d(xaxis, taxis, v, t0_m, theta_m, amp_m, wav)
```

We can now define the spatial locations along which the data has been sampled. In this specific example we will assume that we have access only to 40% of the 'original' locations.

```
perc_subsampling = 0.6
nxsub = int(np.round(par["nx"] * perc_subsampling))

iava = np.sort(np.random.permutation(np.arange(par["nx"]))[:nxsub])

# restriction operator
Rop = pylops.Restriction((par["nx"], par["nt"]), iava, axis=0, dtype="float64")

# data
y = Rop * x.ravel()
y = y.reshape(nxsub, par["nt"])

# mask
ymask = Rop.mask(x.ravel())

# inverse
xinv = Rop / y.ravel()
xinv = xinv.reshape(par["nx"], par["nt"])

fig, axs = plt.subplots(1, 2, sharey=True, figsize=(5, 4))
axs[0].imshow(
    x.T, cmap="gray", vmin=-2, vmax=2, extent=(xaxis[0], xaxis[-1], taxis[-1], taxis[0])
)
axs[0].set_title("Model")
axs[0].axis("tight")
axs[1].imshow(
    ymask.T,
    cmap="gray",
    vmin=-2,
    vmax=2,
    extent=(xaxis[0], xaxis[-1], taxis[-1], taxis[0]),
)
axs[1].set_title("Masked model")
axs[1].axis("tight")
plt.tight_layout()
```



As we can see, inverting the restriction operator is not possible without adding any prior information into the inverse problem. In the following we will consider two possible routes:

- regularized inversion with second derivative along the spatial axis

$$J = \|\mathbf{y} - \mathbf{R}\mathbf{x}\|_2 + \epsilon_\nabla^2 \|\nabla \mathbf{x}\|_2$$

- sparsity-promoting inversion with `pylops.FFT2` operator used as sparsyfing transform

$$J = \|\mathbf{y} - \mathbf{R}\mathbf{F}^H\mathbf{x}\|_2 + \epsilon \|\mathbf{F}^H\mathbf{x}\|_1$$

```
# smooth inversion
D2op = pylops.SecondDerivative((par["nx"], par["nt"]), axis=0, dtype="float64")

xsmooth, _, _ = pylops.waveeqprocessing.SeismicInterpolation(
    y,
    par["nx"],
    iava,
    kind="spatial",
    **dict(epsRs=[np.sqrt(0.1)], damp=np.sqrt(1e-4), iter_lim=50, show=0)
)

# sparse inversion with FFT2
nfft = 2**8
FFTop = pylops.signalprocessing.FFT2D(
    dims=[par["nx"], par["nt"]], nffts=[nfft, nfft], sampling=[par["dx"], par["dt"]]
)
X = FFTop * x.ravel()
```

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```
X = np.reshape(X, (nfft, nfft))

xl1, Xl1, cost = pylops.waveeqprocessing.SeismicInterpolation(
    y,
    par["nx"],
    iava,
    kind="fk",
    nffts=(nfft, nfft),
    sampling=(par["dx"], par["dt"]),
    **dict(niter=50, eps=1e-1)
)

fig, axs = plt.subplots(1, 4, sharey=True, figsize=(13, 4))
axs[0].imshow(
    x.T, cmap="gray", vmin=-2, vmax=2, extent=(xaxis[0], xaxis[-1], taxis[-1], taxis[0])
)
axs[0].set_title("Model")
axs[0].axis("tight")
axs[1].imshow(
    ymask.T,
    cmap="gray",
    vmin=-2,
    vmax=2,
    extent=(xaxis[0], xaxis[-1], taxis[-1], taxis[0]),
)
axs[1].set_title("Masked model")
axs[1].axis("tight")
axs[2].imshow(
    xsmooth.T,
    cmap="gray",
    vmin=-2,
    vmax=2,
    extent=(xaxis[0], xaxis[-1], taxis[-1], taxis[0]),
)
axs[2].set_title("Smoothed model")
axs[2].axis("tight")
axs[3].imshow(
    xl1.T,
    cmap="gray",
    vmin=-2,
    vmax=2,
    extent=(xaxis[0], xaxis[-1], taxis[-1], taxis[0]),
)
axs[3].set_title("L1 model")
axs[3].axis("tight")

fig, axs = plt.subplots(1, 3, figsize=(10, 2))
axs[0].imshow(
    np.fft.fftshift(np.abs(X[:, : nfft // 2 - 1]), axes=0).T,
    extent=(
        np.fft.fftshift(FFTop.f1)[0],
        np.fft.fftshift(FFTop.f1)[-1],
```

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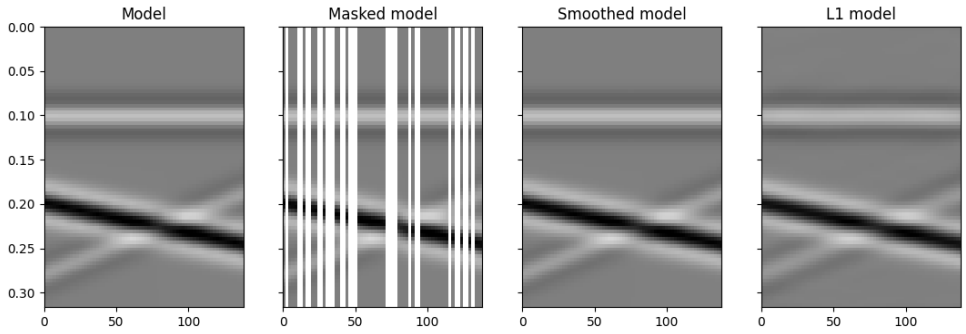
```
        FFTop.f2[nfft // 2 - 1],
        FFTop.f2[0],
    ),
)
axs[0].set_title("Model in f-k domain")
axs[0].axis("tight")
axs[0].set_xlim(-0.1, 0.1)
axs[0].set_ylim(50, 0)
axs[1].imshow(
    np.fft.fftshift(np.abs(Xl1[:, : nfft // 2 - 1]), axes=0).T,
    extent=(
        np.fft.fftshift(FFTop.f1)[0],
        np.fft.fftshift(FFTop.f1)[-1],
        FFTop.f2[nfft // 2 - 1],
        FFTop.f2[0],
    ),
)
axs[1].set_title("Reconstructed model in f-k domain")
axs[1].axis("tight")
axs[1].set_xlim(-0.1, 0.1)
axs[1].set_ylim(50, 0)
axs[2].plot(cost, "k", lw=3)
axs[2].set_title("FISTA convergence")
plt.tight_layout()
```

- 

We see how adding prior information to the inversion can help improving the estimate of the regularized seismic data. Nevertheless, in both cases the reconstructed data is not perfect. A better sparsyfing transform could in fact be chosen here to be the linear *pylops.signalprocessing.Radon2D* transform in spite of the `pylops.FFT2` transform.

```
npx = 40
pxmax = 1e-3
px = np.linspace(-pxmax, pxmax, npx)
```

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```
Radop = pylops.signalprocessing.Radon2D(taxis, xaxis, px, engine="numba")

RRop = Rop * Radop

# adjoint
Xadj_fromx = Radop.H * x.ravel()
Xadj_fromx = Xadj_fromx.reshape(npx, par["nt"])

Xadj = RRop.H * y.ravel()
Xadj = Xadj.reshape(npx, par["nt"])

# L1 inverse
xl1, Xl1, cost = pylops.waveeqprocessing.SeismicInterpolation(
    y,
    par["nx"],
    iava,
    kind="radon-linear",
    spataxis=xaxis,
    taxis=taxis,
    paxis=px,
    centeredh=True,
    **dict(niter=50, eps=1e-1)
)

fig, axs = plt.subplots(2, 3, sharey=True, figsize=(12, 7))
axs[0][0].imshow(
    x.T, cmap="gray", vmin=-2, vmax=2, extent=(xaxis[0], xaxis[-1], taxis[-1], taxis[0])
)
axs[0][0].set_title("Data", fontsize=12)
axs[0][0].axis("tight")
axs[0][1].imshow(
    ymask.T,
    cmap="gray",
    vmin=-2,
    vmax=2,
    extent=(xaxis[0], xaxis[-1], taxis[-1], taxis[0]),
)
axs[0][1].set_title("Masked data", fontsize=12)
axs[0][1].axis("tight")
axs[0][2].imshow(
    xl1.T,
    cmap="gray",
    vmin=-2,
    vmax=2,
    extent=(xaxis[0], xaxis[-1], taxis[-1], taxis[0]),
)
axs[0][2].set_title("Reconstructed data", fontsize=12)
axs[0][2].axis("tight")
axs[1][0].imshow(
    Xadj_fromx.T,
    cmap="gray",
    vmin=-70,
```

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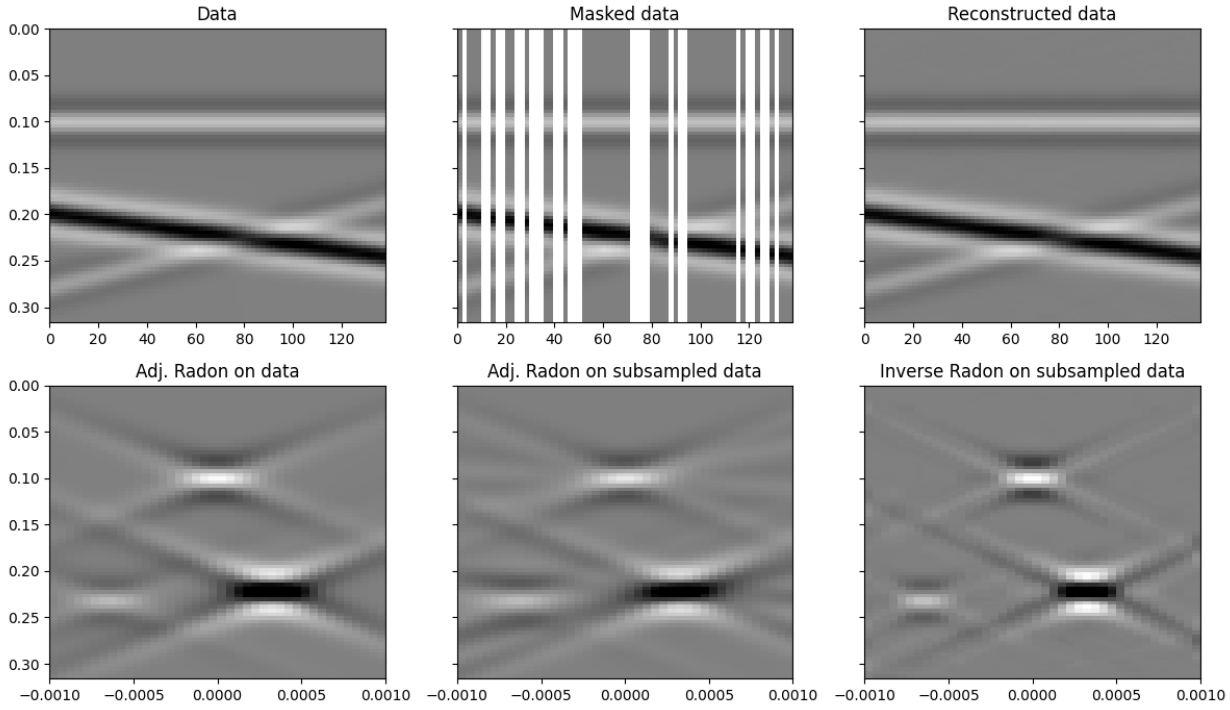
```
    vmax=70,
    extent=(px[0], px[-1], taxis[-1], taxis[0]),
)
axs[1][0].set_title("Adj. Radon on data", fontsize=12)
axs[1][0].axis("tight")
axs[1][1].imshow(
    Xadj.T, cmap="gray", vmin=-50, vmax=50, extent=(px[0], px[-1], taxis[-1], taxis[0])
)
axs[1][1].set_title("Adj. Radon on subsampled data", fontsize=12)
axs[1][1].axis("tight")
axs[1][2].imshow(
    Xl1.T, cmap="gray", vmin=-0.2, vmax=0.2, extent=(px[0], px[-1], taxis[-1], taxis[0])
)
axs[1][2].set_title("Inverse Radon on subsampled data", fontsize=12)
axs[1][2].axis("tight")
plt.tight_layout()
```



Finally, let's take now a more realistic dataset. We will use once again the linear *pylops.signalprocessing.Radon2D* transform but we will take advantnge of the *pylops.signalprocessing.Sliding2D* operator to perform such a transform locally instead of globally to the entire dataset.

```
inputfile = "../testdata/marchenko/input.npz"
inputdata = np.load(inputfile)

x = inputdata["R"][50, :, ::2]
x = x / np.abs(x).max()
taxis, xaxis = inputdata["t"][::2], inputdata["r"][0]

par = {}
```

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```
par["nx"], par["nt"] = x.shape
par["dx"] = inputdata["r"][0, 1] - inputdata["r"][0, 0]
par["dt"] = inputdata["t"][1] - inputdata["t"][0]

# add wavelet
wav = inputdata["wav"][::2]
wav_c = np.argmax(wav)
x = np.apply_along_axis(convolve, 1, x, wav, mode="full")
x = x[:, wav_c:][:, : par["nt"]]

# gain
gain = np.tile((taxis**2)[:, np.newaxis], (1, par["nx"])).T
x = x * gain

# subsampling locations
perc_subsampling = 0.5
Nsub = int(np.round(par["nx"] * perc_subsampling))
iava = np.sort(np.random.permutation(np.arange(par["nx"]))[:Nsub])

# restriction operator
Rop = pylops.Restriction((par["nx"], par["nt"]), iava, axis=0, dtype="float64")

y = Rop * x.ravel()
xadj = Rop.H * y.ravel()

y = y.reshape(Nsub, par["nt"])
xadj = xadj.reshape(par["nx"], par["nt"])

# apply mask
ymask = Rop.mask(x.ravel())

# sliding windows with radon transform
dx = par["dx"]
nwins = 4
nwin = 27
nover = 3
npx = 31
pxmax = 5e-4
px = np.linspace(-pxmax, pxmax, npx)
dimsd = x.shape
dims = (nwins * npx, dimsd[1])

Op = pylops.signalprocessing.Radon2D(
    taxis,
    np.linspace(-par["dx"] * nwin // 2, par["dx"] * nwin // 2, nwin),
    px,
    centeredh=True,
    kind="linear",
    engine="numba",
)
Slidop = pylops.signalprocessing.Sliding2D(
    Op, dims, dimsd, nwin, nover, tapertype="cosine"
```

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```
)

# adjoint
RSop = Rop * Slidop

Xadj_fromx = Slidop.H * x.ravel()
Xadj_fromx = Xadj_fromx.reshape(npx * nwins, par["nt"])

Xadj = RSop.H * y.ravel()
Xadj = Xadj.reshape(npx * nwins, par["nt"])

# inverse
xl1, Xl1, _ = pylops.waveeqprocessing.SeismicInterpolation(
    y,
    par["nx"],
    iava,
    kind="sliding",
    spataxis=xaxis,
    taxis=taxis,
    paxis=px,
    nwins=nwins,
    nwin=nwin,
    nover=nover,
    **dict(niter=50, eps=1e-2)
)

fig, axs = plt.subplots(2, 3, sharey=True, figsize=(12, 14))
axs[0][0].imshow(
    x.T,
    cmap="gray",
    vmin=-0.1,
    vmax=0.1,
    extent=(xaxis[0], xaxis[-1], taxis[-1], taxis[0]),
)
axs[0][0].set_title("Data")
axs[0][0].axis("tight")
axs[0][1].imshow(
    ymask.T,
    cmap="gray",
    vmin=-0.1,
    vmax=0.1,
    extent=(xaxis[0], xaxis[-1], taxis[-1], taxis[0]),
)
axs[0][1].set_title("Masked data")
axs[0][1].axis("tight")
axs[0][2].imshow(
    xl1.T,
    cmap="gray",
    vmin=-0.1,
    vmax=0.1,
    extent=(xaxis[0], xaxis[-1], taxis[-1], taxis[0]),
)
```

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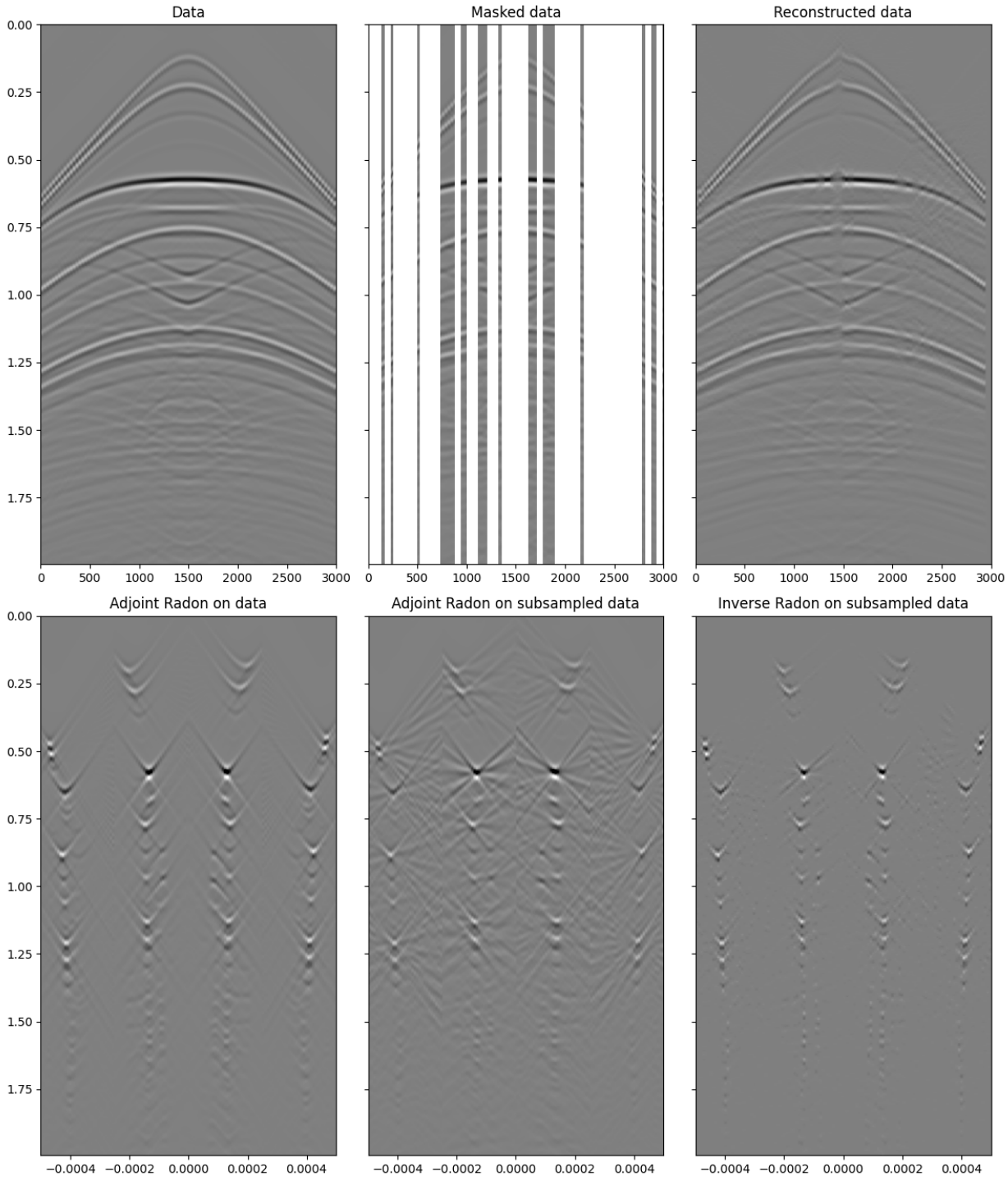
```
axs[0][2].set_title("Reconstructed data")
axs[0][2].axis("tight")
axs[1][0].imshow(
    Xadj_fromx.T,
    cmap="gray",
    vmin=-1,
    vmax=1,
    extent=(px[0], px[-1], taxis[-1], taxis[0]),
)
axs[1][0].set_title("Adjoint Radon on data")
axs[1][0].axis("tight")
axs[1][1].imshow(
    Xadj.T,
    cmap="gray",
    vmin=-0.6,
    vmax=0.6,
    extent=(px[0], px[-1], taxis[-1], taxis[0]),
)
axs[1][1].set_title("Adjoint Radon on subsampled data")
axs[1][1].axis("tight")
axs[1][2].imshow(
    Xl1.T,
    cmap="gray",
    vmin=-0.03,
    vmax=0.03,
    extent=(px[0], px[-1], taxis[-1], taxis[0]),
)
axs[1][2].set_title("Inverse Radon on subsampled data")
axs[1][2].axis("tight")
plt.tight_layout()
```



As expected the linear *pylops.signalprocessing.Radon2D* is able to locally explain events in the input data and leads to a satisfactory recovery. Note that increasing the number of iterations and sliding windows can further refine the result, especially the accuracy of weak events, as shown in this companion notebook.

Total running time of the script: (0 minutes 8.310 seconds)

3.4.14 13. Deghosting

Single-component seismic data can be decomposed in their up- and down-going constituents in a model driven fashion. This task can be achieved by defining an f-k propagator (or ghost model) and solving an inverse problem as described in *pylops.waveeqprocessing.Deghosting*.

```
import matplotlib.pyplot as plt

# sphinx_gallery_thumbnail_number = 3
import numpy as np
from scipy.sparse.linalg import lsqr

import pylops

np.random.seed(0)
plt.close("all")
```

Let's start by loading the input dataset and geometry

```
inputfile = "../testdata/updown/input.npz"
inputdata = np.load(inputfile)

vel_sep = 2400.0  # velocity at separation level
clip = 1e-1  # plotting clip


# Receivers
r = inputdata["r"]
nr = r.shape[1]
dr = r[0, 1] - r[0, 0]

# Sources
s = inputdata["s"]

# Model
rho = inputdata["rho"]

# Axes
t = inputdata["t"]
nt, dt = len(t), t[1] - t[0]
x, z = inputdata["x"], inputdata["z"]
dx, dz = x[1] - x[0], z[1] - z[0]

# Data
p = inputdata["p"].T
p /= p.max()

fig = plt.figure(figsize=(9, 4))
ax1 = plt.subplot2grid((1, 5), (0, 0), colspan=4)
ax2 = plt.subplot2grid((1, 5), (0, 4))
ax1.imshow(rho, cmap="gray", extent=(x[0], x[-1], z[-1], z[0]))
ax1.scatter(r[0, ::5], r[1, ::5], marker="v", s=150, c="b", edgecolors="k")
ax1.scatter(s[0], s[1], marker="*", s=250, c="r", edgecolors="k")
ax1.axis("tight")
```

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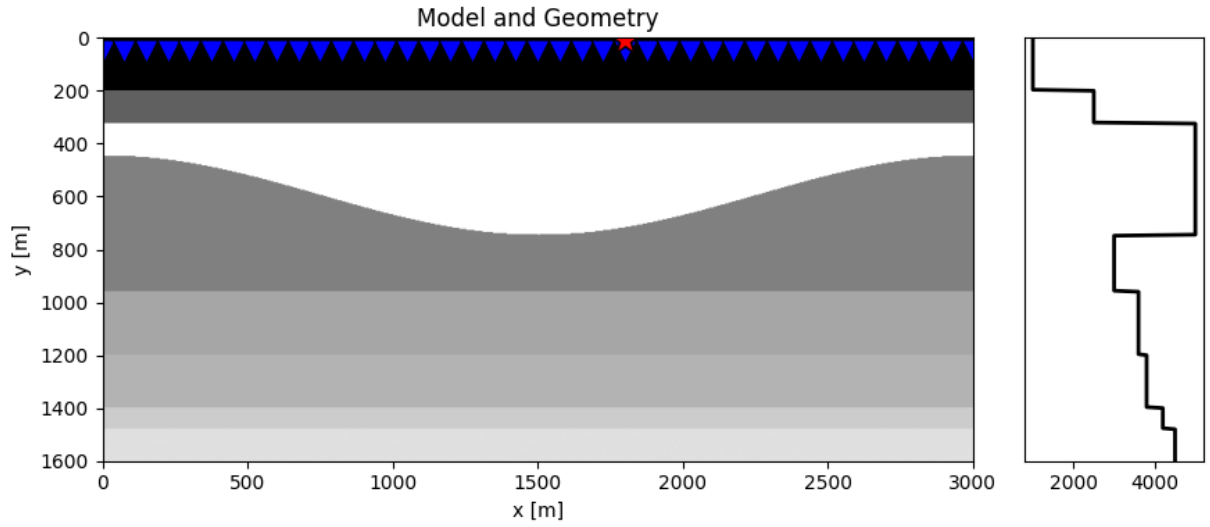
```
ax1.set_xlabel("x [m]")
ax1.set_ylabel("y [m]")
ax1.set_title("Model and Geometry")
ax1.set_xlim(x[0], x[-1])
ax1.set_ylim(z[-1], z[0])
ax2.plot(rho[:, len(x) // 2], z, "k", lw=2)
ax2.set_ylim(z[-1], z[0])
ax2.set_yticks([])
plt.tight_layout()
```



To be able to deghost the input dataset, we need to remove its direct arrival. In this example we will create a mask based on the analytical traveltime of the direct arrival.

```
direct = np.sqrt(np.sum((s[:, np.newaxis] - r) ** 2, axis=0)) / vel_sep

# Window
off = 0.035
direct_off = direct + off
win = np.zeros((nt, nr))
iwin = np.round(direct_off / dt).astype(int)
for i in range(nr):
    win[iwin[i] :, i] = 1

fig, axs = plt.subplots(1, 2, sharey=True, figsize=(8, 7))
axs[0].imshow(
    p.T,
    cmap="gray",
    vmin=-clip * np.abs(p).max(),
    vmax=clip * np.abs(p).max(),
    extent=(r[0, 0], r[0, -1], t[-1], t[0]),
)
axs[0].plot(r[0], direct_off, "r", lw=2)
axs[0].set_title(r"$P$")
axs[0].axis("tight")
```

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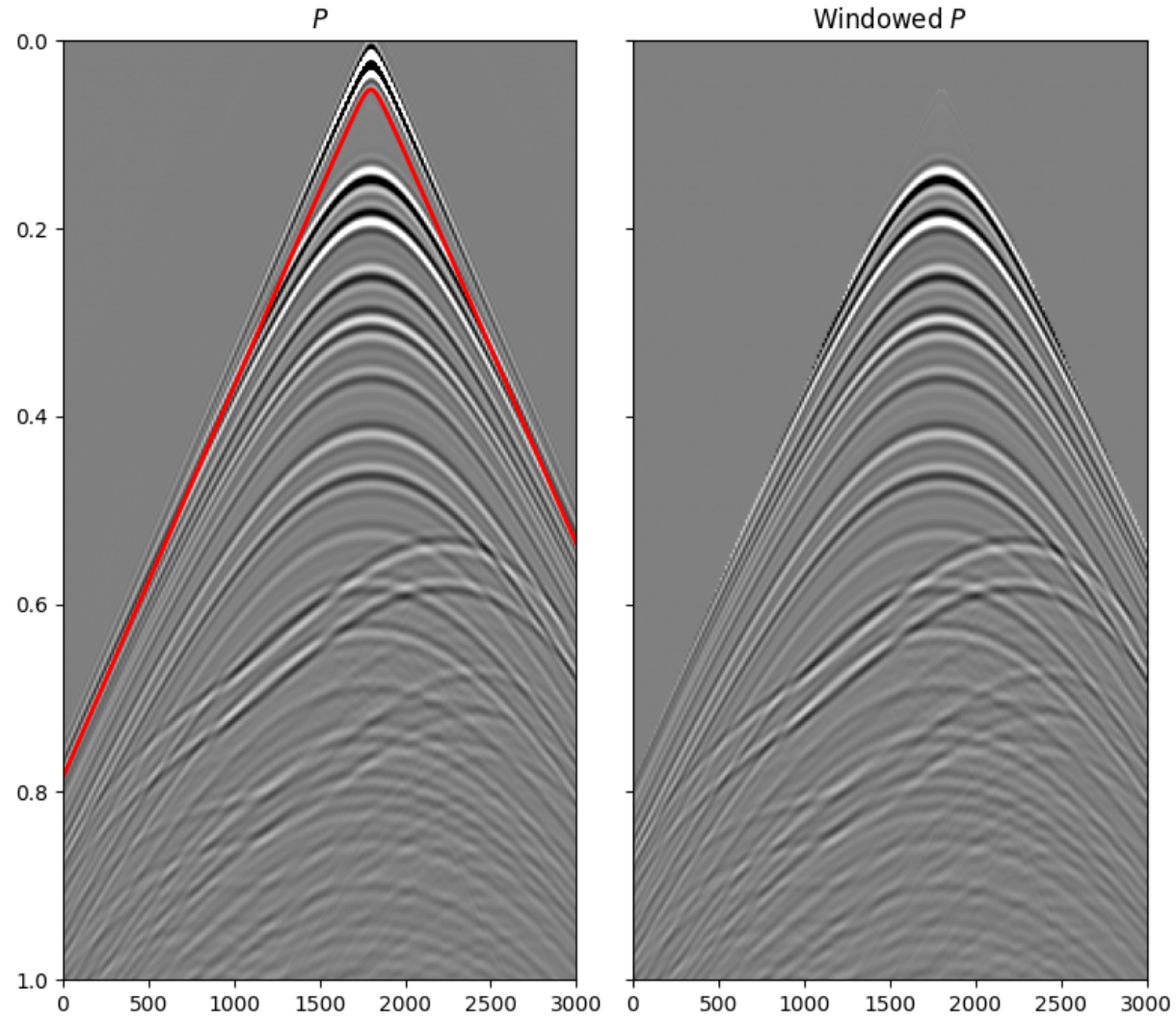
```
axs[1].imshow(
    win * p.T,
    cmap="gray",
    vmin=-clip * np.abs(p).max(),
    vmax=clip * np.abs(p).max(),
    extent=(r[0, 0], r[0, -1], t[-1], t[0]),
)
axs[1].set_title(r"Windowed $P$")
axs[1].axis("tight")
axs[1].set_ylim(1, 0)
plt.tight_layout()
```



We can now perform deghosting

```
pup, pdown = pylops.waveeqprocessing.Deghosting(
    p.T,
    nt,
```

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```
    nr,
    dt,
    dr,
    vel_sep,
    r[1, 0] + dz,
    win=win,
    npad=5,
    ntaper=11,
    solver=lsqr,
    dottest=False,
    dtype="complex128",
    **dict(damp=1e-10, iter_lim=60)
)

fig, axs = plt.subplots(1, 3, sharey=True, figsize=(12, 7))
axs[0].imshow(
    p.T,
    cmap="gray",
    vmin=-clip * np.abs(p).max(),
    vmax=clip * np.abs(p).max(),
    extent=(r[0, 0], r[0, -1], t[-1], t[0]),
)
axs[0].set_title(r"$P$")
axs[0].axis("tight")
axs[1].imshow(
    pup,
    cmap="gray",
    vmin=-clip * np.abs(p).max(),
    vmax=clip * np.abs(p).max(),
    extent=(r[0, 0], r[0, -1], t[-1], t[0]),
)
axs[1].set_title(r"$P^-$")
axs[1].axis("tight")
axs[2].imshow(
    pdown,
    cmap="gray",
    vmin=-clip * np.abs(p).max(),
    vmax=clip * np.abs(p).max(),
    extent=(r[0, 0], r[0, -1], t[-1], t[0]),
)
axs[2].set_title(r"$P^+$")
axs[2].axis("tight")
axs[2].set_ylim(1, 0)

plt.figure(figsize=(14, 3))
plt.plot(t, p[nr // 2], "k", lw=2, label=r"$p$")
plt.plot(t, pup[:, nr // 2], "r", lw=2, label=r"$p^-$")
plt.xlim(0, t[200])
plt.ylim(-0.2, 0.2)
plt.legend()
plt.tight_layout()
```

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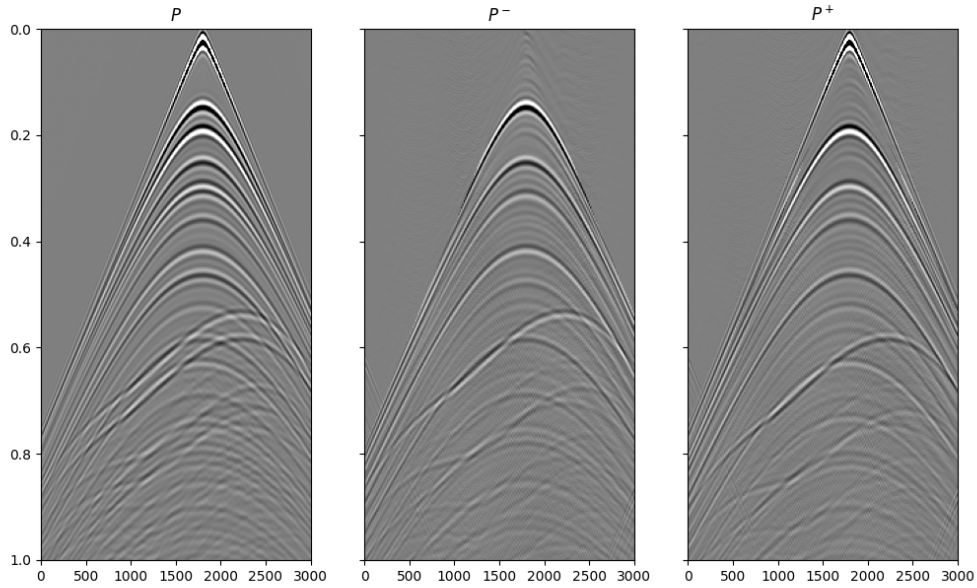
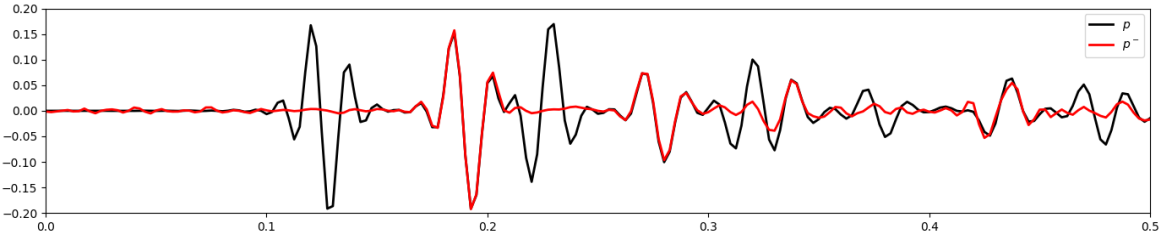
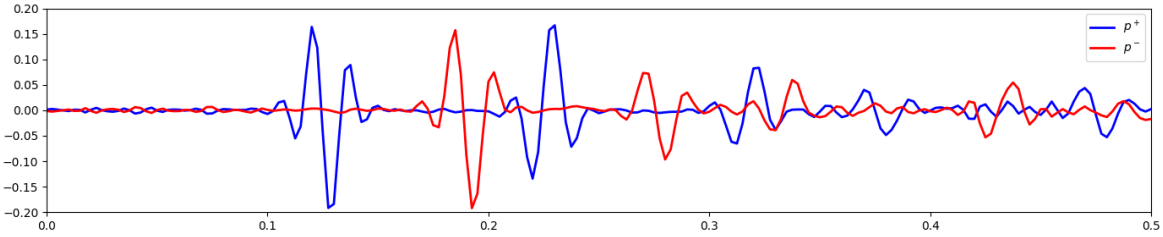
```
plt.figure(figsize=(14, 3))
plt.plot(t, pdown[:, nr // 2], "b", lw=2, label=r"$p^+$")
plt.plot(t, pup[:, nr // 2], "r", lw=2, label=r"$p^-$")
plt.xlim(0, t[200])
plt.ylim(-0.2, 0.2)
plt.legend()
plt.tight_layout()
```



- 

- 

To see more examples head over to the following notebook: notebook1.

Total running time of the script: (0 minutes 6.874 seconds)

3.4.15 14. Seismic wavefield decomposition

Multi-component seismic data can be decomposed in their up- and down-going constituents in a purely data driven fashion. This task can be accurately achieved by linearly combining the input pressure and particle velocity data in the frequency-wavenumber described in details in *pylops.waveeqprocessing.UpDownComposition2D* and *pylops.waveeqprocessing.WavefieldDecomposition*.

In this tutorial we will consider a simple synthetic data composed of six events (three up-going and three down-going). We will first combine them to create pressure and particle velocity data and then show how we can retrieve their directional constituents both by directly combining the input data as well as by setting an inverse problem. The latter approach results vital in case of spatial aliasing, as applying simple scaled summation in the frequency-wavenumber would result in sub-optimal decomposition due to the superposition of different frequency-wavenumber pairs at some (aliased) locations.

```
import matplotlib.pyplot as plt
import numpy as np
from scipy.signal import filtfilt

import pylops
from pylops.utils.seismicevents import hyperbolic2d, makeaxis
from pylops.utils.wavelets import ricker

np.random.seed(0)
plt.close("all")
```

Let's first the input up- and down-going wavefields

```
par = {"ox": -220, "dx": 5, "nx": 89, "ot": 0, "dt": 0.004, "nt": 200, "f0": 40}

t0_plus = np.array([0.2, 0.5, 0.7])
t0_minus = t0_plus + 0.04
vrms = np.array([1400.0, 1500.0, 2000.0])
amp = np.array([1.0, -0.6, 0.5])
vel_sep = 1000.0  # velocity at separation level
rho_sep = 1000.0  # density at separation level

# Create axis
t, t2, x, y = makeaxis(par)

# Create wavelet
wav = ricker(t[:41], f0=par["f0"])[0]

# Create data
_, p_minus = hyperbolic2d(x, t, t0_minus, vrms, amp, wav)
_, p_plus = hyperbolic2d(x, t, t0_plus, vrms, amp, wav)
```

We can now combine them to create pressure and particle velocity data

```
critical = 1.1
ntaper = 51
nfft = 2**10

# 2d fft operator
FFTop = pylops.signalprocessing.FFT2D(
```

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```
    dims=[par["nx"], par["nt"]], nffts=[nfft, nfft], sampling=[par["dx"], par["dt"]]
)

# obliquity factor
[Kx, F] = np.meshgrid(FFTop.f1, FFTop.f2, indexing="ij")
k = F / vel_sep
Kz = np.sqrt((k**2 - Kx**2).astype(np.complex128))
Kz[np.isnan(Kz)] = 0
OBL = rho_sep * (np.abs(F) / Kz)
OBL[Kz == 0] = 0

mask = np.abs(Kx) < critical * np.abs(F) / vel_sep
OBL *= mask
OBL = filtfilt(np.ones(ntaper) / float(ntaper), 1, OBL, axis=0)
OBL = filtfilt(np.ones(ntaper) / float(ntaper), 1, OBL, axis=1)

# composition operator
UPop = pylops.waveeqprocessing.UpDownComposition2D(
    par["nt"],
    par["nx"],
    par["dt"],
    par["dx"],
    rho_sep,
    vel_sep,
    nffts=(nfft, nfft),
    critical=critical * 100.0,
    ntaper=ntaper,
    dtype="complex128",
)

# wavefield modelling
d = UPop * np.concatenate((p_plus.ravel(), p_minus.ravel())).ravel()
d = np.real(d.reshape(2 * par["nx"], par["nt"]))
p, vz = d[: par["nx"]], d[par["nx"] :]

# obliquity scaled vz
VZ = FFTop * vz.ravel()
VZ = VZ.reshape(nfft, nfft)

VZ_obl = OBL * VZ
vz_obl = FFTop.H * VZ_obl.ravel()
vz_obl = np.real(vz_obl.reshape(par["nx"], par["nt"]))

fig, axs = plt.subplots(1, 4, figsize=(10, 5))
axs[0].imshow(
    p.T,
    aspect="auto",
    vmin=-1,
    vmax=1,
    interpolation="nearest",
    cmap="gray",
    extent=(x.min(), x.max(), t.max(), t.min()),
```

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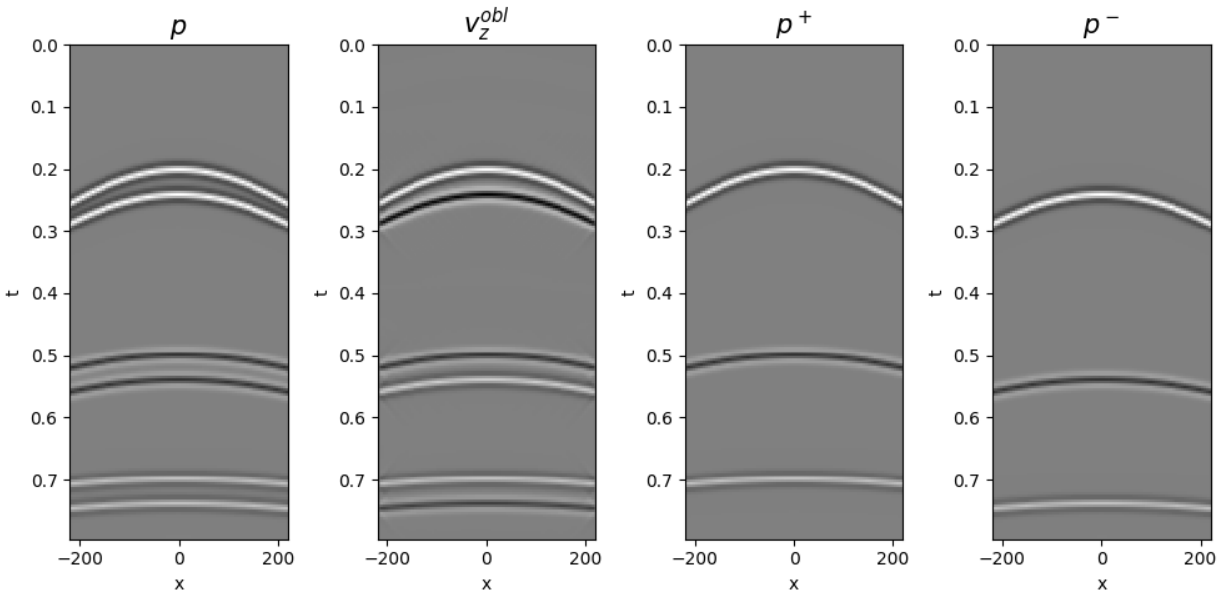
```
)
axs[0].set_title(r"$p$", fontsize=15)
axs[0].set_xlabel("x")
axs[0].set_ylabel("t")
axs[1].imshow(
    vz_obl.T,
    aspect="auto",
    vmin=-1,
    vmax=1,
    interpolation="nearest",
    cmap="gray",
    extent=(x.min(), x.max(), t.max(), t.min()),
)
axs[1].set_title(r"$v_z^{obl}$", fontsize=15)
axs[1].set_xlabel("x")
axs[1].set_ylabel("t")
axs[2].imshow(
    p_plus.T,
    aspect="auto",
    vmin=-1,
    vmax=1,
    interpolation="nearest",
    cmap="gray",
    extent=(x.min(), x.max(), t.max(), t.min()),
)
axs[2].set_title(r"$p^+$", fontsize=15)
axs[2].set_xlabel("x")
axs[2].set_ylabel("t")
axs[3].imshow(
    p_minus.T,
    aspect="auto",
    interpolation="nearest",
    cmap="gray",
    extent=(x.min(), x.max(), t.max(), t.min()),
    vmin=-1,
    vmax=1,
)
axs[3].set_title(r"$p^-$", fontsize=15)
axs[3].set_xlabel("x")
axs[3].set_ylabel("t")
plt.tight_layout()
```



Wavefield separation is first performed using the analytical expression for combining pressure and particle velocity data in the wavenumber-frequency domain

```
pup_sep, pdown_sep = pylops.waveeqprocessing.WavefieldDecomposition(
    p,
    vz,
    par["nt"],
    par["nx"],
    par["dt"],
    par["dx"],
    rho_sep,
    vel_sep,
    nffts=(nfft, nfft),
    kind="analytical",
    critical=critical * 100,
    ntaper=ntaper,
    dtype="complex128",
)
fig = plt.figure(figsize=(12, 5))
axs0 = plt.subplot2grid((2, 5), (0, 0), rowspan=2)
axs1 = plt.subplot2grid((2, 5), (0, 1), rowspan=2)
axs2 = plt.subplot2grid((2, 5), (0, 2), colspan=3)
axs3 = plt.subplot2grid((2, 5), (1, 2), colspan=3)
axs0.imshow(
    pup_sep.T, cmap="gray", vmin=-1, vmax=1, extent=(x.min(), x.max(), t.max(), t.min())
)
axs0.set_title(r"$p^-$ analytical")
axs0.axis("tight")
axs1.imshow(
    pdown_sep.T,
    cmap="gray",
    vmin=-1,
    vmax=1,
```

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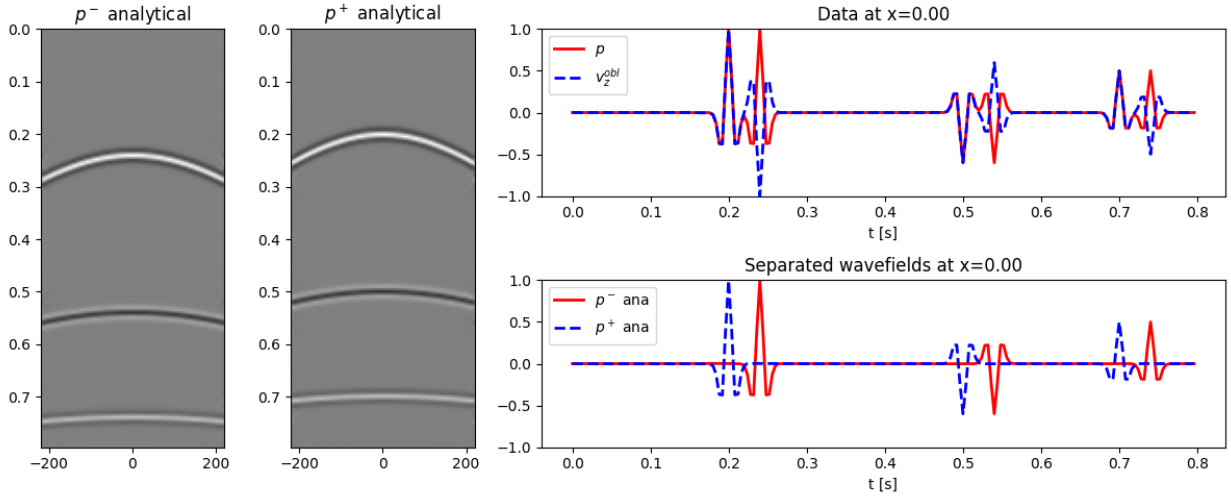
```
    extent=(x.min(), x.max(), t.max(), t.min()),
)
axs1.set_title(r"$p^+$ analytical")
axs1.axis("tight")
axs2.plot(t, p[par["nx"] // 2], "r", lw=2, label=r"$p$")
axs2.plot(t, vz_obl[par["nx"] // 2], "--b", lw=2, label=r"$v_z^{obl}$")
axs2.set_ylim(-1, 1)
axs2.set_title("Data at x=%.2f" % x[par["nx"] // 2])
axs2.set_xlabel("t [s]")
axs2.legend()
axs3.plot(t, pup_sep[par["nx"] // 2], "r", lw=2, label=r"$p^-$ ana")
axs3.plot(t, pdown_sep[par["nx"] // 2], "--b", lw=2, label=r"$p^+$ ana")
axs3.set_title("Separated wavefields at x=%.2f" % x[par["nx"] // 2])
axs3.set_xlabel("t [s]")
axs3.set_ylim(-1, 1)
axs3.legend()
plt.tight_layout()
```



We repeat the same exercise but this time we invert the composition operator *pylops.waveeqprocessing.UpDownComposition2D*

```
pup_inv, pdown_inv = pylops.waveeqprocessing.WavefieldDecomposition(
    p,
    vz,
    par["nt"],
    par["nx"],
    par["dt"],
    par["dx"],
    rho_sep,
    vel_sep,
    nffts=(nfft, nfft),
    kind="inverse",
    critical=critical * 100,
    ntaper=ntaper,
    scaling=1.0 / vz.max(),
```

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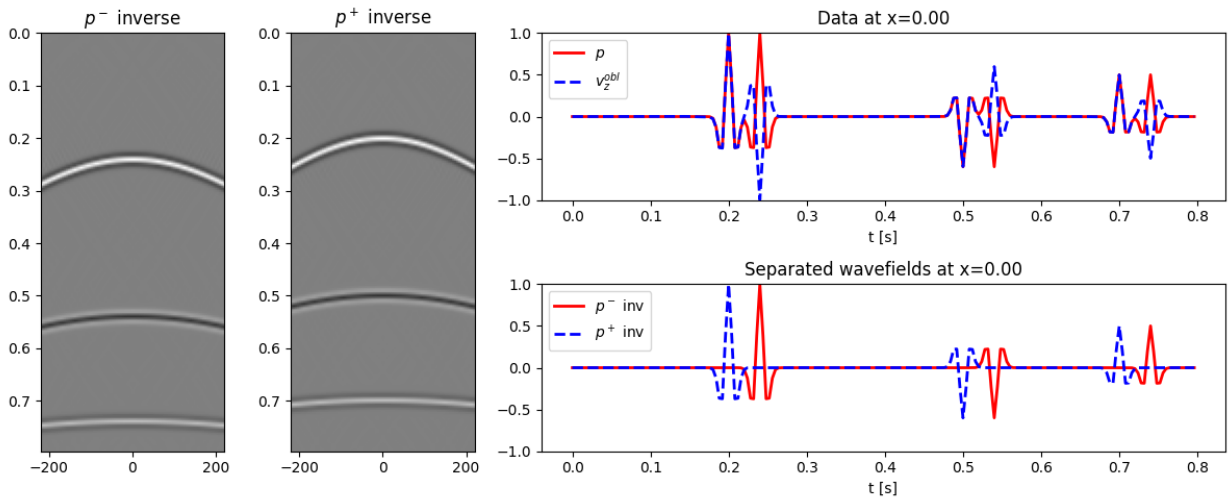
```
    dtype="complex128",
    **dict(damp=1e-10, iter_lim=20)
)

fig = plt.figure(figsize=(12, 5))
axs0 = plt.subplot2grid((2, 5), (0, 0), rowspan=2)
axs1 = plt.subplot2grid((2, 5), (0, 1), rowspan=2)
axs2 = plt.subplot2grid((2, 5), (0, 2), colspan=3)
axs3 = plt.subplot2grid((2, 5), (1, 2), colspan=3)
axs0.imshow(
    pup_inv.T, cmap="gray", vmin=-1, vmax=1, extent=(x.min(), x.max(), t.max(), t.min())
)
axs0.set_title(r"$p^-$ inverse")
axs0.axis("tight")
axs1.imshow(
    pdown_inv.T,
    cmap="gray",
    vmin=-1,
    vmax=1,
    extent=(x.min(), x.max(), t.max(), t.min()),
)
axs1.set_title(r"$p^+$ inverse")
axs1.axis("tight")
axs2.plot(t, p[par["nx"] // 2], "r", lw=2, label=r"$p$")
axs2.plot(t, vz_obl[par["nx"] // 2], "--b", lw=2, label=r"$v_z^{obl}$")
axs2.set_ylim(-1, 1)
axs2.set_title("Data at x=%.2f" % x[par["nx"] // 2])
axs2.set_xlabel("t [s]")
axs2.legend()
axs3.plot(t, pup_inv[par["nx"] // 2], "r", lw=2, label=r"$p^-$ inv")
axs3.plot(t, pdown_inv[par["nx"] // 2], "--b", lw=2, label=r"$p^+$ inv")
axs3.set_title("Separated wavefields at x=%.2f" % x[par["nx"] // 2])
axs3.set_xlabel("t [s]")
axs3.set_ylim(-1, 1)
axs3.legend()
plt.tight_layout()
```



The up- and down-going constituents have been succesfully separated in both cases. Finally, we use the *pylops.waveeqprocessing.UpDownComposition2D* operator to reconstruct the particle velocity wavefield from its up- and down-going pressure constituents

```
PtoVop = pylops.waveeqprocessing.PressureToVelocity(
    par["nt"],
    par["nx"],
    par["dt"],
    par["dx"],
    rho_sep,
    vel_sep,
    nffts=(nfft, nfft),
    critical=critical * 100.0,
    ntaper=ntaper,
    topressure=False,
)

vdown_rec = (PtoVop * pdown_inv.ravel()).reshape(par["nx"], par["nt"])
vup_rec = (PtoVop * pup_inv.ravel()).reshape(par["nx"], par["nt"])
vz_rec = np.real(vdown_rec - vup_rec)

fig, axs = plt.subplots(1, 3, figsize=(13, 6))
axs[0].imshow(
    vz.T,
    cmap="gray",
    vmin=-1e-6,
    vmax=1e-6,
    extent=(x.min(), x.max(), t.max(), t.min()),
)
axs[0].set_title(r"$vz$")
axs[0].axis("tight")
axs[1].imshow(
    vz_rec.T, cmap="gray", vmin=-1e-6, vmax=1e-6, extent=(x.min(), x.max(), t[-1], t[0])
)
axs[1].set_title(r"$vz rec$")
axs[1].axis("tight")
```

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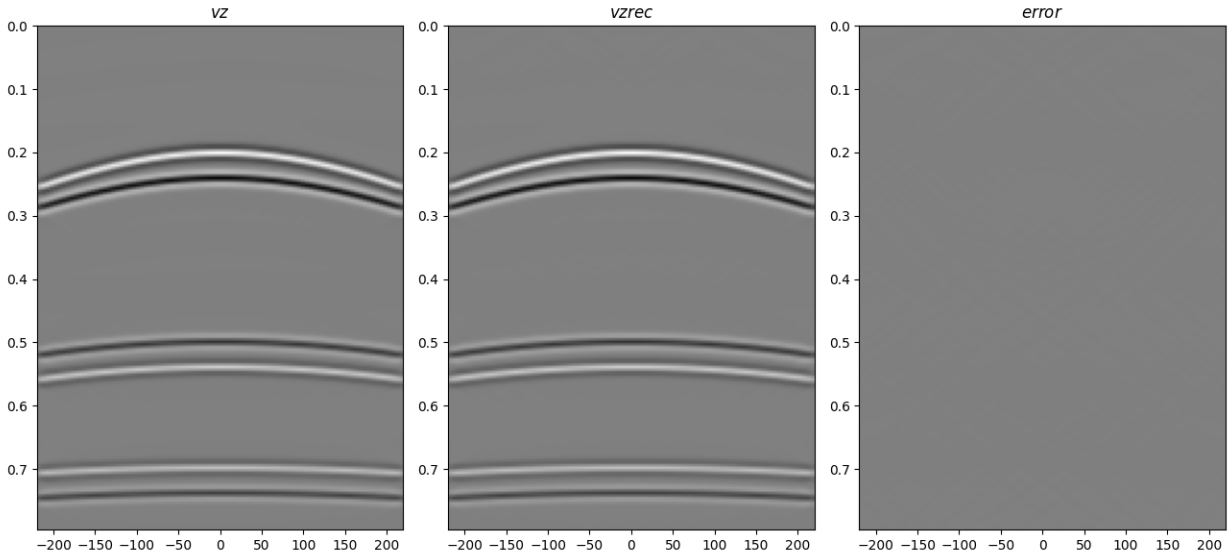
```
axs[2].imshow(
    vz.T - vz_rec.T,
    cmap="gray",
    vmin=-1e-6,
    vmax=1e-6,
    extent=(x.min(), x.max(), t[-1], t[0]),
)
axs[2].set_title(r"$error$")
axs[2].axis("tight")
plt.tight_layout()
```



To see more examples, including applying wavefield separation and regularization simultaneously, as well as 3D examples, head over to the following notebooks: notebook1 and notebook2

Total running time of the script: (0 minutes 15.497 seconds)

3.4.16 15. Least-squares migration

Seismic migration is the process by which seismic data are manipulated to create an image of the subsurface reflectivity.

While traditionally solved as the adjont of the demigration operator, it is becoming more and more common to solve the underlying inverse problem in the quest for more accurate and detailed subsurface images.

Indipendently of the choice of the modelling operator (i.e., ray-based or full wavefield-based), the demigration/migration process can be expressed as a linear operator of such a kind:

$$d(\mathbf{x_r}, \mathbf{x_s}, t) = w(t) * \int_V G(\mathbf{x}, \mathbf{x_s}, t) G(\mathbf{x_r}, \mathbf{x}, t) m(\mathbf{x}) \, \mathrm{d}\mathbf{x}$$

where $m(\mathbf{x})$ is the reflectivity at every location in the subsurface, $G(\mathbf{x}, \mathbf{x_s}, t)$ and $G(\mathbf{x_r}, \mathbf{x}, t)$ are the Green's functions from source-to-subsurface-to-receiver and finally $w(t)$ is the wavelet. Ultimately, while the Green's functions can be computed in many different ways, solving this system of equations for the reflectivity model is what we generally refer to as Least-squares migration (LSM).

In this tutorial we will consider the most simple scenario where we use an eikonal solver to compute the Green's functions and show how we can use the *pylops.waveeqprocessing.LSM* operator to perform LSM.

```
import matplotlib.pyplot as plt
import numpy as np
from scipy.sparse.linalg import lsqr

import pylops

plt.close("all")
np.random.seed(0)
```

To start we create a simple model with 2 interfaces

```
# Velocity Model
nx, nz = 81, 60
dx, dz = 4, 4
x, z = np.arange(nx) * dx, np.arange(nz) * dz
v0 = 1000  # initial velocity
kv = 0.0  # gradient
vel = np.outer(np.ones(nx), v0 + kv * z)

# Reflectivity Model
refl = np.zeros((nx, nz))
refl[:, 30] = -1
refl[:, 50] = 0.5

# Receivers
nr = 11
rx = np.linspace(10 * dx, (nx - 10) * dx, nr)
rz = 20 * np.ones(nr)
recs = np.vstack((rx, rz))
dr = recs[0, 1] - recs[0, 0]

# Sources
ns = 10
sx = np.linspace(dx * 10, (nx - 10) * dx, ns)
sz = 10 * np.ones(ns)
sources = np.vstack((sx, sz))
ds = sources[0, 1] - sources[0, 0]
```

```
plt.figure(figsize=(10, 5))
im = plt.imshow(vel.T, cmap="summer", extent=(x[0], x[-1], z[-1], z[0]))
plt.scatter(recs[0], recs[1], marker="v", s=150, c="b", edgecolors="k")
plt.scatter(sources[0], sources[1], marker="*", s=150, c="r", edgecolors="k")
cb = plt.colorbar(im)
cb.set_label("[m/s]")
plt.axis("tight")
plt.xlabel("x [m]"), plt.ylabel("y [m]")
plt.title("Velocity")
plt.xlim(x[0], x[-1])
plt.tight_layout()

plt.figure(figsize=(10, 5))
im = plt.imshow(refl.T, cmap="gray", extent=(x[0], x[-1], z[-1], z[0]))
plt.scatter(recs[0], recs[1], marker="v", s=150, c="b", edgecolors="k")
```

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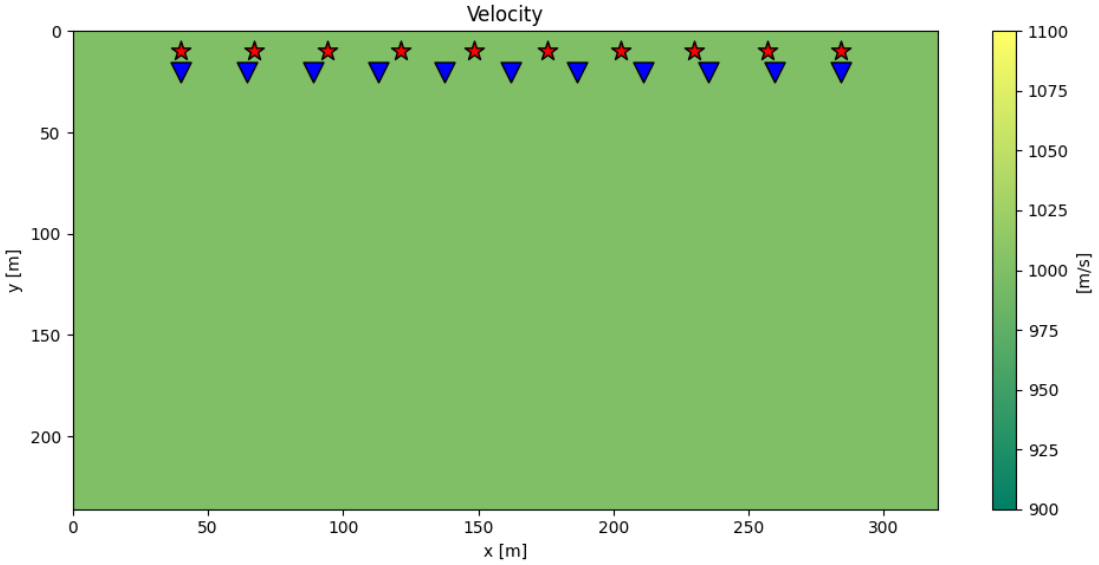
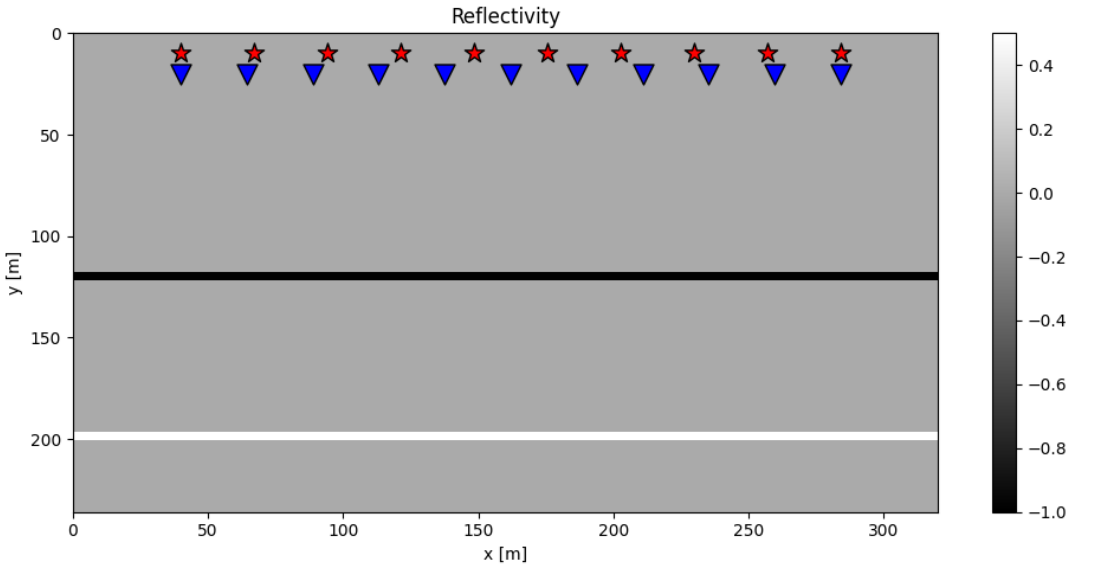
```
plt.scatter(sources[0], sources[1], marker="*", s=150, c="r", edgecolors="k")
plt.colorbar(im)
plt.axis("tight")
plt.xlabel("x [m]"), plt.ylabel("y [m]")
plt.title("Reflectivity")
plt.xlim(x[0], x[-1])
plt.tight_layout()
```





We can now create our LSM object and invert for the reflectivity using two different solvers: `scipy.sparse.linalg.lsqr` (LS solution) and *`pylops.optimization.sparsity.fista`* (LS solution with sparse model).

```
nt = 651
dt = 0.004
t = np.arange(nt) * dt
```

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```
wav, wavt, wavc = pylops.utils.wavelets.ricker(t[:41], f0=20)


lsm = pylops.waveeqprocessing.LSM(
    z,
    x,
    t,
    sources,
    recs,
    v0,
    wav,
    wavc,
    mode="analytic",
    engine="numba",
)

d = lsm.Demop * refl

madj = lsm.Demop.H * d

minv = lsm.solve(d.ravel(), solver=lsqr, **dict(iter_lim=100))
minv = minv.reshape(nx, nz)

minv_sparse = lsm.solve(
    d.ravel(), solver=pylops.optimization.sparsity.fista, **dict(eps=1e2, niter=100)
)
minv_sparse = minv_sparse.reshape(nx, nz)

# demigration
d = d.reshape(ns, nr, nt)

dadj = lsm.Demop * madj  # (ns * nr, nt)
dadj = dadj.reshape(ns, nr, nt)

dinv = lsm.Demop * minv
dinv = dinv.reshape(ns, nr, nt)

dinv_sparse = lsm.Demop * minv_sparse
dinv_sparse = dinv_sparse.reshape(ns, nr, nt)

# sphinx_gallery_thumbnail_number = 2
fig, axs = plt.subplots(2, 2, figsize=(10, 8))
axs[0][0].imshow(refl.T, cmap="gray", vmin=-1, vmax=1)
axs[0][0].axis("tight")
axs[0][0].set_title(r"$m$")
axs[0][1].imshow(madj.T, cmap="gray", vmin=-madj.max(), vmax=madj.max())
axs[0][1].set_title(r"$m_{adj}$")
axs[0][1].axis("tight")
axs[1][0].imshow(minv.T, cmap="gray", vmin=-1, vmax=1)
axs[1][0].axis("tight")
axs[1][0].set_title(r"$m_{inv}$")
axs[1][1].imshow(minv_sparse.T, cmap="gray", vmin=-1, vmax=1)
```

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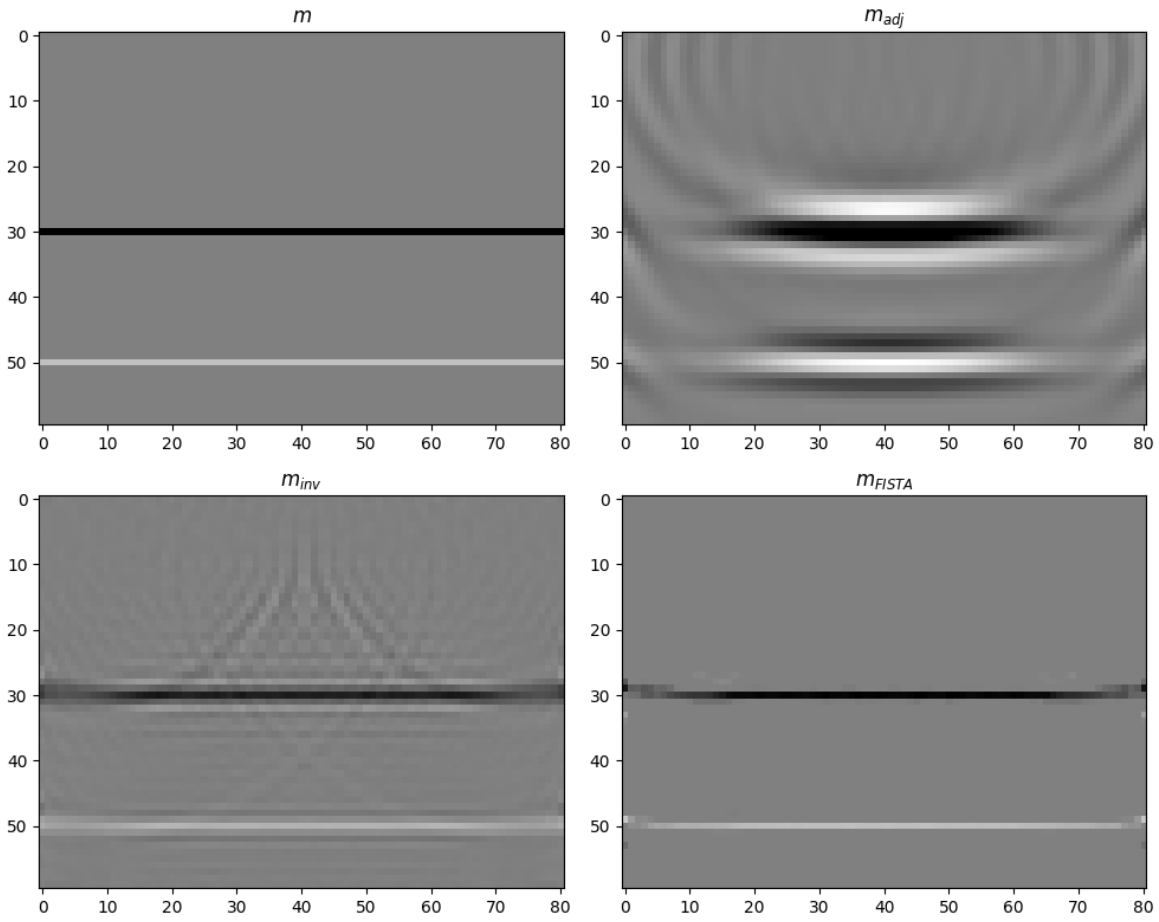
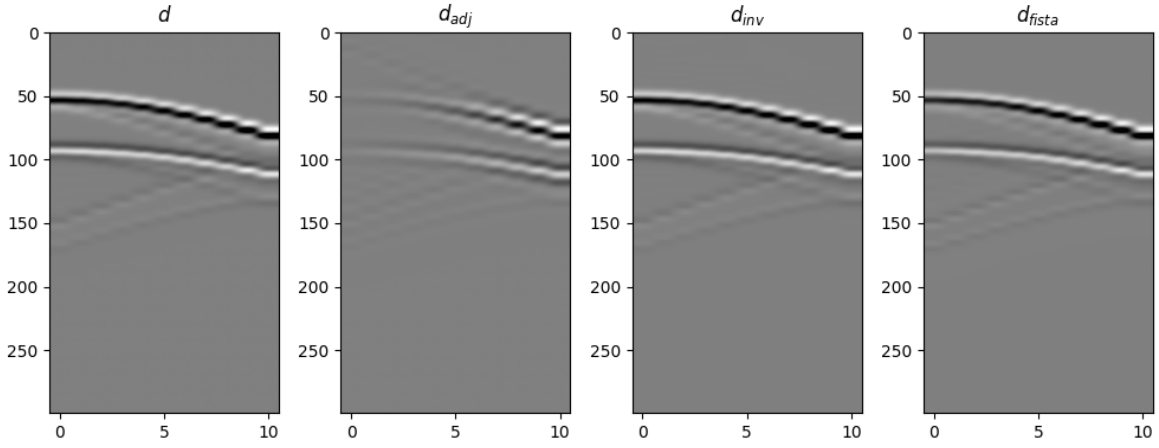
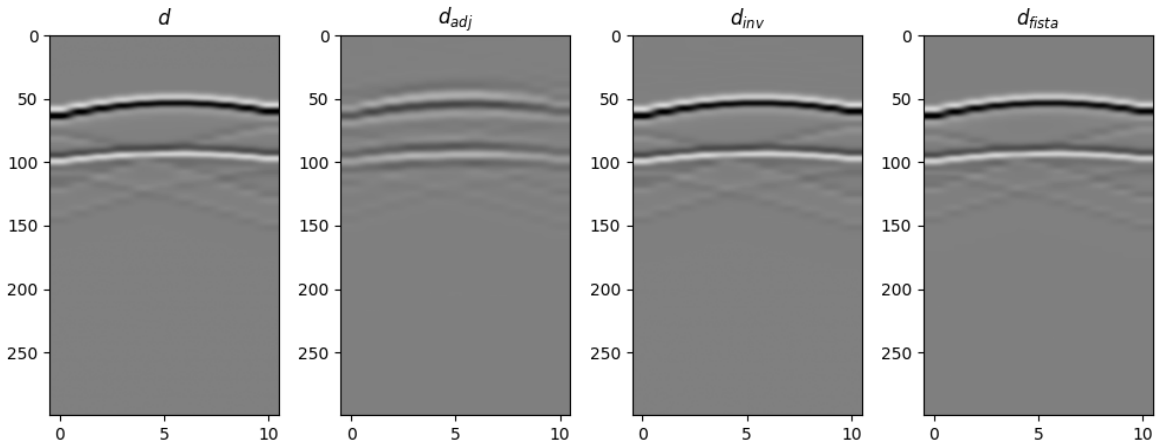
```
axs[1][1].axis("tight")
axs[1][1].set_title(r"$m_{FISTA}$")
plt.tight_layout()

fig, axs = plt.subplots(1, 4, figsize=(10, 4))
axs[0].imshow(d[0, :, :300].T, cmap="gray", vmin=-d.max(), vmax=d.max())
axs[0].set_title(r"$d$")
axs[0].axis("tight")
axs[1].imshow(dadj[0, :, :300].T, cmap="gray", vmin=-dadj.max(), vmax=dadj.max())
axs[1].set_title(r"$d_{adj}$")
axs[1].axis("tight")
axs[2].imshow(dinv[0, :, :300].T, cmap="gray", vmin=-d.max(), vmax=d.max())
axs[2].set_title(r"$d_{inv}$")
axs[2].axis("tight")
axs[3].imshow(dinv_sparse[0, :, :300].T, cmap="gray", vmin=-d.max(), vmax=d.max())
axs[3].set_title(r"$d_{fista}$")
axs[3].axis("tight")
plt.tight_layout()

fig, axs = plt.subplots(1, 4, figsize=(10, 4))
axs[0].imshow(d[ns // 2, :, :300].T, cmap="gray", vmin=-d.max(), vmax=d.max())
axs[0].set_title(r"$d$")
axs[0].axis("tight")
axs[1].imshow(dadj[ns // 2, :, :300].T, cmap="gray", vmin=-dadj.max(), vmax=dadj.max())
axs[1].set_title(r"$d_{adj}$")
axs[1].axis("tight")
axs[2].imshow(dinv[ns // 2, :, :300].T, cmap="gray", vmin=-d.max(), vmax=d.max())
axs[2].set_title(r"$d_{inv}$")
axs[2].axis("tight")
axs[3].imshow(dinv_sparse[ns // 2, :, :300].T, cmap="gray", vmin=-d.max(), vmax=d.max())
axs[3].set_title(r"$d_{fista}$")
axs[3].axis("tight")
plt.tight_layout()
```

- 
m
m_{adj}
m_{inv}
m_{FISTA}

- 
d
d_{adj}
d_{inv}
d_{fista}



This was just a short teaser, for a more advanced set of examples of 2D and 3D traveltime-based LSM head over to this notebook.

Total running time of the script: (0 minutes 7.017 seconds)

3.4.17 16. CT Scan Imaging

This tutorial considers a very well-known inverse problem from the field of medical imaging.

We will be using the *pylops.signalprocessing.Radon2D* operator to model a *sinogram*, which is a graphic representation of the raw data obtained from a CT scan. The sinogram is further inverted using both a L2 solver and a TV-regularized solver like Split-Bregman.

```
import matplotlib.pyplot as plt

# sphinx_gallery_thumbnail_number = 2
import numpy as np
from numba import jit

import pylops

plt.close("all")
np.random.seed(10)
```

Let's start by loading the Shepp-Logan phantom model. We can then construct the sinogram by providing a custom-made function to the *pylops.signalprocessing.Radon2D* that samples parametric curves of such a type:

$$t(r, \theta; x) = \tan(90 - \theta)x + \frac{r}{\sin(\theta)}$$

where θ is the angle between the x-axis (x) and the perpendicular to the summation line and r is the distance from the origin of the summation line.

```
@jit(nopython=True)
def radoncurve(x, r, theta):
    return (
        (r - ny // 2) / (np.sin(theta) + 1e-15)
        + np.tan(np.pi / 2.0 - theta) * x
        + ny // 2
```

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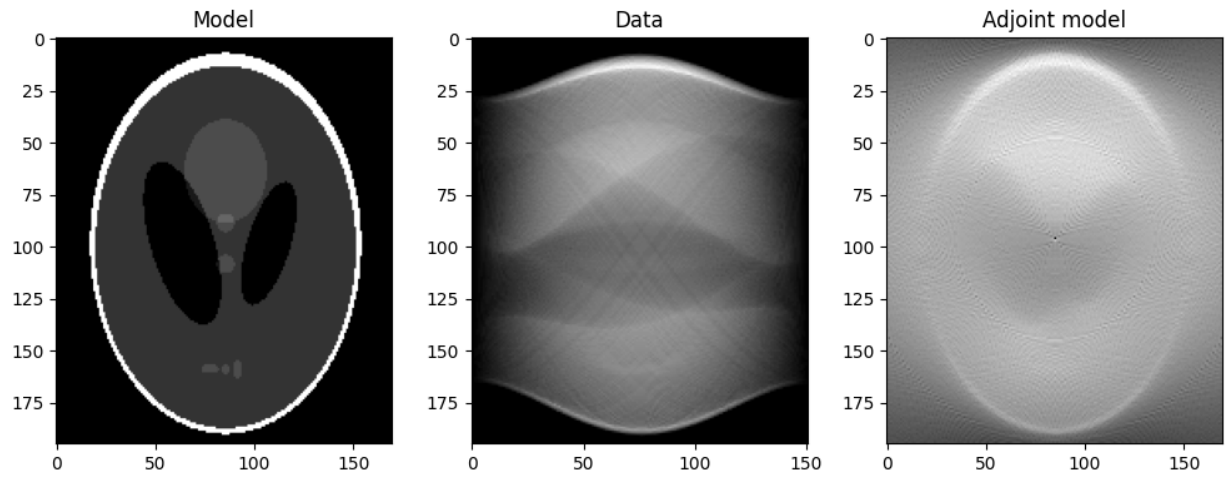
```
    )


x = np.load("../testdata/optimization/shepp_logan_phantom.npy").T
x = x / x.max()
nx, ny = x.shape

ntheta = 151
theta = np.linspace(0.0, np.pi, ntheta, endpoint=False)

RLop = pylops.signalprocessing.Radon2D(
    np.arange(ny),
    np.arange(nx),
    theta,
    kind=radoncurve,
    centeredh=True,
    interp=False,
    engine="numba",
    dtype="float64",
)

y = RLop.H * x
```

We can now first perform the adjoint, which in the medical imaging literature is also referred to as back-projection.

This is the first step of a common reconstruction technique, named filtered back-projection, which simply applies a correction filter in the frequency domain to the adjoint model.

```
xrec = RLop * y

fig, axs = plt.subplots(1, 3, figsize=(10, 4))
axs[0].imshow(x.T, vmin=0, vmax=1, cmap="gray")
axs[0].set_title("Model")
axs[0].axis("tight")
axs[1].imshow(y.T, cmap="gray")
axs[1].set_title("Data")
axs[1].axis("tight")
axs[2].imshow(xrec.T, cmap="gray")
axs[2].set_title("Adjoint model")
axs[2].axis("tight")
fig.tight_layout()
```



Finally we take advantage of our different solvers and try to invert the modelling operator both in a least-squares sense and using TV-reg.

```
Dop = [
    pylops.FirstDerivative(
        (nx, ny), axis=0, edge=True, kind="backward", dtype=np.float64
    ),
    pylops.FirstDerivative(
        (nx, ny), axis=1, edge=True, kind="backward", dtype=np.float64
    ),
]
D2op = pylops.Laplacian(dims=(nx, ny), edge=True, dtype=np.float64)

# L2
xinv_sm = pylops.optimization.leastsquares.regularized_inversion(
    RLop.H, y.ravel(), [D2op], epsRs=[1e1], **dict(iter_lim=20)
)[0]
xinv_sm = np.real(xinv_sm.reshape(nx, ny))

# TV
mu = 1.5
lamda = [1.0, 1.0]
niter = 3
niterinner = 4

xinv = pylops.optimization.sparsity.splitbregman(
    RLop.H,
    y.ravel(),
    Dop,
    niter_outer=niter,
    niter_inner=niterinner,
    mu=mu,
    epsRL1s=lamda,
    tol=1e-4,
    tau=1.0,
    show=False,
    **dict(iter_lim=20, damp=1e-2)
```

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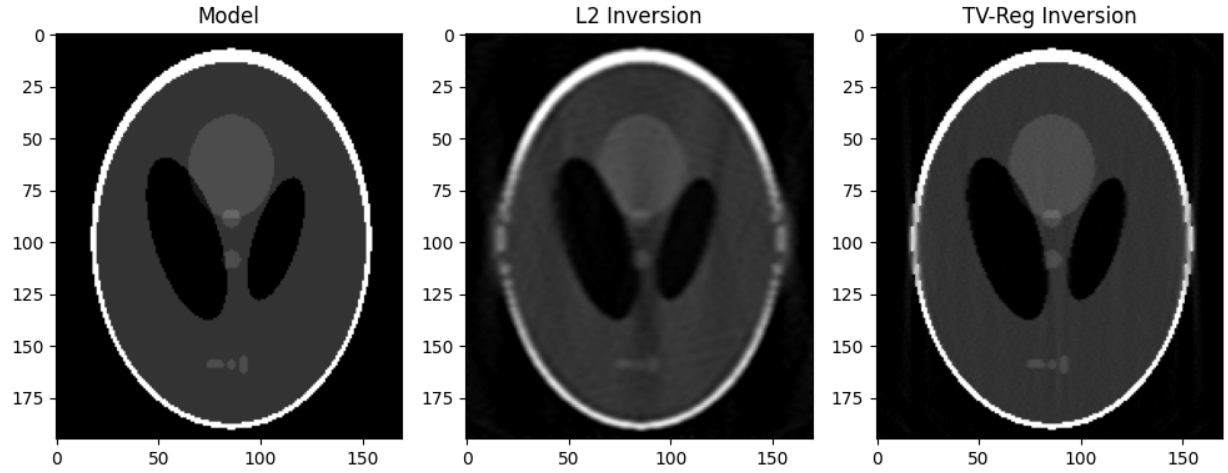
```
)[0]
xinv = np.real(xinv.reshape(nx, ny))

fig, axs = plt.subplots(1, 3, figsize=(10, 4))
axs[0].imshow(x.T, vmin=0, vmax=1, cmap="gray")
axs[0].set_title("Model")
axs[0].axis("tight")
axs[1].imshow(xinv_sm.T, vmin=0, vmax=1, cmap="gray")
axs[1].set_title("L2 Inversion")
axs[1].axis("tight")
axs[2].imshow(xinv.T, vmin=0, vmax=1, cmap="gray")
axs[2].set_title("TV-Reg Inversion")
axs[2].axis("tight")
fig.tight_layout()
```



Total running time of the script: (0 minutes 12.445 seconds)

3.4.18 17. Real/Complex Inversion

In this tutorial we will discuss two equivalent approaches to the solution of inverse problems with real-valued model vector and complex-valued data vector. In other words, we consider a modelling operator $\mathbf{A} : \mathbb{F}^m \to \mathbb{C}^n$ (which could be the case for example for the real FFT).

Mathematically speaking, this problem can be solved equivalently by inverting the complex-valued problem:

$$\mathbf{y} = \mathbf{A}\mathbf{x}$$

or the real-valued augmented system

$$ReIm \begin{bmatrix} (\mathbf{y}) \\ (\mathbf{y}) \end{bmatrix} = \begin{bmatrix} (\mathbf{A}) \\ (\mathbf{A}) \end{bmatrix} \mathbf{x}$$

Whilst we already know how to solve the first problem, let's see how we can solve the second one by taking advantage of the `real` method of the *pylops.LinearOperator* object. We will also wrap our linear operator into a *pylops.MemoizeOperator* which remembers the last N model and data vectors and by-passes the computation of the forward and/or adjoint pass whenever the same pair reappears. This is very useful in our case when we want to compute the real and the imag components of

```
import matplotlib.pyplot as plt
import numpy as np

import pylops

plt.close("all")
np.random.seed(0)
```

To start we create the forward problem

```
n = 5
x = np.arange(n) + 1.0

# make A
Ar = np.random.normal(0, 1, (n, n))
Ai = np.random.normal(0, 1, (n, n))
A = Ar + 1j * Ai
Aop = pylops.MatrixMult(A, dtype=np.complex128)
y = Aop @ x
```

Let's check we can solve this problem using the first formulation

```
A1op = Aop.toreal(forw=False, adj=True)
xinv = A1op.div(y)

print(f"xinv={xinv}\n")
```

```
xinv=[1.+1.83186799e-15j 2.-8.88178420e-16j 3.-8.88178420e-16j
 4.-2.22044605e-16j 5.+6.66133815e-16j]
```

Let's now see how we formulate the second problem

```
Amop = pylops.MemoizeOperator(Aop, max_neval=10)
Arop = Amop.toreal()
Aiop = Amop.toimag()

A1op = pylops.VStack([Arop, Aiop])
y1 = np.concatenate([np.real(y), np.imag(y)])
xinv1 = np.real(A1op.div(y1))

print(f"xinv1={xinv1}\n")
```

```
xinv1=[1. 2. 3. 4. 5.]
```

Total running time of the script: (0 minutes 0.010 seconds)

3.4.19 18. Deblending

The cocktail party problem arises when sounds from different sources mix before reaching our ears (or any recording device), requiring the brain (or any hardware in the recording device) to estimate individual sources from the received mixture. In seismic acquisition, an analog problem is present when multiple sources are fired simultaneously. This family of acquisition methods is usually referred to as simultaneous shooting and the problem of separating the blendend shot gathers into their individual components is called deblending. Whilst various firing strategies can be adopted, in this example we consider the continuos blending problem where a single source is fired sequentially at an interval shorter than the amount of time required for waves to travel into the Earth and come back.

Simply stated the forward problem can be written as:

$$\mathbf{d}^b = \mathbf{\Phi}\mathbf{d}$$

Here $\mathbf{d} = [\mathbf{d}_1^T, \mathbf{d}_2^T, \ldots, \mathbf{d}_N^T]^T$ is a stack of N individual shot gathers, $\mathbf{\Phi} = [\mathbf{\Phi}_1, \mathbf{\Phi}_2, \ldots, \mathbf{\Phi}_N]$ is the blending operator, $\mathbf{d}^b$ is the so-called supergather than contains all shots superimposed to each other.

In order to successfully invert this severely underdetermined problem, two key ingredients must be introduced:

- the firing time of each source (i.e., shifts of the blending operator) must be chosen to be dithered around a nominal regular, periodic firing interval. In our case, we consider shots of duration $T = 4s$, regular firing time of $T_s = 2s$ and a dithering code as follows $\Delta t = U(-1, 1)$;
- prior information about the data to reconstruct, either in the form of regularization or preconditioning must be introduced. In our case we will use a patch-FK transform as preconditioner and solve the problem imposing sparsity in the transformed domain.

In other words, we aim to solve the following problem:

$$J = \|\mathbf{d}^b - \mathbf{\Phi}\mathbf{S}^H\mathbf{x}\|_2 + \epsilon\|\mathbf{x}\|_1$$

for which we will use the *pylops.optimization.sparsity.fista* solver.

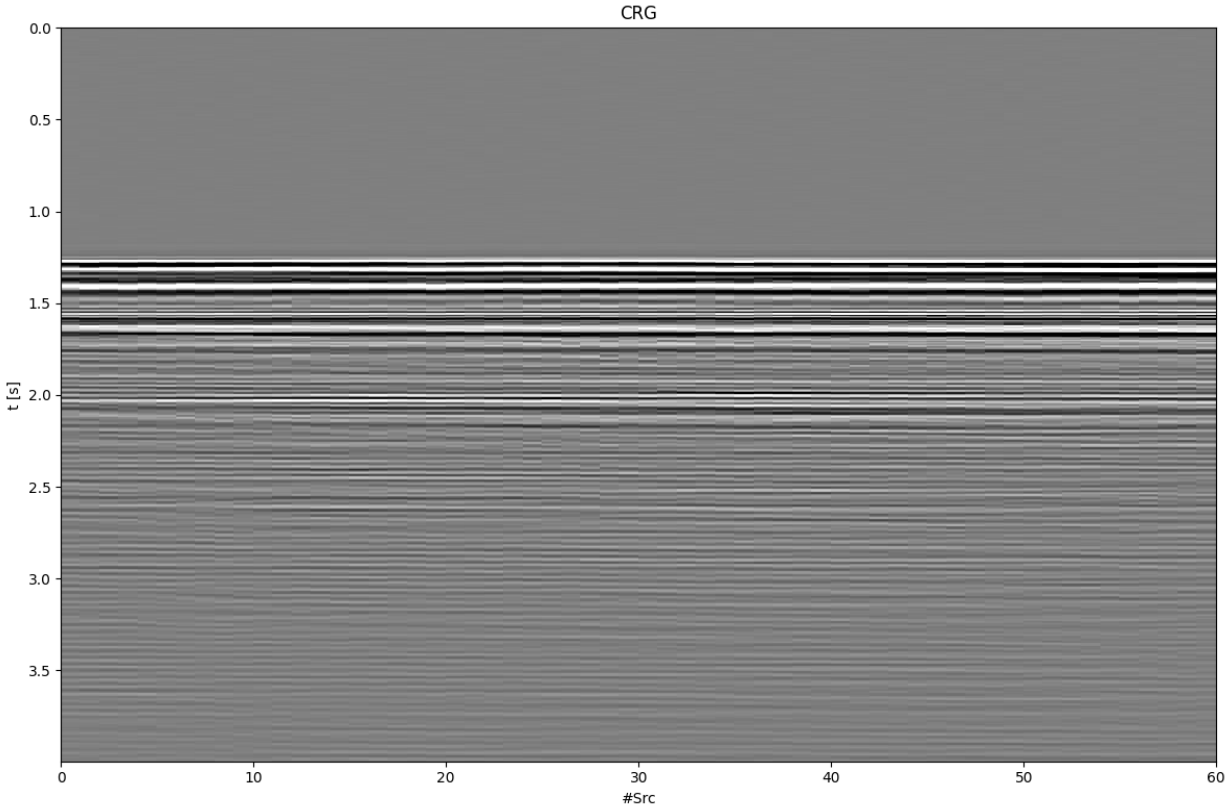
```
import matplotlib.pyplot as plt
import numpy as np
from scipy.sparse.linalg import lobpcg as sp_lobpcg

import pylops

np.random.seed(10)
plt.close("all")
```

Let's start by defining a blending operator

```
def Blending(nt, ns, dt, overlap, times, dtype="float64"):
    """Blending operator"""
    pad = int(overlap * nt)
    OpShiftPad = []
    for i in range(ns):
        PadOp = pylops.Pad(nt, (pad * i, pad * (ns - 1 - i)), dtype=dtype)
        ShiftOp = pylops.signalprocessing.Shift(
            pad * (ns - 1) + nt, times[i], axis=0, sampling=dt, real=False, dtype=dtype
        )
        OpShiftPad.append(ShiftOp * PadOp)
    return pylops.HStack(OpShiftPad)
```

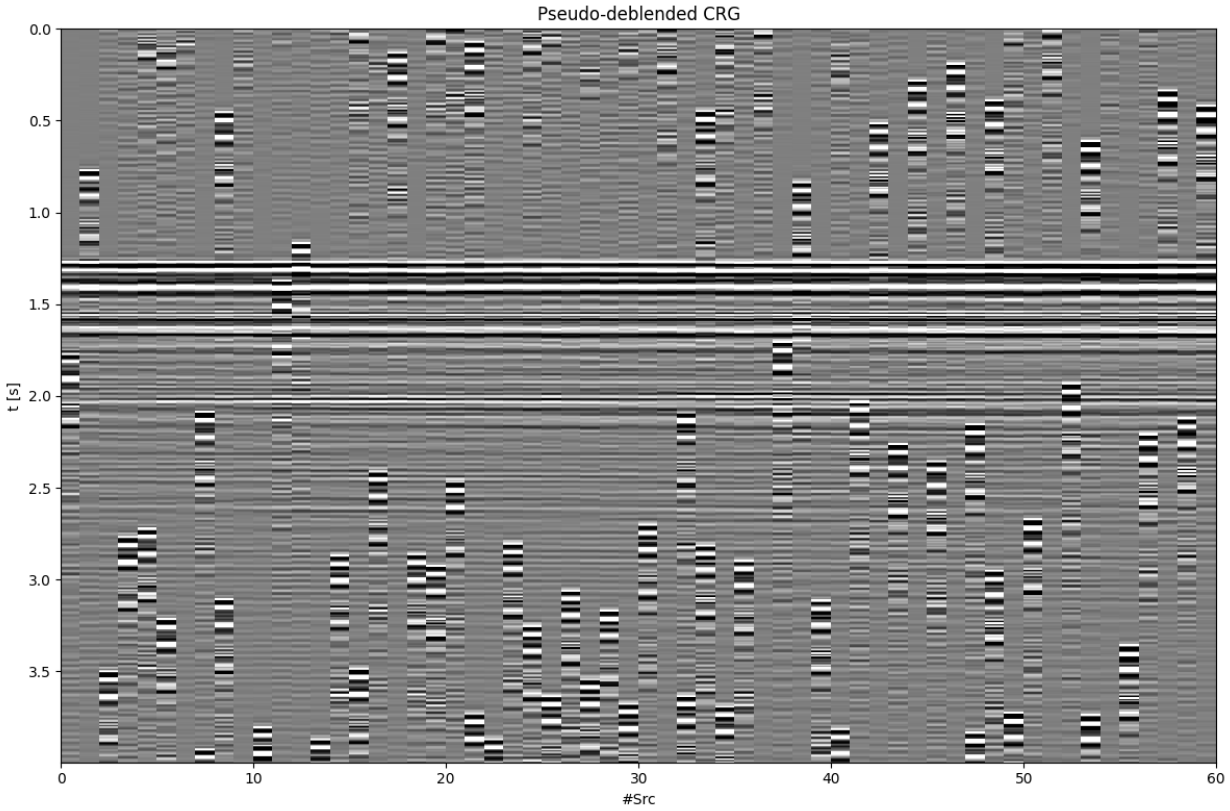
We can now load and display a small portion of the MobilAVO dataset composed of 60 shots and a single receiver. This data is unblended.

```
data = np.load("../testdata/deblending/mobil.npy")
ns, nt = data.shape

dt = 0.004
t = np.arange(nt) * dt

fig, ax = plt.subplots(1, 1, figsize=(12, 8))
ax.imshow(
    data.T,
    cmap="gray",
    vmin=-50,
    vmax=50,
    extent=(0, ns, t[-1], 0),
    interpolation="none",
)
ax.set_title("CRG")
ax.set_xlabel("#Src")
ax.set_ylabel("t [s]")
ax.axis("tight")
plt.tight_layout()
```



We are now ready to define the blending operator, blend our data, and apply the adjoint of the blending operator to it. This is usually referred as pseudo-deblending: as we will see brings back each source to its own nominal firing time, but since sources partially overlap in time, it will also generate some burst like noise in the data. Deblending can hopefully fix this.

```
overlap = 0.5
pad = int(overlap * nt)
ignition_times = 2.0 * np.random.rand(ns) - 1.0
Bop = Blending(nt, ns, dt, overlap, ignition_times, dtype="complex128")
data_blended = Bop * data.ravel()
data_pseudo = Bop.H * data_blended.ravel()
data_pseudo = data_pseudo.reshape(ns, nt)

fig, ax = plt.subplots(1, 1, figsize=(12, 8))
ax.imshow(
    data_pseudo.T.real,
    cmap="gray",
    vmin=-50,
    vmax=50,
    extent=(0, ns, t[-1], 0),
    interpolation="none",
)
ax.set_title("Pseudo-deblended CRG")
ax.set_xlabel("#Src")
ax.set_ylabel("t [s]")
ax.axis("tight")
plt.tight_layout()
```



We are finally ready to solve our deblending inverse problem

```
# Patched FK
```

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```
dimsd = data.shape
nwin = (20, 80)
nover = (10, 40)
nop = (128, 128)
nop1 = (128, 65)
nwins = (5, 24)
dims = (nwins[0] * nop1[0], nwins[1] * nop1[1])

Fop = pylops.signalprocessing.FFT2D(nwin, nffts=nop, real=True)
Sop = pylops.signalprocessing.Patch2D(
    Fop.H, dims, dimsd, nwin, nover, nop1, tapertype="hanning"
)
# Overall operator
Op = Bop * Sop

# Compute max eigenvalue (we do this explicitly to be able to run this fast)
Op1 = pylops.LinearOperator(Op.H * Op, explicit=False)
X = np.random.rand(Op1.shape[0], 1).astype(Op1.dtype)
maxeig = sp_lobpcg(Op1, X=X, maxiter=5, tol=1e-10)[0][0]
alpha = 1.0 / maxeig

# Deblend
niter = 60
decay = (np.exp(-0.05 * np.arange(niter)) + 0.2) / 1.2

with pylops.disabled_ndarray_multiplication():
    p_inv = pylops.optimization.sparsity.fista(
        Op,
        data_blended.ravel(),
        niter=niter,
        eps=5e0,
        alpha=alpha,
        decay=decay,
        show=True,
    )[0]
data_inv = Sop * p_inv
data_inv = data_inv.reshape(ns, nt)

fig, axs = plt.subplots(1, 4, sharey=False, figsize=(12, 8))
axs[0].imshow(
    data.T.real,
    cmap="gray",
    extent=(0, ns, t[-1], 0),
    vmin=-50,
    vmax=50,
    interpolation="none",
)
axs[0].set_title("CRG")
axs[0].set_xlabel("#Src")
axs[0].set_ylabel("t [s]")
axs[0].axis("tight")
axs[1].imshow(
```

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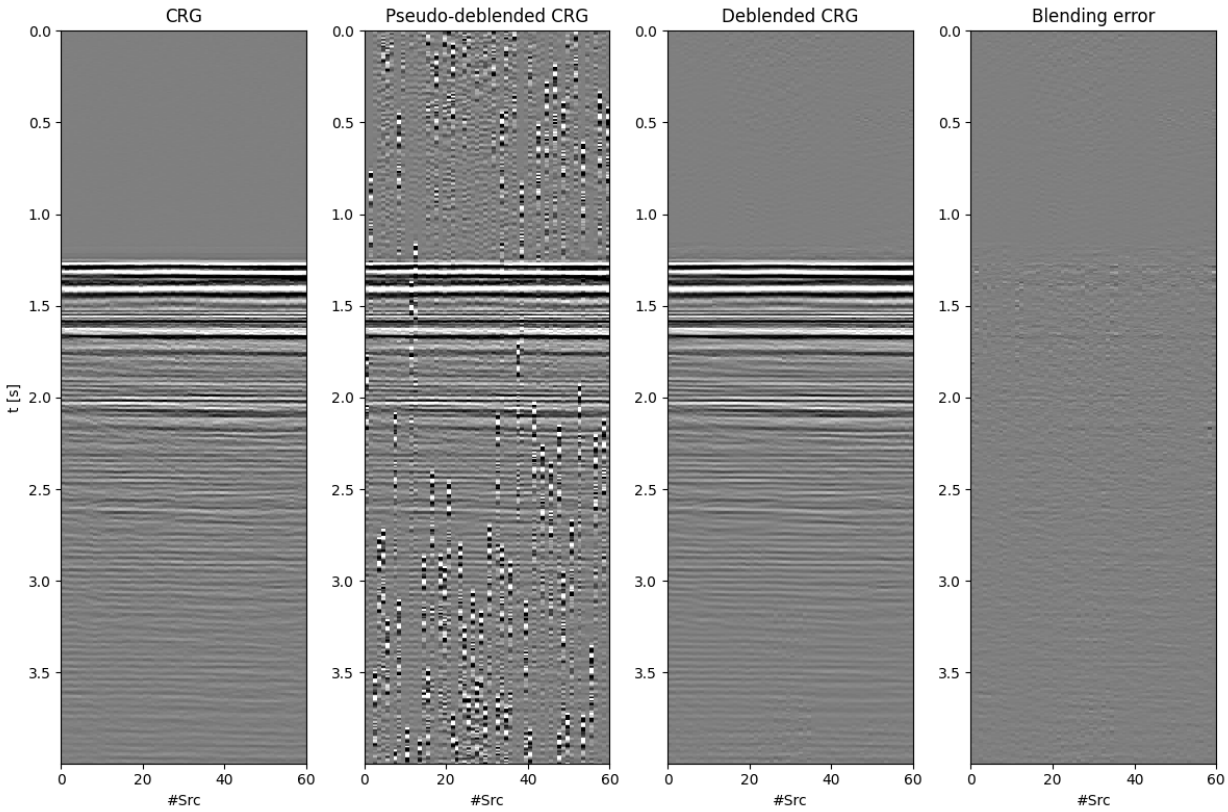
```
    data_pseudo.T.real,
    cmap="gray",
    extent=(0, ns, t[-1], 0),
    vmin=-50,
    vmax=50,
    interpolation="none",
)
axs[1].set_title("Pseudo-deblended CRG")
axs[1].set_xlabel("#Src")
axs[1].axis("tight")
axs[2].imshow(
    data_inv.T.real,
    cmap="gray",
    extent=(0, ns, t[-1], 0),
    vmin=-50,
    vmax=50,
    interpolation="none",
)
axs[2].set_xlabel("#Src")
axs[2].set_title("Deblended CRG")
axs[2].axis("tight")
axs[3].imshow(
    data.T.real - data_inv.T.real,
    cmap="gray",
    extent=(0, ns, t[-1], 0),
    vmin=-50,
    vmax=50,
    interpolation="none",
)
axs[3].set_xlabel("#Src")
axs[3].set_title("Blending error")
axs[3].axis("tight")
plt.tight_layout()
```




```
FISTA (soft thresholding)
-----------------------------------------------------------------------------------
The Operator Op has 30500 rows and 998400 cols
eps = 5.000000e+00      tol = 1.000000e-10      niter = 60
alpha = 3.261681e-01    thresh = 8.154204e-01
-----------------------------------------------------------------------------------
   Itn          x[0]              r2norm     r12norm     xupdate
     1   0.00e+00+0.00e+00j    3.463e+06   5.081e+06   1.251e+03
     2   0.00e+00+0.00e+00j    1.955e+06   4.282e+06   6.844e+02
     3   0.00e+00+0.00e+00j    1.134e+06   3.898e+06   5.635e+02
     4   0.00e+00+0.00e+00j    7.080e+05   3.681e+06   4.576e+02
     5   0.00e+00+0.00e+00j    4.873e+05   3.510e+06   3.831e+02
     6   0.00e+00+0.00e+00j    3.688e+05   3.348e+06   3.357e+02
     7   0.00e+00+0.00e+00j    3.014e+05   3.192e+06   3.048e+02
     8   0.00e+00+0.00e+00j    2.600e+05   3.048e+06   2.826e+02
     9   0.00e+00+0.00e+00j    2.316e+05   2.923e+06   2.642e+02
    10   0.00e+00+0.00e+00j    2.105e+05   2.816e+06   2.479e+02
    11   0.00e+00+0.00e+00j    1.934e+05   2.727e+06   2.327e+02
    21   0.00e+00+0.00e+00j    1.028e+05   2.435e+06   1.202e+02
    31   -0.00e+00+0.00e+00j    6.359e+04   2.450e+06   8.024e+01
    41   -0.00e+00+0.00e+00j    4.281e+04   2.490e+06   6.262e+01
    51   -0.00e+00+0.00e+00j    3.179e+04   2.520e+06   5.211e+01
    52   -0.00e+00+0.00e+00j    3.101e+04   2.523e+06   5.126e+01
    53   -0.00e+00+0.00e+00j    3.027e+04   2.525e+06   5.040e+01
    54   -0.00e+00+0.00e+00j    2.957e+04   2.527e+06   4.960e+01
    55   -0.00e+00+0.00e+00j    2.890e+04   2.529e+06   4.879e+01
```

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```
    56   -0.00e+00+0.00e+00j    2.827e+04   2.531e+06   4.804e+01
    57   -0.00e+00+0.00e+00j    2.768e+04   2.533e+06   4.733e+01
    58   -0.00e+00+0.00e+00j    2.712e+04   2.535e+06   4.663e+01
    59   -0.00e+00+0.00e+00j    2.660e+04   2.536e+06   4.598e+01
    60   -0.00e+00+0.00e+00j    2.610e+04   2.538e+06   4.531e+01

Iterations = 60        Total time (s) = 44.24
--------------------------------------------------------------------------------
```

Finally, let's look a bit more at what really happened under the hood. We display a number of patches and their associated FK spectrum

```
Sop1 = pylops.signalprocessing.Patch2D(
    Fop.H, dims, dimsd, nwin, nover, nop1, tapertype=None
)

# Original
p = Sop1.H * data.ravel()
preshape = p.reshape(nwins[0], nwins[1], nop1[0], nop1[1])

ix = 16
fig, axs = plt.subplots(2, 4, figsize=(12, 5))
fig.suptitle("Data patches")
for i in range(4):
    axs[0][i].imshow(np.fft.fftshift(np.abs(preshape[i, ix]).T, axes=1))
    axs[0][i].axis("tight")
    axs[1][i].imshow(
        np.real((Fop.H * preshape[i, ix].ravel()).reshape(nwin)).T,
        cmap="gray",
        vmin=-30,
        vmax=30,
        interpolation="none",
    )
    axs[1][i].axis("tight")
plt.tight_layout()

# Pseudo-deblended
p_pseudo = Sop1.H * data_pseudo.ravel()
p_pseudoreshape = p_pseudo.reshape(nwins[0], nwins[1], nop1[0], nop1[1])

ix = 16
fig, axs = plt.subplots(2, 4, figsize=(12, 5))
fig.suptitle("Pseudo-deblended patches")
for i in range(4):
    axs[0][i].imshow(np.fft.fftshift(np.abs(p_pseudoreshape[i, ix]).T, axes=1))
    axs[0][i].axis("tight")
    axs[1][i].imshow(
        np.real((Fop.H * p_pseudoreshape[i, ix].ravel()).reshape(nwin)).T,
        cmap="gray",
        vmin=-30,
        vmax=30,
        interpolation="none",
```

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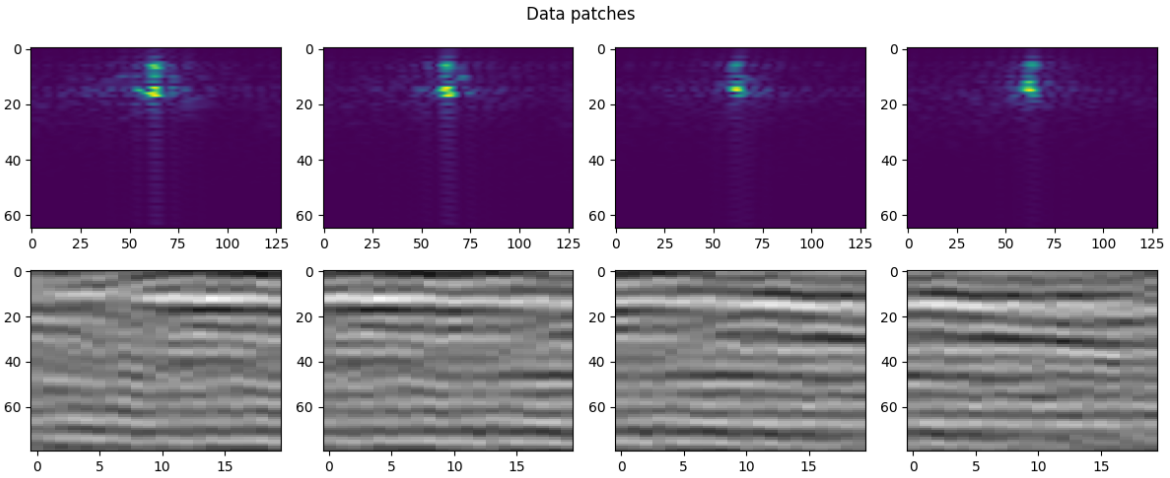
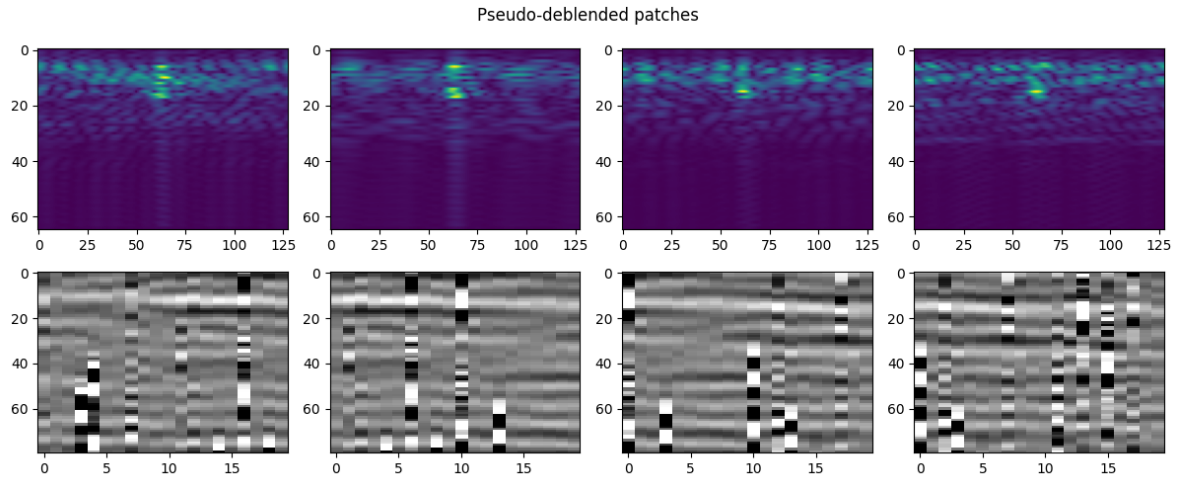
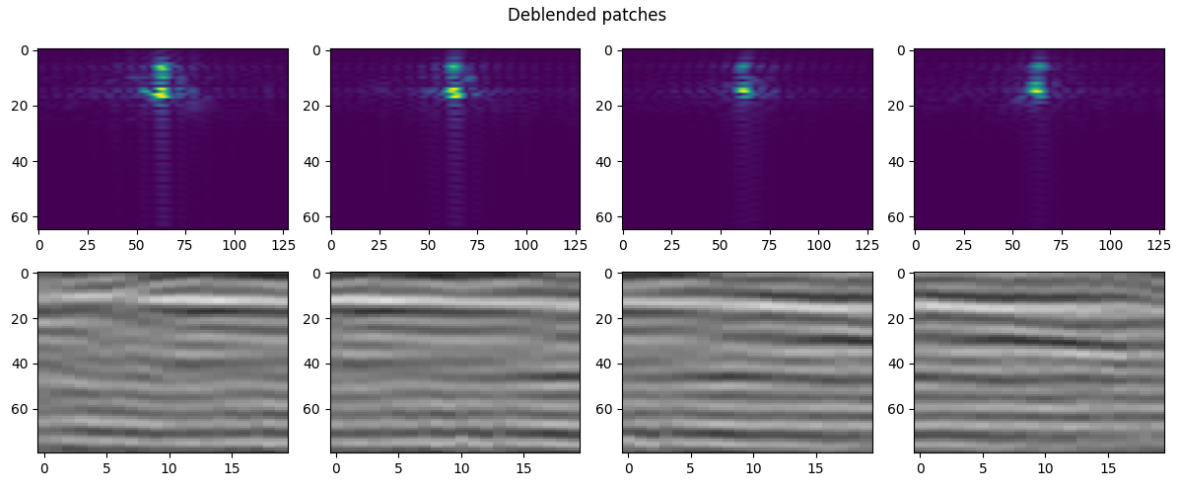
```
    )
    axs[1][i].axis("tight")
plt.tight_layout()

# Deblended
p_inv = Sop1.H * data_inv.ravel()
p_invreshape = p_inv.reshape(nwins[0], nwins[1], nop1[0], nop1[1])

ix = 16
fig, axs = plt.subplots(2, 4, figsize=(12, 5))
fig.suptitle("Deblended patches")
for i in range(4):
    axs[0][i].imshow(np.fft.fftshift(np.abs(p_invreshape[i, ix]).T, axes=1))
    axs[0][i].axis("tight")
    axs[1][i].imshow(
        np.real((Fop.H * p_invreshape[i, ix].ravel()).reshape(nwin)).T,
        cmap="gray",
        vmin=-30,
        vmax=30,
        interpolation="none",
    )
    axs[1][i].axis("tight")
plt.tight_layout()
```



- 

- 

Total running time of the script: (0 minutes 51.398 seconds)

3.4.20 19. Automatic Differentiation

This tutorial focuses on the use of *pylops.TorchOperator* to allow performing Automatic Differentiation (AD) on chains of operators which can be:

- native PyTorch mathematical operations (e.g., `torch.log`, `torch.sin`, `torch.tan`, `torch.pow`, ...)
- neural network operators in `torch.nn`
- PyLops linear operators

This opens up many opportunities, such as easily including linear regularization terms to nonlinear cost functions or using linear preconditioners with nonlinear modelling operators.

```
import matplotlib.pyplot as plt
import numpy as np
import torch
import torch.nn as nn
from torch.autograd import gradcheck
```

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```
import pylops

plt.close("all")
np.random.seed(10)
torch.manual_seed(10)
```

```
<torch._C.Generator object at 0x7f686f92f450>
```

In this example we consider a simple multidimensional functional:

$$\mathbf{y} = \mathbf{A}sin(\mathbf{x})$$

and we use AD to compute the gradient with respect to the input vector evaluated at $\mathbf{x} = \mathbf{x}_0$: $\mathbf{g} = d\mathbf{y}/d\mathbf{x}|_{\mathbf{x}=\mathbf{x}_0}$.

Let's start by defining the Jacobian:

$$\mathbf{J} = \begin{bmatrix} dy_1/dx_1 & ... & dy_1/dx_M \\ ... & ... & ... \\ dy_N/dx_1 & ... & dy_N/dx_M \end{bmatrix} = \begin{bmatrix} a_{11}cos(x_1) & ... & a_{1M}cos(x_M) \\ ... & ... & ... \\ a_{N1}cos(x_1) & ... & a_{NM}cos(x_M) \end{bmatrix} = \mathbf{A}cos(\mathbf{x})$$

Since both input and output are multidimensional, PyTorch `backward` actually computes the product between the transposed Jacobian and a vector $\mathbf{v}$: $\mathbf{g} = \mathbf{J}^\mathbf{T}\mathbf{v}$.

To validate the correctness of the AD result, we can in this simple case also compute the Jacobian analytically and apply it to the same vector $\mathbf{v}$ that we have provided to PyTorch `backward`.

```
nx, ny = 10, 6
x0 = torch.arange(nx, dtype=torch.double, requires_grad=True)

# Forward
A = np.random.normal(0.0, 1.0, (ny, nx))
At = torch.from_numpy(A)
Aop = pylops.TorchOperator(pylops.MatrixMult(A))
y = Aop.apply(torch.sin(x0))

# AD
v = torch.ones(ny, dtype=torch.double)
y.backward(v, retain_graph=True)
adgrad = x0.grad

# Analytical
J = At * torch.cos(x0)
anagrad = torch.matmul(J.T, v)

print("Input: ", x0)
print("AD gradient: ", adgrad)
print("Analytical gradient: ", anagrad)
```

```
Input:  tensor([0., 1., 2., 3., 4., 5., 6., 7., 8., 9.], dtype=torch.float64,
       requires_grad=True)
AD gradient:  tensor([ 0.1539, -0.2356,  1.4944, -3.1160, -2.1699,  0.4492,  1.6168,  1.
↪9101,
```

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```
        -0.4259,  1.7154], dtype=torch.float64)
Analytical gradient:  tensor([ 0.1539, -0.2356,  1.4944, -3.1160, -2.1699,  0.4492,  1.
↪6168,  1.9101,
        -0.4259,  1.7154], dtype=torch.float64, grad_fn=<MvBackward>)
```

Similarly we can use the `torch.autograd.gradcheck` directly from PyTorch. Note that doubles must be used for this to succeed with very small *eps* and *atol*

```
input = (
    torch.arange(nx, dtype=torch.double, requires_grad=True),
    Aop.matvec,
    Aop.rmatvec,
    Aop.device,
    "cpu",
)
test = gradcheck(Aop.Top, input, eps=1e-6, atol=1e-4)
print(test)
```

```
True
```

Note that while matrix-vector multiplication could have been performed using the native PyTorch operator `torch.matmul`, in this case we have shown that we are also able to use a PyLops operator wrapped in *pylops.TorchOperator*. As already mentioned, this gives us the ability to use much more complex linear operators provided by PyLops within a chain of mixed linear and nonlinear AD-enabled operators. To conclude, let's see how we can chain a torch convolutional network with PyLops *pylops.Smoothing2D* operator. First of all, we consider a single training sample.

```
class Network(nn.Module):
    def __init__(self, input_channels):
        super(Network, self).__init__()
        self.conv1 = nn.Conv2d(
            input_channels, input_channels // 2, kernel_size=3, padding=1
        )
        self.conv2 = nn.Conv2d(
            input_channels // 2, input_channels // 4, kernel_size=3, padding=1
        )
        self.activation = nn.LeakyReLU(0.2)
        self.maxpool = nn.MaxPool2d(kernel_size=2, stride=2)

    def forward(self, x):
        x = self.conv1(x)
        x = self.activation(x)
        x = self.conv2(x)
        x = self.activation(x)
        return x


net = Network(4)
Cop = pylops.TorchOperator(pylops.Smoothing2D((5, 5), dims=(32, 32)))

# Forward
x = torch.randn(1, 4, 32, 32).requires_grad_()
```

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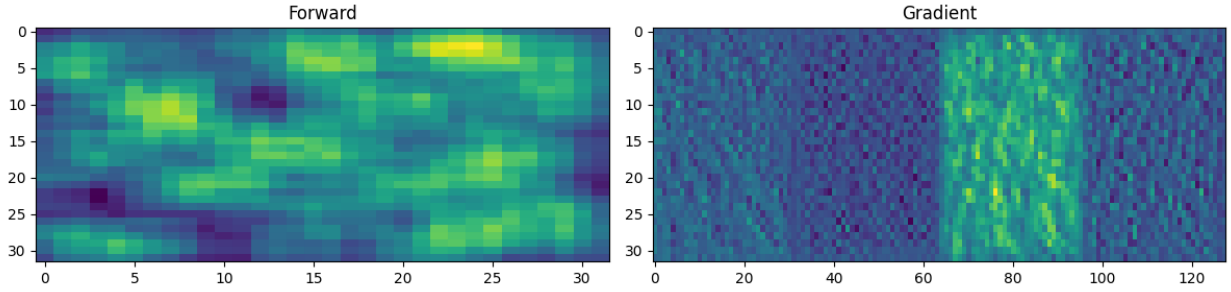
```
y = Cop.apply(net(x).view(-1)).reshape(32, 32)

# Backward
loss = y.sum()
loss.backward()

fig, axs = plt.subplots(1, 2, figsize=(12, 3))
axs[0].imshow(y.detach().numpy())
axs[0].set_title("Forward")
axs[0].axis("tight")
axs[1].imshow(x.grad.reshape(4 * 32, 32).T)
axs[1].set_title("Gradient")
axs[1].axis("tight")
plt.tight_layout()
```



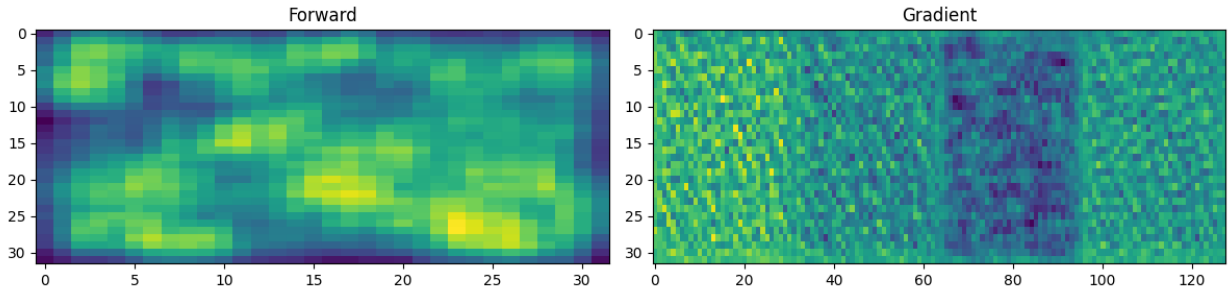
And finally we do the same with a batch of 3 training samples.

```
net = Network(4)
Cop = pylops.TorchOperator(pylops.Smoothing2D((5, 5), dims=(32, 32)), batch=True)

# Forward
x = torch.randn(3, 4, 32, 32).requires_grad_()
y = Cop.apply(net(x).reshape(3, 32 * 32)).reshape(3, 32, 32)

# Backward
loss = y.sum()
loss.backward()

fig, axs = plt.subplots(1, 2, figsize=(12, 3))
axs[0].imshow(y[0].detach().numpy())
axs[0].set_title("Forward")
axs[0].axis("tight")
axs[1].imshow(x.grad[0].reshape(4 * 32, 32).T)
axs[1].set_title("Gradient")
axs[1].axis("tight")
plt.tight_layout()
```



Total running time of the script: (0 minutes 0.531 seconds)

3.5 Gallery

Below is a gallery of examples which use PyLops operators and utilities.

3.5.1 1D Smoothing

This example shows how to use the *pylops.Smoothing1D* operator to smooth an input signal along a given axis.

Derivative (or roughening) operators are generally used *regularization* in inverse problems. Smoothing has the opposite effect of roughening and it can be employed as *preconditioning* in inverse problems.

A smoothing operator is a simple compact filter on lenght n_{smooth} and each elements is equal to $1/n_{smooth}$.

```
import matplotlib.pyplot as plt
import numpy as np

import pylops

plt.close("all")
```

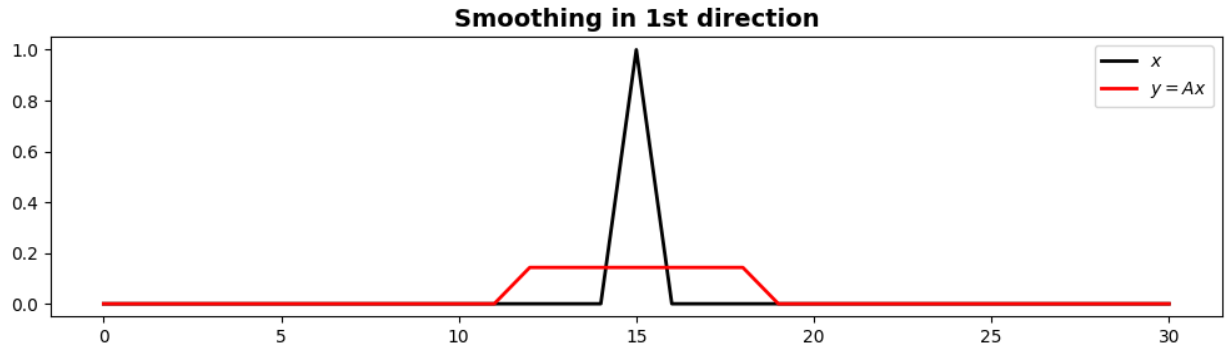
Define the input parameters: number of samples of input signal (`N`) and lenght of the smoothing filter regression coefficients (n_{smooth}). In this first case the input signal is one at the center and zero elsewhere.

```
N = 31
nsmooth = 7
x = np.zeros(N)
x[int(N / 2)] = 1

Sop = pylops.Smoothing1D(nsmooth=nsmooth, dims=[N], dtype="float32")

y = Sop * x
xadj = Sop.H * y

fig, ax = plt.subplots(1, 1, figsize=(10, 3))
ax.plot(x, "k", lw=2, label=r"$x$")
ax.plot(y, "r", lw=2, label=r"$y=Ax$")
ax.set_title("Smoothing in 1st direction", fontsize=14, fontweight="bold")
ax.legend()
plt.tight_layout()
```

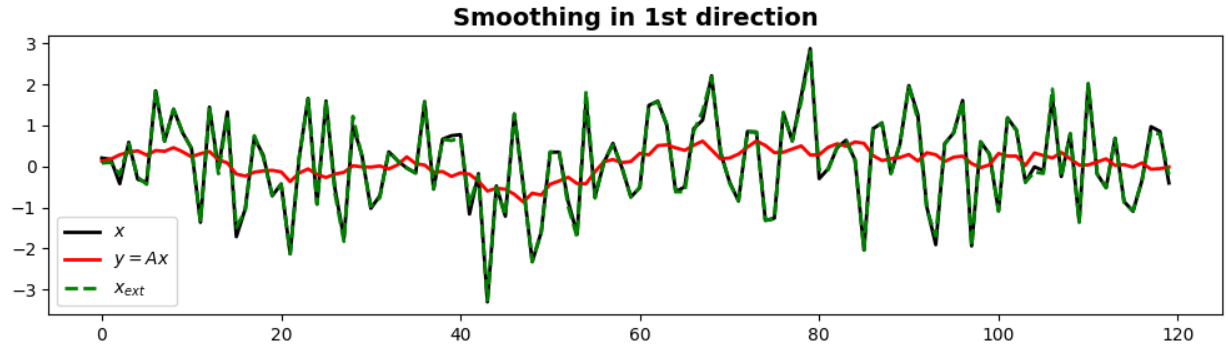


Let's repeat the same exercise with a random signal as input. After applying smoothing, we will also try to invert it.

```
N = 120
nsmooth = 13
x = np.random.normal(0, 1, N)
Sop = pylops.Smoothing1D(nsmooth=13, dims=(N), dtype="float32")

y = Sop * x
xest = Sop / y

fig, ax = plt.subplots(1, 1, figsize=(10, 3))
ax.plot(x, "k", lw=2, label=r"$x$")
ax.plot(y, "r", lw=2, label=r"$y=Ax$")
ax.plot(xest, "--g", lw=2, label=r"$x_{ext}$")
ax.set_title("Smoothing in 1st direction", fontsize=14, fontweight="bold")
ax.legend()
plt.tight_layout()
```



Finally we show that the same operator can be applied to multi-dimensional data along a chosen axis.

```
A = np.zeros((11, 21))
A[5, 10] = 1

Sop = pylops.Smoothing1D(nsmooth=5, dims=(11, 21), axis=0, dtype="float64")
B = Sop * A

fig, axs = plt.subplots(1, 2, figsize=(10, 3))
fig.suptitle(
    "Smoothing in 1st direction for 2d data", fontsize=14, fontweight="bold", y=0.95
```

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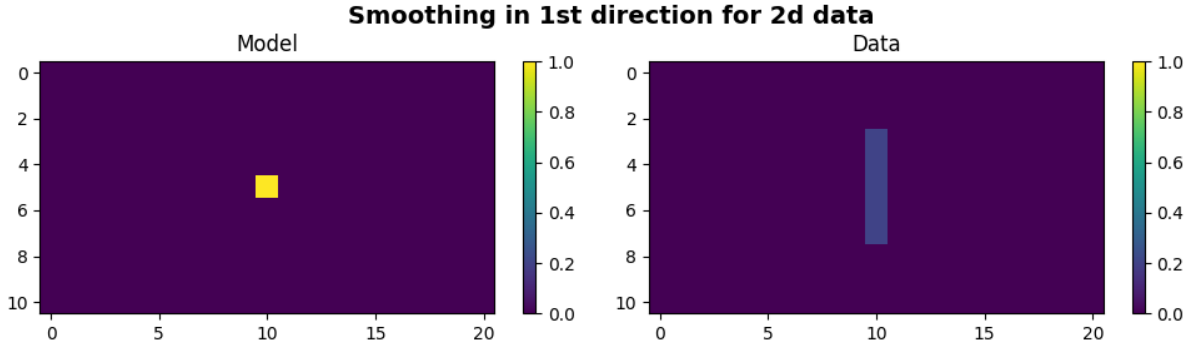
```
)
im = axs[0].imshow(A, interpolation="nearest", vmin=0, vmax=1)
axs[0].axis("tight")
axs[0].set_title("Model")
plt.colorbar(im, ax=axs[0])
im = axs[1].imshow(B, interpolation="nearest", vmin=0, vmax=1)
axs[1].axis("tight")
axs[1].set_title("Data")
plt.colorbar(im, ax=axs[1])
plt.tight_layout()
plt.subplots_adjust(top=0.8)
```



Total running time of the script: (0 minutes 0.954 seconds)

3.5.2 1D, 2D and 3D Sliding

This example shows how to use the *pylops.signalprocessing.Sliding1D*, *pylops.signalprocessing.Sliding2D* and *pylops.signalprocessing.Sliding3D* operators to perform repeated transforms over small strides of a 1-, 2- or 3-dimensional array.

For the 1-d case, the transform that we apply in this example is the *pylops.signalprocessing.FFT*.

For the 2- and 3-d cases, the transform that we apply in this example is the *pylops.signalprocessing.Radon2D* (and *pylops.signalprocessing.Radon3D*) but this operator has been design to allow a variety of transforms as long as they operate with signals that are 2 or 3-dimensional in nature, respectively.

```
import matplotlib.pyplot as plt
import numpy as np

import pylops

plt.close("all")
```

Let's start by creating a 1-dimensional array of size n_t and create a sliding operator to compute its transformed representation.

```
nwins = 4
nwin = 26
nover = 3
nop = 64
```

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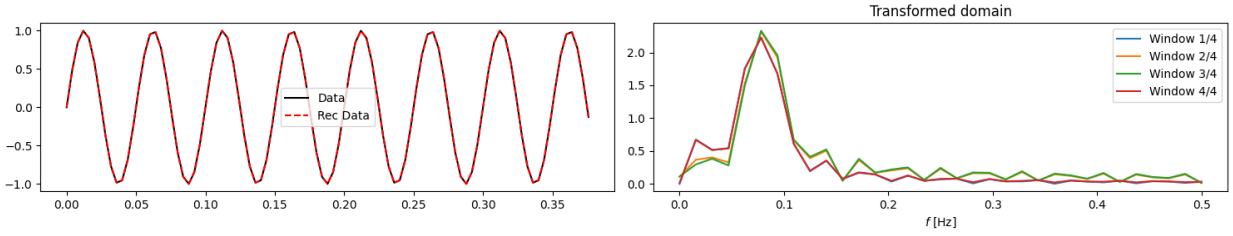
```
dimd = nwin * nwins - 3 * nover

t = np.arange(dimd) * 0.004
data = np.sin(2 * np.pi * 20 * t)

Op = pylops.signalprocessing.FFT(nwin, nfft=nop, real=True)

nwins, dim, mwin_inends, dwin_inends = pylops.signalprocessing.sliding1d_design(
    dimd, nwin, nover, (nop + 2) // 2
)
Slid = pylops.signalprocessing.Sliding1D(
    Op.H,
    dim,
    dimd,
    nwin,
    nover,
    tapertype=None,
)

x = Slid.H * data
```

We now create a similar operator but we also add a taper to the overlapping parts of the patches and use it to reconstruct the original signal. This is done by simply using the adjoint of the *pylops.signalprocessing.Sliding1D* operator. Note that for non- orthogonal operators, this must be replaced by an inverse.

```
Slid = pylops.signalprocessing.Sliding1D(
    Op.H, dim, dimd, nwin, nover, tapertype="cosine"
)

reconstructed_data = Slid * x

fig, axs = plt.subplots(1, 2, figsize=(15, 3))
axs[0].plot(t, data, "k", label="Data")
axs[0].plot(t, reconstructed_data.real, "--r", label="Rec Data")
axs[0].legend()
axs[1].set(xlabel=r"$t$ [s]", title="Original domain")
for i in range(nwins):
    axs[1].plot(Op.f, np.abs(x[i, :]), label=f"Window {i+1}/{nwins}")
axs[1].set(xlabel=r"$f$ [Hz]", title="Transformed domain")
axs[1].legend()
plt.tight_layout()
```



We now create a 2-dimensional array of size $n_x \times n_t$ composed of 3 parabolic events

```
par = {"ox": -140, "dx": 2, "nx": 140, "ot": 0, "dt": 0.004, "nt": 200, "f0": 20}

v = 1500
t0 = [0.2, 0.4, 0.5]
px = [0, 0, 0]
pxx = [1e-5, 5e-6, 1e-20]
amp = [1.0, -2, 0.5]

# Create axis
t, t2, x, y = pylops.utils.seismicevents.makeaxis(par)

# Create wavelet
wav = pylops.utils.wavelets.ricker(t[:41], f0=par["f0"])[0]

# Generate model
_, data = pylops.utils.seismicevents.parabolic2d(x, t, t0, px, pxx, amp, wav)
```

We want to divide this 2-dimensional data into small overlapping patches in the spatial direction and apply the adjoint of the *pylops.signalprocessing.Radon2D* operator to each patch. This is done by simply using the adjoint of the *pylops.signalprocessing.Sliding2D* operator

```
winsize = 36
overlap = 10
npx = 61
px = np.linspace(-5e-3, 5e-3, npx)
dimsd = data.shape

# Sliding window transform without taper
Op = pylops.signalprocessing.Radon2D(
    t,
    np.linspace(-par["dx"] * winsize // 2, par["dx"] * winsize // 2, winsize),
    px,
    centeredh=True,
    kind="linear",
    engine="numba",
)

nwins, dims, mwin_inends, dwin_inends = pylops.signalprocessing.sliding2d_design(
    dimsd, winsize, overlap, (npx, par["nt"])
)
Slid = pylops.signalprocessing.Sliding2D(
    Op, dims, dimsd, winsize, overlap, tapertype=None
)

radon = Slid.H * data
```

We now create a similar operator but we also add a taper to the overlapping parts of the patches.

```
Slid = pylops.signalprocessing.Sliding2D(
    Op, dims, dimsd, winsize, overlap, tapertype="cosine"
)
reconstructed_data = Slid * radon
```

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```
# Reshape for plotting
radon = radon.reshape(dims)
reconstructed_data = reconstructed_data.reshape(dimsd)
```

We will see that our reconstructed signal presents some small artifacts. This is because we have not inverted our operator but simply applied the adjoint to estimate the representation of the input data in the Radon domain. We can do better if we use the inverse instead.

```
radoninv = pylops.LinearOperator(Slid, explicit=False).div(data.ravel(), niter=10)
reconstructed_datainv = Slid * radoninv.ravel()

radoninv = radoninv.reshape(dims)
reconstructed_datainv = reconstructed_datainv.reshape(dimsd)
```

Let's finally visualize all the intermediate results as well as our final data reconstruction after inverting the *pylops.signalprocessing.Sliding2D* operator.

```
fig, axs = plt.subplots(2, 3, sharey=True, figsize=(12, 10))
im = axs[0][0].imshow(data.T, cmap="gray")
axs[0][0].set_title("Original data")
plt.colorbar(im, ax=axs[0][0])
axs[0][0].axis("tight")
im = axs[0][1].imshow(radon.T, cmap="gray")
axs[0][1].set_title("Adjoint Radon")
plt.colorbar(im, ax=axs[0][1])
axs[0][1].axis("tight")
im = axs[0][2].imshow(reconstructed_data.T, cmap="gray")
axs[0][2].set_title("Reconstruction from adjoint")
plt.colorbar(im, ax=axs[0][2])
axs[0][2].axis("tight")
axs[1][0].axis("off")
im = axs[1][1].imshow(radoninv.T, cmap="gray")
axs[1][1].set_title("Inverse Radon")
plt.colorbar(im, ax=axs[1][1])
axs[1][1].axis("tight")
im = axs[1][2].imshow(reconstructed_datainv.T, cmap="gray")
axs[1][2].set_title("Reconstruction from inverse")
plt.colorbar(im, ax=axs[1][2])
axs[1][2].axis("tight")

for i in range(0, 114, 24):
    axs[0][0].axvline(i, color="w", lw=1, ls="--")
    axs[0][0].axvline(i + winsize, color="k", lw=1, ls="--")
    axs[0][0].text(
        i + winsize // 2,
        par["nt"] - 10,
        "w" + str(i // 24),
        ha="center",
        va="center",
        weight="bold",
        color="w",
    )
```

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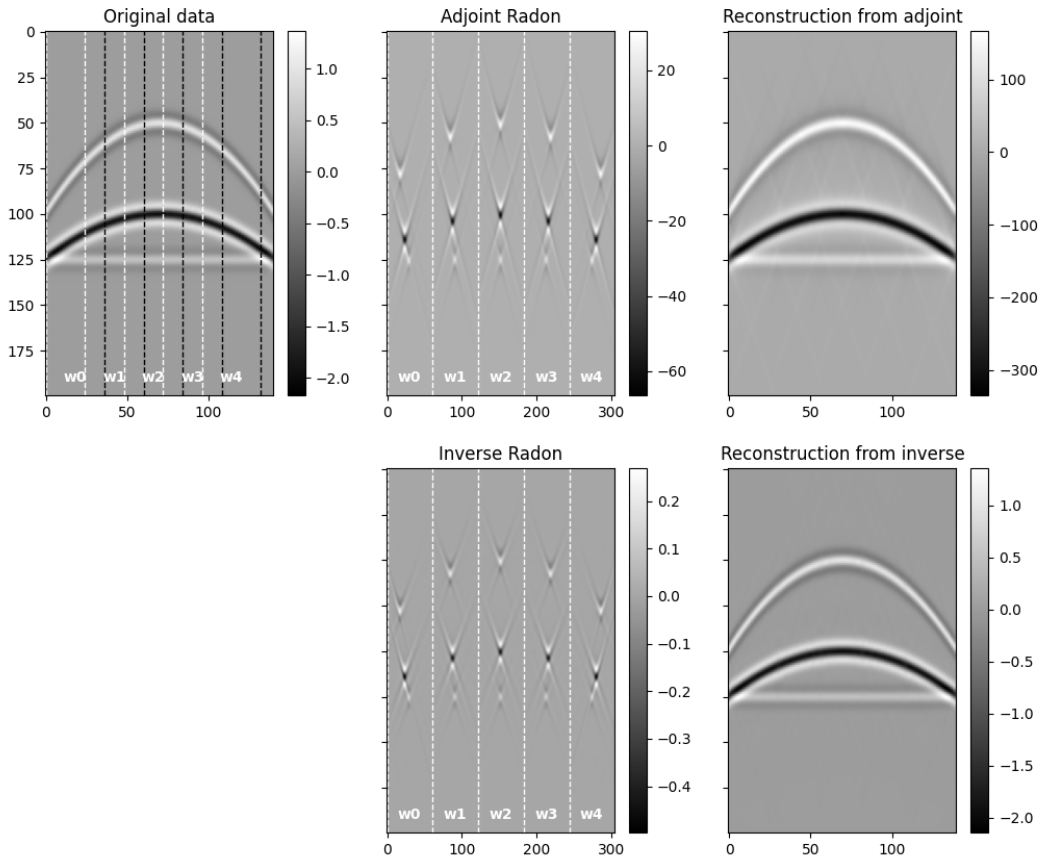
```
for i in range(0, 305, 61):
    axs[0][1].axvline(i, color="w", lw=1, ls="--")
    axs[0][1].text(
        i + npx // 2,
        par["nt"] - 10,
        "w" + str(i // 61),
        ha="center",
        va="center",
        weight="bold",
        color="w",
    )
    axs[1][1].axvline(i, color="w", lw=1, ls="--")
    axs[1][1].text(
        i + npx // 2,
        par["nt"] - 10,
        "w" + str(i // 61),
        ha="center",
        va="center",
        weight="bold",
        color="w",
    )
```



We notice two things, i)provided small enough patches and a transform that can explain data *locally*, we have been able reconstruct our original data almost to perfection. ii) inverse is betten than adjoint as expected as the adjoin does not only introduce small artifacts but also does not respect the original amplitudes of the data.

An appropriate transform alongside with a sliding window approach will result a very good approach for interpolation (or *regularization*) or irregularly sampled seismic data.

Finally we do the same for a 3-dimensional array of size $n_y \times n_x \times n_t$ composed of 3 hyperbolic events

```
par = {
    "oy": -15,
    "dy": 2,
    "ny": 14,
    "ox": -18,
    "dx": 2,
    "nx": 18,
    "ot": 0,
    "dt": 0.004,
    "nt": 50,
    "f0": 30,
}
```

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```
vrms = [200, 200]
t0 = [0.05, 0.1]
amp = [1.0, -2]

# Create axis
t, t2, x, y = pylops.utils.seismicevents.makeaxis(par)

# Create wavelet
wav = pylops.utils.wavelets.ricker(t[:41], f0=par["f0"])[0]

# Generate model
_, data = pylops.utils.seismicevents.hyperbolic3d(x, y, t, t0, vrms, vrms, amp, wav)

# Sliding window plan
winsize = (5, 6)
overlap = (2, 3)
npx = 21
px = np.linspace(-5e-3, 5e-3, npx)
dimsd = data.shape

# Sliding window transform without taper
Op = pylops.signalprocessing.Radon3D(
    t,
    np.linspace(-par["dy"] * winsize[0] // 2, par["dy"] * winsize[0] // 2, winsize[0]),
    np.linspace(-par["dx"] * winsize[1] // 2, par["dx"] * winsize[1] // 2, winsize[1]),
    px,
    px,
    centeredh=True,
    kind="linear",
    engine="numba",
)

nwins, dims, mwin_inends, dwin_inends = pylops.signalprocessing.sliding3d_design(
    dimsd, winsize, overlap, (npx, npx, par["nt"])
)
Slid = pylops.signalprocessing.Sliding3D(
    Op, dims, dimsd, winsize, overlap, (npx, npx), tapertype=None
)

radon = Slid.H * data

Slid = pylops.signalprocessing.Sliding3D(
    Op, dims, dimsd, winsize, overlap, (npx, npx), tapertype="cosine"
)

reconstructed_data = Slid * radon

radoninv = pylops.LinearOperator(Slid, explicit=False).div(data.ravel(), niter=10)
radoninv = radoninv.reshape(Slid.dims)
reconstructed_datainv = Slid * radoninv
```

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```
fig, axs = plt.subplots(2, 3, sharey=True, figsize=(12, 7))
im = axs[0][0].imshow(data[par["ny"] // 2].T, cmap="gray", vmin=-2, vmax=2)
axs[0][0].set_title("Original data")
plt.colorbar(im, ax=axs[0][0])
axs[0][0].axis("tight")
im = axs[0][1].imshow(
    radon[nwins[0] // 2, :, :, npx // 2].reshape(nwins[1] * npx, par["nt"]).T,
    cmap="gray",
    vmin=-25,
    vmax=25,
)
axs[0][1].set_title("Adjoint Radon")
plt.colorbar(im, ax=axs[0][1])
axs[0][1].axis("tight")
im = axs[0][2].imshow(
    reconstructed_data[par["ny"] // 2].T, cmap="gray", vmin=-1000, vmax=1000
)
axs[0][2].set_title("Reconstruction from adjoint")
plt.colorbar(im, ax=axs[0][2])
axs[0][2].axis("tight")
axs[1][0].axis("off")
im = axs[1][1].imshow(
    radoninv[nwins[0] // 2, :, :, npx // 2].reshape(nwins[1] * npx, par["nt"]).T,
    cmap="gray",
    vmin=-0.025,
    vmax=0.025,
)
axs[1][1].set_title("Inverse Radon")
plt.colorbar(im, ax=axs[1][1])
axs[1][1].axis("tight")
im = axs[1][2].imshow(
    reconstructed_datainv[par["ny"] // 2].T, cmap="gray", vmin=-2, vmax=2
)
axs[1][2].set_title("Reconstruction from inverse")
plt.colorbar(im, ax=axs[1][2])
axs[1][2].axis("tight")


fig, axs = plt.subplots(2, 3, figsize=(12, 7))
im = axs[0][0].imshow(data[:, :, 25], cmap="gray", vmin=-2, vmax=2)
axs[0][0].set_title("Original data")
plt.colorbar(im, ax=axs[0][0])
axs[0][0].axis("tight")
im = axs[0][1].imshow(
    radon[nwins[0] // 2, :, :, :, 25].reshape(nwins[1] * npx, npx).T,
    cmap="gray",
    vmin=-25,
    vmax=25,
)
axs[0][1].set_title("Adjoint Radon")
plt.colorbar(im, ax=axs[0][1])
axs[0][1].axis("tight")
```

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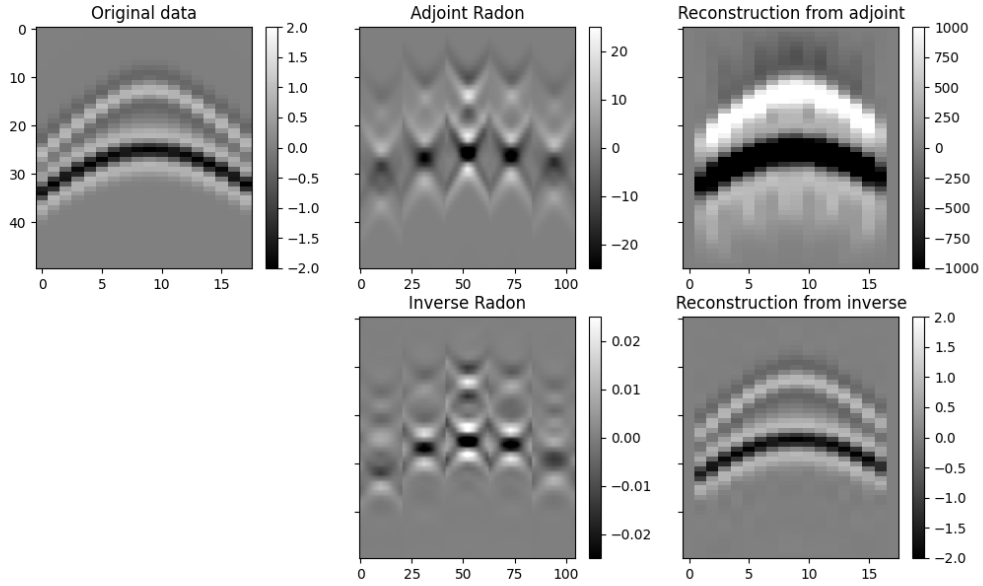
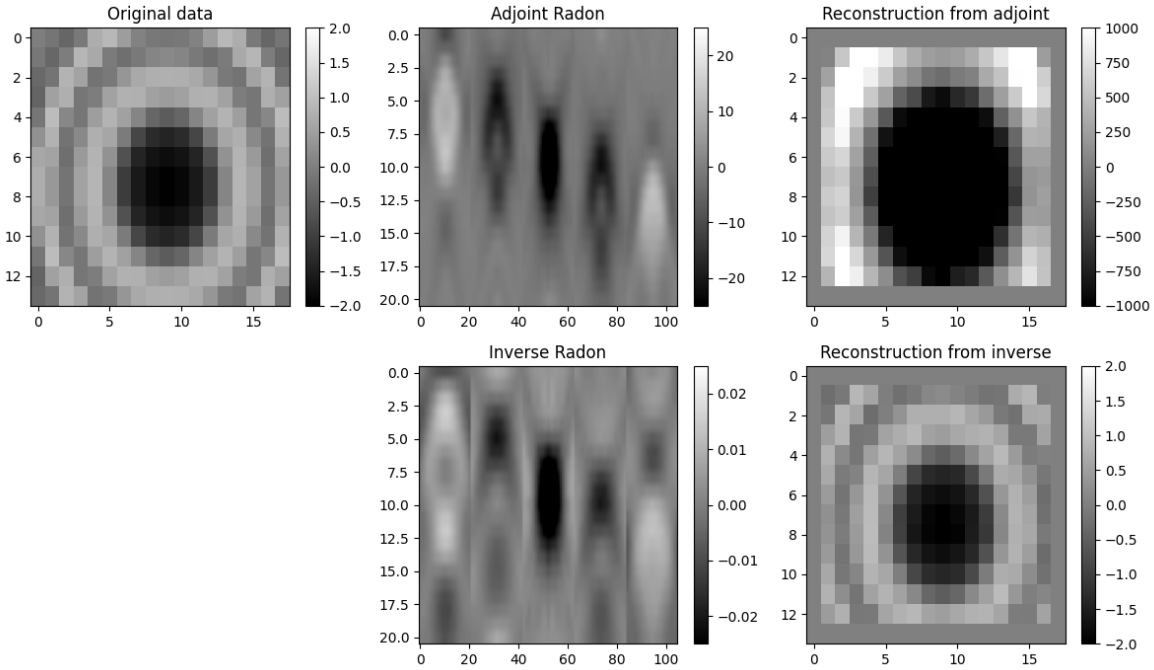
```
im = axs[0][2].imshow(reconstructed_data[:, :, 25], cmap="gray", vmin=-1000, vmax=1000)
axs[0][2].set_title("Reconstruction from adjoint")
plt.colorbar(im, ax=axs[0][2])
axs[0][2].axis("tight")
axs[1][0].axis("off")
im = axs[1][1].imshow(
    radoninv[nwins[0] // 2, :, :, :, 25].reshape(nwins[1] * npx, npx).T,
    cmap="gray",
    vmin=-0.025,
    vmax=0.025,
)
axs[1][1].set_title("Inverse Radon")
plt.colorbar(im, ax=axs[1][1])
axs[1][1].axis("tight")
im = axs[1][2].imshow(reconstructed_datainv[:, :, 25], cmap="gray", vmin=-2, vmax=2)
axs[1][2].set_title("Reconstruction from inverse")
plt.colorbar(im, ax=axs[1][2])
axs[1][2].axis("tight")
plt.tight_layout()
```





-

Total running time of the script: (0 minutes 9.120 seconds)

3.5.3 2D Smoothing

This example shows how to use the *pylops.Smoothing2D* operator to smooth a multi-dimensional input signal along two given axes.

```
import matplotlib.pyplot as plt
import numpy as np

import pylops

plt.close("all")
```

Define the input parameters: number of samples of input signal (`N` and `M`) and lenght of the smoothing filter regression coefficients ($n_{smooth,1}$ and $n_{smooth,2}$). In this first case the input signal is one at the center and zero elsewhere.

```
N, M = 11, 21
nsmooth1, nsmooth2 = 5, 3
A = np.zeros((N, M))
A[5, 10] = 1

Sop = pylops.Smoothing2D(nsmooth=[nsmooth1, nsmooth2], dims=[N, M], dtype="float64")
B = Sop * A
```

After applying smoothing, we will also try to invert it.

```
Aest = (Sop / B.ravel()).reshape(Sop.dims)

fig, axs = plt.subplots(1, 3, figsize=(10, 3))
im = axs[0].imshow(A, interpolation="nearest", vmin=0, vmax=1)
```

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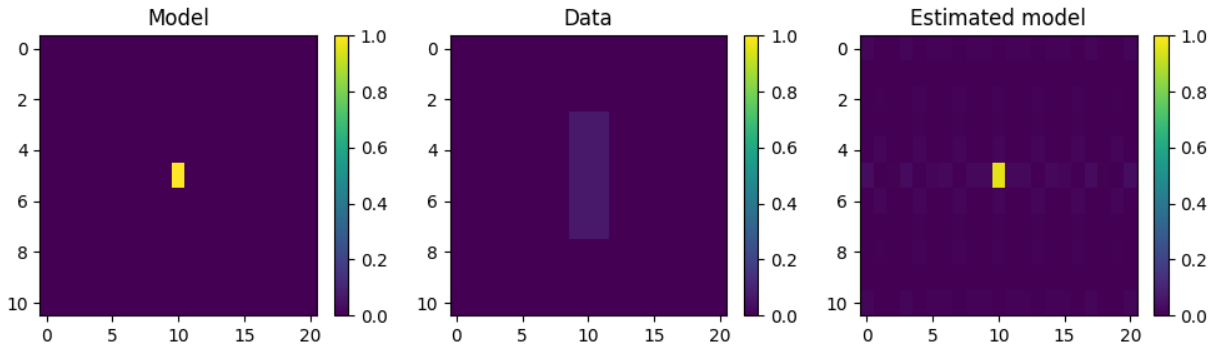
```
axs[0].axis("tight")
axs[0].set_title("Model")
plt.colorbar(im, ax=axs[0])
im = axs[1].imshow(B, interpolation="nearest", vmin=0, vmax=1)
axs[1].axis("tight")
axs[1].set_title("Data")
plt.colorbar(im, ax=axs[1])
im = axs[2].imshow(Aest, interpolation="nearest", vmin=0, vmax=1)
axs[2].axis("tight")
axs[2].set_title("Estimated model")
plt.colorbar(im, ax=axs[2])
plt.tight_layout()
```



Total running time of the script: (0 minutes 0.459 seconds)

3.5.4 AVO modelling

This example shows how to create pre-stack angle gathers using the *pylops.avo.avo.AVOLinearModelling* operator.

```
import matplotlib.pyplot as plt
import numpy as np
from mpl_toolkits.axes_grid1.axes_divider import make_axes_locatable
from scipy.signal import filtfilt

import pylops
from pylops.utils.wavelets import ricker

plt.close("all")
np.random.seed(0)
```

Let's start by creating the input elastic property profiles

```
nt0 = 501
dt0 = 0.004
ntheta = 21

t0 = np.arange(nt0) * dt0
thetamin, thetamax = 0.0, 40.0
```

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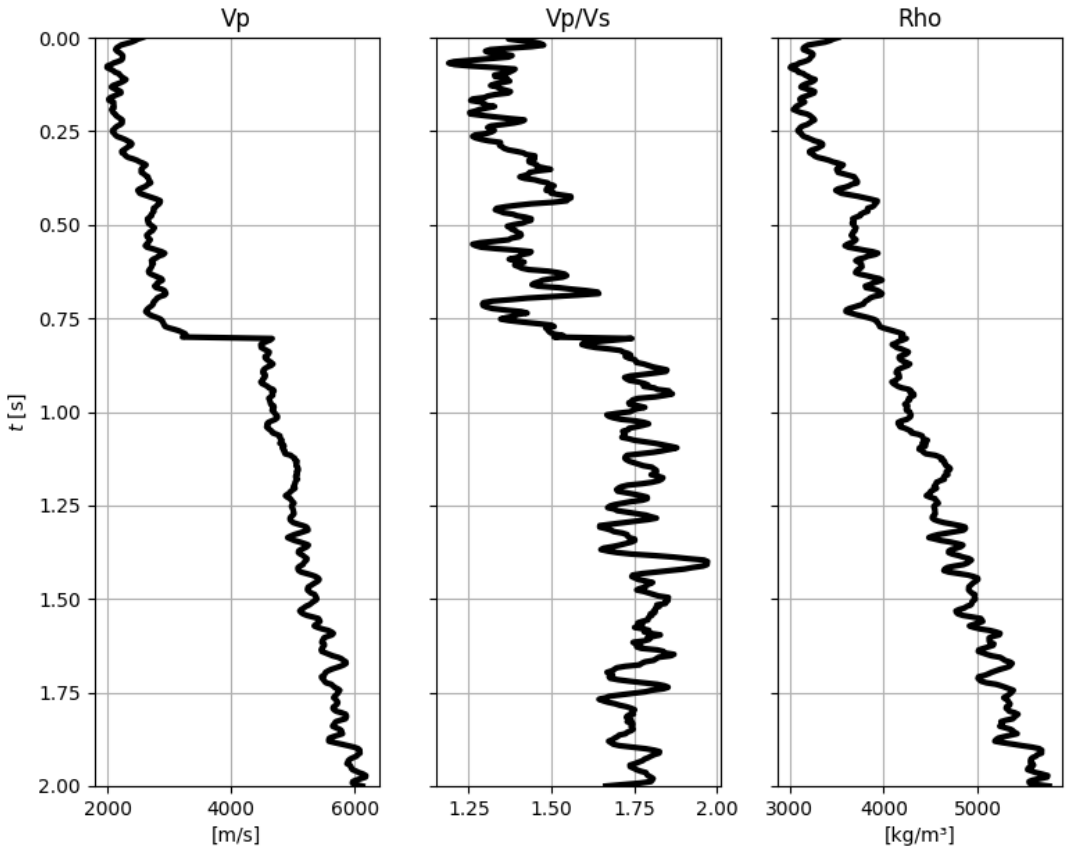
```
theta = np.linspace(thetamin, thetamax, ntheta)

# Elastic property profiles
vp = (
    2000
    + 5 * np.arange(nt0)
    + 2 * filtfilt(np.ones(5) / 5.0, 1, np.random.normal(0, 160, nt0))
)
vs = 600 + vp / 2 + 3 * filtfilt(np.ones(5) / 5.0, 1, np.random.normal(0, 100, nt0))
rho = 1000 + vp + filtfilt(np.ones(5) / 5.0, 1, np.random.normal(0, 120, nt0))
vp[201:] += 1500
vs[201:] += 500
rho[201:] += 100

# Wavelet
ntwav = 41
wavoff = 10
wav, twav, wavc = ricker(t0[: ntwav // 2 + 1], 20)
wav_phase = np.hstack((wav[wavoff:], np.zeros(wavoff)))

# vs/vp profile
vsvp = 0.5
vsvp_z = vs / vp

# Model
m = np.stack((np.log(vp), np.log(vs), np.log(rho)), axis=1)

fig, axs = plt.subplots(1, 3, figsize=(9, 7), sharey=True)
axs[0].plot(vp, t0, "k", lw=3)
axs[0].set(xlabel="[m/s]", ylabel=r"$t$ [s]", ylim=[t0[0], t0[-1]], title="Vp")
axs[0].grid()
axs[1].plot(vp / vs, t0, "k", lw=3)
axs[1].set(title="Vp/Vs")
axs[1].grid()
axs[2].plot(rho, t0, "k", lw=3)
axs[2].set(xlabel="[kg/m$^3$]", title="Rho")
axs[2].invert_yaxis()
axs[2].grid()
```



We create now the operators to model the AVO responses for a set of elastic profiles

```
# constant vsvp
PPop_const = pylops.avo.avo.AVOLinearModelling(
    theta, vsvp=vsvp, nt0=nt0, linearization="akirich", dtype=np.float64
)

# depth-variant vsvp
PPop_variant = pylops.avo.avo.AVOLinearModelling(
    theta, vsvp=vsvp_z, linearization="akirich", dtype=np.float64
)
```

We can then apply those operators to the elastic model and create some synthetic reflection responses

```
dPP_const = PPop_const * m
dPP_variant = PPop_variant * m
```

To visualize these responses, we will plot their anomaly - how much they deveiate from their mean

```
mean_dPP_const = dPP_const.mean()
dPP_const -= mean_dPP_const
mean_dPP_variant = dPP_variant.mean()
dPP_variant -= mean_dPP_variant
```

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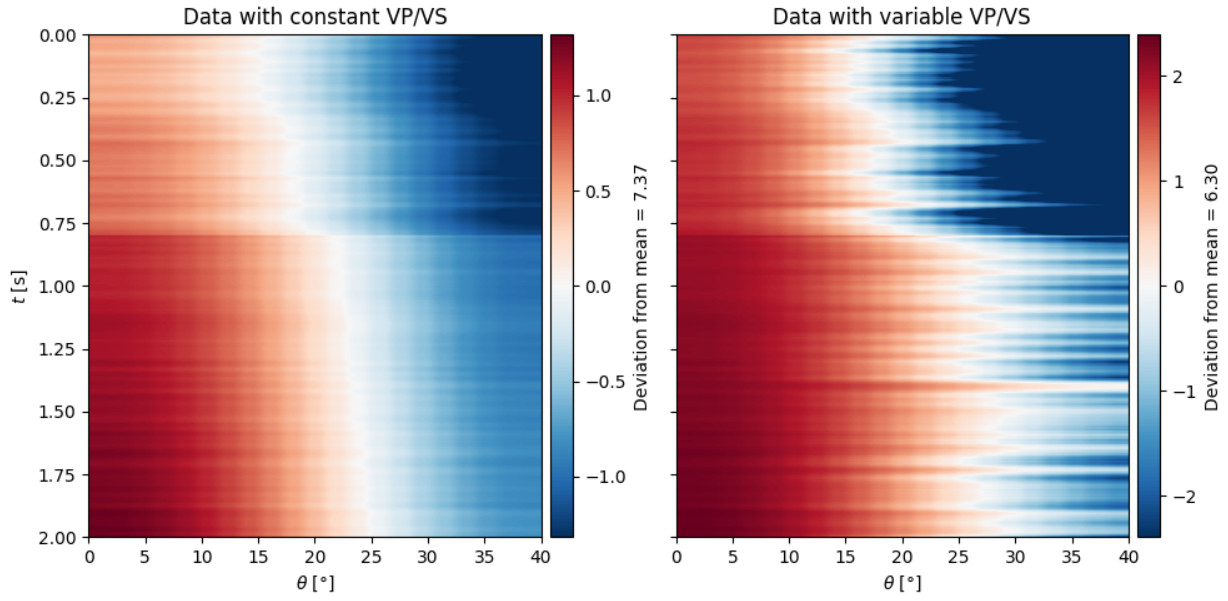
```
fig, axs = plt.subplots(1, 2, figsize=(10, 5), sharey=True)
im = axs[0].imshow(
    dPP_const,
    cmap="RdBu_r",
    extent=(theta[0], theta[-1], t0[-1], t0[0]),
    vmin=-dPP_const.max(),
    vmax=dPP_const.max(),
)
cax = make_axes_locatable(axs[0]).append_axes("right", size="5%", pad="2%")
cb = fig.colorbar(im, cax=cax)
cb.set_label(f"Deviation from mean = {mean_dPP_const:.2f}")
axs[0].set(xlabel=r"$\theta$ [°]", ylabel=r"$t$ [s]", title="Data with constant VP/VS")
axs[0].axis("tight")

im = axs[1].imshow(
    dPP_variant,
    cmap="RdBu_r",
    extent=(theta[0], theta[-1], t0[-1], t0[0]),
    vmin=-dPP_variant.max(),
    vmax=dPP_variant.max(),
)
cax = make_axes_locatable(axs[1]).append_axes("right", size="5%", pad="2%")
cb = fig.colorbar(im, cax=cax)
cb.set_label(f"Deviation from mean = {mean_dPP_variant:.2f}")
axs[1].set(xlabel=r"$\theta$ [°]", title="Data with variable VP/VS")
axs[1].axis("tight")
plt.tight_layout()
```



Finally we can also model the PS response by simply changing the `linearization` choice as follows

```
PSop = pylops.avo.avo.AVOLinearModelling(
```

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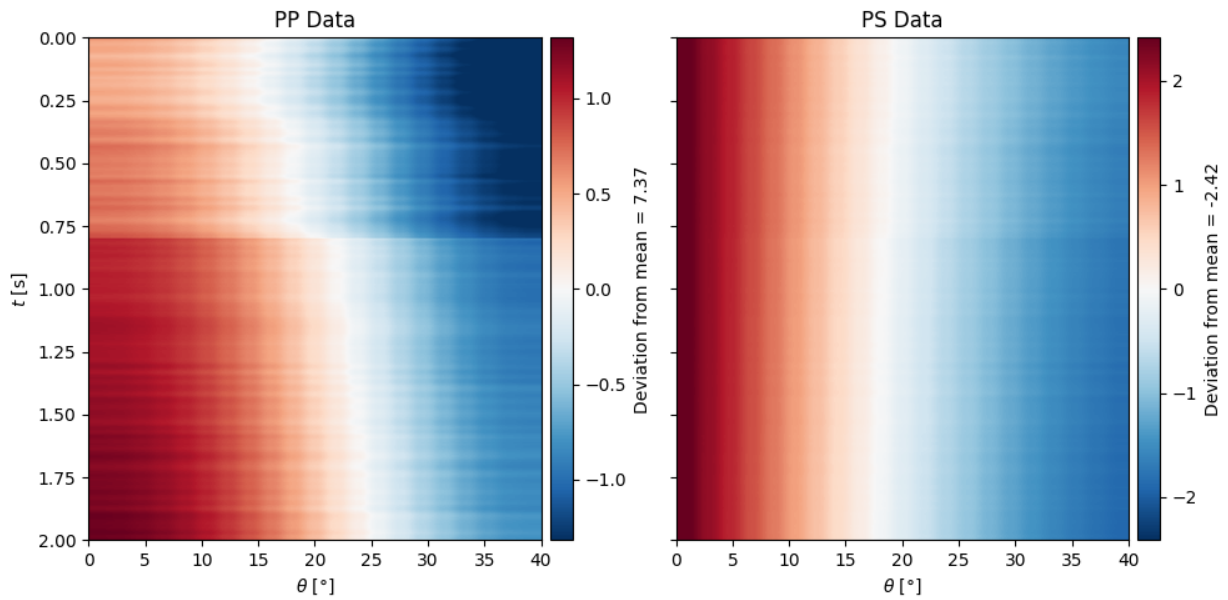
```
    theta, vsvp=vsvp, nt0=nt0, linearization="ps", dtype=np.float64
)
```

We can then apply those operators to the elastic model and create some synthetic reflection responses

```
dPS = PSop * m
mean_dPS = dPS.mean()
dPS -= mean_dPS

fig, axs = plt.subplots(1, 2, figsize=(10, 5), sharey=True)
im = axs[0].imshow(
    dPP_const,
    cmap="RdBu_r",
    extent=(theta[0], theta[-1], t0[-1], t0[0]),
    vmin=-dPP_const.max(),
    vmax=dPP_const.max(),
)
cax = make_axes_locatable(axs[0]).append_axes("right", size="5%", pad="2%")
cb = fig.colorbar(im, cax=cax)
cb.set_label(f"Deviation from mean = {mean_dPP_const:.2f}")
axs[0].set(xlabel=r"$\theta$ [°]", ylabel=r"$t$ [s]", title="PP Data")
axs[0].axis("tight")

im = axs[1].imshow(
    dPS,
    cmap="RdBu_r",
    extent=(theta[0], theta[-1], t0[-1], t0[0]),
    vmin=-dPS.max(),
    vmax=dPS.max(),
)
cax = make_axes_locatable(axs[1]).append_axes("right", size="5%", pad="2%")
cb = fig.colorbar(im, cax=cax)
cb.set_label(f"Deviation from mean = {mean_dPS:.2f}")
axs[1].set(xlabel=r"$\theta$ [°]", title="PS Data")
axs[1].axis("tight")
plt.tight_layout()
```



Total running time of the script: (0 minutes 1.095 seconds)

3.5.5 Bilinear Interpolation

This example shows how to use the `pylops.signalprocessing.Bilinar` operator to perform bilinear interpolation to a 2-dimensional input vector.

```
import matplotlib.pyplot as plt
import numpy as np
from scipy import misc

import pylops

plt.close("all")
np.random.seed(0)
```

First of all, we create a 2-dimensional input vector containing an image from the `scipy.misc` family.

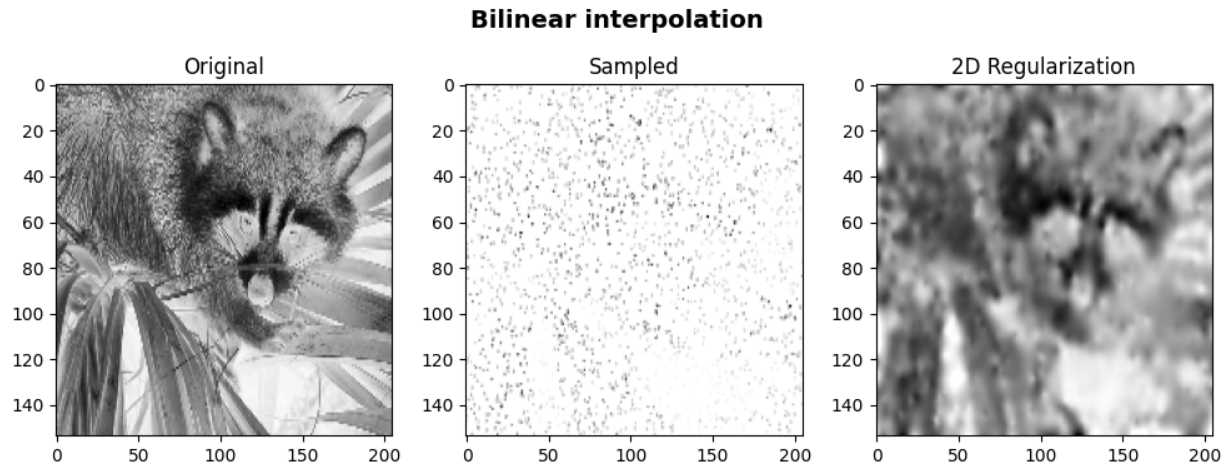
```
x = misc.face()[::5, ::5, 0]
nz, nx = x.shape
```

We can now define a set of available samples in the first and second direction of the array and apply bilinear interpolation.

```
nsamples = 2000
iava = np.vstack(
    (np.random.uniform(0, nz - 1, nsamples), np.random.uniform(0, nx - 1, nsamples))
)

Bop = pylops.signalprocessing.Bilinear(iava, (nz, nx))
y = Bop * x
```

At this point we try to reconstruct the input signal imposing a smooth solution by means of a regularization term that minimizes the Laplacian of the solution.

```
D2op = pylops.Laplacian((nz, nx), weights=(1, 1), dtype="float64")

xadj = Bop.H * y
xinv = pylops.optimization.leastsquares.normal_equations_inversion(
    Bop, y, [D2op], epsRs=[np.sqrt(0.1)], **dict(maxiter=100)
)[0]
xadj = xadj.reshape(nz, nx)
xinv = xinv.reshape(nz, nx)

fig, axs = plt.subplots(1, 3, figsize=(10, 4))
fig.suptitle("Bilinear interpolation", fontsize=14, fontweight="bold", y=0.95)
axs[0].imshow(x, cmap="gray_r", vmin=0, vmax=250)
axs[0].axis("tight")
axs[0].set_title("Original")
axs[1].imshow(xadj, cmap="gray_r", vmin=0, vmax=250)
axs[1].axis("tight")
axs[1].set_title("Sampled")
axs[2].imshow(xinv, cmap="gray_r", vmin=0, vmax=250)
axs[2].axis("tight")
axs[2].set_title("2D Regularization")
plt.tight_layout()
plt.subplots_adjust(top=0.8)
```



Total running time of the script: (0 minutes 1.372 seconds)

3.5.6 CGLS and LSQR Solvers

This example shows how to use the `pylops.optimization.leastsquares.cgls` and `pylops.optimization.leastsquares.lsqr` PyLops solvers to minimize the following cost function:

$$J = \|\mathbf{y} - \mathbf{A}\mathbf{x}\|_2^2 + \epsilon\|\mathbf{x}\|_2^2$$

Note that the LSQR solver behaves in the same way as the scipy's `scipy.sparse.linalg.lsqr` solver. However, our solver is also able to operate on cupy arrays and perform computations on a GPU.

```
import warnings

import matplotlib.pyplot as plt
import numpy as np

import pylops

plt.close("all")
warnings.filterwarnings("ignore")
```

Let's define a matrix $\mathbf{A}$ or size (N and M) and fill the matrix with random numbers

```
N, M = 20, 10
A = np.random.normal(0, 1, (N, M))
Aop = pylops.MatrixMult(A, dtype="float64")

x = np.ones(M)
```

We can now use the cgls solver to invert this matrix

```
y = Aop * x
xest, istop, nit, r1norm, r2norm, cost_cgls = pylops.optimization.basic.cgls(
    Aop, y, x0=np.zeros_like(x), niter=10, tol=1e-10, show=True
)

print(f"x= {x}")
print(f"cgls solution xest= {xest}")
```

```
CGLS
---------------------------------------------------------------------
The Operator Op has 20 rows and 10 cols
damp = 0.000000e+00     tol = 1.000000e-10      niter = 10
---------------------------------------------------------------------

    Itn          x[0]              r1norm         r2norm
     1        9.1362e-01         3.5210e+00     3.5210e+00
     2        1.1328e+00         1.9174e+00     1.9174e+00
     3        1.1030e+00         7.9210e-01     7.9210e-01
     4        1.0366e+00         3.9919e-01     3.9919e-01
     5        1.0086e+00         1.4627e-01     1.4627e-01
     6        1.0069e+00         8.0987e-02     8.0987e-02
     7        9.9981e-01         3.8979e-02     3.8979e-02
     8        9.9936e-01         1.9302e-02     1.9302e-02
     9        1.0006e+00         3.0820e-03     3.0820e-03
    10        1.0000e+00         3.6146e-15     3.6146e-15

Iterations = 10        Total time (s) = 0.00
---------------------------------------------------------------------

x= [1. 1. 1. 1. 1. 1. 1. 1. 1. 1.]
cgls solution xest= [1. 1. 1. 1. 1. 1. 1. 1. 1. 1.]
```

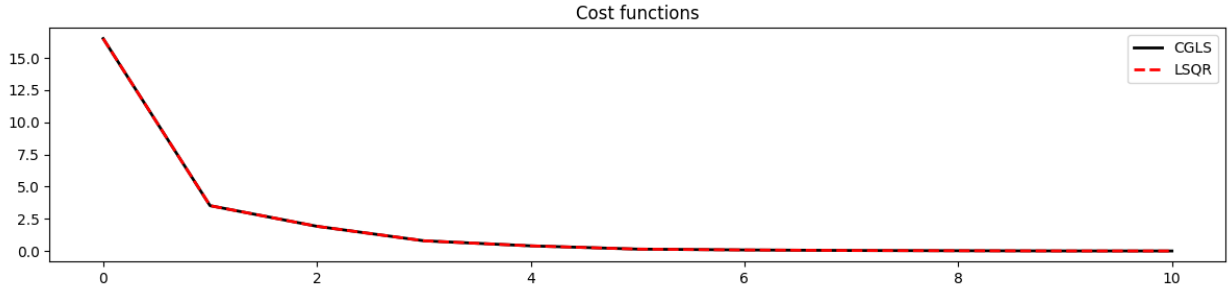
And the lsqr solver to invert this matrix

```
y = Aop * x
(
    xest,
    istop,
    itn,
    r1norm,
    r2norm,
    anorm,
    acond,
    arnorm,
    xnorm,
    var,
    cost_lsqr,
) = pylops.optimization.basic.lsqr(Aop, y, x0=np.zeros_like(x), niter=10, show=True)

print(f"x= {x}")
print(f"lsqr solution xest= {xest}")
```

```
LSQR
------------------------------------------------------------------------------------------
↪-
The Operator Op has 20 rows and 10 cols
damp = 0.00000000000000e+00     calc_var =      1
atol = 1.00e-08                 conlim = 1.00e+08
btol = 1.00e-08                 niter =       10
------------------------------------------------------------------------------------------
↪-
   Itn     x[0]      r1norm     r2norm   Compatible   LS     Norm A   Cond A
     0  0.0000e+00  1.650e+01  1.650e+01   1.0e+00  3.4e-01
     1  9.1362e-01  3.521e+00  3.521e+00   2.1e-01  1.4e-01  5.7e+00  1.0e+00
     2  1.1328e+00  1.917e+00  1.917e+00   1.2e-01  8.8e-02  7.3e+00  2.1e+00
     3  1.1030e+00  7.921e-01  7.921e-01   4.8e-02  3.9e-02  9.0e+00  3.5e+00
     4  1.0366e+00  3.992e-01  3.992e-01   2.4e-02  1.3e-02  1.1e+01  4.7e+00
     5  1.0086e+00  1.463e-01  1.463e-01   8.9e-03  5.1e-03  1.1e+01  6.2e+00
     6  1.0069e+00  8.099e-02  8.099e-02   4.9e-03  3.3e-03  1.2e+01  7.4e+00
     7  9.9981e-01  3.898e-02  3.898e-02   2.4e-03  1.5e-03  1.3e+01  8.8e+00
     8  9.9936e-01  1.930e-02  1.930e-02   1.2e-03  7.2e-04  1.4e+01  1.0e+01
     9  1.0006e+00  3.082e-03  3.082e-03   1.9e-04  8.3e-05  1.4e+01  1.2e+01
    10  1.0000e+00  4.480e-15  4.480e-15   2.7e-16  3.1e-16  1.4e+01  1.3e+01

LSQR finished, Op x - b is small enough, given atol, btol

istop =       1   r1norm = 4.5e-15   anorm = 1.4e+01   arnorm = 2.8e-14
itn   =      10   r2norm = 4.5e-15   acond = 1.3e+01   xnorm  = 3.2e+00
Total time (s) = 0.00
------------------------------------------------------------------------------------------
↪-

x= [1. 1. 1. 1. 1. 1. 1. 1. 1. 1.]
lsqr solution xest= [1. 1. 1. 1. 1. 1. 1. 1. 1. 1.]
```

Finally we show that the L2 norm of the residual of the two solvers decays in the same way, as LSQR is algebrically equivalent to CG on the normal equations and CGLS

```
plt.figure(figsize=(12, 3))
plt.plot(cost_cgls, "k", lw=2, label="CGLS")
plt.plot(cost_lsqr, "--r", lw=2, label="LSQR")
plt.title("Cost functions")
plt.legend()
plt.tight_layout()
```



Note that while we used a dense matrix here, any other linear operator can be fed to cgls and lsqr as is the case for any other PyLops solver.

Total running time of the script: (0 minutes 0.185 seconds)

3.5.7 Causal Integration

This example shows how to use the *pylops.CausalIntegration* operator to integrate an input signal (in forward mode) and to apply a smooth, regularized derivative (in inverse mode). This is a very interesting by-product of this operator which may result very useful when the data to which you want to apply a numerical derivative is noisy.

```
import matplotlib.pyplot as plt
import numpy as np

import pylops

plt.close("all")
```

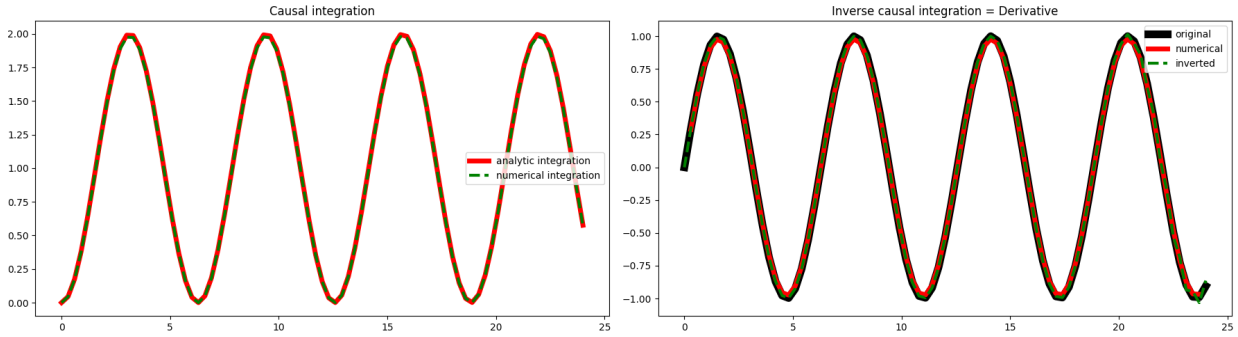
Let's start with a 1D example. Define the input parameters: number of samples of input signal (`nt`), sampling step (`dt`) as well as the input signal which will be equal to $x(t) = \sin(t)$:

```
nt = 81
dt = 0.3
t = np.arange(nt) * dt
x = np.sin(t)
```

We can now create our causal integration operator and apply it to the input signal. We can also compute the analytical integral $y(t) = \int \sin(t)\,\mathrm{d}t = -\cos(t)$ and compare the results. We can also invert the integration operator and by remembering that this is equivalent to a first order derivative, we will compare our inverted model with the result obtained by simply applying the *pylops.FirstDerivative* forward operator to the same data.

Note that, as explained in details in *pylops.CausalIntegration*, integration has no unique solution, as any constant c can be added to the integrated signal y, for example if $x(t) = t^2$ the $y(t) = \int t^2\,\mathrm{d}t = \frac{t^3}{3} + c$. We thus subtract first sample from the analytical integral to obtain the same result as the numerical one.

```
Cop = pylops.CausalIntegration(nt, sampling=dt, kind="half")

yana = -np.cos(t) + np.cos(t[0])
y = Cop * x
xinv = Cop / y

# Numerical derivative
Dop = pylops.FirstDerivative(nt, sampling=dt)
xder = Dop * y

# Visualize data and inversion
fig, axs = plt.subplots(1, 2, figsize=(18, 5))
axs[0].plot(t, yana, "r", lw=5, label="analytic integration")
axs[0].plot(t, y, "--g", lw=3, label="numerical integration")
axs[0].legend()
axs[0].set_title("Causal integration")

axs[1].plot(t, x, "k", lw=8, label="original")
axs[1].plot(t[1:-1], xder[1:-1], "r", lw=5, label="numerical")
axs[1].plot(t, xinv, "--g", lw=3, label="inverted")
axs[1].legend()
axs[1].set_title("Inverse causal integration = Derivative")
plt.tight_layout()
```



As expected we obtain the same result. Let's see what happens if we now add some random noise to our data.

```
# Add noise
yn = y + np.random.normal(0, 4e-1, y.shape)

# Numerical derivative
Dop = pylops.FirstDerivative(nt, sampling=dt)
xder = Dop * yn

# Regularized derivative
Rop = pylops.SecondDerivative(nt)
xreg = pylops.optimization.leastsquares.regularized_inversion(
    Cop, yn, [Rop], epsRs=[1e0], **dict(iter_lim=100, atol=1e-5)
)[0]

# Preconditioned derivative
Sop = pylops.Smoothing1D(41, nt)
```

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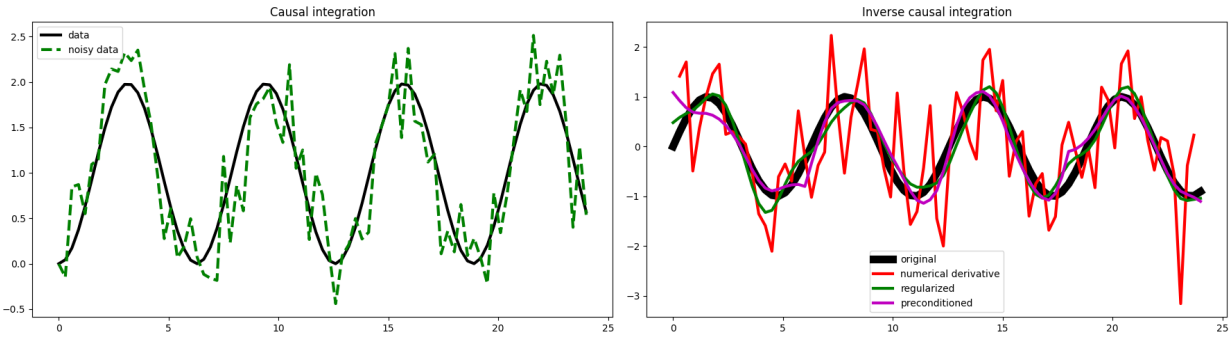
```
xp = pylops.optimization.leastsquares.preconditioned_inversion(
    Cop, yn, Sop, **dict(iter_lim=10, atol=1e-3)
)[0]

# Visualize data and inversion
fig, axs = plt.subplots(1, 2, figsize=(18, 5))
axs[0].plot(t, y, "k", lw=3, label="data")
axs[0].plot(t, yn, "--g", lw=3, label="noisy data")
axs[0].legend()
axs[0].set_title("Causal integration")
axs[1].plot(t, x, "k", lw=8, label="original")
axs[1].plot(t[1:-1], xder[1:-1], "r", lw=3, label="numerical derivative")
axs[1].plot(t, xreg, "g", lw=3, label="regularized")
axs[1].plot(t, xp, "m", lw=3, label="preconditioned")
axs[1].legend()
axs[1].set_title("Inverse causal integration")
plt.tight_layout()
```



We can see here the great advantage of framing our numerical derivative as an inverse problem, and more specifically as the inverse of the causal integration operator.

Let's conclude with a 2d example where again the integration/derivative will be performed along the first axis

```
nt, nx = 41, 11
dt = 0.3
ot = 0
t = np.arange(nt) * dt + ot
x = np.outer(np.sin(t), np.ones(nx))

Cop = pylops.CausalIntegration(dims=(nt, nx), sampling=dt, axis=0, kind="half")

y = Cop * x
yn = y + np.random.normal(0, 4e-1, y.shape)

# Numerical derivative
Dop = pylops.FirstDerivative(dims=(nt, nx), axis=0, sampling=dt)
xder = Dop * yn

# Regularized derivative
Rop = pylops.Laplacian(dims=(nt, nx))
xreg = pylops.optimization.leastsquares.regularized_inversion(
```

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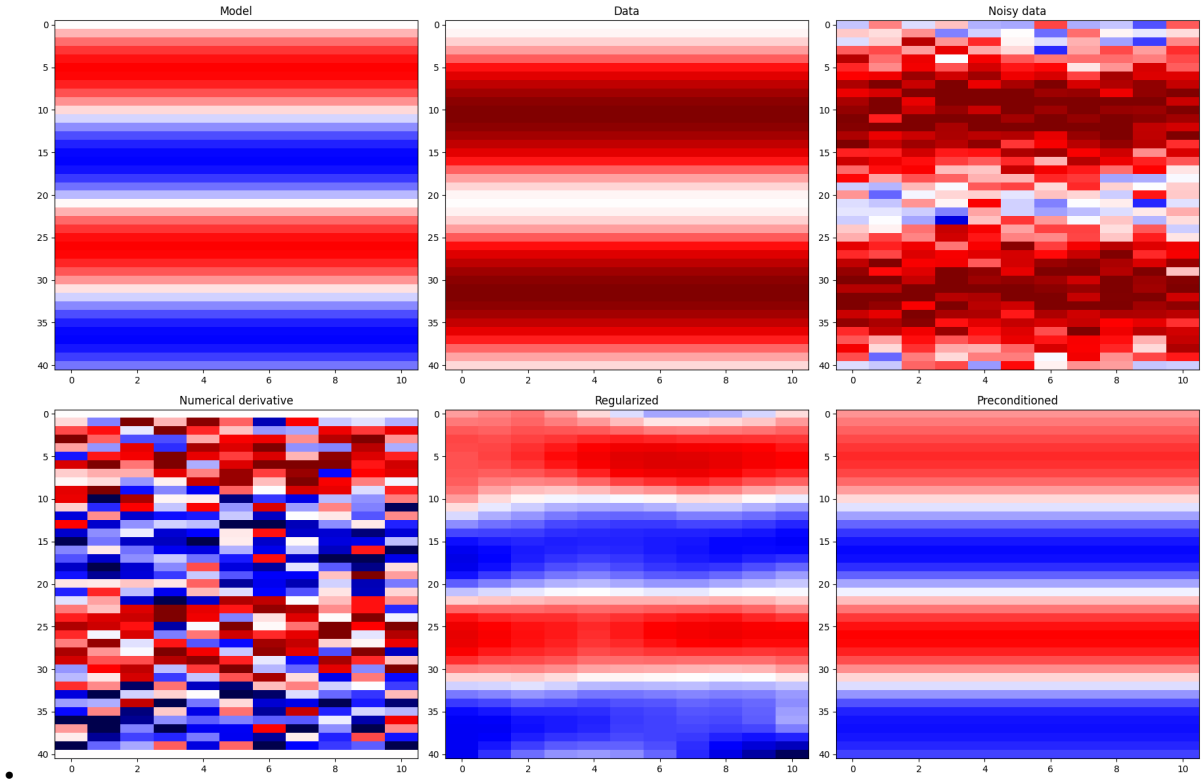
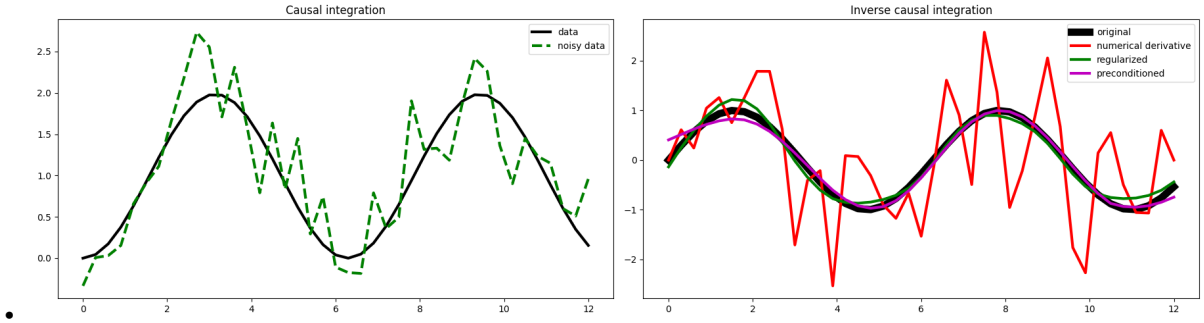
```
    Cop, yn.ravel(), [Rop], epsRs=[1e0], **dict(iter_lim=100, atol=1e-5)
)[0]
xreg = xreg.reshape(nt, nx)

# Preconditioned derivative
Sop = pylops.Smoothing2D((11, 21), dims=(nt, nx))
xp = pylops.optimization.leastsquares.preconditioned_inversion(
    Cop, yn.ravel(), Sop, **dict(iter_lim=10, atol=1e-2)
)[0]
xp = xp.reshape(nt, nx)

# Visualize data and inversion
vmax = 2 * np.max(np.abs(x))
fig, axs = plt.subplots(2, 3, figsize=(18, 12))
axs[0][0].imshow(x, cmap="seismic", vmin=-vmax, vmax=vmax)
axs[0][0].set_title("Model")
axs[0][0].axis("tight")
axs[0][1].imshow(y, cmap="seismic", vmin=-vmax, vmax=vmax)
axs[0][1].set_title("Data")
axs[0][1].axis("tight")
axs[0][2].imshow(yn, cmap="seismic", vmin=-vmax, vmax=vmax)
axs[0][2].set_title("Noisy data")
axs[0][2].axis("tight")
axs[1][0].imshow(xder, cmap="seismic", vmin=-vmax, vmax=vmax)
axs[1][0].set_title("Numerical derivative")
axs[1][0].axis("tight")
axs[1][1].imshow(xreg, cmap="seismic", vmin=-vmax, vmax=vmax)
axs[1][1].set_title("Regularized")
axs[1][1].axis("tight")
axs[1][2].imshow(xp, cmap="seismic", vmin=-vmax, vmax=vmax)
axs[1][2].set_title("Preconditioned")
axs[1][2].axis("tight")
plt.tight_layout()

# Visualize data and inversion at a chosen xlocation
fig, axs = plt.subplots(1, 2, figsize=(18, 5))
axs[0].plot(t, y[:, nx // 2], "k", lw=3, label="data")
axs[0].plot(t, yn[:, nx // 2], "--g", lw=3, label="noisy data")
axs[0].legend()
axs[0].set_title("Causal integration")
axs[1].plot(t, x[:, nx // 2], "k", lw=8, label="original")
axs[1].plot(t, xder[:, nx // 2], "r", lw=3, label="numerical derivative")
axs[1].plot(t, xreg[:, nx // 2], "g", lw=3, label="regularized")
axs[1].plot(t, xp[:, nx // 2], "m", lw=3, label="preconditioned")
axs[1].legend()
axs[1].set_title("Inverse causal integration")
plt.tight_layout()
```





Total running time of the script: (0 minutes 1.737 seconds)

3.5.8 Chirp Radon Transform

This example shows how to use the *pylops.signalprocessing.ChirpRadon2D* and *pylops.signalprocessing.ChirpRadon3D* operators to apply the linear Radon Transform to 2-dimensional or 3-dimensional signals, respectively.

When working with the linear Radon transform, this is a faster implementation compared to in *pylops.signalprocessing.Radon2D* and *pylops.signalprocessing.Radon3D* and should be preferred. This method provides also an analytical inverse.

Note that the forward and adjoint definitions in these two pairs of operators are swapped.

```
import matplotlib.pyplot as plt
import numpy as np
```

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```
import pylops

plt.close("all")
```

Let's start by creating a empty 2d matrix of size $n_x \times n_t$ with a single linear event.

```
par = {
    "ot": 0,
    "dt": 0.004,
    "nt": 51,
    "ox": -250,
    "dx": 10,
    "nx": 51,
    "oy": -250,
    "dy": 10,
    "ny": 51,
    "f0": 40,
}
theta = [0.0]
t0 = [0.1]
amp = [1.0]

# Create axes
t, t2, x, y = pylops.utils.seismicevents.makeaxis(par)
dt, dx, dy = par["dt"], par["dx"], par["dy"]

# Create wavelet
wav, _, wav_c = pylops.utils.wavelets.ricker(t[:41], f0=par["f0"])

# Generate data
_, d = pylops.utils.seismicevents.linear2d(x, t, 1500.0, t0, theta, amp, wav)
```

We can now define our operators and apply the forward, adjoint and inverse steps.

```
npx, pxmax = par["nx"], 5e-4
px = np.linspace(-pxmax, pxmax, npx)

R2Op = pylops.signalprocessing.ChirpRadon2D(t, x, pxmax * dx / dt, dtype="float64")
dL_chirp = R2Op * d
dadj_chirp = R2Op.H * dL_chirp
dinv_chirp = R2Op.inverse(dL_chirp).reshape(R2Op.dimsd)

fig, axs = plt.subplots(1, 4, figsize=(12, 4), sharey=True)
axs[0].imshow(d.T, vmin=-1, vmax=1, cmap="bwr_r", extent=(x[0], x[-1], t[-1], t[0]))
axs[0].set(xlabel=r"$x$ [m]", ylabel=r"$t$ [s]", title="Input model")
axs[0].axis("tight")
axs[1].imshow(
    dL_chirp.T,
    cmap="bwr_r",
    vmin=-dL_chirp.max(),
    vmax=dL_chirp.max(),
    extent=(1e3 * px[0], 1e3 * px[-1], t[-1], t[0]),
```

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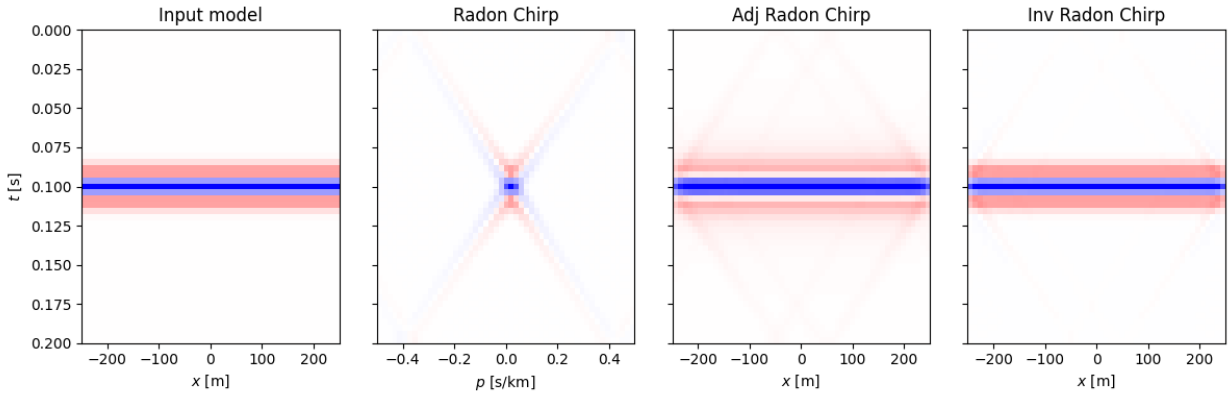
```
)
axs[1].set(xlabel=r"$p$ [s/km]", title="Radon Chirp")
axs[1].axis("tight")
axs[2].imshow(
    dadj_chirp.T,
    cmap="bwr_r",
    vmin=-dadj_chirp.max(),
    vmax=dadj_chirp.max(),
    extent=(x[0], x[-1], t[-1], t[0]),
)
axs[2].set(xlabel=r"$x$ [m]", title="Adj Radon Chirp")
axs[2].axis("tight")
axs[3].imshow(
    dinv_chirp.T,
    cmap="bwr_r",
    vmin=-d.max(),
    vmax=d.max(),
    extent=(x[0], x[-1], t[-1], t[0]),
)
axs[3].set(xlabel=r"$x$ [m]", title="Inv Radon Chirp")
axs[3].axis("tight")
plt.tight_layout()
```



Finally we repeat the same exercise with 3d data.

```
par = {
    "ot": 0,
    "dt": 0.004,
    "nt": 51,
    "ox": -400,
    "dx": 10,
    "nx": 81,
    "oy": -600,
    "dy": 10,
    "ny": 61,
    "f0": 20,
}
theta = [10]
```

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```
phi = [0]
t0 = [0.1]
amp = [1.0]

# Create axes
t, t2, x, y = pylops.utils.seismicevents.makeaxis(par)
dt, dx, dy = par["dt"], par["dx"], par["dy"]

# Generate data
_, d = pylops.utils.seismicevents.linear3d(x, y, t, 1500.0, t0, theta, phi, amp, wav)

npy, pymax = par["ny"], 3e-4
npx, pxmax = par["nx"], 5e-4
py = np.linspace(-pymax, pymax, npy)
px = np.linspace(-pxmax, pxmax, npx)

R3Op = pylops.signalprocessing.ChirpRadon3D(
    t, y, x, (pymax * dy / dt, pxmax * dx / dt), dtype="float64"
)
dL_chirp = R3Op * d
dadj_chirp = R3Op.H * dL_chirp
dinv_chirp = R3Op.inverse(dL_chirp).reshape(R3Op.dimsd)

fig, axs = plt.subplots(1, 4, figsize=(12, 4), sharey=True)
axs[0].imshow(
    d[par["ny"] // 2].T,
    vmin=-1,
    vmax=1,
    cmap="bwr_r",
    extent=(x[0], x[-1], t[-1], t[0]),
)
axs[0].set(xlabel=r"$x$ [m]", ylabel=r"$t$ [s]", title="Input model")
axs[0].axis("tight")
axs[1].imshow(
    dL_chirp[par["ny"] // 2].T,
    cmap="bwr_r",
    vmin=-dL_chirp.max(),
    vmax=dL_chirp.max(),
    extent=(1e3 * px[0], 1e3 * px[-1], t[-1], t[0]),
)
axs[1].set(xlabel=r"$p_x$ [s/km]", title="Radon Chirp")
axs[1].axis("tight")
axs[2].imshow(
    dadj_chirp[par["ny"] // 2].T,
    cmap="bwr_r",
    vmin=-dadj_chirp.max(),
    vmax=dadj_chirp.max(),
    extent=(x[0], x[-1], t[-1], t[0]),
)
axs[2].set(xlabel=r"$x$ [m]", title="Adj Radon Chirp")
axs[2].axis("tight")
axs[3].imshow(
```

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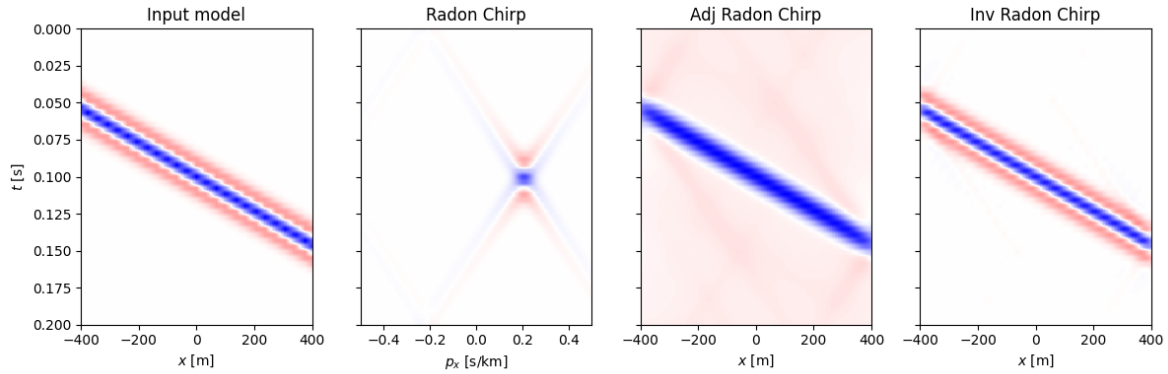
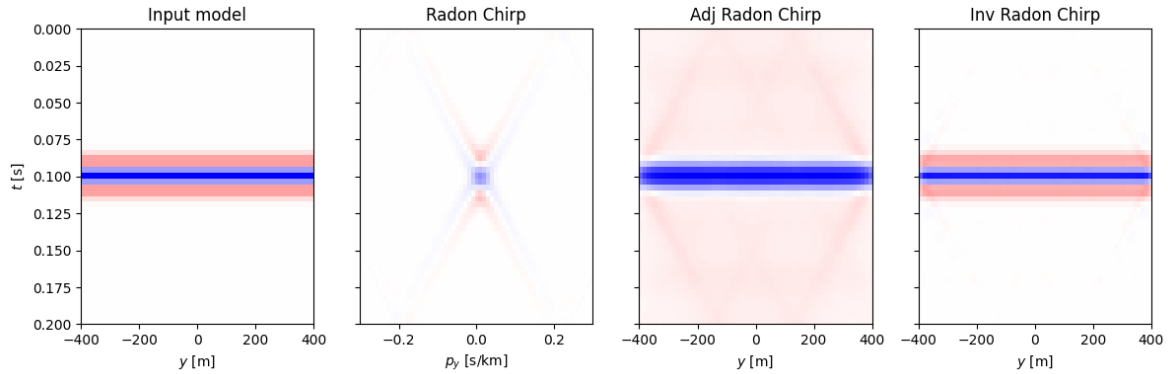
```
    dinv_chirp[par["ny"] // 2].T,
    cmap="bwr_r",
    vmin=-d.max(),
    vmax=d.max(),
    extent=(x[0], x[-1], t[-1], t[0]),
)
axs[3].set(xlabel=r"$x$ [m]", title="Inv Radon Chirp")
axs[3].axis("tight")
plt.tight_layout()

fig, axs = plt.subplots(1, 4, figsize=(12, 4), sharey=True)
axs[0].imshow(
    d[:, par["nx"] // 2].T,
    vmin=-1,
    vmax=1,
    cmap="bwr_r",
    extent=(x[0], x[-1], t[-1], t[0]),
)
axs[0].set(xlabel=r"$y$ [m]", ylabel=r"$t$ [s]", title="Input model")
axs[0].axis("tight")
axs[1].imshow(
    dL_chirp[:, 2 * par["nx"] // 3].T,
    cmap="bwr_r",
    vmin=-dL_chirp.max(),
    vmax=dL_chirp.max(),
    extent=(1e3 * py[0], 1e3 * py[-1], t[-1], t[0]),
)
axs[1].set(xlabel=r"$p_y$ [s/km]", title="Radon Chirp")
axs[1].axis("tight")
axs[2].imshow(
    dadj_chirp[:, par["nx"] // 2].T,
    cmap="bwr_r",
    vmin=-dadj_chirp.max(),
    vmax=dadj_chirp.max(),
    extent=(x[0], x[-1], t[-1], t[0]),
)
axs[2].set(xlabel=r"$y$ [m]", title="Adj Radon Chirp")
axs[2].axis("tight")
axs[3].imshow(
    dinv_chirp[:, par["nx"] // 2].T,
    cmap="bwr_r",
    vmin=-d.max(),
    vmax=d.max(),
    extent=(x[0], x[-1], t[-1], t[0]),
)
axs[3].set(xlabel=r"$y$ [m]", title="Inv Radon Chirp")
axs[3].axis("tight")
plt.tight_layout()
```

- 

- 

Total running time of the script: (0 minutes 1.630 seconds)

3.5.9 Conj

This example shows how to use the `pylops.basicoperators.Conj` operator. This operator returns the complex conjugate in both forward and adjoint modes (it is self adjoint).

```
import matplotlib.pyplot as plt
import numpy as np

import pylops

plt.close("all")
```

Let's define a Conj operator to get the complex conjugate of the input.

```
M = 5
x = np.arange(M) + 1j * np.arange(M)[::-1]
Rop = pylops.basicoperators.Conj(M, dtype="complex128")

y = Rop * x
xadj = Rop.H * y

_, axs = plt.subplots(1, 3, figsize=(10, 4))
axs[0].plot(np.real(x), lw=2, label="Real")
axs[0].plot(np.imag(x), lw=2, label="Imag")
axs[0].legend()
```

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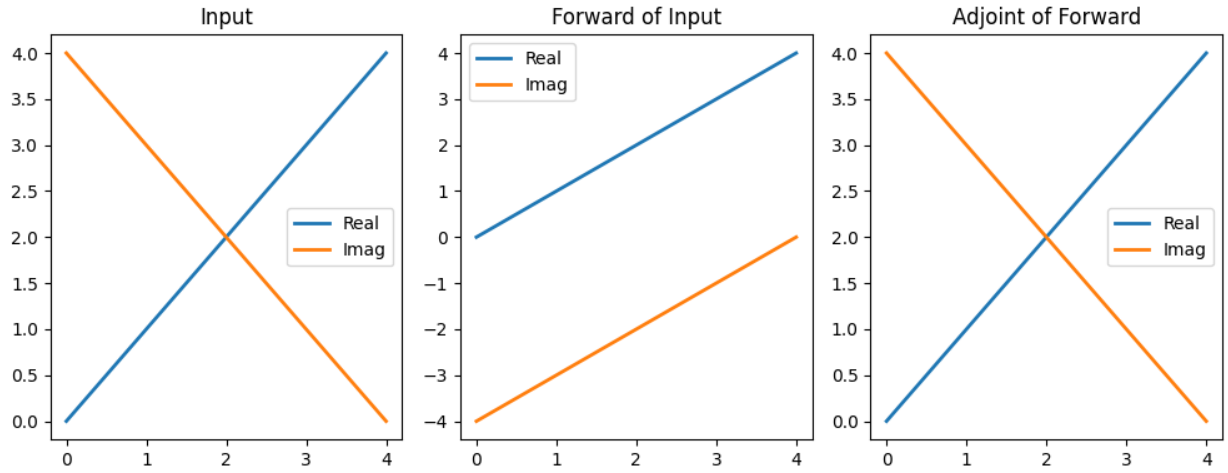
```
axs[0].set_title("Input")
axs[1].plot(np.real(y), lw=2, label="Real")
axs[1].plot(np.imag(y), lw=2, label="Imag")
axs[1].legend()
axs[1].set_title("Forward of Input")
axs[2].plot(np.real(xadj), lw=2, label="Real")
axs[2].plot(np.imag(xadj), lw=2, label="Imag")
axs[2].legend()
axs[2].set_title("Adjoint of Forward")
plt.tight_layout()
```



Total running time of the script: (0 minutes 0.364 seconds)

3.5.10 Convolution

This example shows how to use the *pylops.signalprocessing.Convolve1D*, *pylops.signalprocessing.Convolve2D* and *pylops.signalprocessing.ConvolveND* operators to perform convolution between two signals.

Such operators can be used in the forward model of several common application in signal processing that require filtering of an input signal for the instrument response. Similarly, removing the effect of the instrument response from signal is equivalent to solving linear system of equations based on Convolve1D, Convolve2D or ConvolveND operators. This problem is generally referred to as *Deconvolution*.

A very practical example of deconvolution can be found in the geophysical processing of seismic data where the effect of the source response (i.e., airgun or vibroseis) should be removed from the recorded signal to be able to better interpret the response of the subsurface. Similar examples can be found in telecommunication and speech analysis.

```
import matplotlib.pyplot as plt
import numpy as np
from scipy.sparse.linalg import lsqr

import pylops
from pylops.utils.wavelets import ricker

plt.close("all")
```

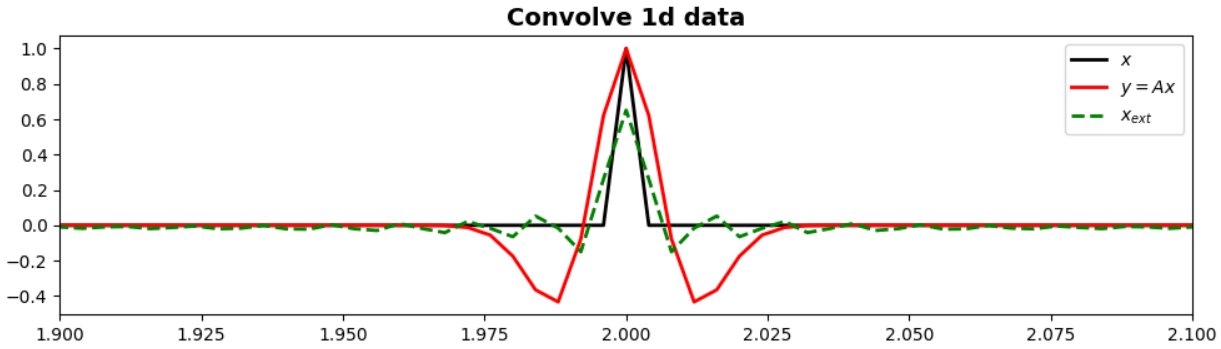
We will start by creating a zero signal of lenght nt and we will place a unitary spike at its center. We also create our filter to be applied by means of *pylops.signalprocessing.Convolve1D* operator. Following the seismic example mentioned above, the filter is a Ricker wavelet with dominant frequency $f_0 = 30Hz$.

```
nt = 1001
dt = 0.004
t = np.arange(nt) * dt
x = np.zeros(nt)
x[int(nt / 2)] = 1
h, th, hcenter = ricker(t[:101], f0=30)

Cop = pylops.signalprocessing.Convolve1D(nt, h=h, offset=hcenter, dtype="float32")
y = Cop * x

xinv = Cop / y

fig, ax = plt.subplots(1, 1, figsize=(10, 3))
ax.plot(t, x, "k", lw=2, label=r"$x$")
ax.plot(t, y, "r", lw=2, label=r"$y=Ax$")
ax.plot(t, xinv, "--g", lw=2, label=r"$x_{ext}$")
ax.set_title("Convolve 1d data", fontsize=14, fontweight="bold")
ax.legend()
ax.set_xlim(1.9, 2.1)
plt.tight_layout()
```



We show now that also a filter with mixed phase (i.e., not centered around zero) can be applied and inverted for using the *pylops.signalprocessing.Convolve1D* operator.

```
Cop = pylops.signalprocessing.Convolve1D(nt, h=h, offset=hcenter - 3, dtype="float32")
y = Cop * x
y1 = Cop.H * x
xinv = Cop / y

fig, ax = plt.subplots(1, 1, figsize=(10, 3))
ax.plot(t, x, "k", lw=2, label=r"$x$")
ax.plot(t, y, "r", lw=2, label=r"$y=Ax$")
ax.plot(t, y1, "b", lw=2, label=r"$y=A^Hx$")
ax.plot(t, xinv, "--g", lw=2, label=r"$x_{ext}$")
ax.set_title(
    "Convolve 1d data with non-zero phase filter", fontsize=14, fontweight="bold"
)
```

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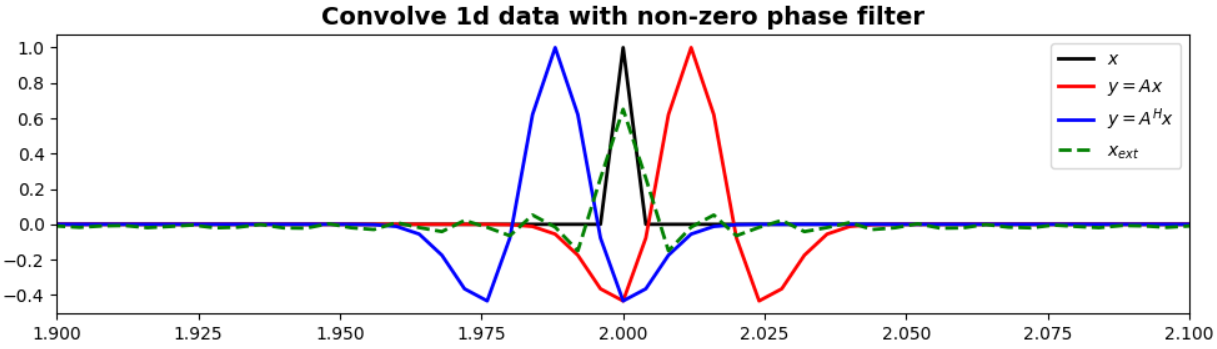
```
ax.set_xlim(1.9, 2.1)
ax.legend()
plt.tight_layout()
```



We repeat a similar exercise but using two dimensional signals and filters taking advantage of the *pylops.signalprocessing.Convolve2D* operator.

```
nt = 51
nx = 81
dt = 0.004
t = np.arange(nt) * dt
x = np.zeros((nt, nx))
x[int(nt / 2), int(nx / 2)] = 1

nh = [11, 5]
h = np.ones((nh[0], nh[1]))

Cop = pylops.signalprocessing.Convolve2D(
    (nt, nx),
    h=h,
    offset=(int(nh[0]) / 2, int(nh[1]) / 2),
    dtype="float32",
)
y = Cop * x
xinv = (Cop / y.ravel()).reshape(Cop.dims)

fig, axs = plt.subplots(1, 3, figsize=(10, 3))
fig.suptitle("Convolve 2d data", fontsize=14, fontweight="bold", y=0.95)
axs[0].imshow(x, cmap="gray", vmin=-1, vmax=1)
axs[1].imshow(y, cmap="gray", vmin=-1, vmax=1)
axs[2].imshow(xinv, cmap="gray", vmin=-1, vmax=1)
axs[0].set_title("x")
axs[0].axis("tight")
axs[1].set_title("y")
axs[1].axis("tight")
axs[2].set_title("xlsqr")
axs[2].axis("tight")
plt.tight_layout()
plt.subplots_adjust(top=0.8)
```

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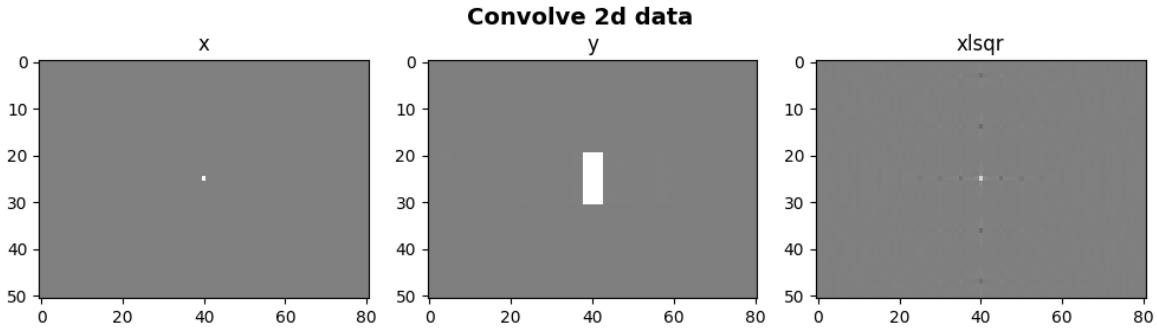
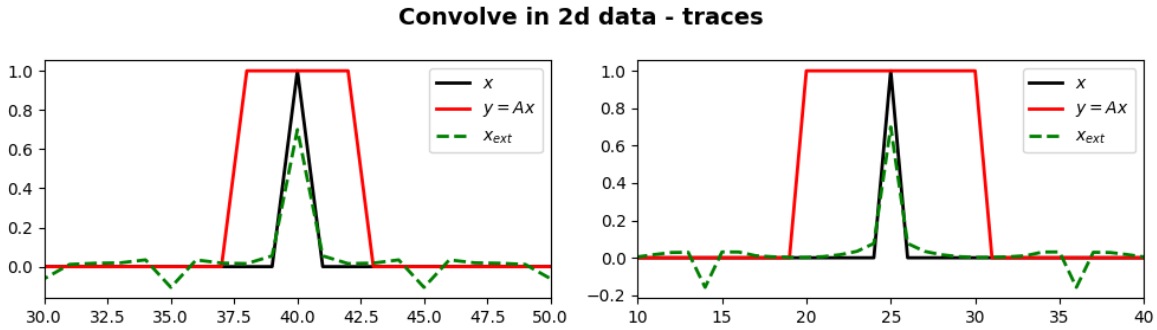
```
fig, ax = plt.subplots(1, 2, figsize=(10, 3))
fig.suptitle("Convolve in 2d data - traces", fontsize=14, fontweight="bold", y=0.95)
ax[0].plot(x[int(nt / 2), :], "k", lw=2, label=r"$x$")
ax[0].plot(y[int(nt / 2), :], "r", lw=2, label=r"$y=Ax$")
ax[0].plot(xinv[int(nt / 2), :], "--g", lw=2, label=r"$x_{ext}$")
ax[1].plot(x[:, int(nx / 2)], "k", lw=2, label=r"$x$")
ax[1].plot(y[:, int(nx / 2)], "r", lw=2, label=r"$y=Ax$")
ax[1].plot(xinv[:, int(nx / 2)], "--g", lw=2, label=r"$x_{ext}$")
ax[0].legend()
ax[0].set_xlim(30, 50)
ax[1].legend()
ax[1].set_xlim(10, 40)
plt.tight_layout()
plt.subplots_adjust(top=0.8)
```

- 

- 

Finally we do the same using three dimensional signals and filters taking advantage of the *pylops.signalprocessing.ConvolveND* operator.

```
ny, nx, nz = 13, 10, 7
x = np.zeros((ny, nx, nz))
x[ny // 3, nx // 2, nz // 4] = 1
h = np.ones((3, 5, 3))
offset = [1, 2, 1]

Cop = pylops.signalprocessing.ConvolveND(
    dims=(ny, nx, nz), h=h, offset=offset, axes=(0, 1, 2), dtype="float32"
)
y = Cop * x
xlsqr = lsqr(Cop, y.ravel(), damp=0, iter_lim=300, show=0)[0]
```

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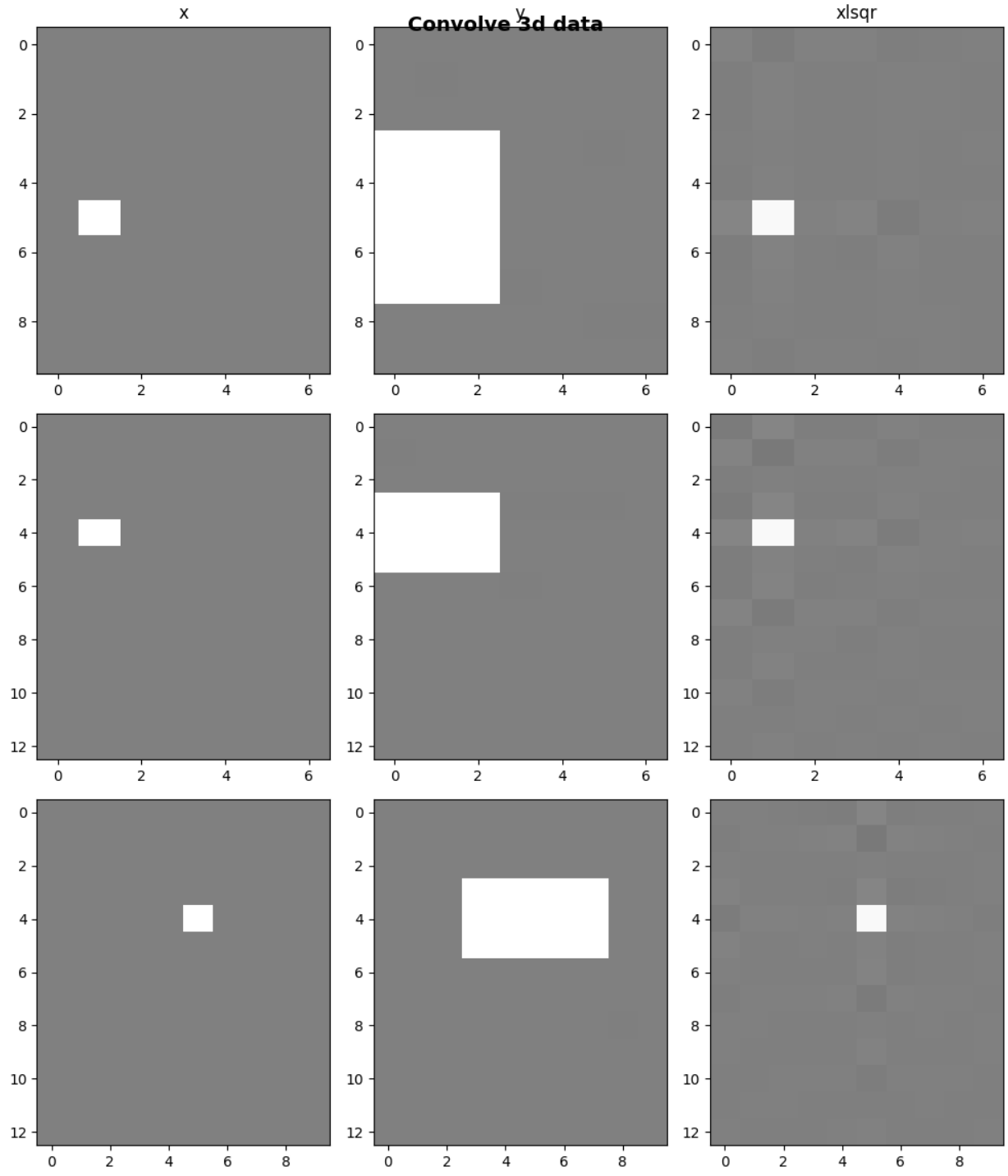
```
xlsqr = xlsqr.reshape(Cop.dims)

fig, axs = plt.subplots(3, 3, figsize=(10, 12))
fig.suptitle("Convolve 3d data", y=0.95, fontsize=14, fontweight="bold")
axs[0][0].imshow(x[ny // 3], cmap="gray", vmin=-1, vmax=1)
axs[0][1].imshow(y[ny // 3], cmap="gray", vmin=-1, vmax=1)
axs[0][2].imshow(xlsqr[ny // 3], cmap="gray", vmin=-1, vmax=1)
axs[0][0].set_title("x")
axs[0][0].axis("tight")
axs[0][1].set_title("y")
axs[0][1].axis("tight")
axs[0][2].set_title("xlsqr")
axs[0][2].axis("tight")
axs[1][0].imshow(x[:, nx // 2], cmap="gray", vmin=-1, vmax=1)
axs[1][1].imshow(y[:, nx // 2], cmap="gray", vmin=-1, vmax=1)
axs[1][2].imshow(xlsqr[:, nx // 2], cmap="gray", vmin=-1, vmax=1)
axs[1][0].axis("tight")
axs[1][1].axis("tight")
axs[1][2].axis("tight")
axs[2][0].imshow(x[..., nz // 4], cmap="gray", vmin=-1, vmax=1)
axs[2][1].imshow(y[..., nz // 4], cmap="gray", vmin=-1, vmax=1)
axs[2][2].imshow(xlsqr[..., nz // 4], cmap="gray", vmin=-1, vmax=1)
axs[2][0].axis("tight")
axs[2][1].axis("tight")
axs[2][2].axis("tight")
plt.tight_layout()
```



Total running time of the script: (0 minutes 2.024 seconds)

3.5.11 Derivatives

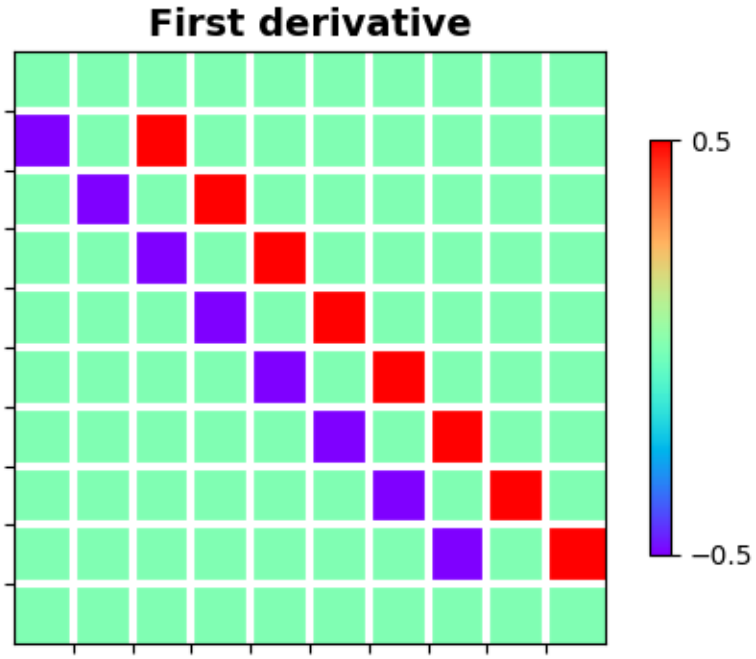
This example shows how to use the suite of derivative operators, namely *pylops.FirstDerivative*, *pylops.SecondDerivative*, *pylops.Laplacian* and *pylops.Gradient*, *pylops.FirstDirectionalDerivative* and *pylops.SecondDirectionalDerivative*.

The derivative operators are very useful when the model to be inverted for is expect to be smooth in one or more directions. As shown in the *Optimization* tutorial, these operators will be used as part of the regularization term to obtain a smooth solution.

```
import matplotlib.pyplot as plt
import numpy as np

import pylops

plt.close("all")
np.random.seed(0)
```

Let's start by looking at a simple first-order centered derivative and how could implement it naively by creating a dense matrix. Note that we will not apply the derivative where the stencil is partially outside of the range of the input signal (i.e., at the edge of the signal)

```
nx = 10

D = np.diag(0.5 * np.ones(nx - 1), k=1) - np.diag(0.5 * np.ones(nx - 1), -1)
D[0] = D[-1] = 0

fig, ax = plt.subplots(1, 1, figsize=(6, 4))
im = plt.imshow(D, cmap="rainbow", vmin=-0.5, vmax=0.5)
ax.set_title("First derivative", size=14, fontweight="bold")
ax.set_xticks(np.arange(nx - 1) + 0.5)
ax.set_yticks(np.arange(nx - 1) + 0.5)
ax.grid(linewidth=3, color="white")
ax.xaxis.set_ticklabels([])
ax.yaxis.set_ticklabels([])
fig.colorbar(im, ax=ax, ticks=[-0.5, 0.5], shrink=0.7)
```




```
<matplotlib.colorbar.Colorbar object at 0x7f6840c540d0>
```

We now create a signal filled with zero and a single one at its center and apply the derivative matrix by means of a dot product

```
x = np.zeros(nx)
x[int(nx / 2)] = 1

y_dir = np.dot(D, x)
xadj_dir = np.dot(D.T, y_dir)
```

Let's now do the same using the *pylops.FirstDerivative* operator and compare its outputs after applying the forward and adjoint operators to those from the dense matrix.

```
D1op = pylops.FirstDerivative(nx, dtype="float32")

y_lop = D1op * x
xadj_lop = D1op.H * y_lop

fig, axs = plt.subplots(3, 1, figsize=(13, 8), sharex=True)
axs[0].stem(np.arange(nx), x, linefmt="k", markerfmt="ko")
axs[0].set_title("Input", size=20, fontweight="bold")
axs[1].stem(np.arange(nx), y_dir, linefmt="k", markerfmt="ko", label="direct")
axs[1].stem(np.arange(nx), y_lop, linefmt="--r", markerfmt="ro", label="lop")
axs[1].set_title("Forward", size=20, fontweight="bold")
axs[1].legend()
axs[2].stem(np.arange(nx), xadj_dir, linefmt="k", markerfmt="ko", label="direct")
axs[2].stem(np.arange(nx), xadj_lop, linefmt="--r", markerfmt="ro", label="lop")
axs[2].set_title("Adjoint", size=20, fontweight="bold")
```

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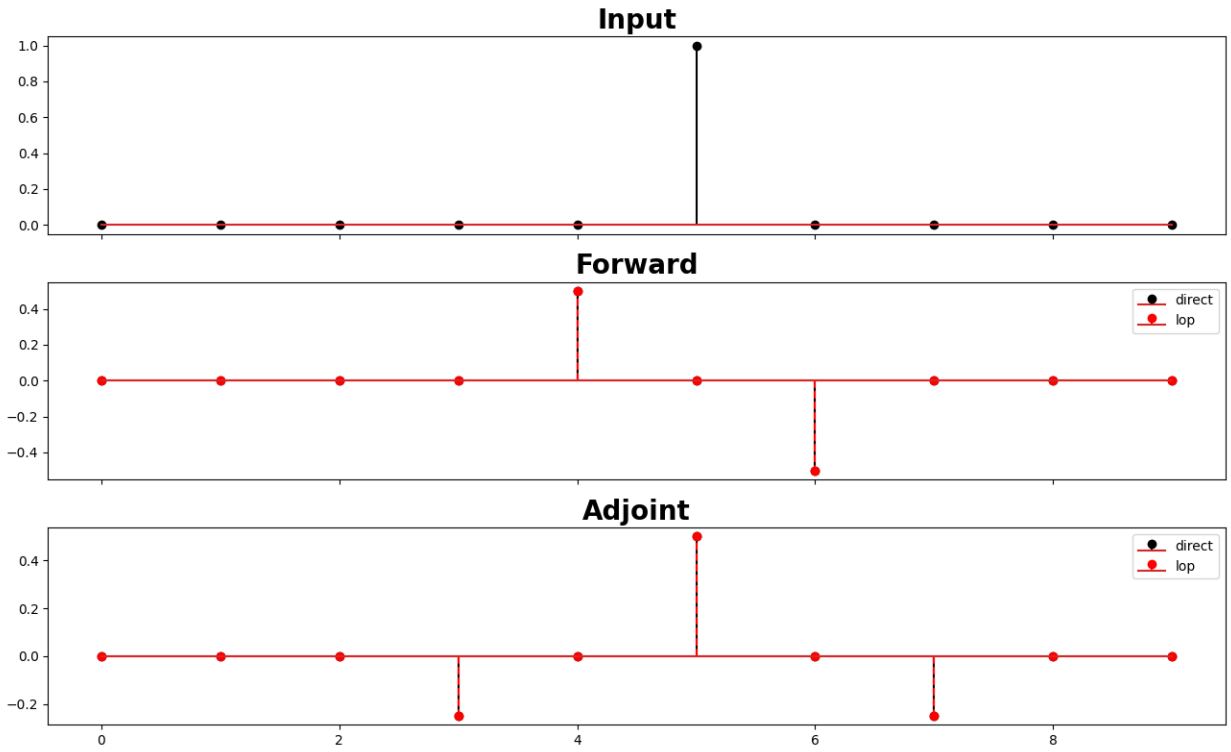
```
axs[2].legend()
plt.tight_layout()
```



As expected we obtain the same result, with the only difference that in the second case we did not need to explicitly create a matrix, saving memory and computational time.

Let's move onto applying the same first derivative to a 2d array in the first direction

```
nx, ny = 11, 21
A = np.zeros((nx, ny))
A[nx // 2, ny // 2] = 1.0

D1op = pylops.FirstDerivative((nx, ny), axis=0, dtype="float64")
B = D1op * A

fig, axs = plt.subplots(1, 2, figsize=(10, 3), sharey=True)
fig.suptitle(
    "First Derivative in 1st direction", fontsize=12, fontweight="bold", y=0.95
)
im = axs[0].imshow(A, interpolation="nearest", cmap="rainbow")
axs[0].axis("tight")
axs[0].set_title("x")
plt.colorbar(im, ax=axs[0])
im = axs[1].imshow(B, interpolation="nearest", cmap="rainbow")
axs[1].axis("tight")
axs[1].set_title("y")
plt.colorbar(im, ax=axs[1])
plt.tight_layout()
```

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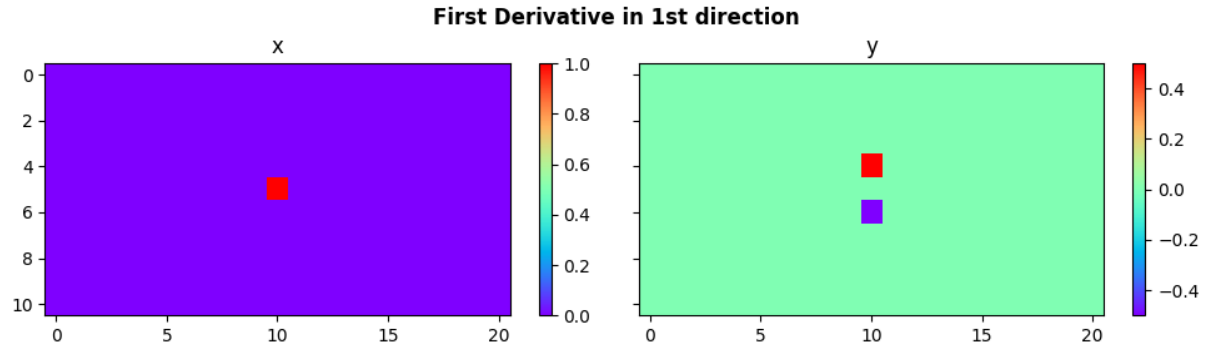
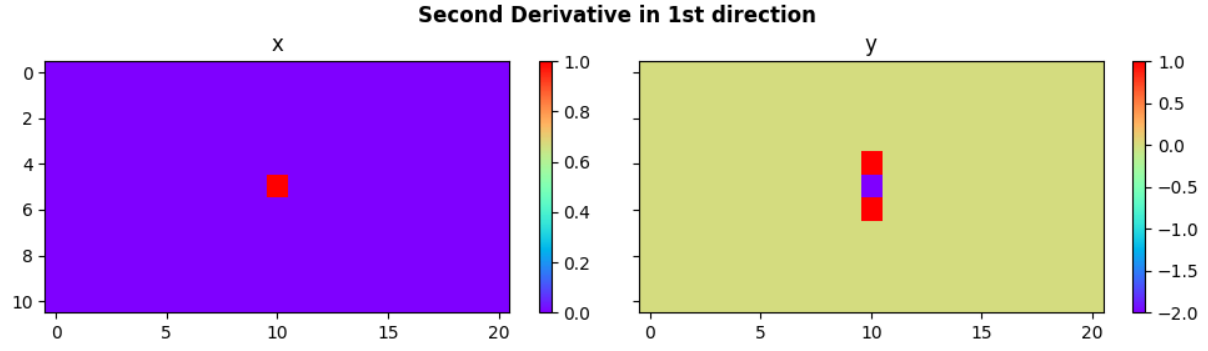
```
plt.subplots_adjust(top=0.8)
```



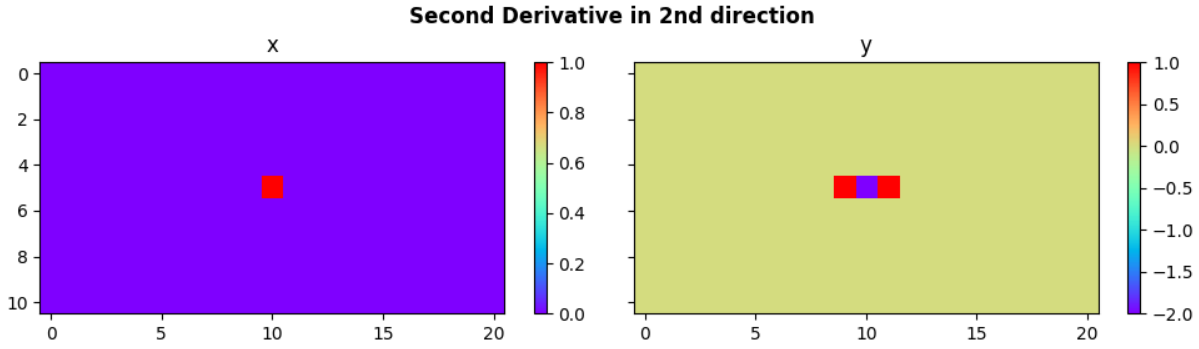
We can now do the same for the second derivative

```
A = np.zeros((nx, ny))
A[nx // 2, ny // 2] = 1.0

D2op = pylops.SecondDerivative(dims=(nx, ny), axis=0, dtype="float64")
B = D2op * A

fig, axs = plt.subplots(1, 2, figsize=(10, 3), sharey=True)
fig.suptitle(
    "Second Derivative in 1st direction", fontsize=12, fontweight="bold", y=0.95
)
im = axs[0].imshow(A, interpolation="nearest", cmap="rainbow")
axs[0].axis("tight")
axs[0].set_title("x")
plt.colorbar(im, ax=axs[0])
im = axs[1].imshow(B, interpolation="nearest", cmap="rainbow")
axs[1].axis("tight")
axs[1].set_title("y")
plt.colorbar(im, ax=axs[1])
plt.tight_layout()
plt.subplots_adjust(top=0.8)
```



We can also apply the second derivative to the second direction of our data (`axis=1`)

```
D2op = pylops.SecondDerivative(dims=(nx, ny), axis=1, dtype="float64")
B = D2op * A

fig, axs = plt.subplots(1, 2, figsize=(10, 3), sharey=True)
fig.suptitle(
    "Second Derivative in 2nd direction", fontsize=12, fontweight="bold", y=0.95
)
im = axs[0].imshow(A, interpolation="nearest", cmap="rainbow")
axs[0].axis("tight")
axs[0].set_title("x")
plt.colorbar(im, ax=axs[0])
im = axs[1].imshow(B, interpolation="nearest", cmap="rainbow")
axs[1].axis("tight")
axs[1].set_title("y")
plt.colorbar(im, ax=axs[1])
plt.tight_layout()
plt.subplots_adjust(top=0.8)
```



We use the symmetrical Laplacian operator as well as a asymmetrical version of it (by adding more weight to the derivative along one direction)

```
# symmetrical
L2symop = pylops.Laplacian(dims=(nx, ny), weights=(1, 1), dtype="float64")

# asymmetrical
L2asymop = pylops.Laplacian(dims=(nx, ny), weights=(3, 1), dtype="float64")

Bsym = L2symop * A
Basym = L2asymop * A

fig, axs = plt.subplots(1, 3, figsize=(10, 3), sharey=True)
fig.suptitle("Laplacian", fontsize=12, fontweight="bold", y=0.95)
im = axs[0].imshow(A, interpolation="nearest", cmap="rainbow")
axs[0].axis("tight")
axs[0].set_title("x")
plt.colorbar(im, ax=axs[0])
im = axs[1].imshow(Bsym, interpolation="nearest", cmap="rainbow")
axs[1].axis("tight")
axs[1].set_title("y sym")
plt.colorbar(im, ax=axs[1])
```

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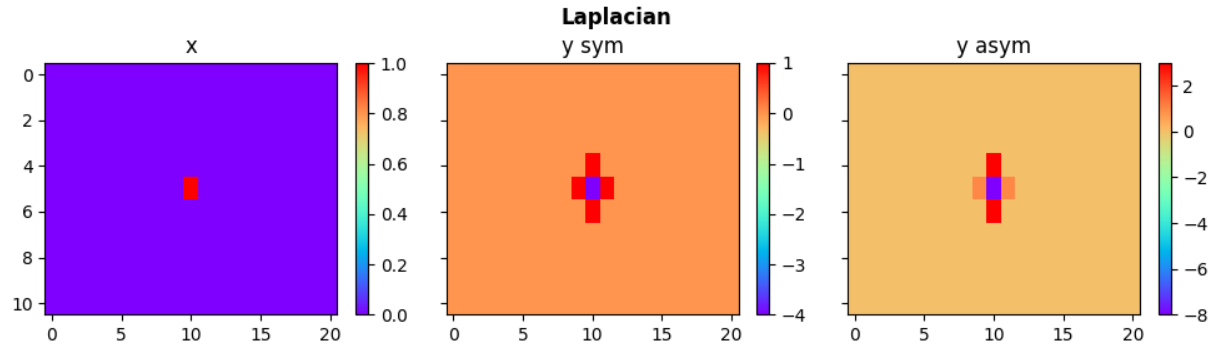
```
im = axs[2].imshow(Basym, interpolation="nearest", cmap="rainbow")
axs[2].axis("tight")
axs[2].set_title("y asym")
plt.colorbar(im, ax=axs[2])
plt.tight_layout()
plt.subplots_adjust(top=0.8)
```

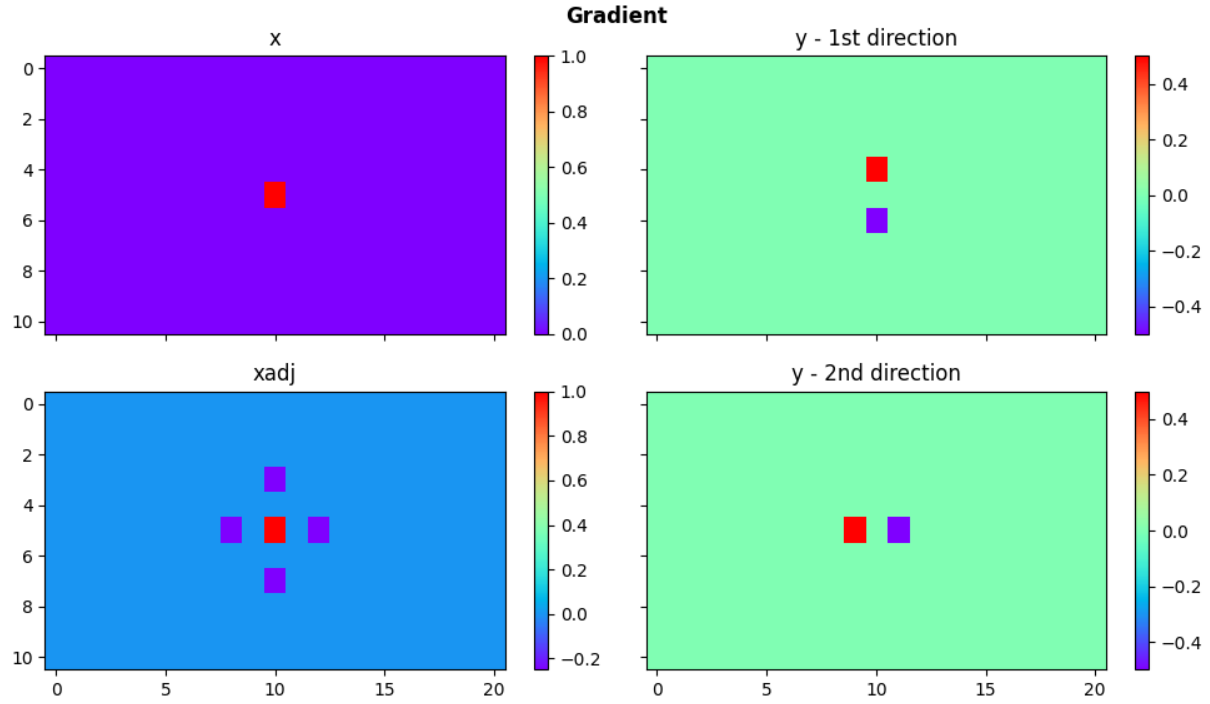


We consider now the gradient operator. Given a 2-dimensional array, this operator applies first-order derivatives on both dimensions and concatenates them.

```
Gop = pylops.Gradient(dims=(nx, ny), dtype="float64")

B = Gop * A
C = Gop.H * B

fig, axs = plt.subplots(2, 2, figsize=(10, 6), sharex=True, sharey=True)
fig.suptitle("Gradient", fontsize=12, fontweight="bold", y=0.95)
im = axs[0, 0].imshow(A, interpolation="nearest", cmap="rainbow")
axs[0, 0].axis("tight")
axs[0, 0].set_title("x")
plt.colorbar(im, ax=axs[0, 0])
im = axs[0, 1].imshow(B[0, ...], interpolation="nearest", cmap="rainbow")
axs[0, 1].axis("tight")
axs[0, 1].set_title("y - 1st direction")
plt.colorbar(im, ax=axs[0, 1])
im = axs[1, 1].imshow(B[1, ...], interpolation="nearest", cmap="rainbow")
axs[1, 1].axis("tight")
axs[1, 1].set_title("y - 2nd direction")
plt.colorbar(im, ax=axs[1, 1])
im = axs[1, 0].imshow(C, interpolation="nearest", cmap="rainbow")
axs[1, 0].axis("tight")
axs[1, 0].set_title("xadj")
plt.colorbar(im, ax=axs[1, 0])
plt.tight_layout()
```



Finally we use the Gradient operator to compute directional derivatives. We create a model which has some layering in the horizontal and vertical directions and show how the direction derivatives differs from standard derivatives

```
nx, nz = 60, 40

horlayers = np.cumsum(np.random.uniform(2, 10, 20).astype(int))
horlayers = horlayers[horlayers < nz // 2]
nhorlayers = len(horlayers)

vertlayers = np.cumsum(np.random.uniform(2, 20, 10).astype(int))
vertlayers = vertlayers[vertlayers < nx]
nvertlayers = len(vertlayers)

A = 1500 * np.ones((nz, nx))
for top, base in zip(horlayers[:-1], horlayers[1:]):
    A[top:base] = np.random.normal(2000, 200)
for top, base in zip(vertlayers[:-1], vertlayers[1:]):
    A[horlayers[-1] :, top:base] = np.random.normal(2000, 200)

v = np.zeros((2, nz, nx))
v[0, : horlayers[-1]] = 1
v[1, horlayers[-1] :] = 1

Ddop = pylops.FirstDirectionalDerivative((nz, nx), v=v, sampling=(nz, nx))
D2dop = pylops.SecondDirectionalDerivative((nz, nx), v=v, sampling=(nz, nx))

dirder = Ddop * A
dir2der = D2dop * A
```

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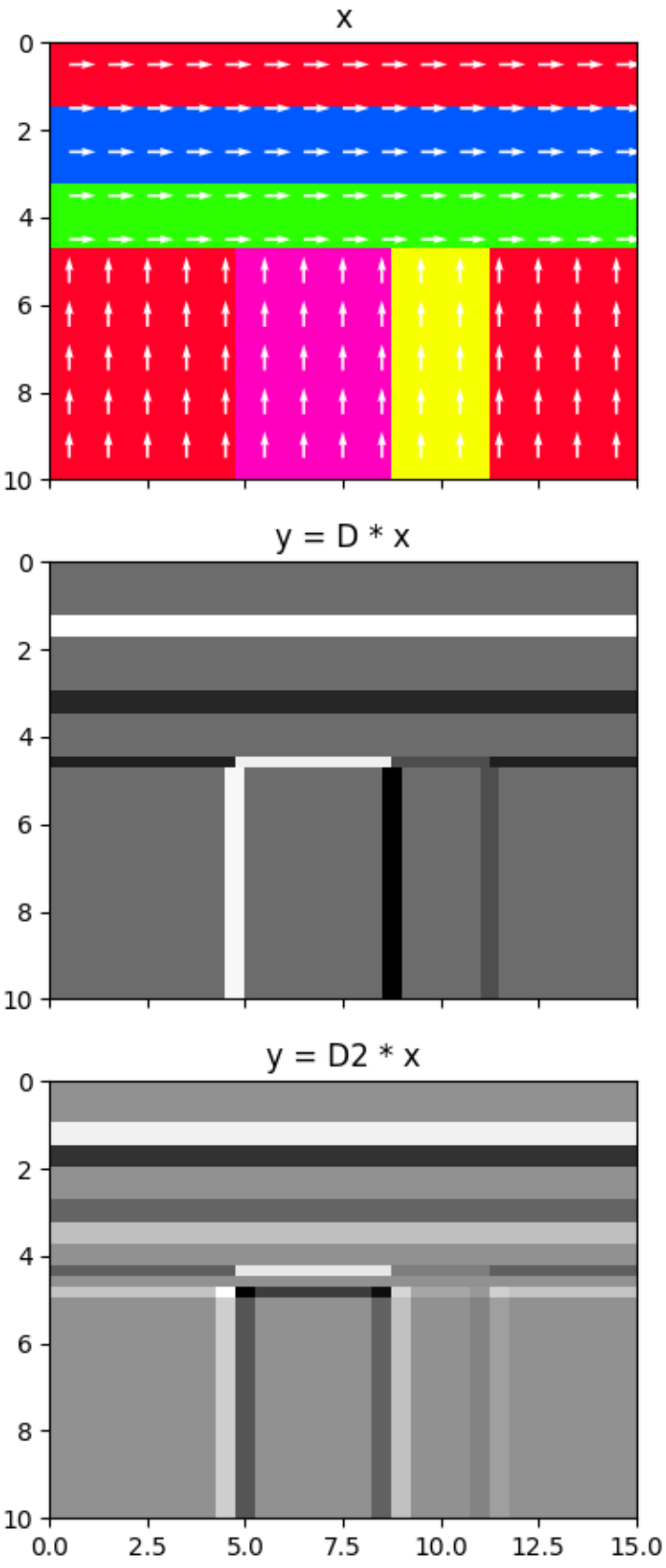
```
jump = 4
fig, axs = plt.subplots(3, 1, figsize=(4, 9), sharex=True)
im = axs[0].imshow(A, cmap="gist_rainbow", extent=(0, nx // jump, nz // jump, 0))
q = axs[0].quiver(
    np.arange(nx // jump) + 0.5,
    np.arange(nz // jump) + 0.5,
    np.flipud(v[1, ::jump, ::jump]),
    np.flipud(v[0, ::jump, ::jump]),
    color="w",
    linewidths=20,
)
axs[0].set_title("x")
axs[0].axis("tight")
axs[1].imshow(dirder, cmap="gray", extent=(0, nx // jump, nz // jump, 0))
axs[1].set_title("y = D * x")
axs[1].axis("tight")
axs[2].imshow(dir2der, cmap="gray", extent=(0, nx // jump, nz // jump, 0))
axs[2].set_title("y = D2 * x")
axs[2].axis("tight")
plt.tight_layout()
```


x
y = D * x
y = D2 * x
0
2
4
6
8
10
0.0
2.5
5.0
7.5
10.0
12.5
15.0

Total running time of the script: (0 minutes 2.968 seconds)

3.5.12 Describe

This example focuses on the usage of the *pylops.utils.describe.describe* method, which allows expressing any PyLops operator into its equivalent mathematical representation. This is done with the aid of sympy, a Python library for symbolic computing

```
import matplotlib.pyplot as plt
import numpy as np

import pylops
from pylops.utils.describe import describe

plt.close("all")
```

Let's start by defining 3 PyLops operators. Note that once an operator is defined we can attach a name to the operator; by doing so, this name will be used in the mathematical description of the operator. Alternatively, the describe method will randomly choose a name for us.

```
A = pylops.MatrixMult(np.ones((10, 5)))
A.name = "A"
B = pylops.Diagonal(np.ones(5))
B.name = "A"
C = pylops.MatrixMult(np.ones((10, 5)))

# Simple operator
describe(A)

# Transpose
AT = A.T
describe(AT)

# Adjoint
AH = A.H
describe(AH)

# Scaled
A3 = 3 * A
describe(A3)

# Sum
D = A + C
describe(D)
```

```
A
where: {'A': 'MatrixMult'}
A.T
where: {'A': 'MatrixMult'}
Adjoint(A)
where: {'A': 'MatrixMult'}
3*A
```

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```
where: {'A': 'MatrixMult'}
A + M
where: {'A': 'MatrixMult', 'M': 'MatrixMult'}
```

So far so good. Let's see what happens if we accidentally call two different operators with the same name. You will see that PyLops catches that and changes the name for us (and provides us with a nice warning!)

```
D = A * B
describe(D)
```

```
A*K
where: {'A': 'MatrixMult', 'K': 'Diagonal'}
```

We can move now to something more complicated using various composition operators

```
H = pylops.HStack((A * B, C * B))
describe(H)

H = pylops.Block([[A * B, C], [A, A]])
describe(H)
```

```
Matrix([[A*K, M*K]])
where: {'A': 'MatrixMult', 'K': 'Diagonal', 'M': 'MatrixMult'}
Matrix([
[Matrix([[A*K, M]])],
[  Matrix([[A, A]])]])
where: {'A': 'MatrixMult', 'K': 'Diagonal', 'M': 'MatrixMult'}
```

Finally, note that you can get the best out of the describe method if working inside a Jupyter notebook. There, the mathematical expression will be rendered using a LeTex format! See an example notebook.

Total running time of the script: (0 minutes 0.323 seconds)

3.5.13 Diagonal

This example shows how to use the *pylops.Diagonal* operator to perform *Element-wise multiplication* between the input vector and a vector $\mathbf{d}$.

In other words, the operator acts as a diagonal operator $\mathbf{D}$ whose elements along the diagonal are the elements of the vector $\mathbf{d}$.

```
import matplotlib.gridspec as pltgs
import matplotlib.pyplot as plt
import numpy as np

import pylops

plt.close("all")
```

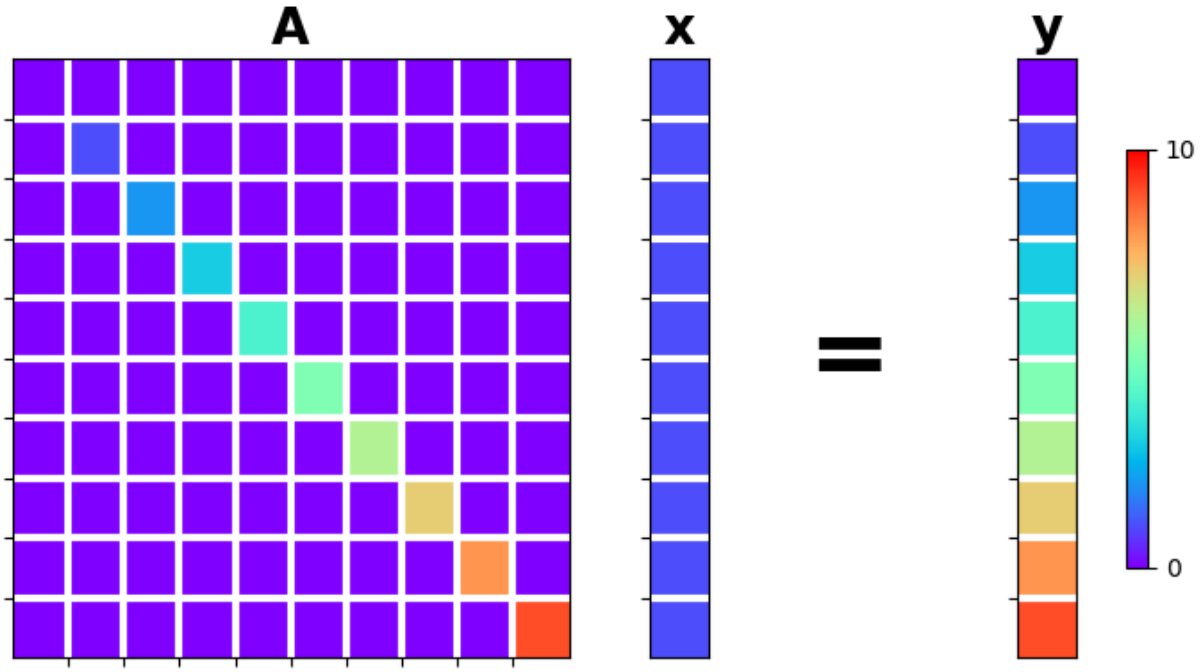
Let's define a diagonal operator $\mathbf{d}$ with increasing numbers from `0` to `N` and a unitary model $\mathbf{x}$.

```
N = 10
d = np.arange(N)
x = np.ones(N)

Dop = pylops.Diagonal(d)

y = Dop * x
y1 = Dop.H * x

gs = pltgs.GridSpec(1, 6)
fig = plt.figure(figsize=(7, 4))
ax = plt.subplot(gs[0, 0:3])
im = ax.imshow(Dop.matrix(), cmap="rainbow", vmin=0, vmax=N)
ax.set_title("A", size=20, fontweight="bold")
ax.set_xticks(np.arange(N - 1) + 0.5)
ax.set_yticks(np.arange(N - 1) + 0.5)
ax.grid(linewidth=3, color="white")
ax.xaxis.set_ticklabels([])
ax.yaxis.set_ticklabels([])
ax.axis("tight")
ax = plt.subplot(gs[0, 3])
ax.imshow(x[:, np.newaxis], cmap="rainbow", vmin=0, vmax=N)
ax.set_title("x", size=20, fontweight="bold")
ax.set_xticks([])
ax.set_yticks(np.arange(N - 1) + 0.5)
ax.grid(linewidth=3, color="white")
ax.xaxis.set_ticklabels([])
ax.yaxis.set_ticklabels([])
ax = plt.subplot(gs[0, 4])
ax.text(
    0.35,
    0.5,
    "=",
    horizontalalignment="center",
    verticalalignment="center",
    size=40,
    fontweight="bold",
)
ax.axis("off")
ax = plt.subplot(gs[0, 5])
ax.imshow(y[:, np.newaxis], cmap="rainbow", vmin=0, vmax=N)
ax.set_title("y", size=20, fontweight="bold")
ax.set_xticks([])
ax.set_yticks(np.arange(N - 1) + 0.5)
ax.grid(linewidth=3, color="white")
ax.xaxis.set_ticklabels([])
ax.yaxis.set_ticklabels([])
fig.colorbar(im, ax=ax, ticks=[0, N], pad=0.3, shrink=0.7)
plt.tight_layout()
```



Similarly we can consider the input model as composed of two or more dimensions. In this case the diagonal operator can be still applied to each element or broadcasted along a specific direction. Let's start with the simplest case where each element is multipled by a different value

```
nx, ny = 3, 5
x = np.ones((nx, ny))
print(f"x =\n{x}")

d = np.arange(nx * ny).reshape(nx, ny)
Dop = pylops.Diagonal(d)

y = Dop * x.ravel()
y1 = Dop.H * x.ravel()

print(f"y = D*x =\n{y.reshape(nx, ny)}")
print(f"xadj = D'*x =\n{y1.reshape(nx, ny)}")
```

```
x =
[[1. 1. 1. 1. 1.]
 [1. 1. 1. 1. 1.]
 [1. 1. 1. 1. 1.]]
y = D*x =
[[ 0.  1.  2.  3.  4.]
 [ 5.  6.  7.  8.  9.]
 [10. 11. 12. 13. 14.]]
xadj = D'*x =
[[ 0.  1.  2.  3.  4.]
 [ 5.  6.  7.  8.  9.]
 [10. 11. 12. 13. 14.]]
```

And we now broadcast

```
nx, ny = 3, 5
x = np.ones((nx, ny))
print(f"x =\n{x}")

# 1st dim
d = np.arange(nx)
Dop = pylops.Diagonal(d, dims=(nx, ny), axis=0)

y = Dop * x.ravel()
y1 = Dop.H * x.ravel()

print(f"1st dim: y = D*x =\n{y.reshape(nx, ny)}")
print(f"1st dim: xadj = D'*x =\n{y1.reshape(nx, ny)}")

# 2nd dim
d = np.arange(ny)
Dop = pylops.Diagonal(d, dims=(nx, ny), axis=1)

y = Dop * x.ravel()
y1 = Dop.H * x.ravel()

print(f"2nd dim: y = D*x =\n{y.reshape(nx, ny)}")
print(f"2nd dim: xadj = D'*x =\n{y1.reshape(nx, ny)}")
```

```
x =
[[1. 1. 1. 1. 1.]
 [1. 1. 1. 1. 1.]
 [1. 1. 1. 1. 1.]]
1st dim: y = D*x =
[[0. 0. 0. 0. 0.]
 [1. 1. 1. 1. 1.]
 [2. 2. 2. 2. 2.]]
1st dim: xadj = D'*x =
[[0. 0. 0. 0. 0.]
 [1. 1. 1. 1. 1.]
 [2. 2. 2. 2. 2.]]
2nd dim: y = D*x =
[[0. 1. 2. 3. 4.]
 [0. 1. 2. 3. 4.]
 [0. 1. 2. 3. 4.]]
2nd dim: xadj = D'*x =
[[0. 1. 2. 3. 4.]
 [0. 1. 2. 3. 4.]
 [0. 1. 2. 3. 4.]]
```

Total running time of the script: (0 minutes 0.261 seconds)

3.5.14 Flip along an axis

This example shows how to use the *pylops.Flip* operator to simply flip an input signal along an axis.

```
import matplotlib.pyplot as plt
import numpy as np

import pylops

plt.close("all")
```

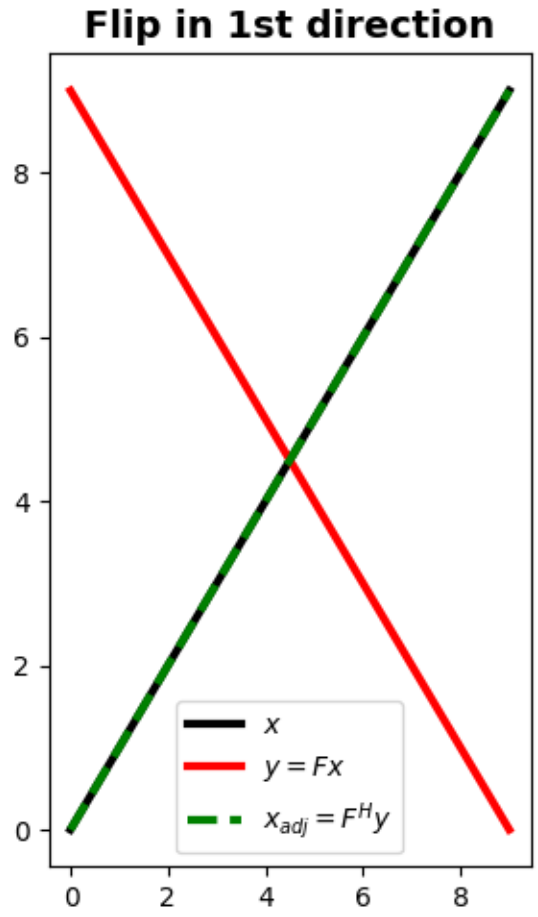
Let's start with a 1D example. Define an input signal composed of `nt` samples

```
nt = 10
x = np.arange(nt)
```

We can now create our flip operator and apply it to the input signal. We can also apply the adjoint to the flipped signal and we can see how for this operator the adjoint is effectively equivalent to the inverse.

```
Fop = pylops.Flip(nt)
y = Fop * x
xadj = Fop.H * y

plt.figure(figsize=(3, 5))
plt.plot(x, "k", lw=3, label=r"$x$")
plt.plot(y, "r", lw=3, label=r"$y=Fx$")
plt.plot(xadj, "--g", lw=3, label=r"$x_{adj} = F^H y$")
plt.title("Flip in 1st direction", fontsize=14, fontweight="bold")
plt.legend()
plt.tight_layout()
```



Let's now repeat the same exercise on a two dimensional signal. We will first flip the model along the first axis and then along the second axis

```
nt, nx = 10, 5
x = np.outer(np.arange(nt), np.ones(nx))
Fop = pylops.Flip((nt, nx), axis=0)
y = Fop * x
xadj = Fop.H * y

fig, axs = plt.subplots(1, 3, figsize=(7, 3))
fig.suptitle(
    "Flip in 1st direction for 2d data", fontsize=14, fontweight="bold", y=0.95
)
axs[0].imshow(x, cmap="rainbow")
axs[0].set_title(r"$x$")
axs[0].axis("tight")
axs[1].imshow(y, cmap="rainbow")
axs[1].set_title(r"$y = F x$")
axs[1].axis("tight")
axs[2].imshow(xadj, cmap="rainbow")
axs[2].set_title(r"$x_{adj} = F^H y$")
axs[2].axis("tight")
plt.tight_layout()
```

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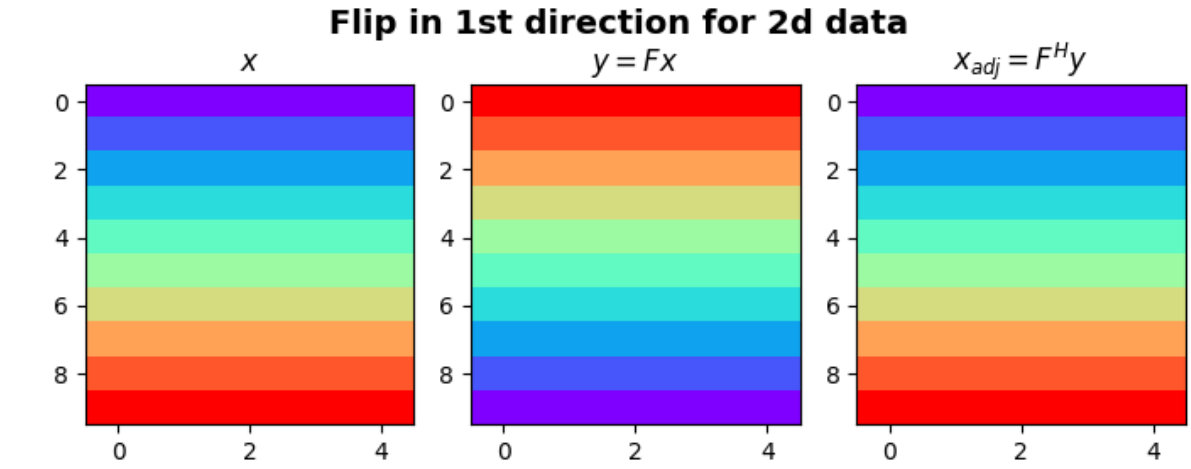
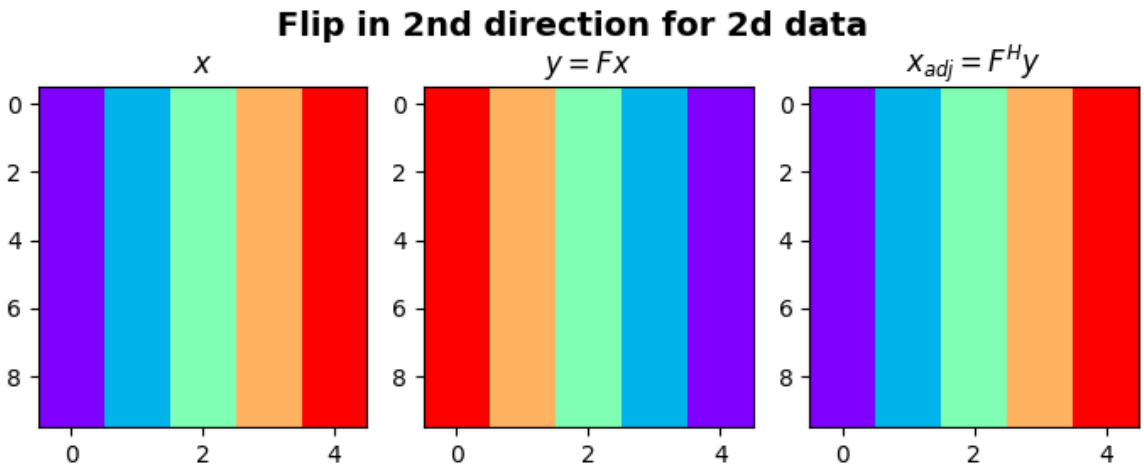
```
plt.subplots_adjust(top=0.8)


x = np.outer(np.ones(nt), np.arange(nx))
Fop = pylops.Flip(dims=(nt, nx), axis=1)
y = Fop * x
xadj = Fop.H * y

# sphinx_gallery_thumbnail_number = 3
fig, axs = plt.subplots(1, 3, figsize=(7, 3))
fig.suptitle(
    "Flip in 2nd direction for 2d data", fontsize=14, fontweight="bold", y=0.95
)
axs[0].imshow(x, cmap="rainbow")
axs[0].set_title(r"$x$")
axs[0].axis("tight")
axs[1].imshow(y, cmap="rainbow")
axs[1].set_title(r"$y = F x$")
axs[1].axis("tight")
axs[2].imshow(xadj, cmap="rainbow")
axs[2].set_title(r"$x_{adj} = F^H y$")
axs[2].axis("tight")
plt.tight_layout()
plt.subplots_adjust(top=0.8)
```





Total running time of the script: (0 minutes 0.729 seconds)

3.5.15 Fourier Transform

This example shows how to use the *pylops.signalprocessing.FFT*, *pylops.signalprocessing.FFT2D* and *pylops.signalprocessing.FFTND* operators to apply the Fourier Transform to the model and the inverse Fourier Transform to the data.

```
import matplotlib.pyplot as plt
import numpy as np

import pylops

plt.close("all")
```

Let's start by applying the one dimensional FFT to a one dimensional sinusoidal signal $d(t) = sin(2\pi f_0 t)$ using a time axis of lenght nt and sampling dt

```
dt = 0.005
nt = 100
t = np.arange(nt) * dt
f0 = 10
nfft = 2**10
d = np.sin(2 * np.pi * f0 * t)

FFTop = pylops.signalprocessing.FFT(dims=nt, nfft=nfft, sampling=dt, engine="numpy")
D = FFTop * d

# Adjoint = inverse for FFT
dinv = FFTop.H * D
dinv = FFTop / D

fig, axs = plt.subplots(1, 2, figsize=(10, 4))
axs[0].plot(t, d, "k", lw=2, label="True")
axs[0].plot(t, dinv.real, "--r", lw=2, label="Inverted")
axs[0].legend()
```

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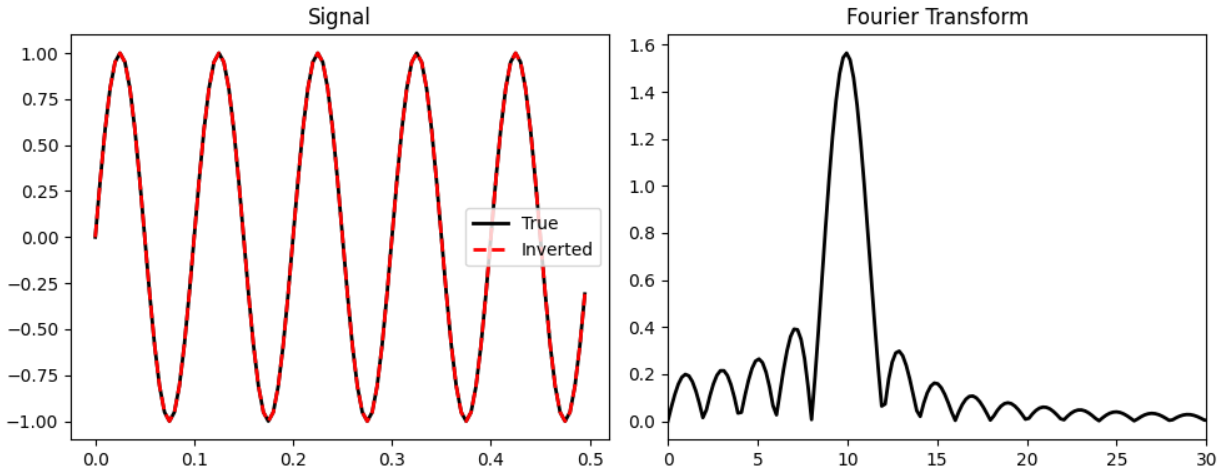
```
axs[0].set_title("Signal")
axs[1].plot(FFTop.f[: int(FFTop.nfft / 2)], np.abs(D[: int(FFTop.nfft / 2)]), "k", lw=2)
axs[1].set_title("Fourier Transform")
axs[1].set_xlim([0, 3 * f0])
plt.tight_layout()
```

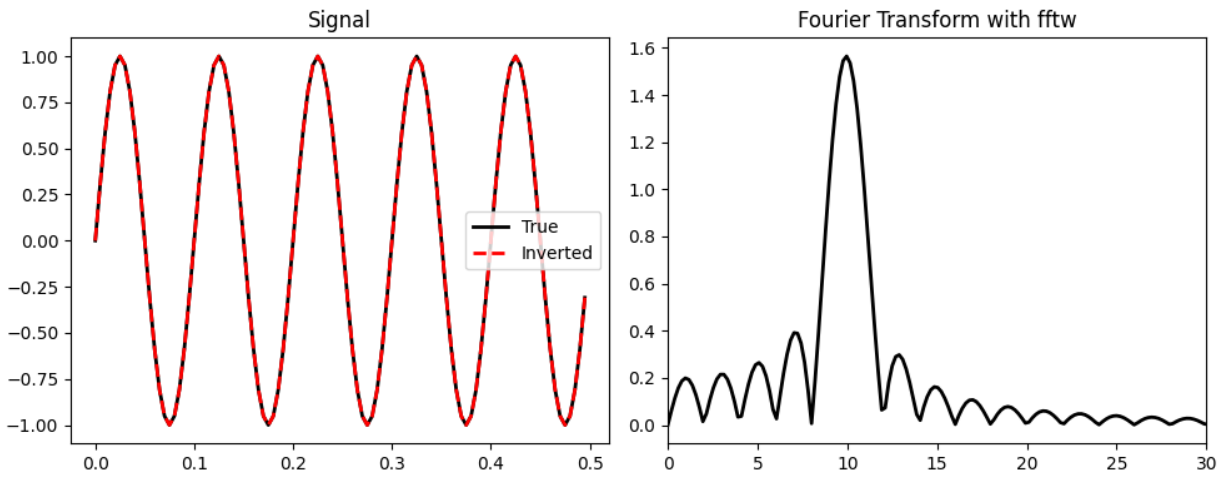


In this example we used numpy as our engine for the `fft` and `ifft`. PyLops implements a second engine (`engine='fftw'`) which uses the well-known FFTW via the python wrapper `pyfftw.FFTW`. This optimized fft tends to outperform the one from numpy in many cases but it is not inserted in the mandatory requirements of PyLops. If interested to use `FFTW` backend, read the *fft routines* section at *Advanced installation*.

```
FFTop = pylops.signalprocessing.FFT(dims=nt, nfft=nfft, sampling=dt, engine="fftw")
D = FFTop * d

# Adjoint = inverse for FFT
dinv = FFTop.H * D
dinv = FFTop / D

fig, axs = plt.subplots(1, 2, figsize=(10, 4))
axs[0].plot(t, d, "k", lw=2, label="True")
axs[0].plot(t, dinv.real, "--r", lw=2, label="Inverted")
axs[0].legend()
axs[0].set_title("Signal")
axs[1].plot(FFTop.f[: int(FFTop.nfft / 2)], np.abs(D[: int(FFTop.nfft / 2)]), "k", lw=2)
axs[1].set_title("Fourier Transform with fftw")
axs[1].set_xlim([0, 3 * f0])
plt.tight_layout()
```



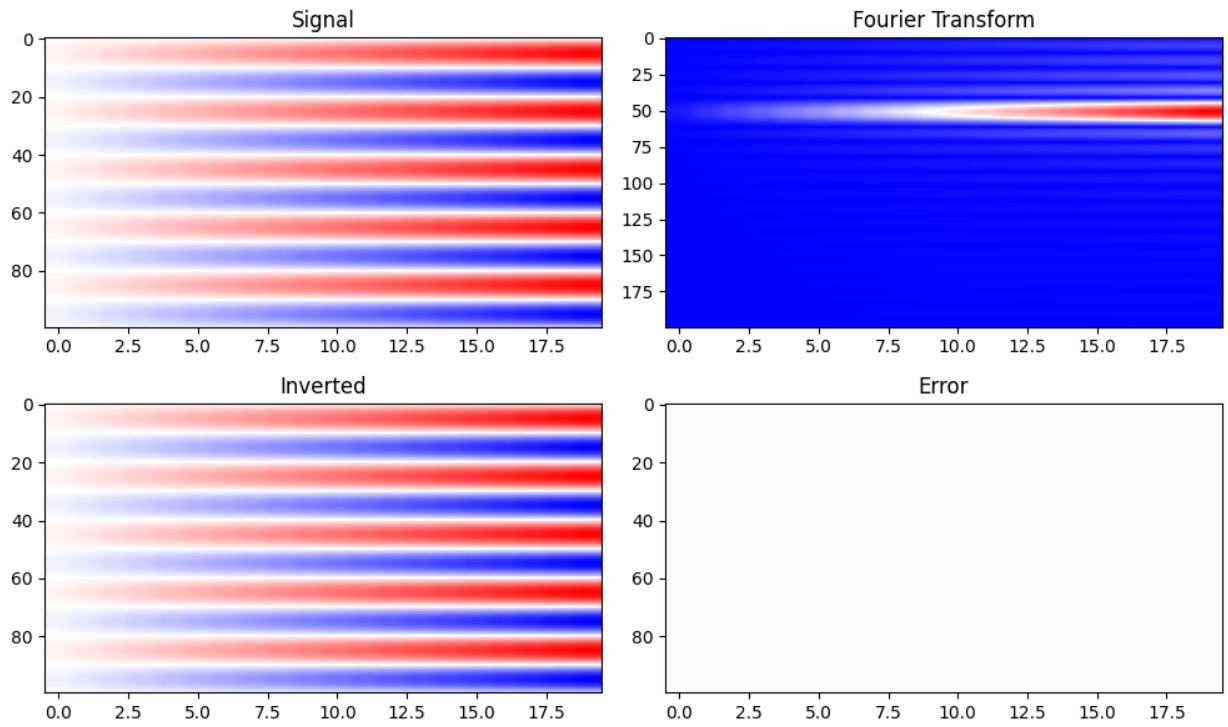
We can also apply the one dimensional FFT to to a two-dimensional signal (along one of the first axis)

```
dt = 0.005
nt, nx = 100, 20
t = np.arange(nt) * dt
f0 = 10
nfft = 2**10
d = np.outer(np.sin(2 * np.pi * f0 * t), np.arange(nx) + 1)

FFTop = pylops.signalprocessing.FFT(dims=(nt, nx), axis=0, nfft=nfft, sampling=dt)
D = FFTop * d.ravel()

# Adjoint = inverse for FFT
dinv = FFTop.H * D
dinv = FFTop / D
dinv = np.real(dinv).reshape(nt, nx)

fig, axs = plt.subplots(2, 2, figsize=(10, 6))
axs[0][0].imshow(d, vmin=-20, vmax=20, cmap="bwr")
axs[0][0].set_title("Signal")
axs[0][0].axis("tight")
axs[0][1].imshow(np.abs(D.reshape(nfft, nx)[:200, :]), cmap="bwr")
axs[0][1].set_title("Fourier Transform")
axs[0][1].axis("tight")
axs[1][0].imshow(dinv, vmin=-20, vmax=20, cmap="bwr")
axs[1][0].set_title("Inverted")
axs[1][0].axis("tight")
axs[1][1].imshow(d - dinv, vmin=-20, vmax=20, cmap="bwr")
axs[1][1].set_title("Error")
axs[1][1].axis("tight")
fig.tight_layout()
```



We can also apply the two dimensional FFT to to a two-dimensional signal

```
dt, dx = 0.005, 5
nt, nx = 100, 201
t = np.arange(nt) * dt
x = np.arange(nx) * dx
f0 = 10
nfft = 2**10
d = np.outer(np.sin(2 * np.pi * f0 * t), np.arange(nx) + 1)

FFTop = pylops.signalprocessing.FFT2D(
    dims=(nt, nx), nffts=(nfft, nfft), sampling=(dt, dx)
)
D = FFTop * d.ravel()

dinv = FFTop.H * D
dinv = FFTop / D
dinv = np.real(dinv).reshape(nt, nx)

fig, axs = plt.subplots(2, 2, figsize=(10, 6))
axs[0][0].imshow(d, vmin=-100, vmax=100, cmap="bwr")
axs[0][0].set_title("Signal")
axs[0][0].axis("tight")
axs[0][1].imshow(
    np.abs(np.fft.fftshift(D.reshape(nfft, nfft), axes=1)[:200, :]), cmap="bwr"
)
axs[0][1].set_title("Fourier Transform")
axs[0][1].axis("tight")
axs[1][0].imshow(dinv, vmin=-100, vmax=100, cmap="bwr")
```

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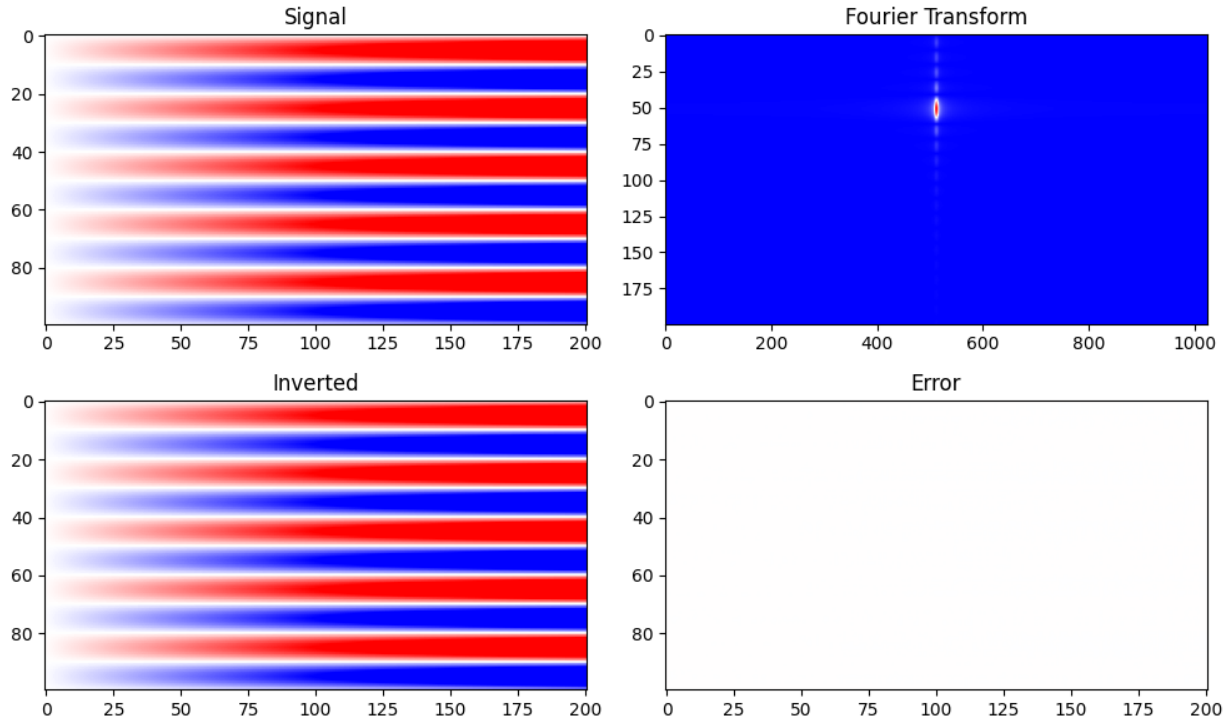
```
axs[1][0].set_title("Inverted")
axs[1][0].axis("tight")
axs[1][1].imshow(d - dinv, vmin=-100, vmax=100, cmap="bwr")
axs[1][1].set_title("Error")
axs[1][1].axis("tight")
fig.tight_layout()
```



Finally can apply the three dimensional FFT to to a three-dimensional signal

```
dt, dx, dy = 0.005, 5, 3
nt, nx, ny = 30, 21, 11
t = np.arange(nt) * dt
x = np.arange(nx) * dx
y = np.arange(nx) * dy
f0 = 10
nfft = 2**6
nfftk = 2**5

d = np.outer(np.sin(2 * np.pi * f0 * t), np.arange(nx) + 1)
d = np.tile(d[:, :, np.newaxis], [1, 1, ny])

FFTop = pylops.signalprocessing.FFTND(
    dims=(nt, nx, ny), nffts=(nfft, nfftk, nfftk), sampling=(dt, dx, dy)
)
D = FFTop * d.ravel()

dinv = FFTop.H * D
dinv = FFTop / D
```

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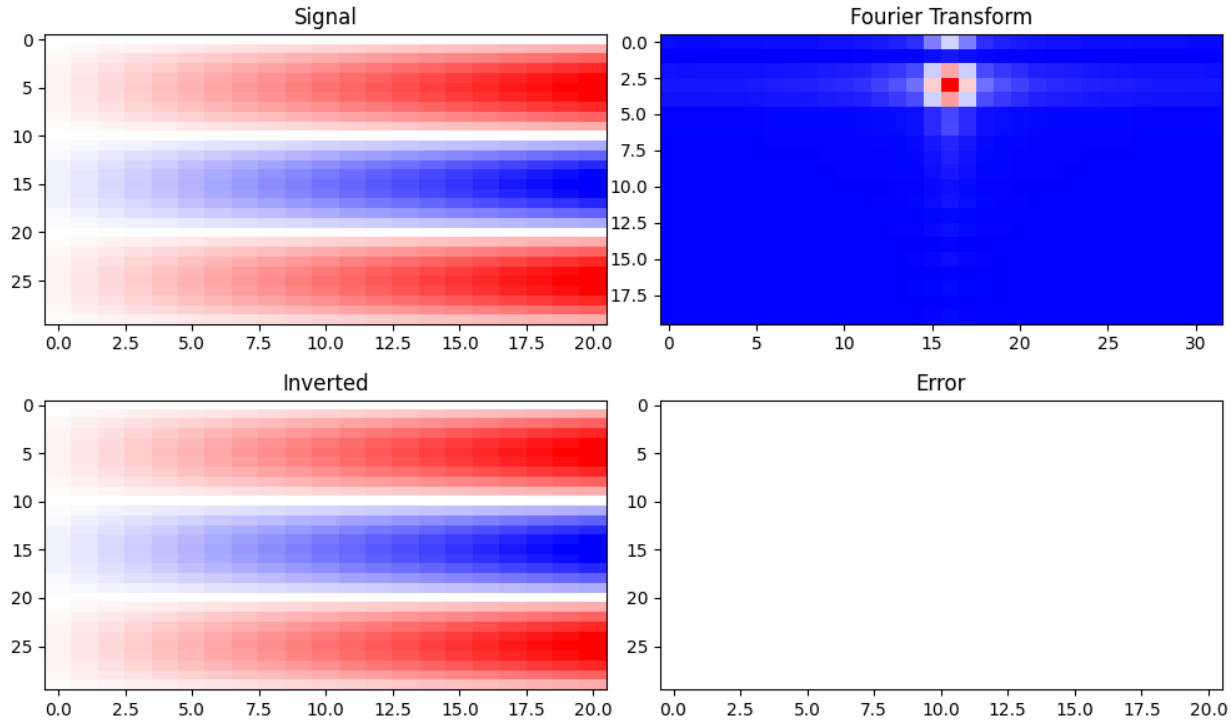
```
dinv = np.real(dinv).reshape(nt, nx, ny)

fig, axs = plt.subplots(2, 2, figsize=(10, 6))
axs[0][0].imshow(d[:, :, ny // 2], vmin=-20, vmax=20, cmap="bwr")
axs[0][0].set_title("Signal")
axs[0][0].axis("tight")
axs[0][1].imshow(
    np.abs(np.fft.fftshift(D.reshape(nfft, nfftk, nfftk), axes=1)[:20, :, nfftk // 2]),
    cmap="bwr",
)
axs[0][1].set_title("Fourier Transform")
axs[0][1].axis("tight")
axs[1][0].imshow(dinv[:, :, ny // 2], vmin=-20, vmax=20, cmap="bwr")
axs[1][0].set_title("Inverted")
axs[1][0].axis("tight")
axs[1][1].imshow(d[:, :, ny // 2] - dinv[:, :, ny // 2], vmin=-20, vmax=20, cmap="bwr")
axs[1][1].set_title("Error")
axs[1][1].axis("tight")
fig.tight_layout()
```



Total running time of the script: (0 minutes 1.977 seconds)

3.5.16 Identity

This example shows how to use the *pylops.Identity* operator to transfer model into data and viceversa.

```
import matplotlib.gridspec as pltgs
import matplotlib.pyplot as plt
import numpy as np

import pylops

plt.close("all")
```

Let's define an identity operator $\mathbf{Iop}$ with same number of elements for data and model ($N = M$).

```
N, M = 5, 5
x = np.arange(M)
Iop = pylops.Identity(M, dtype="int")

y = Iop * x
xadj = Iop.H * y

gs = pltgs.GridSpec(1, 6)
fig = plt.figure(figsize=(7, 4))
ax = plt.subplot(gs[0, 0:3])
im = ax.imshow(np.eye(N), cmap="rainbow")
ax.set_title("A", size=20, fontweight="bold")
ax.set_xticks(np.arange(N - 1) + 0.5)
ax.set_yticks(np.arange(M - 1) + 0.5)
ax.grid(linewidth=3, color="white")
ax.xaxis.set_ticklabels([])
ax.yaxis.set_ticklabels([])
ax = plt.subplot(gs[0, 3])
ax.imshow(x[:, np.newaxis], cmap="rainbow")
ax.set_title("x", size=20, fontweight="bold")
ax.set_xticks([])
ax.set_yticks(np.arange(M - 1) + 0.5)
ax.grid(linewidth=3, color="white")
ax.xaxis.set_ticklabels([])
ax.yaxis.set_ticklabels([])
ax = plt.subplot(gs[0, 4])
ax.text(
    0.35,
    0.5,
    "=",
    horizontalalignment="center",
    verticalalignment="center",
    size=40,
    fontweight="bold",
)
ax.axis("off")
ax = plt.subplot(gs[0, 5])
ax.imshow(y[:, np.newaxis], cmap="rainbow")
ax.set_title("y", size=20, fontweight="bold")
ax.set_xticks([])
```

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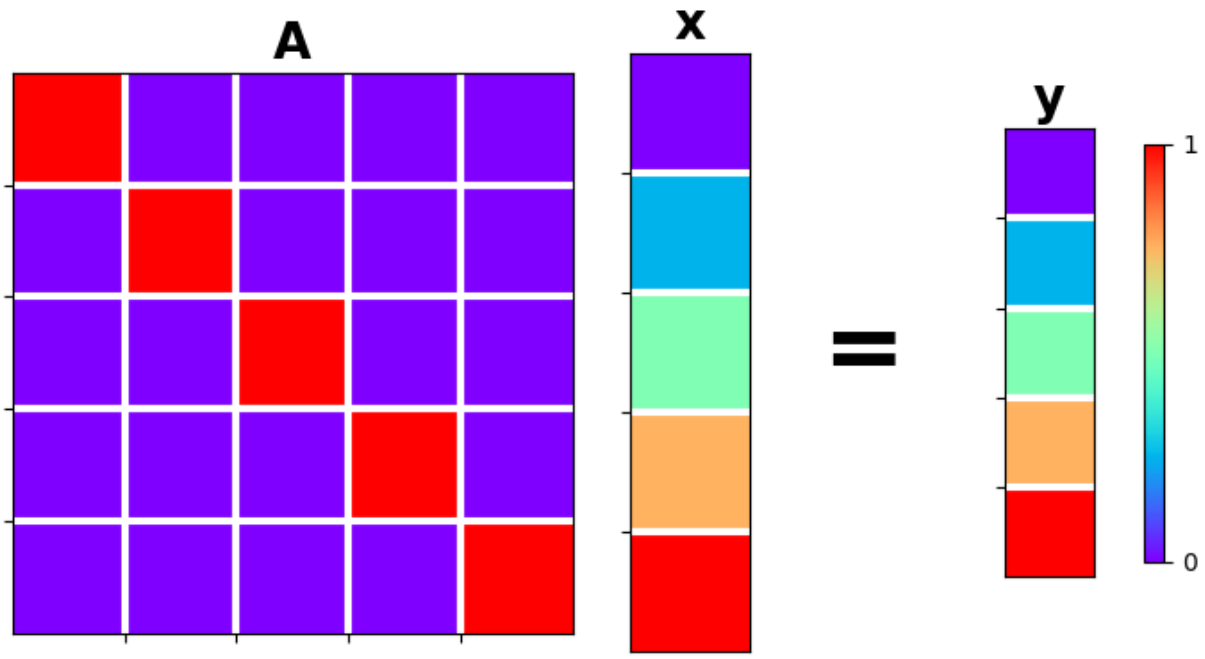
```
ax.set_yticks(np.arange(N - 1) + 0.5)
ax.grid(linewidth=3, color="white")
ax.xaxis.set_ticklabels([])
ax.yaxis.set_ticklabels([])
fig.colorbar(im, ax=ax, ticks=[0, 1], pad=0.3, shrink=0.7)
plt.tight_layout()
```



Similarly we can consider the case with data bigger than model

```
N, M = 10, 5
x = np.arange(M)
Iop = pylops.Identity(N, M, dtype="int")

y = Iop * x
xadj = Iop.H * y

print(f"x = {x} ")
print(f"I*x = {y} ")
print(f"I'*y = {xadj} ")
```

```
x = [0 1 2 3 4]
I*x = [0 1 2 3 4 0 0 0 0 0]
I'*y = [0 1 2 3 4]
```

and model bigger than data

```
N, M = 5, 10
x = np.arange(M)
Iop = pylops.Identity(N, M, dtype="int")
```

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```
y = Iop * x
xadj = Iop.H * y

print(f"x = {x} ")
print(f"I*x = {y} ")
print(f"I'*y = {xadj} ")
```

```
x = [0 1 2 3 4 5 6 7 8 9]
I*x = [0 1 2 3 4]
I'*y = [0 1 2 3 4 0 0 0 0 0]
```

Note that this operator can be useful in many real-life applications when for example we want to manipulate a subset of the model array and keep intact the rest of the array. For example:

$$\begin{bmatrix} \mathbf{A} & \mathbf{I} \end{bmatrix} \begin{bmatrix} \mathbf{x_1} \\ \mathbf{x_2} \end{bmatrix} = \mathbf{A}\mathbf{x_1} + \mathbf{x_2}$$

Refer to the tutorial on *Optimization* for more details on this.

Total running time of the script: (0 minutes 0.207 seconds)

3.5.17 Imag

This example shows how to use the `pylops.basicoperators.Imag` operator. This operator returns the imaginary part of the data as a real value in forward mode, and the real part of the model as an imaginary value in adjoint mode (with zero real part).

```
import matplotlib.pyplot as plt
import numpy as np

import pylops

plt.close("all")
```

Let's define a Imag operator $\Im$ to extract the imaginary component of the input.

```
M = 5
x = np.arange(M) + 1j * np.arange(M)[::-1]
Rop = pylops.basicoperators.Imag(M, dtype="complex128")

y = Rop * x
xadj = Rop.H * y

_, axs = plt.subplots(1, 3, figsize=(10, 4))
axs[0].plot(np.real(x), lw=2, label="Real")
axs[0].plot(np.imag(x), lw=2, label="Imag")
axs[0].legend()
axs[0].set_title("Input")
axs[1].plot(np.real(y), lw=2, label="Real")
```

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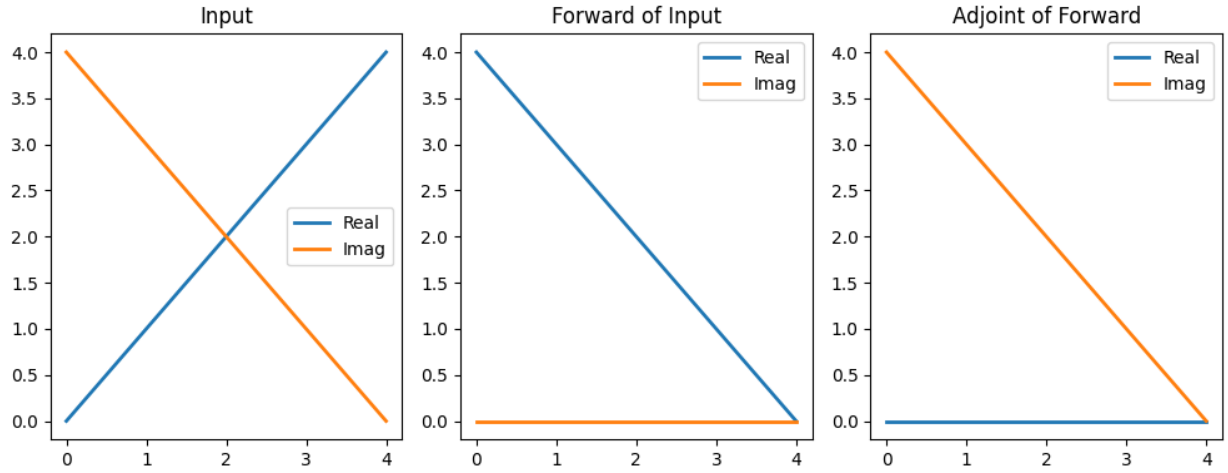
```
axs[1].plot(np.imag(y), lw=2, label="Imag")
axs[1].legend()
axs[1].set_title("Forward of Input")
axs[2].plot(np.real(xadj), lw=2, label="Real")
axs[2].plot(np.imag(xadj), lw=2, label="Imag")
axs[2].legend()
axs[2].set_title("Adjoint of Forward")
plt.tight_layout()
```



Total running time of the script: (0 minutes 0.356 seconds)

3.5.18 Linear Regression

This example shows how to use the *pylops.LinearRegression* operator to perform *Linear regression analysis*.

In short, linear regression is the problem of finding the best fitting coefficients, namely intercept $\mathbf{x_0}$ and gradient $\mathbf{x_1}$, for this equation:

$$y_i = x_0 + x_1 t_i \qquad \forall i = 0, 1, \ldots, N-1$$

As we can express this problem in a matrix form:

$$\mathbf{y} = \mathbf{A}\mathbf{x}$$

our solution can be obtained by solving the following optimization problem:

$$J = \|\mathbf{y} - \mathbf{A}\mathbf{x}\|_2$$

See documentation of *pylops.LinearRegression* for more detailed definition of the forward problem.

```
import matplotlib.pyplot as plt
import numpy as np

import pylops

plt.close("all")
np.random.seed(10)
```

Define the input parameters: number of samples along the t-axis (`N`), linear regression coefficients (`x`), and standard deviation of noise to be added to data (`sigma`).

```
N = 30
x = np.array([1.0, 2.0])
sigma = 1
```

Let's create the time axis and initialize the *pylops.LinearRegression* operator

```
t = np.arange(N, dtype="float64")
LRop = pylops.LinearRegression(t, dtype="float64")
```

We can then apply the operator in forward mode to compute our data points along the x-axis (`y`). We will also generate some random gaussian noise and create a noisy version of the data (`yn`).

```
y = LRop * x
yn = y + np.random.normal(0, sigma, N)
```

We are now ready to solve our problem. As we are using an operator from the *pylops.LinearOperator* family, we can simply use `/`, which in this case will solve the system by means of an iterative solver (i.e., `scipy.sparse.linalg.lsqr`).

```
xest = LRop / y
xnest = LRop / yn
```

Let's plot the best fitting line for the case of noise free and noisy data

```
plt.figure(figsize=(5, 7))
plt.plot(
    np.array([t.min(), t.max()]),
    np.array([t.min(), t.max()]) * x[1] + x[0],
    "k",
    lw=4,
    label=rf"true: $x_0$ = {x[0]:.2f}, $x_1$ = {x[1]:.2f}",
)
plt.plot(
    np.array([t.min(), t.max()]),
    np.array([t.min(), t.max()]) * xest[1] + xest[0],
    "--r",
    lw=4,
    label=rf"est noise-free: $x_0$ = {xest[0]:.2f}, $x_1$ = {xest[1]:.2f}",
)
plt.plot(
    np.array([t.min(), t.max()]),
    np.array([t.min(), t.max()]) * xnest[1] + xnest[0],
    "--g",
```

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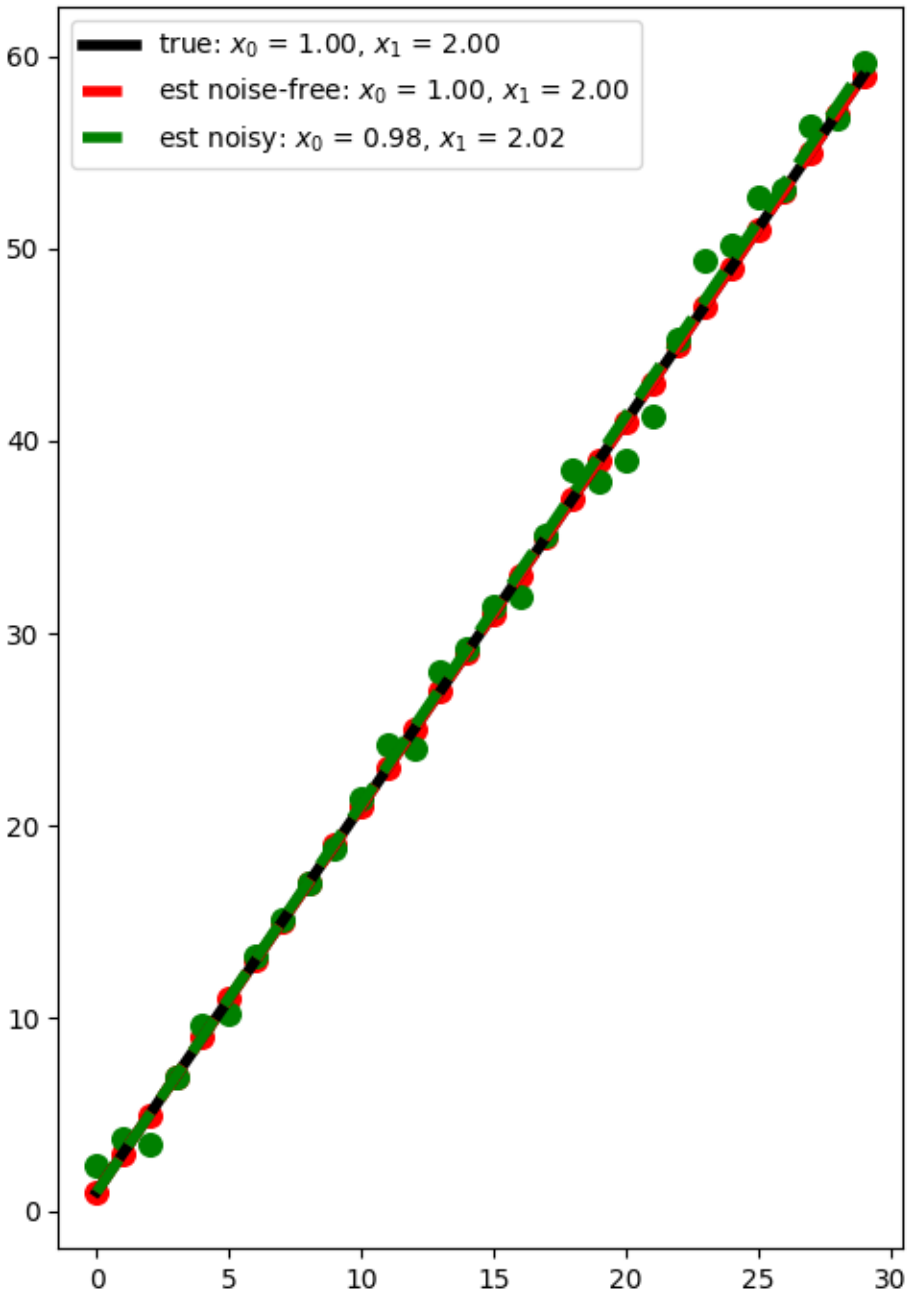
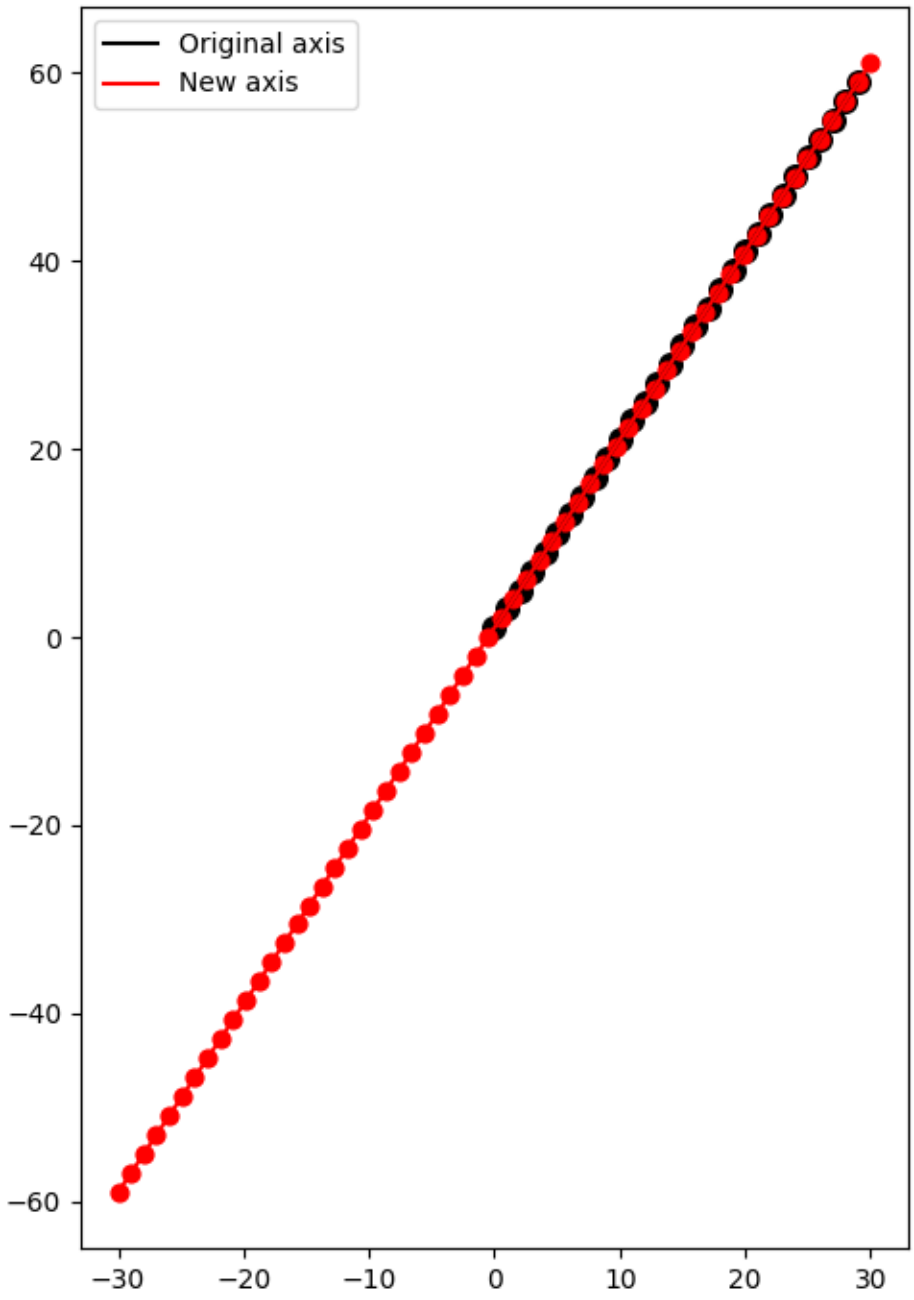
```
    lw=4,
    label=rf"est noisy: $x_0$ = {xnest[0]:.2f}, $x_1$ = {xnest[1]:.2f}",
)
plt.scatter(t, y, c="r", s=70)
plt.scatter(t, yn, c="g", s=70)
plt.legend()
plt.tight_layout()
```



Once that we have estimated the best fitting coefficients $\mathbf{x}$ we can now use them to compute the *y values* for a different set of values along the *t-axis*.

```
t1 = np.linspace(-N, N, 2 * N, dtype="float64")
y1 = LRop.apply(t1, xest)

plt.figure(figsize=(5, 7))
plt.plot(t, y, "k", label="Original axis")
plt.plot(t1, y1, "r", label="New axis")
plt.scatter(t, y, c="k", s=70)
plt.scatter(t1, y1, c="r", s=40)
plt.legend()
plt.tight_layout()
```



We consider now the case where some of the observations have large errors. Such elements are generally referred to as *outliers* and can affect the quality of the least-squares solution if not treated with care. In this example we will see how using a L1 solver such as `pylops.optimization.sparsity.IRLS` can drammatically improve the quality of the estimation of intercept and gradient.

```
class CallbackIRLS(pylops.optimization.callback.Callbacks):
    def __init__(self, n):
        self.n = n
        self.xirls_hist = []
        self.rw_hist = []

    def on_step_end(self, solver, x):
        print(solver.iiter)
        if solver.iiter > 1:
            self.xirls_hist.append(x)
            self.rw_hist.append(solver.rw)
        else:
            self.rw_hist.append(np.ones(self.n))


# Add outliers
yn[1] += 40
yn[N - 2] -= 20

# IRLS
nouter = 20
epsR = 1e-2
epsI = 0
tolIRLS = 1e-2

xnest = LRop / yn

cb = CallbackIRLS(N)
irlssolve = pylops.optimization.sparsity.IRLS(
    LRop,
    [
        cb,
    ],
)
xirls, nouter = irlssolve.solve(
    yn, nouter=nouter, threshR=False, epsR=epsR, epsI=epsI, tolIRLS=tolIRLS
)
xirls_hist, rw_hist = np.array(cb.xirls_hist), cb.rw_hist
print(f"IRLS converged at {nouter} iterations...")

plt.figure(figsize=(5, 7))
plt.plot(
    np.array([t.min(), t.max()]),
    np.array([t.min(), t.max()]) * x[1] + x[0],
    "k",
    lw=4,
    label=rf"true: $x_0$ = {x[0]:.2f}, $x_1$ = {x[1]:.2f}",
)
```

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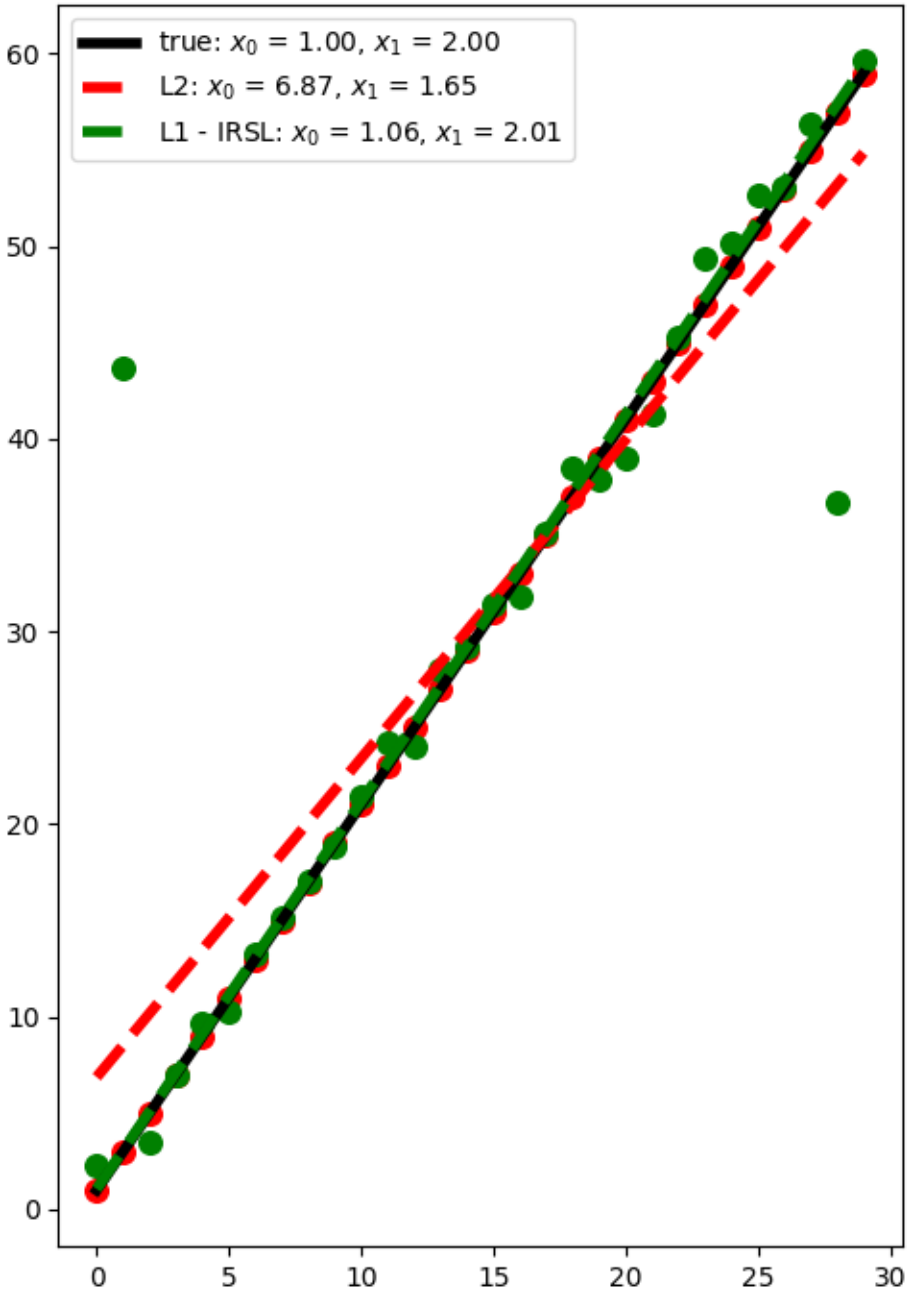
```
plt.plot(
    np.array([t.min(), t.max()]),
    np.array([t.min(), t.max()]) * xnest[1] + xnest[0],
    "--r",
    lw=4,
    label=rf"L2: $x_0$ = {xnest[0]:.2f}, $x_1$ = {xnest[1]:.2f}",
)
plt.plot(
    np.array([t.min(), t.max()]),
    np.array([t.min(), t.max()]) * xirls[1] + xirls[0],
    "--g",
    lw=4,
    label=rf"L1 - IRSL: $x_0$ = {xirls[0]:.2f}, $x_1$ = {xirls[1]:.2f}",
)
plt.scatter(t, y, c="r", s=70)
plt.scatter(t, yn, c="g", s=70)
plt.legend()
plt.tight_layout()
```




```
1
2
3
4
5
6
7
8
9
10
```

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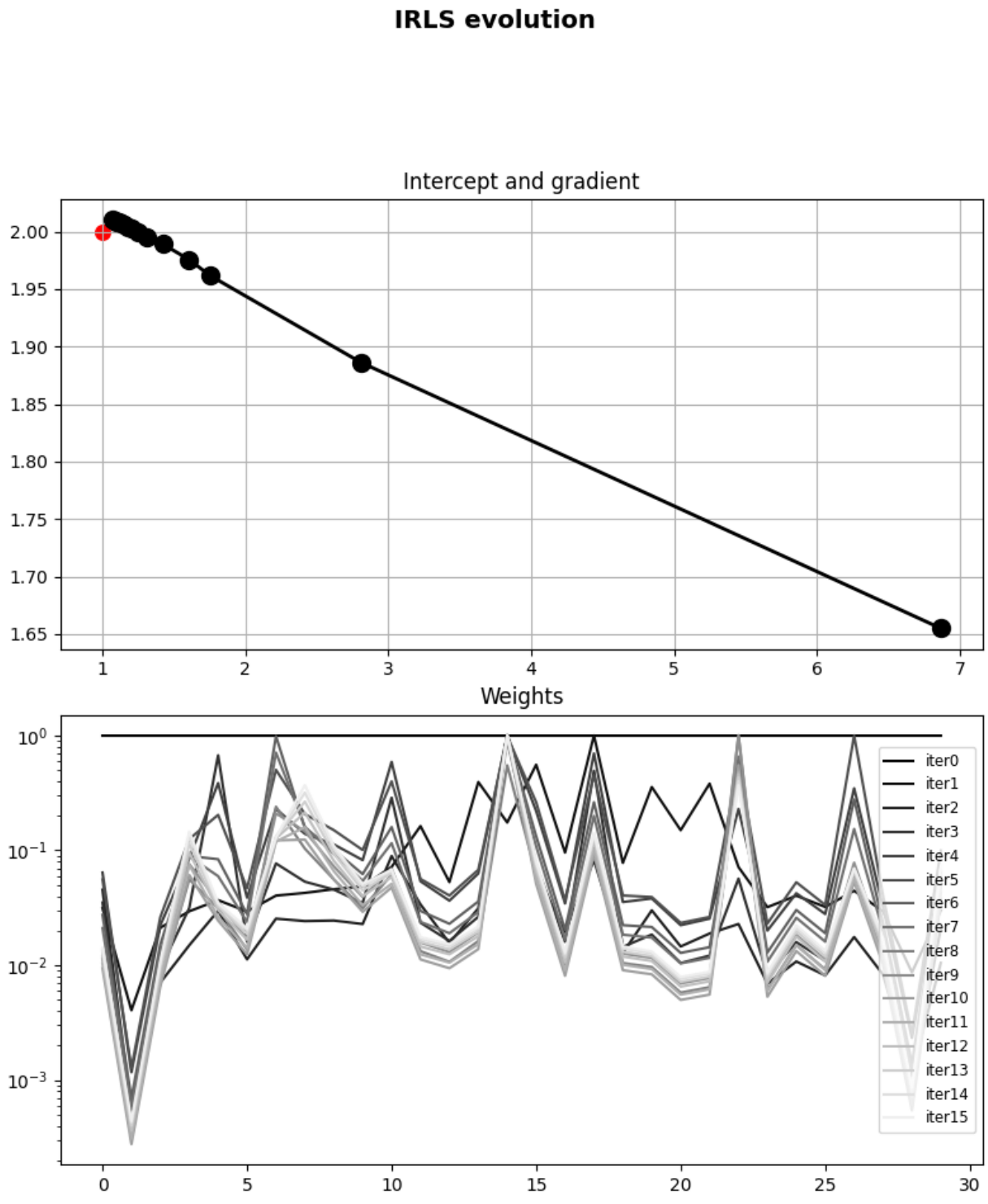
```
11
12
13
14
15
16
IRLS converged at 16 iterations...
```

Let's finally take a look at the convergence of IRLS. First we visualize the evolution of intercept and gradient

```
fig, axs = plt.subplots(2, 1, figsize=(8, 10))
fig.suptitle("IRLS evolution", fontsize=14, fontweight="bold", y=0.95)
axs[0].plot(xirls_hist[:, 0], xirls_hist[:, 1], ".-k", lw=2, ms=20)
axs[0].scatter(x[0], x[1], c="r", s=70)
axs[0].set_title("Intercept and gradient")
axs[0].grid()
for iiter in range(nouter):
    axs[1].semilogy(
        rw_hist[iiter],
        color=(iiter / nouter, iiter / nouter, iiter / nouter),
        label="iter%d" % iiter,
    )
axs[1].set_title("Weights")
axs[1].legend(loc=5, fontsize="small")
plt.tight_layout()
plt.subplots_adjust(top=0.8)
```



Total running time of the script: (0 minutes 1.131 seconds)

3.5.19 MP, OMP, ISTA and FISTA

This example shows how to use the *pylops.optimization.sparsity.omp*, *pylops.optimization.sparsity.irls*, *pylops.optimization.sparsity.ista*, and *pylops.optimization.sparsity.fista* solvers.

These solvers can be used when the model to retrieve is supposed to have a sparse representation in a certain domain. MP and OMP use a L0 norm and mathematically translates to solving the following constrained problem:

$$\|\mathbf{Opx} - \mathbf{b}\|_2 <= \sigma,$$

while IRLS, ISTA and FISTA solve an uncostrained problem with a L1 regularization term:

$$J = \|\mathbf{d} - \mathbf{Opx}\|_2 + \epsilon\|\mathbf{x}\|_1$$

```
import matplotlib.pyplot as plt
import numpy as np

import pylops

plt.close("all")
np.random.seed(0)
```

Let's start with a simple example, where we create a dense mixing matrix and a sparse signal and we use OMP and ISTA to recover such a signal. Note that the mixing matrix leads to an underdetermined system of equations ($N < M$) so being able to add some extra prior information regarding the sparsity of our desired model is essential to be able to invert such a system.

```
N, M = 15, 20
A = np.random.randn(N, M)
A = A / np.linalg.norm(A, axis=0)
Aop = pylops.MatrixMult(A)

x = np.random.rand(M)
x[x < 0.9] = 0
y = Aop * x

# MP/OMP
eps = 1e-2
maxit = 500
x_mp = pylops.optimization.sparsity.omp(
    Aop, y, niter_outer=maxit, niter_inner=0, sigma=1e-4
)[0]
x_omp = pylops.optimization.sparsity.omp(Aop, y, niter_outer=maxit, sigma=1e-4)[0]

# IRLS
x_irls = pylops.optimization.sparsity.irls(
    Aop, y, nouter=50, epsI=1e-5, kind="model", **dict(iter_lim=10)
)[0]

# ISTA
x_ista = pylops.optimization.sparsity.ista(
    Aop,
    y,
    niter=maxit,
```

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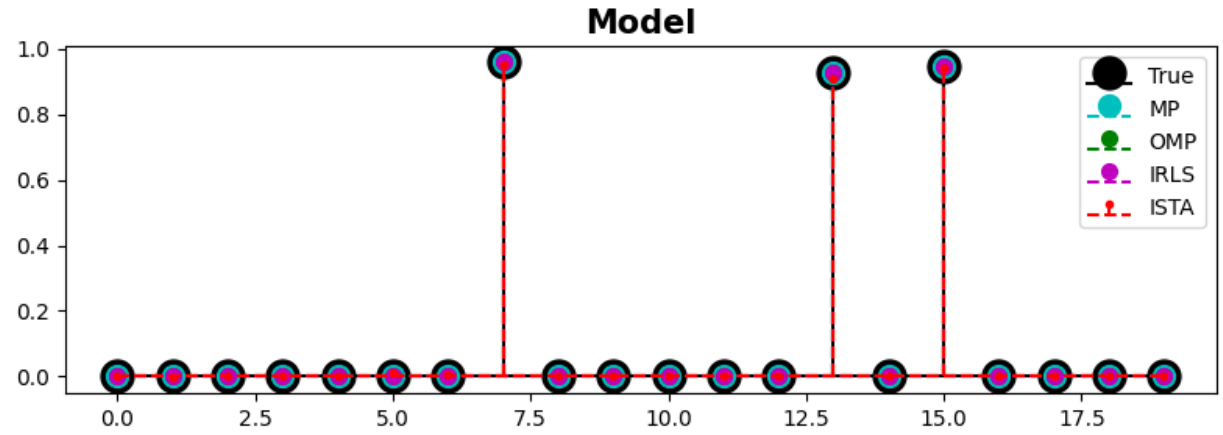
```
    eps=eps,
    tol=1e-3,
)[0]

fig, ax = plt.subplots(1, 1, figsize=(8, 3))
m, s, b = ax.stem(x, linefmt="k", basefmt="k", markerfmt="ko", label="True")
plt.setp(m, markersize=15)
m, s, b = ax.stem(x_mp, linefmt="--c", basefmt="--c", markerfmt="co", label="MP")
plt.setp(m, markersize=10)
m, s, b = ax.stem(x_omp, linefmt="--g", basefmt="--g", markerfmt="go", label="OMP")
plt.setp(m, markersize=7)
m, s, b = ax.stem(x_irls, linefmt="--m", basefmt="--m", markerfmt="mo", label="IRLS")
plt.setp(m, markersize=7)
m, s, b = ax.stem(x_ista, linefmt="--r", basefmt="--r", markerfmt="ro", label="ISTA")
plt.setp(m, markersize=3)
ax.set_title("Model", size=15, fontweight="bold")
ax.legend()
plt.tight_layout()
```



We now consider a more interesting problem problem, *wavelet deconvolution* from a signal that we assume being composed by a train of spikes convolved with a certain wavelet. We will see how solving such a problem with a least-squares solver such as *pylops.optimization.leastsquares.regularized_inversion* does not produce the expected results (especially in the presence of noisy data), conversely using the *pylops.optimization.sparsity.ista* and *pylops.optimization.sparsity.fista* solvers allows us to succesfully retrieve the input signal even in the presence of noise. *pylops.optimization.sparsity.fista* shows faster convergence which is particularly useful for this problem.

```
nt = 61
dt = 0.004
t = np.arange(nt) * dt
x = np.zeros(nt)
x[10] = -0.4
x[int(nt / 2)] = 1
x[nt - 20] = 0.5

h, th, hcenter = pylops.utils.wavelets.ricker(t[:101], f0=20)
```

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```
Cop = pylops.signalprocessing.Convolve1D(nt, h=h, offset=hcenter, dtype="float32")
y = Cop * x
yn = y + np.random.normal(0, 0.1, y.shape)

# noise free
xls = Cop / y

xomp, nitero, costo = pylops.optimization.sparsity.omp(
    Cop, y, niter_outer=200, sigma=1e-8
)

xista, niteri, costi = pylops.optimization.sparsity.ista(
    Cop,
    y,
    niter=400,
    eps=5e-1,
    tol=1e-8,
)

fig, ax = plt.subplots(1, 1, figsize=(8, 3))
ax.plot(t, x, "k", lw=8, label=r"$x$")
ax.plot(t, y, "r", lw=4, label=r"$y=Ax$")
ax.plot(t, xls, "--g", lw=4, label=r"$x_{LS}$")
ax.plot(t, xomp, "--b", lw=4, label=r"$x_{OMP} (niter=%d)$" % nitero)
ax.plot(t, xista, "--m", lw=4, label=r"$x_{ISTA} (niter=%d)$" % niteri)
ax.set_title("Noise-free deconvolution", fontsize=14, fontweight="bold")
ax.legend()
plt.tight_layout()

# noisy
xls = pylops.optimization.leastsquares.regularized_inversion(
    Cop, yn, [], **dict(damp=1e-1, atol=1e-3, iter_lim=100, show=0)
)[0]

xista, niteri, costi = pylops.optimization.sparsity.ista(
    Cop,
    yn,
    niter=100,
    eps=5e-1,
    tol=1e-5,
)

xfista, niterf, costf = pylops.optimization.sparsity.fista(
    Cop,
    yn,
    niter=100,
    eps=5e-1,
    tol=1e-5,
)

fig, ax = plt.subplots(1, 1, figsize=(8, 3))
ax.plot(t, x, "k", lw=8, label=r"$x$")
```

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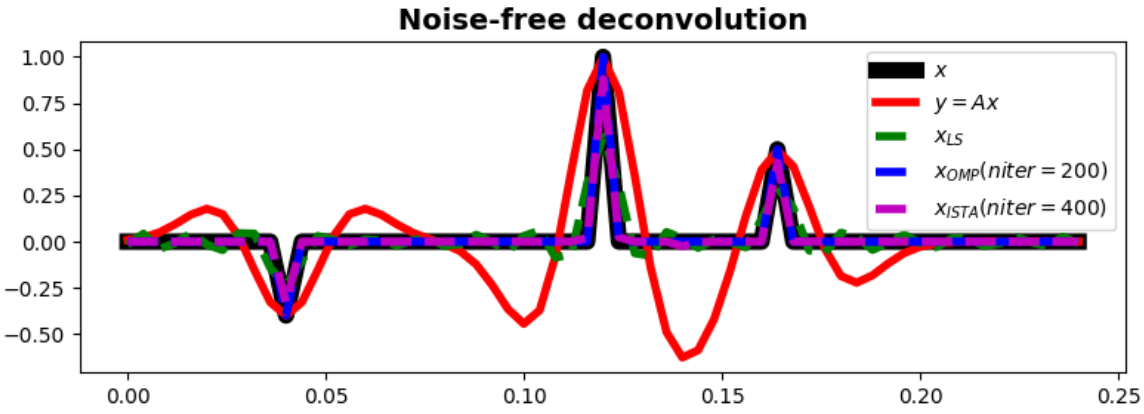
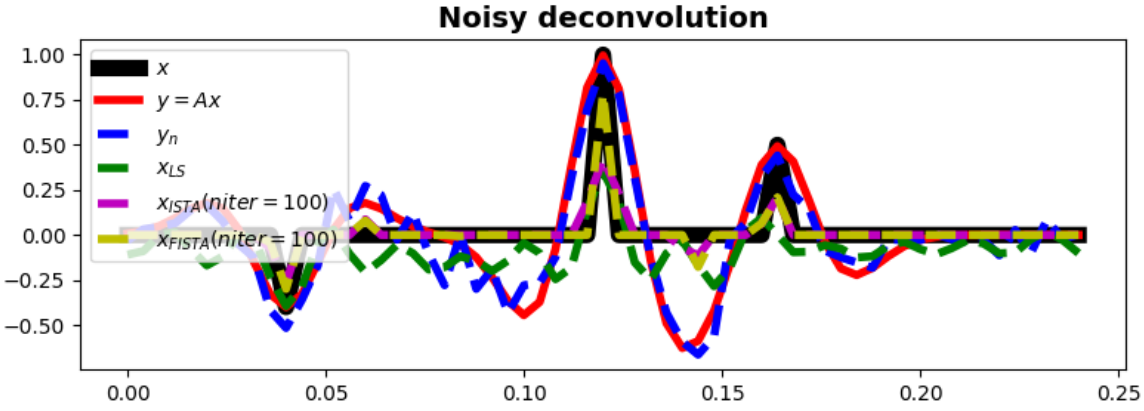
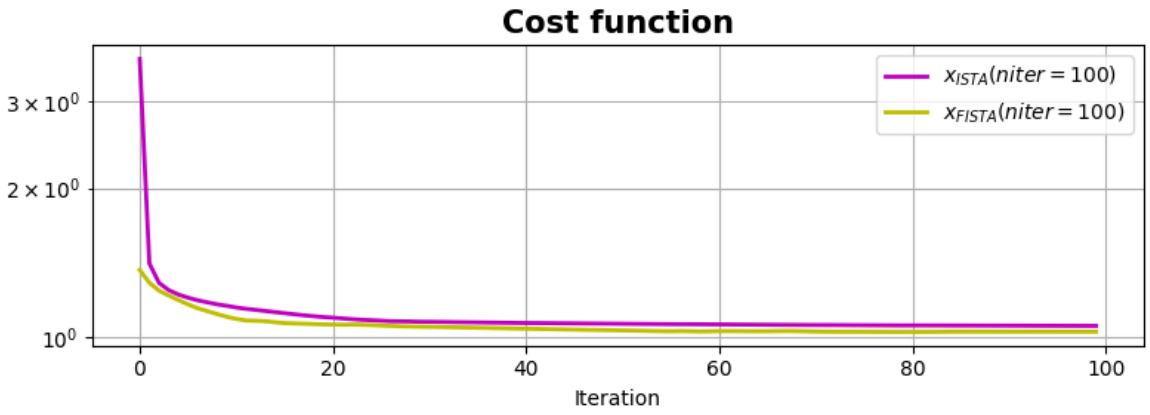
```
ax.plot(t, y, "r", lw=4, label=r"$y=Ax$")
ax.plot(t, yn, "--b", lw=4, label=r"$y_n$")
ax.plot(t, xls, "--g", lw=4, label=r"$x_{LS}$")
ax.plot(t, xista, "--m", lw=4, label=r"$x_{ISTA} (niter=%d)$" % niteri)
ax.plot(t, xfista, "--y", lw=4, label=r"$x_{FISTA} (niter=%d)$" % niterf)
ax.set_title("Noisy deconvolution", fontsize=14, fontweight="bold")
ax.legend()
plt.tight_layout()

fig, ax = plt.subplots(1, 1, figsize=(8, 3))
ax.semilogy(costi, "m", lw=2, label=r"$x_{ISTA} (niter=%d)$" % niteri)
ax.semilogy(costf, "y", lw=2, label=r"$x_{FISTA} (niter=%d)$" % niterf)
ax.set_title("Cost function", size=15, fontweight="bold")
ax.set_xlabel("Iteration")
ax.legend()
ax.grid(True, which="both")
plt.tight_layout()
```

- 

- 



-

Total running time of the script: (0 minutes 1.817 seconds)

3.5.20 Matrix Multiplication

This example shows how to use the *pylops.MatrixMult* operator to perform *Matrix inversion* of the following linear system.

$$\mathbf{y} = \mathbf{A}\mathbf{x}$$

You will see that since this operator is a simple overloading to a `numpy.ndarray` object, the solution of the linear system can be obtained via both direct inversion (i.e., by means explicit solver such as `scipy.linalg.solve` or `scipy.linalg.lstsq`) and iterative solver (i.e., `from scipy.sparse.linalg.lsqr`).

Note that in case of rectangular $\mathbf{A}$, an exact inverse does not exist and a least-square solution is computed instead.

```
import warnings

import matplotlib.gridspec as pltgs
import matplotlib.pyplot as plt
import numpy as np
from scipy.sparse import rand
from scipy.sparse.linalg import lsqr

import pylops

plt.close("all")
warnings.filterwarnings("ignore")
# sphinx_gallery_thumbnail_number = 2
```

Let's define the sizes of the matrix $\mathbf{A}$ (`N` and `M`) and fill the matrix with random numbers

```
N, M = 20, 20
A = np.random.normal(0, 1, (N, M))
Aop = pylops.MatrixMult(A, dtype="float64")

x = np.ones(M)
```

We can now apply the forward operator to create the data vector $\mathbf{y}$ and use `/` to solve the system by means of an explicit solver.

```
y = Aop * x
xest = Aop / y
```

Let's visually plot the system of equations we just solved.

```
gs = pltgs.GridSpec(1, 6)
fig = plt.figure(figsize=(7, 3))
ax = plt.subplot(gs[0, 0])
ax.imshow(y[:, np.newaxis], cmap="rainbow")
ax.set_title("y", size=20, fontweight="bold")
ax.set_xticks([])
ax.set_yticks(np.arange(N - 1) + 0.5)
ax.grid(linewidth=3, color="white")
ax.xaxis.set_ticklabels([])
ax.yaxis.set_ticklabels([])
ax = plt.subplot(gs[0, 1])
ax.text(
    0.35,
    0.5,
    "=",
    horizontalalignment="center",
    verticalalignment="center",
    size=40,
    fontweight="bold",
)
ax.axis("off")
ax = plt.subplot(gs[0, 2:5])
ax.imshow(Aop.A, cmap="rainbow")
ax.set_title("A", size=20, fontweight="bold")
ax.set_xticks(np.arange(N - 1) + 0.5)
ax.set_yticks(np.arange(M - 1) + 0.5)
ax.grid(linewidth=3, color="white")
ax.xaxis.set_ticklabels([])
ax.yaxis.set_ticklabels([])
ax = plt.subplot(gs[0, 5])
ax.imshow(x[:, np.newaxis], cmap="rainbow")
ax.set_title("x", size=20, fontweight="bold")
ax.set_xticks([])
ax.set_yticks(np.arange(M - 1) + 0.5)
ax.grid(linewidth=3, color="white")
ax.xaxis.set_ticklabels([])
ax.yaxis.set_ticklabels([])
plt.tight_layout()

gs = pltgs.GridSpec(1, 6)
fig = plt.figure(figsize=(7, 3))
ax = plt.subplot(gs[0, 0])
ax.imshow(x[:, np.newaxis], cmap="rainbow")
ax.set_title("xest", size=20, fontweight="bold")
ax.set_xticks([])
ax.set_yticks(np.arange(M - 1) + 0.5)
ax.grid(linewidth=3, color="white")
ax.xaxis.set_ticklabels([])
```

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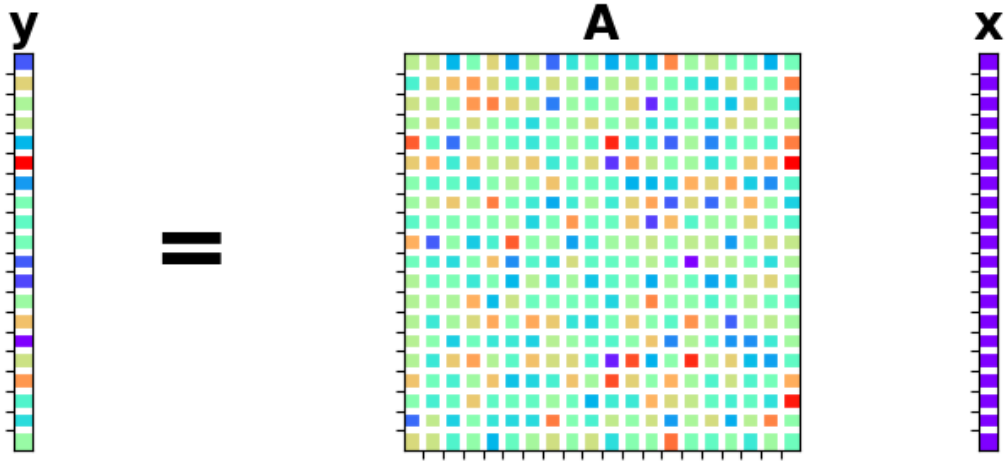
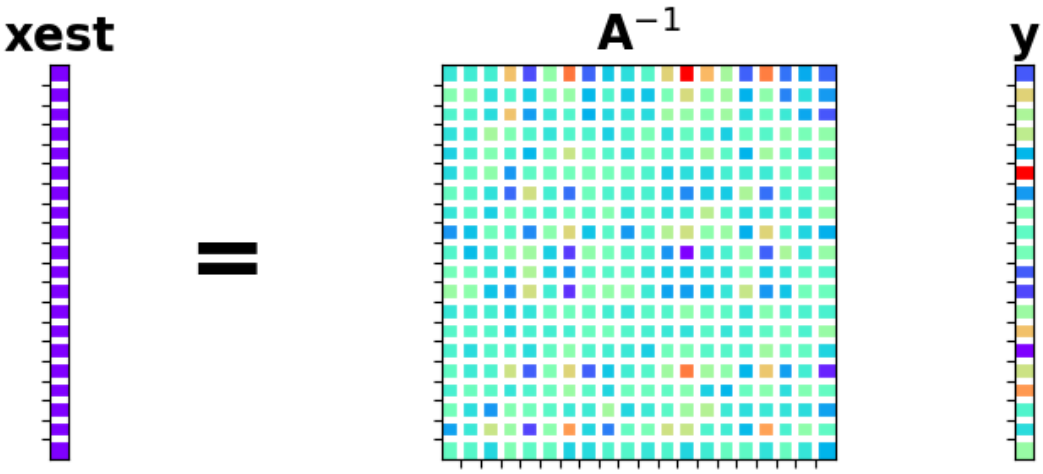
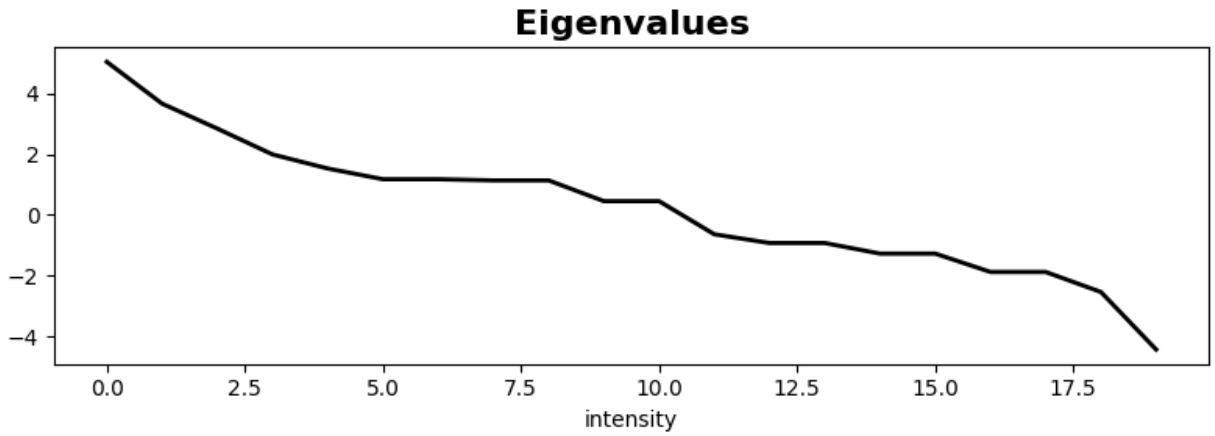
```
ax.yaxis.set_ticklabels([])
ax = plt.subplot(gs[0, 1])
ax.text(
    0.35,
    0.5,
    "=",
    horizontalalignment="center",
    verticalalignment="center",
    size=40,
    fontweight="bold",
)
ax.axis("off")
ax = plt.subplot(gs[0, 2:5])
ax.imshow(Aop.inv(), cmap="rainbow")
ax.set_title(r"A$^{-1}$", size=20, fontweight="bold")
ax.set_xticks(np.arange(N - 1) + 0.5)
ax.set_yticks(np.arange(M - 1) + 0.5)
ax.grid(linewidth=3, color="white")
ax.xaxis.set_ticklabels([])
ax.yaxis.set_ticklabels([])
ax = plt.subplot(gs[0, 5])
ax.imshow(y[:, np.newaxis], cmap="rainbow")
ax.set_title("y", size=20, fontweight="bold")
ax.set_xticks([])
ax.set_yticks(np.arange(N - 1) + 0.5)
ax.grid(linewidth=3, color="white")
ax.xaxis.set_ticklabels([])
ax.yaxis.set_ticklabels([])
plt.tight_layout()
```





Let's also plot the matrix eigenvalues

```
plt.figure(figsize=(8, 3))
plt.plot(Aop.eigs(), "k", lw=2)
plt.title("Eigenvalues", size=16, fontweight="bold")
plt.xlabel("#eigenvalue")
plt.xlabel("intensity")
plt.tight_layout()
```



We can also repeat the same exercise for a non-square matrix

```
N, M = 200, 50
A = np.random.normal(0, 1, (N, M))
x = np.ones(M)

Aop = pylops.MatrixMult(A, dtype="float64")
y = Aop * x
yn = y + np.random.normal(0, 0.3, N)

xest = Aop / y
xnest = Aop / yn
```

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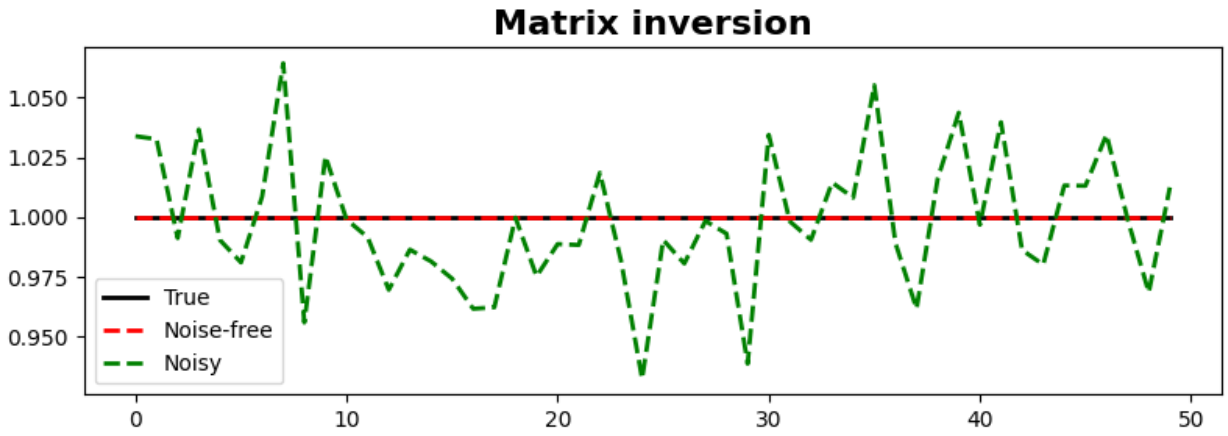
```
plt.figure(figsize=(8, 3))
plt.plot(x, "k", lw=2, label="True")
plt.plot(xest, "--r", lw=2, label="Noise-free")
plt.plot(xnest, "--g", lw=2, label="Noisy")
plt.title("Matrix inversion", size=16, fontweight="bold")
plt.legend()
plt.tight_layout()
```



And we can also use a sparse matrix from the `scipy.sparse` family of sparse matrices.

```
N, M = 5, 5
A = rand(N, M, density=0.75)
x = np.ones(M)

Aop = pylops.MatrixMult(A, dtype="float64")
y = Aop * x
xest = Aop / y

print(f"A= {Aop.A.todense()}")
print(f"A^-1= {Aop.inv().todense()}")
print(f"eigs= {Aop.eigs()}")
print(f"x= {x}")
print(f"y= {y}")
print(f"lsqr solution xest= {xest}")
```

```
A= [[0.09004394 0.07465359 0.         0.87688481 0.47238513]
 [0.79938802 0.85640696 0.71661781 0.85258904 0.        ]
 [0.         0.14734131 0.56655652 0.16975416 0.        ]
 [0.29282169 0.46547813 0.90723735 0.62796526 0.        ]
 [0.50711649 0.9563702  0.77100806 0.01124646 0.        ]]
A^-1= [[  0.          13.07319194  29.81875979 -25.7426719   -3.77140228]
 [  0.         -10.01880875 -25.46927387  20.41578795   4.00446478]
 [  0.           3.89143177  12.16350133  -8.55004722  -1.19720943]
 [  0.          -4.29168649 -12.5984235   10.81561744   0.51995029]
 [  2.11691676   7.05799755  21.72748426 -18.39641109  -0.87913918]]
eigs= [ 1.91944139+0.j          0.50019806+0.j         -0.17658613-0.24554532j]
x= [1. 1. 1. 1. 1.]
```

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```
y= [1.51396747 3.22500183 0.88365199 2.29350244 2.24574121]
lsqr solution xest= [1. 1. 1. 1. 1.]
```

Finally, in several circumstances the input model $\mathbf{x}$ may be more naturally arranged as a matrix or a multi-dimensional array and it may be desirable to apply the same matrix to every columns of the model. This can be mathematically expressed as:

$$\mathbf{y} = \begin{bmatrix} \mathbf{A} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{A} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{A} \end{bmatrix} \begin{bmatrix} \mathbf{x_1} \\ \mathbf{x_2} \\ \mathbf{x_3} \end{bmatrix}$$

and apply it using the same implementation of the *pylops.MatrixMult* operator by simply telling the operator how we want the model to be organized through the `otherdims` input parameter.

```
A = np.array([[1.0, 2.0], [4.0, 5.0]])
x = np.array([[1.0, 1.0], [2.0, 2.0], [3.0, 3.0]])

Aop = pylops.MatrixMult(A, otherdims=(3,), dtype="float64")
y = Aop * x.ravel()

xest, istop, itn, r1norm, r2norm = lsqr(Aop, y, damp=1e-10, iter_lim=10, show=0)[0:5]
xest = xest.reshape(3, 2)

print(f"A= {A}")
print(f"x= {x}")
print(f"y={y}")
print(f"lsqr solution xest= {xest}")
```

```
A= [[1. 2.]
 [4. 5.]]
x= [[1. 1.]
 [2. 2.]
 [3. 3.]]
y=[ 5.  7.  8. 14. 19. 23.]
lsqr solution xest= [[1. 1.]
 [2. 2.]
 [3. 3.]]
```

Total running time of the script: (0 minutes 1.044 seconds)

3.5.21 Multi-Dimensional Convolution

This example shows how to use the *pylops.waveeqprocessing.MDC* operator to convolve a 3D kernel with an input seismic data. The resulting data is a blurred version of the input data and the problem of removing such blurring is reffered to as *Multi-dimensional Deconvolution (MDD)* and its implementation is discussed in more details in the **MDD** tutorial.

```
import matplotlib.pyplot as plt
import numpy as np
```

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```
import pylops
from pylops.utils.seismicevents import hyperbolic2d, makeaxis
from pylops.utils.tapers import taper3d
from pylops.utils.wavelets import ricker

plt.close("all")
```

Let's start by creating a set of hyperbolic events to be used as our MDC kernel

```
# Input parameters
par = {
    "ox": -300,
    "dx": 10,
    "nx": 61,
    "oy": -500,
    "dy": 10,
    "ny": 101,
    "ot": 0,
    "dt": 0.004,
    "nt": 400,
    "f0": 20,
    "nfmax": 200,
}

t0_m = 0.2
vrms_m = 1100.0
amp_m = 1.0

t0_G = (0.2, 0.5, 0.7)
vrms_G = (1200.0, 1500.0, 2000.0)
amp_G = (1.0, 0.6, 0.5)

# Taper
tap = taper3d(par["nt"], (par["ny"], par["nx"]), (5, 5), tapertype="hanning")

# Create axis
t, t2, x, y = makeaxis(par)

# Create wavelet
wav = ricker(t[:41], f0=par["f0"])[0]

# Generate model
m, mwav = hyperbolic2d(x, t, t0_m, vrms_m, amp_m, wav)

# Generate operator
G, Gwav = np.zeros((par["ny"], par["nx"], par["nt"])), np.zeros(
    (par["ny"], par["nx"], par["nt"])
)
for iy, y0 in enumerate(y):
    G[iy], Gwav[iy] = hyperbolic2d(x - y0, t, t0_G, vrms_G, amp_G, wav)
G, Gwav = G * tap, Gwav * tap
```

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```
# Add negative part to data and model
m = np.concatenate((np.zeros((par["nx"], par["nt"] - 1)), m), axis=-1)
mwav = np.concatenate((np.zeros((par["nx"], par["nt"] - 1)), mwav), axis=-1)
Gwav2 = np.concatenate((np.zeros((par["ny"], par["nx"], par["nt"] - 1)), Gwav), axis=-1)

# Define MDC linear operator
Gwav_fft = np.fft.rfft(Gwav2, 2 * par["nt"] - 1, axis=-1)
Gwav_fft = Gwav_fft[..., : par["nfmax"]]

# Move frequency/time to first axis
m, mwav = m.T, mwav.T
Gwav_fft = Gwav_fft.transpose(2, 0, 1)

# Create operator
MDCop = pylops.waveeqprocessing.MDC(
    Gwav_fft,
    nt=2 * par["nt"] - 1,
    nv=1,
    dt=0.004,
    dr=1.0,
)

# Create data
d = MDCop * m.ravel()
d = d.reshape(2 * par["nt"] - 1, par["ny"])

# Apply adjoint operator to data
madj = MDCop.H * d.ravel()
madj = madj.reshape(2 * par["nt"] - 1, par["nx"])
```

Finally let's display the operator, input model, data and adjoint model

```
fig, axs = plt.subplots(1, 2, figsize=(9, 6))
axs[0].imshow(
    Gwav2[int(par["ny"] / 2)].T,
    aspect="auto",
    interpolation="nearest",
    cmap="gray",
    vmin=-Gwav2.max(),
    vmax=Gwav2.max(),
    extent=(x.min(), x.max(), t2.max(), t2.min()),
)
axs[0].set_title("G - inline view", fontsize=15)
axs[0].set_xlabel("r")
axs[1].set_ylabel("t")
axs[1].imshow(
    Gwav2[:, int(par["nx"] / 2)].T,
    aspect="auto",
    interpolation="nearest",
    cmap="gray",
    vmin=-Gwav2.max(),
    vmax=Gwav2.max(),
```

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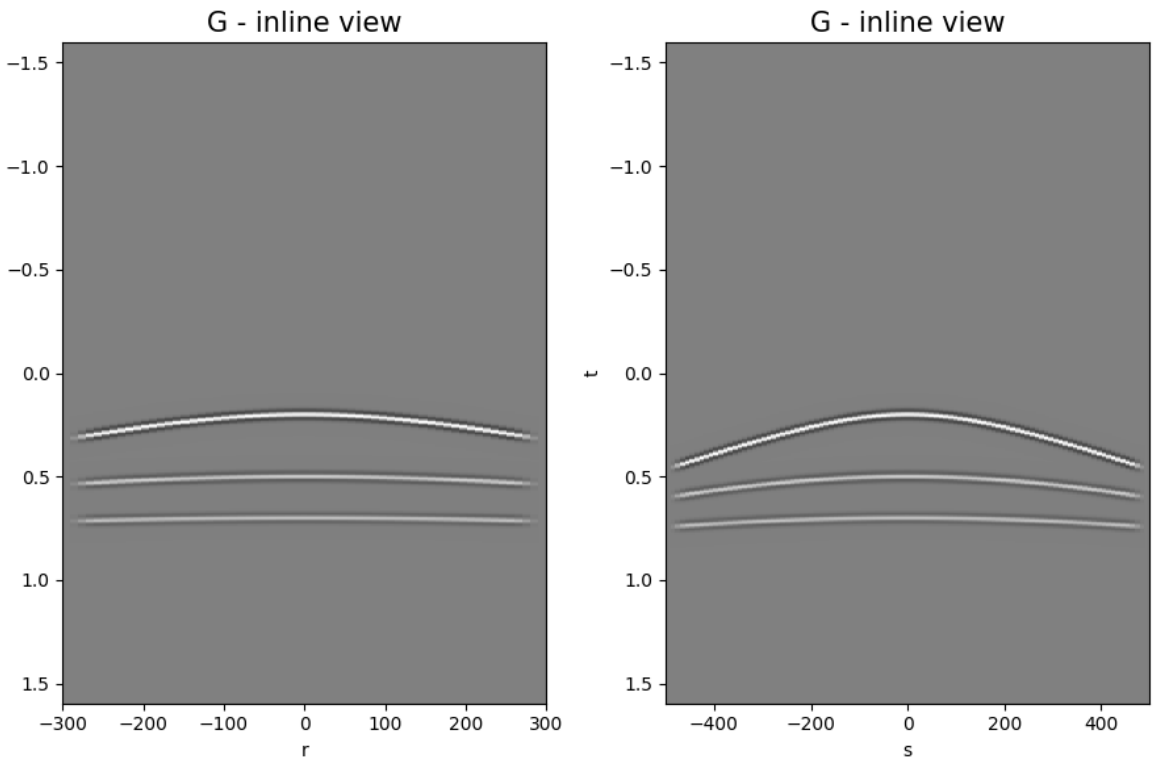
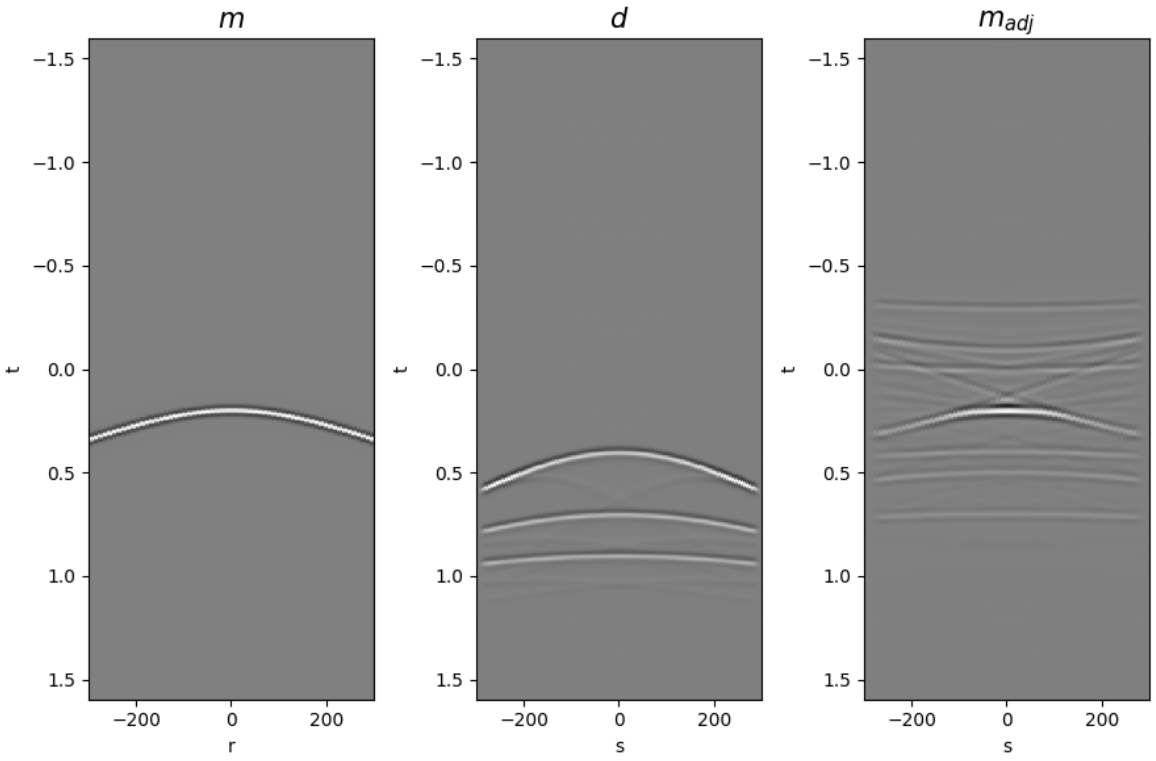
```
    extent=(y.min(), y.max(), t2.max(), t2.min()),
)
axs[1].set_title("G - inline view", fontsize=15)
axs[1].set_xlabel("s")
axs[1].set_ylabel("t")
fig.tight_layout()

fig, axs = plt.subplots(1, 3, figsize=(9, 6))
axs[0].imshow(
    mwav,
    aspect="auto",
    interpolation="nearest",
    cmap="gray",
    vmin=-mwav.max(),
    vmax=mwav.max(),
    extent=(x.min(), x.max(), t2.max(), t2.min()),
)
axs[0].set_title(r"$m$", fontsize=15)
axs[0].set_xlabel("r")
axs[0].set_ylabel("t")
axs[1].imshow(
    d,
    aspect="auto",
    interpolation="nearest",
    cmap="gray",
    vmin=-d.max(),
    vmax=d.max(),
    extent=(x.min(), x.max(), t2.max(), t2.min()),
)
axs[1].set_title(r"$d$", fontsize=15)
axs[1].set_xlabel("s")
axs[1].set_ylabel("t")
axs[2].imshow(
    madj,
    aspect="auto",
    interpolation="nearest",
    cmap="gray",
    vmin=-madj.max(),
    vmax=madj.max(),
    extent=(x.min(), x.max(), t2.max(), t2.min()),
)
axs[2].set_title(r"$m_{adj}$", fontsize=15)
axs[2].set_xlabel("s")
axs[2].set_ylabel("t")
fig.tight_layout()
```





Total running time of the script: (0 minutes 1.161 seconds)

3.5.22 Normal Moveout (NMO) Correction

This example shows how to create your own operator for performing normal moveout (NMO) correction to a seismic record. We will perform classic NMO using an operator created from scratch, as well as using the *pylops.Spread* operator.

```
from math import floor
from time import time

import matplotlib.pyplot as plt
import numpy as np
from mpl_toolkits.axes_grid1 import ImageGrid, make_axes_locatable
from numba import jit, prange
from scipy.interpolate import griddata
from scipy.ndimage import gaussian_filter

from pylops import LinearOperator, Spread
from pylops.utils import dottest
from pylops.utils.decorators import reshaped
from pylops.utils.seismicevents import hyperbolic2d, makeaxis
from pylops.utils.wavelets import ricker


def create_colorbar(im, ax):
    divider = make_axes_locatable(ax)
    cax = divider.append_axes("right", size="5%", pad=0.1)
    cb = fig.colorbar(im, cax=cax, orientation="vertical")
    return cax, cb
```

Given a common-shot or common-midpoint (CMP) record, the objective of NMO correction is to "flatten" events, that is, align events at later offsets to that of the zero offset. NMO has long been a staple of seismic data processing, used even today for initial velocity analysis and QC purposes. In addition, it can be the domain of choice for many useful processing steps, such as angle muting.

To get started, let us create a 2D seismic dataset containing some hyperbolic events representing reflections from flat reflectors. Events are created with a true RMS velocity, which we will be using as if we picked them from, for example, a semblance panel.

```
par = dict(ox=0, dx=40, nx=80, ot=0, dt=0.004, nt=520)
t, _, x, _ = makeaxis(par)

t0s_true = np.array([0.5, 1.22, 1.65])
vrms_true = np.array([2000.0, 2400.0, 2500.0])
amps = np.array([1, 0.2, 0.5])

freq = 10  # Hz
wav, *_ = ricker(t[:41], f0=freq)

_, data = hyperbolic2d(x, t, t0s_true, vrms_true, amp=amps, wav=wav)
```

```
# NMO correction plot
pclip = 0.5
dmax = np.max(np.abs(data))
opts = dict(
```

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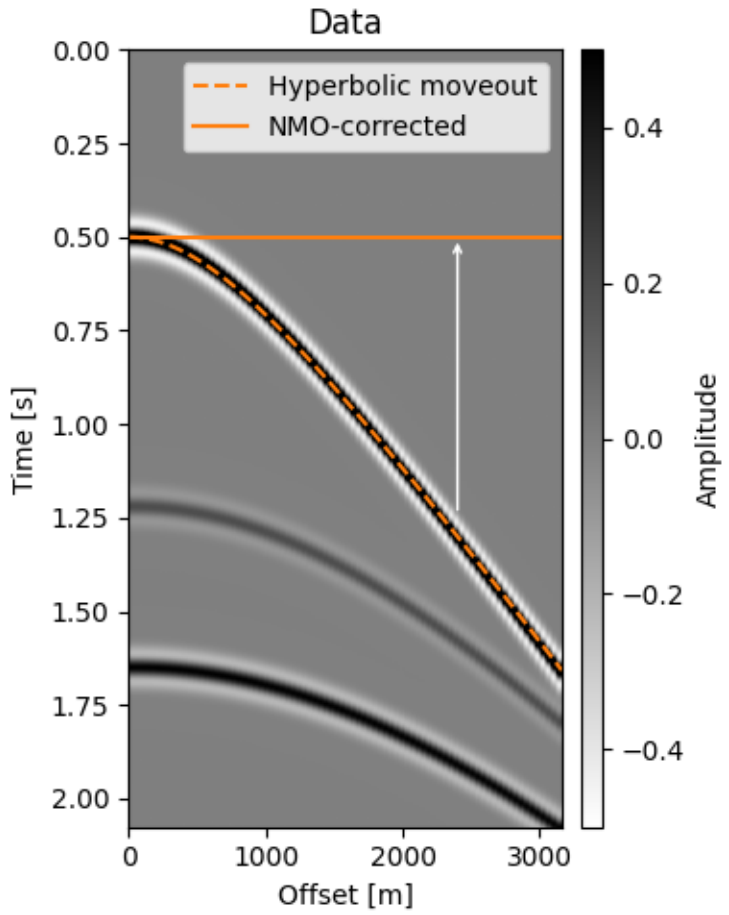
```
    cmap="gray_r",
    extent=[x[0], x[-1], t[-1], t[0]],
    aspect="auto",
    vmin=-pclip * dmax,
    vmax=pclip * dmax,
)

# Offset-dependent traveltime of the first hyperbolic event
t_nmo_ev1 = np.sqrt(t0s_true[0] ** 2 + (x / vrms_true[0]) ** 2)

fig, ax = plt.subplots(figsize=(4, 5))
vmax = np.max(np.abs(data))
im = ax.imshow(data.T, **opts)
ax.plot(x, t_nmo_ev1, "C1--", label="Hyperbolic moveout")
ax.plot(x, t0s_true[0] + x * 0, "C1", label="NMO-corrected")
idx = 3 * par["nx"] // 4
ax.annotate(
    "",
    xy=(x[idx], t0s_true[0]),
    xycoords="data",
    xytext=(x[idx], t_nmo_ev1[idx]),
    textcoords="data",
    fontsize=7,
    arrowprops=dict(edgecolor="w", arrowstyle="->", shrinkA=10),
)
ax.set(title="Data", xlabel="Offset [m]", ylabel="Time [s]")
cax, _ = create_colorbar(im, ax)
cax.set_ylabel("Amplitude")
ax.legend()
fig.tight_layout()
```



NMO correction consists of applying an offset- and time-dependent shift to each sample of the trace in such a way that all events corresponding to the same reflection will be located at the same time intercept after correction.

An arbitrary hyperbolic event at position (t, h) is linked to its zero-offset traveltime t_0 by the following equation

$$t(x) = \sqrt{t_0^2 + \frac{h^2}{v_{\text{rms}}^2(t_0)}}$$

Our strategy in applying the correction is to loop over our time axis (which we will associate to t_0) and respective RMS velocities and, for each offset, move the sample at $t(x)$ to location $t_0(x) \equiv t_0$. In the figure above, we are considering a single $t_0 = 0.5$s which would have values along the dotted curve (i.e., $t(x)$) moved to t_0 for every offset.

Notice that we need NMO velocities for each sample of our time axis. In this example, we actually only have 3 samples, when we need `nt` samples. In practice, we would have many more samples, but probably not one for each `nt`. To resolve this issue, we will interpolate these 3 samples to all samples of our time axis (or, more accurately, their slownesses to preserve traveltimes).

```
def interpolate_vrms(t0_picks, vrms_picks, taxis, smooth=None):
    assert len(t0_picks) == len(vrms_picks)

    # Sampled points in time axis
    points = np.zeros((len(t0_picks) + 2,))
    points[0] = taxis[0]
    points[-1] = taxis[-1]
```

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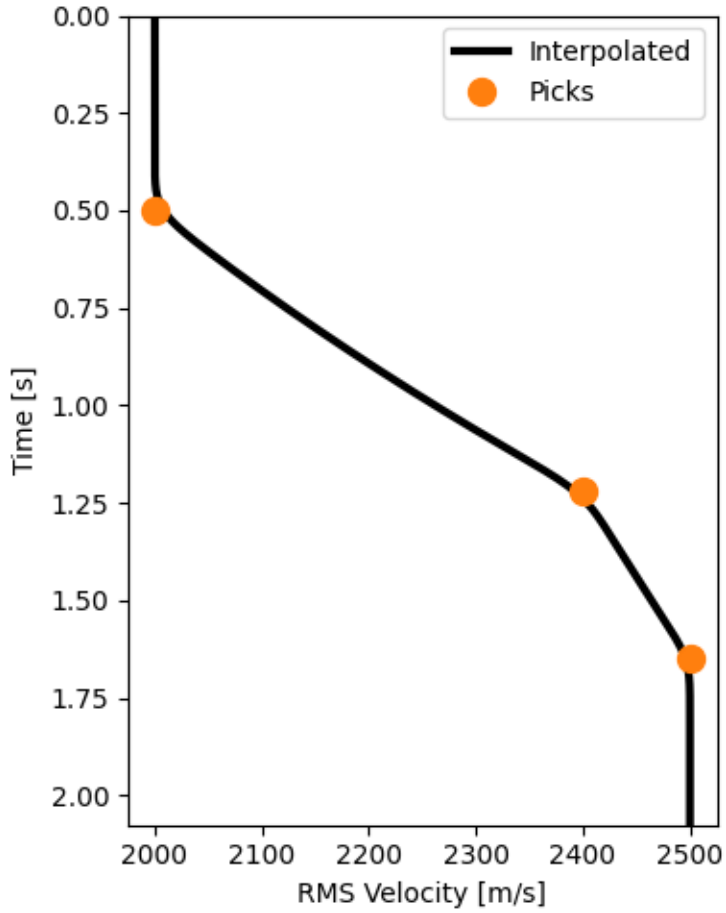
```
    points[1:-1] = t0_picks

    # Sampled values of slowness (in s/km)
    values = np.zeros((len(vrms_picks) + 2,))
    values[0] = 1000.0 / vrms_picks[0]  # Use first slowness before t0_picks[0]
    values[-1] = 1000.0 / vrms_picks[-1]  # Use the last slowness after t0_picks[-1]
    values[1:-1] = 1000.0 / vrms_picks

    slowness = griddata(points, values, taxis, method="linear")
    if smooth is not None:
        slowness = gaussian_filter(slowness, sigma=smooth)

    return 1000.0 / slowness


vel_t = interpolate_vrms(t0s_true, vrms_true, t, smooth=11)
```

```
# Plot interpolated RMS velocities which will be used for NMO
fig, ax = plt.subplots(figsize=(4, 5))
ax.plot(vel_t, t, "k", lw=3, label="Interpolated", zorder=-1)
ax.plot(vrms_true, t0s_true, "C1o", markersize=10, label="Picks")
ax.invert_yaxis()
ax.set(xlabel="RMS Velocity [m/s]", ylabel="Time [s]", ylim=[t[-1], t[0]])
ax.legend()
fig.tight_layout()
```



NMO from scratch

We are very close to building our NMO correction, we just need to take care of one final issue. When moving the sample from $t(x)$ to t_0, we know that, by definition, t_0 is always on our time axis grid. In contrast, $t(x)$ may not fall exactly on a multiple of `dt` (our time axis sampling). Suppose its nearest sample smaller than itself (floor) is `i`. Instead of moving only sample *i*, we will be moving samples both samples `i` and `i+1` with an appropriate weight to account for how far $t(x)$ is from `i*dt` and `(i+1)*dt`.

```
@jit(nopython=True, fastmath=True, nogil=True, parallel=True)
def nmo_forward(data, taxis, haxis, vels_rms):
    dt = taxis[1] - taxis[0]
    ot = taxis[0]
    nt = len(taxis)
    nh = len(haxis)

    dnmo = np.zeros_like(data)

    # Parallel outer loop on slow axis
    for ih in prange(nh):
        h = haxis[ih]
        for it0, (t0, vrms) in enumerate(zip(taxis, vels_rms)):
```

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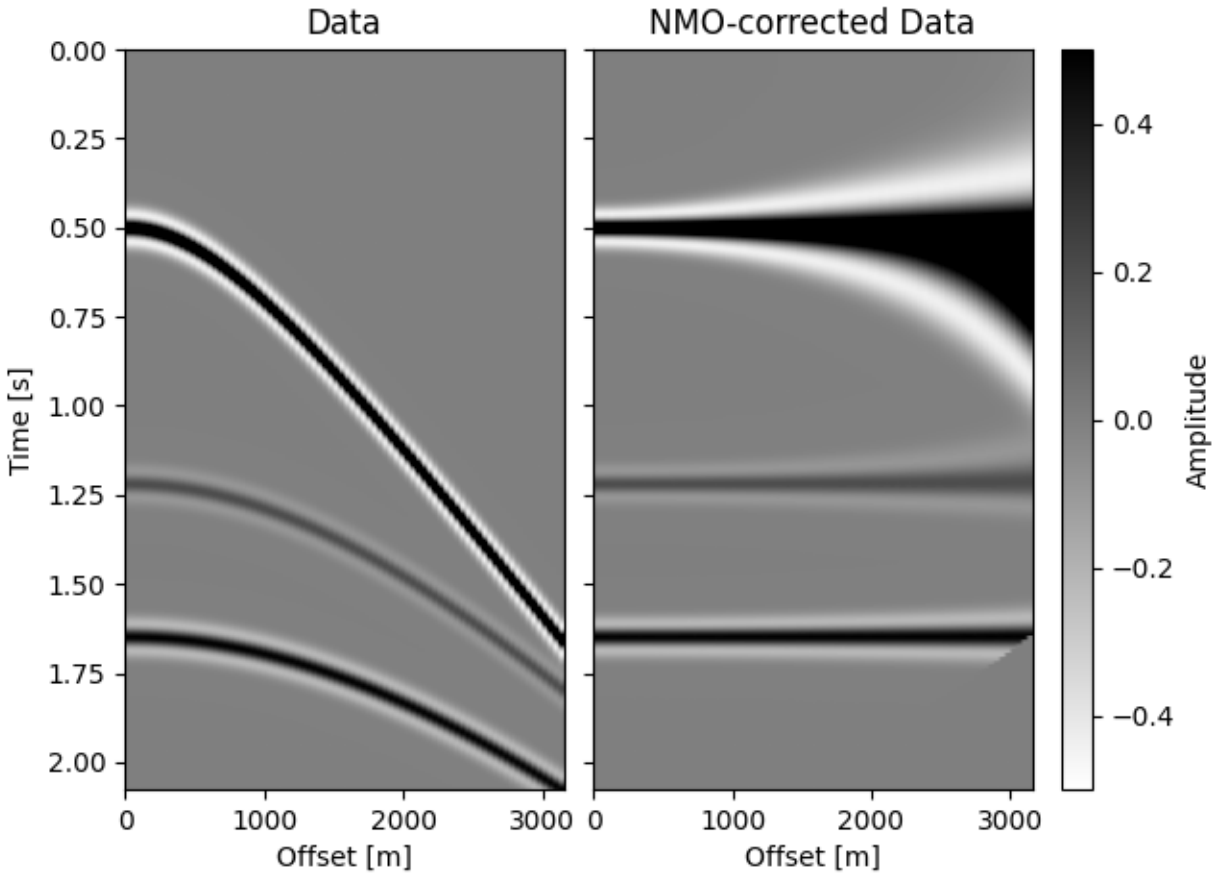
```
            # Compute NMO traveltime
            tx = np.sqrt(t0**2 + (h / vrms) ** 2)
            it_frac = (tx - ot) / dt  # Fractional index
            it_floor = floor(it_frac)
            it_ceil = it_floor + 1
            w = it_frac - it_floor
            if 0 <= it_floor and it_ceil < nt:  # it_floor and it_ceil must be valid
                # Linear interpolation
                dnmo[ih, it0] += (1 - w) * data[ih, it_floor] + w * data[ih, it_ceil]
    return dnmo


dnmo = nmo_forward(data, t, x, vel_t)  # Compile and run

# Time execution
start = time()
nmo_forward(data, t, x, vel_t)
end = time()

print(f"Ran in {1e6*(end-start):.0f} s")
```

```
Ran in 325 s
```

```
# Plot Data and NMO-corrected data
fig = plt.figure(figsize=(6.5, 5))
grid = ImageGrid(
    fig,
    111,
    nrows_ncols=(1, 2),
    axes_pad=0.15,
    cbar_location="right",
    cbar_mode="single",
    cbar_size="7%",
    cbar_pad=0.15,
    aspect=False,
    share_all=True,
)
im = grid[0].imshow(data.T, **opts)
grid[0].set(title="Data", xlabel="Offset [m]", ylabel="Time [s]")
grid[0].cax.colorbar(im)
grid[0].cax.set_ylabel("Amplitude")

grid[1].imshow(dnmo.T, **opts)
grid[1].set(title="NMO-corrected Data", xlabel="Offset [m]")
plt.show()
```



Now that we know how to compute the forward, we'll compute the adjoint pass. With these two functions, we can create a `LinearOperator` and ensure that it passes the dot-test.

```
@jit(nopython=True, fastmath=True, nogil=True, parallel=True)
def nmo_adjoint(dnmo, taxis, haxis, vels_rms):
    dt = taxis[1] - taxis[0]
    ot = taxis[0]
    nt = len(taxis)
    nh = len(haxis)

    data = np.zeros_like(dnmo)

    # Parallel outer loop on slow axis; use range if Numba is not installed
    for ih in prange(nh):
        h = haxis[ih]
        for it0, (t0, vrms) in enumerate(zip(taxis, vels_rms)):
            # Compute NMO traveltime
            tx = np.sqrt(t0**2 + (h / vrms) ** 2)
            it_frac = (tx - ot) / dt  # Fractional index
            it_floor = floor(it_frac)
            it_ceil = it_floor + 1
            w = it_frac - it_floor
            if 0 <= it_floor and it_ceil < nt:
```

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```
                # Linear interpolation
                # In the adjoint, we must spread the same it0 to both it_floor and
                # it_ceil, since in the forward pass, both of these samples were
                # pushed onto it0
                data[ih, it_floor] += (1 - w) * dnmo[ih, it0]
                data[ih, it_ceil] += w * dnmo[ih, it0]
    return data
```

Finally, we can create our linear operator. To exemplify the class-based interface we will subclass *pylops.LinearOperator* and implement the required methods: `_matvec` which will compute the forward and `_rmatvec` which will compute the adjoint. Note the use of the `reshaped` decorator which allows us to pass `x` directly into our auxiliary function without having to do `x.reshape(self.dims)` and to output without having to call `ravel()`.

```
class NMO(LinearOperator):
    def __init__(self, taxis, haxis, vels_rms, dtype=None):
        self.taxis = taxis
        self.haxis = haxis
        self.vels_rms = vels_rms

        dims = (len(haxis), len(taxis))
        if dtype is None:
            dtype = np.result_type(taxis.dtype, haxis.dtype, vels_rms.dtype)
        super().__init__(dims=dims, dimsd=dims, dtype=dtype)

    @reshaped
    def _matvec(self, x):
        return nmo_forward(x, self.taxis, self.haxis, self.vels_rms)

    @reshaped
    def _rmatvec(self, y):
        return nmo_adjoint(y, self.taxis, self.haxis, self.vels_rms)
```

With our new `NMO` linear operator, we can instantiate it with our current example and ensure that it passes the dot test which proves that our forward and adjoint transforms truly are adjoints of each other.

```
NMOOp = NMO(t, x, vel_t)
dottest(NMOOp, rtol=1e-4, verb=True)
```

```
Dot test passed, v^H(Opu)=-172.4466288315369 - u^H(Op^Hv)=-172.44662883153708

True
```

NMO using `pylops.Spread`

We learned how to implement an NMO correction and its adjoint from scratch. The adjoint has an interesting pattern, where energy taken from one domain is "spread" along a previously-defined parametric curve (the NMO hyperbola in this case). This pattern is very common in many algorithms, including Radon transform, Kirchhoff migration (also known as Total Focusing Method in ultrasonics) and many others.

For these classes of operators, PyLops offers a *pylops.Spread* constructor, which we will leverage to implement a version of the NMO correction. The *pylops.Spread* operator will take a value in the "input" domain, and spread it along a parametric curve, defined in the "output" domain. In our case, the spreading operation is the *adjoint* of the NMO, so our "input" domain is the NMO domain, and the "output" domain is the original data domain.

In order to use *pylops.Spread*, we need to define the parametric curves. This can be done through the use of a table with shape (n_{x_i}, n_t, n_{x_o}), where n_{x_i} and n_t represent the 2d dimensions of the "input" domain (NMO domain) and n_{x_o} and n_t the 2d dimensions of the "output" domain. In our NMO case, $n_{x_i} = n_{x_o} = n_h$ represents the number of offsets. Following the documentation of *pylops.Spread*, the table will be used in the following manner:

```
d_out[ix_o, table[ix_i, it, ix_o]] += d_in[ix_i, it]
```

In our case, `ix_o = ix_i = ih`, and comparing with our NMO adjoint, `it` refers to t_0 while `table[ix, it, ix]` should then provide the appropriate index for $t(x)$. In our implementation we will also be constructing a second table containing the weights to be used for linear interpolation.

```
def create_tables(taxis, haxis, vels_rms):
    dt = taxis[1] - taxis[0]
    ot = taxis[0]
    nt = len(taxis)
    nh = len(haxis)

    # NaN values will be not be spread.
    # Using np.zeros has the same result but much slower.
    table = np.full((nh, nt, nh), fill_value=np.nan)
    dtable = np.full((nh, nt, nh), fill_value=np.nan)

    for ih, h in enumerate(haxis):
        for it0, (t0, vrms) in enumerate(zip(taxis, vels_rms)):
            # Compute NMO traveltime
            tx = np.sqrt(t0**2 + (h / vrms) ** 2)
            it_frac = (tx - ot) / dt
            it_floor = floor(it_frac)
            w = it_frac - it_floor
            # Both it_floor and it_floor + 1 must be valid indices for taxis
            # when using two tables (interpolation).
            if 0 <= it_floor and it_floor + 1 < nt:
                table[ih, it0, ih] = it_floor
                dtable[ih, it0, ih] = w
    return table, dtable


nmo_table, nmo_dtable = create_tables(t, x, vel_t)
```

```
SpreadNMO = Spread(
    dims=data.shape,  # "Input" shape: NMO-ed data shape
    dimsd=data.shape,  # "Output" shape: original data shape
    table=nmo_table,  # Table of time indices
```

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```
    dtable=nmo_dtable,  # Table of weights for linear interpolation
    engine="numba",  # numba or numpy
).H  # To perform NMO *correction*, we need the adjoint
dottest(SpreadNMO, rtol=1e-4)
```

```
True
```

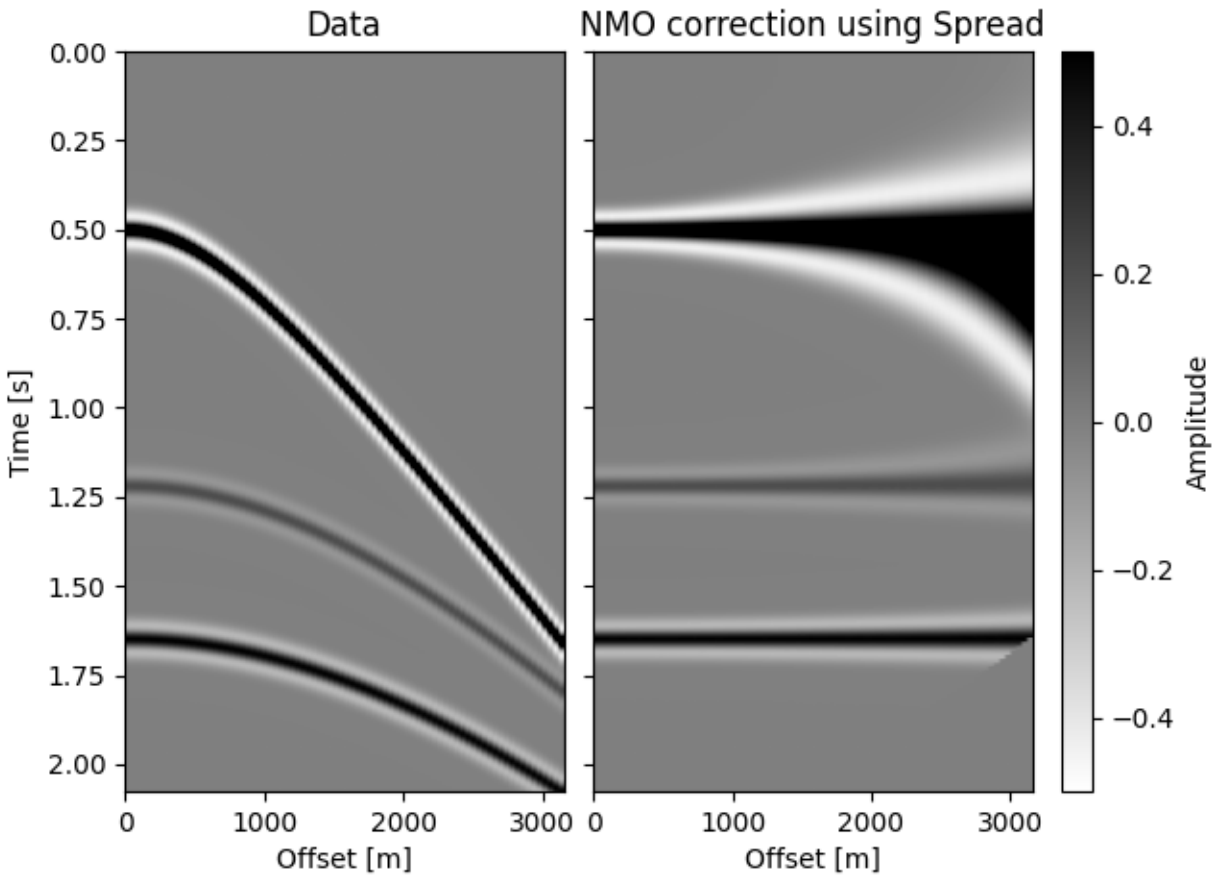
We see it passes the dot test, but are the results right? Let's find out.

```
dnmo_spr = SpreadNMO @ data

start = time()
SpreadNMO @ data
end = time()

print(f"Ran in {1e6*(end-start):.0f} s")
```

```
Ran in 14426 s
```

Note that since v2.0, we do not need to pass a flattened array. Consequently, the output will not be flattened, but will have `SpreadNMO.dimsd` as shape.

```
# Plot Data and NMO-corrected data
fig = plt.figure(figsize=(6.5, 5))
grid = ImageGrid(
    fig,
    111,
    nrows_ncols=(1, 2),
    axes_pad=0.15,
    cbar_location="right",
    cbar_mode="single",
    cbar_size="7%",
    cbar_pad=0.15,
    aspect=False,
    share_all=True,
)
im = grid[0].imshow(data.T, **opts)
grid[0].set(title="Data", xlabel="Offset [m]", ylabel="Time [s]")
grid[0].cax.colorbar(im)
grid[0].cax.set_ylabel("Amplitude")

grid[1].imshow(dnmo_spr.T, **opts)
grid[1].set(title="NMO correction using Spread", xlabel="Offset [m]")
plt.show()
```



Not as blazing fast as out original implementation, but pretty good (try the "numpy" backend for comparison!). In fact, using the `Spread` operator for NMO will always have a speed disadvantage. While iterating over the table, it must loop over the offsets twice: one for the "input" offsets and one for the "output" offsets. We know they are the same for NMO, but since `Spread` is a generic operator, it does not know that. So right off the bat we can expect an 80x slowdown (nh = 80). We diminished this cost to about 30x by setting values where `ix_i != ix_o` to NaN, but nothing beats the custom implementation. Despite this, we can still produce the same result to numerical accuracy:

```
np.allclose(dnmo, dnmo_spr)
```

```
True
```

Total running time of the script: (0 minutes 3.968 seconds)

3.5.23 Operators concatenation

This example shows how to use 'stacking' operators such as *pylops.VStack*, *pylops.HStack*, *pylops.Block*, *pylops.BlockDiag*, and *pylops.Kronecker*.

These operators allow for different combinations of multiple linear operators in a single operator. Such functionalities are used within PyLops as the basis for the creation of complex operators as well as in the definition of various types of optimization problems with regularization or preconditioning.

Some of this operators naturally lend to embarassingly parallel computations. Within PyLops we leverage the multiprocessing module to run multiple processes at the same time evaluating a subset of the operators involved in one of the stacking operations.

```
import matplotlib.pyplot as plt
import numpy as np

import pylops

plt.close("all")
```

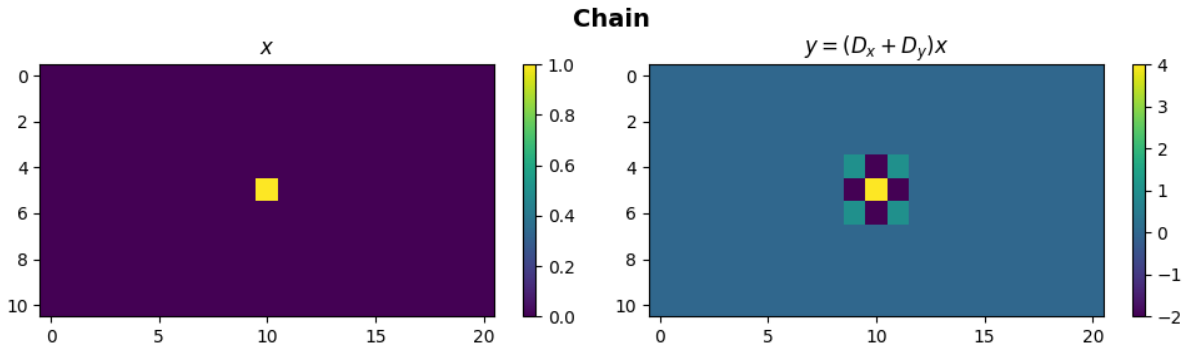
Let's start by defining two second derivatives *pylops.SecondDerivative* that we will be using in this example.

```
D2hop = pylops.SecondDerivative(dims=(11, 21), axis=1, dtype="float32")
D2vop = pylops.SecondDerivative(dims=(11, 21), axis=0, dtype="float32")
```

Chaining of operators represents the simplest concatenation that can be performed between two or more linear operators. This can be easily achieved using the * symbol

$$\mathbf{D_{cat}} = \mathbf{D_v D_h}$$

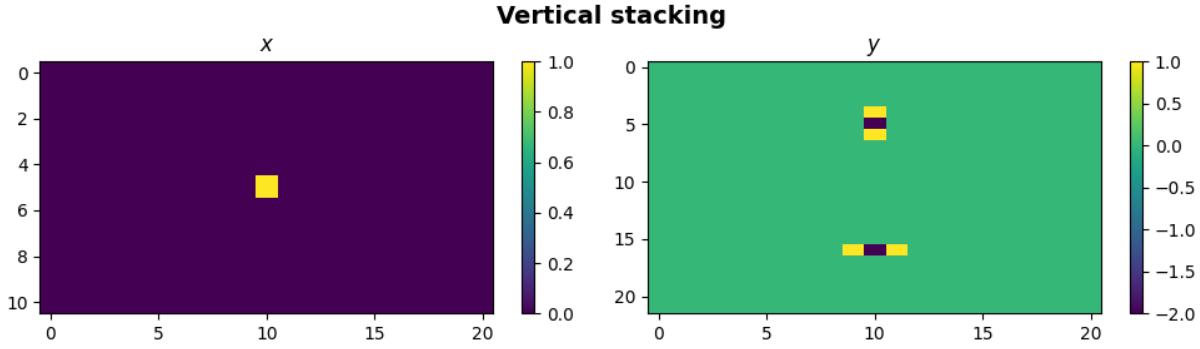
```
Nv, Nh = 11, 21
X = np.zeros((Nv, Nh))
X[int(Nv / 2), int(Nh / 2)] = 1

D2op = D2vop * D2hop
Y = D2op * X

fig, axs = plt.subplots(1, 2, figsize=(10, 3))
fig.suptitle("Chain", fontsize=14, fontweight="bold", y=0.95)
im = axs[0].imshow(X, interpolation="nearest")
axs[0].axis("tight")
axs[0].set_title(r"$x$")
plt.colorbar(im, ax=axs[0])
im = axs[1].imshow(Y, interpolation="nearest")
axs[1].axis("tight")
axs[1].set_title(r"$y=(D_x+D_y) x$")
plt.colorbar(im, ax=axs[1])
plt.tight_layout()
plt.subplots_adjust(top=0.8)
```



We now want to *vertically stack* three operators

$$\mathbf{D_{Vstack}} = \begin{bmatrix} \mathbf{D_v} \\ \mathbf{D_h} \end{bmatrix}, \qquad \mathbf{y} = \begin{bmatrix} \mathbf{D_v x} \\ \mathbf{D_h x} \end{bmatrix}$$

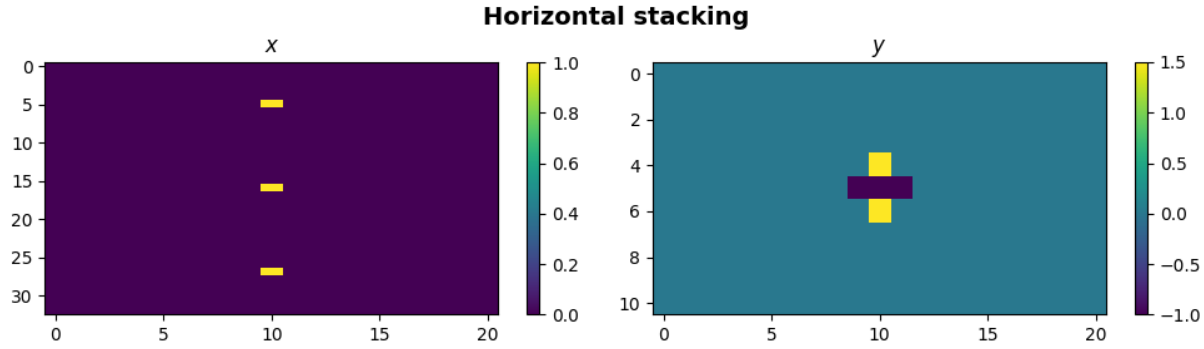
```
Nv, Nh = 11, 21
X = np.zeros((Nv, Nh))
X[int(Nv / 2), int(Nh / 2)] = 1
Dstack = pylops.VStack([D2vop, D2hop])

Y = np.reshape(Dstack * X.ravel(), (Nv * 2, Nh))

fig, axs = plt.subplots(1, 2, figsize=(10, 3))
fig.suptitle("Vertical stacking", fontsize=14, fontweight="bold", y=0.95)
im = axs[0].imshow(X, interpolation="nearest")
axs[0].axis("tight")
axs[0].set_title(r"$x$")
plt.colorbar(im, ax=axs[0])
im = axs[1].imshow(Y, interpolation="nearest")
axs[1].axis("tight")
axs[1].set_title(r"$y$")
plt.colorbar(im, ax=axs[1])
plt.tight_layout()
plt.subplots_adjust(top=0.8)
```



Similarly we can now *horizontally stack* three operators

$$\mathbf{D_{Hstack}} = \begin{bmatrix} \mathbf{D_v} & 0.5 * \mathbf{D_v} & -1 * \mathbf{D_h} \end{bmatrix}, \qquad \mathbf{y} = \mathbf{D_v x_1} + 0.5 * \mathbf{D_v x_2} - \mathbf{D_h x_3}$$

```
Nv, Nh = 11, 21
X = np.zeros((Nv * 3, Nh))
X[int(Nv / 2), int(Nh / 2)] = 1
X[int(Nv / 2) + Nv, int(Nh / 2)] = 1
X[int(Nv / 2) + 2 * Nv, int(Nh / 2)] = 1

Hstackop = pylops.HStack([D2vop, 0.5 * D2vop, -1 * D2hop])
Y = np.reshape(Hstackop * X.ravel(), (Nv, Nh))

fig, axs = plt.subplots(1, 2, figsize=(10, 3))
fig.suptitle("Horizontal stacking", fontsize=14, fontweight="bold", y=0.95)
im = axs[0].imshow(X, interpolation="nearest")
axs[0].axis("tight")
axs[0].set_title(r"$x$")
plt.colorbar(im, ax=axs[0])
im = axs[1].imshow(Y, interpolation="nearest")
axs[1].axis("tight")
axs[1].set_title(r"$y$")
plt.colorbar(im, ax=axs[1])
plt.tight_layout()
plt.subplots_adjust(top=0.8)
```



We can even stack them both *horizontally* and *vertically* such that we create a *block* operator

$$\mathbf{D_{Block}} = \begin{bmatrix} \mathbf{D_v} & 0.5 * \mathbf{D_v} & -1 * \mathbf{D_h} \\ \mathbf{D_h} & 2 * \mathbf{D_h} & \mathbf{D_v} \end{bmatrix}, \qquad \mathbf{y} = \begin{bmatrix} \mathbf{D_v x_1} + 0.5 * \mathbf{D_v x_2} - \mathbf{D_h x_3} \\ \mathbf{D_h x_1} + 2 * \mathbf{D_h x_2} + \mathbf{D_v x_3} \end{bmatrix}$$

```
Bop = pylops.Block([[D2vop, 0.5 * D2vop, -1 * D2hop], [D2hop, 2 * D2hop, D2vop]])
Y = np.reshape(Bop * X.ravel(), (2 * Nv, Nh))

fig, axs = plt.subplots(1, 2, figsize=(10, 3))
fig.suptitle("Block", fontsize=14, fontweight="bold", y=0.95)
im = axs[0].imshow(X, interpolation="nearest")
axs[0].axis("tight")
axs[0].set_title(r"$x$")
```

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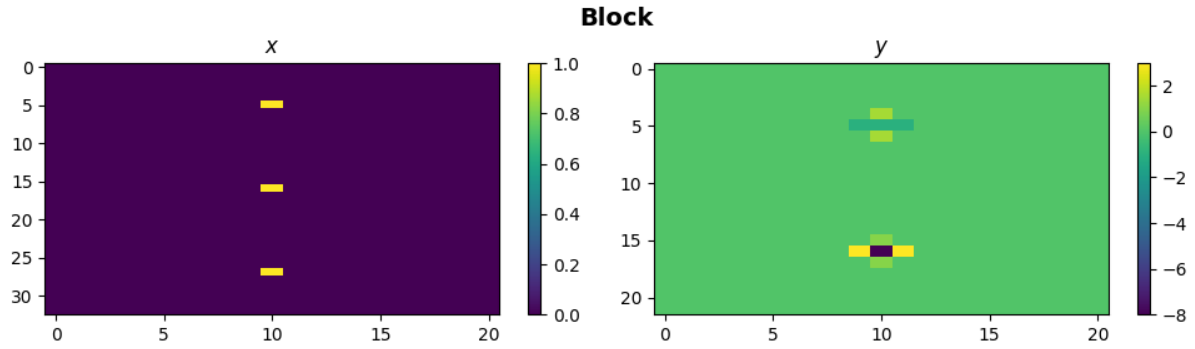
```
plt.colorbar(im, ax=axs[0])
im = axs[1].imshow(Y, interpolation="nearest")
axs[1].axis("tight")
axs[1].set_title(r"$y$")
plt.colorbar(im, ax=axs[1])
plt.tight_layout()
plt.subplots_adjust(top=0.8)
```

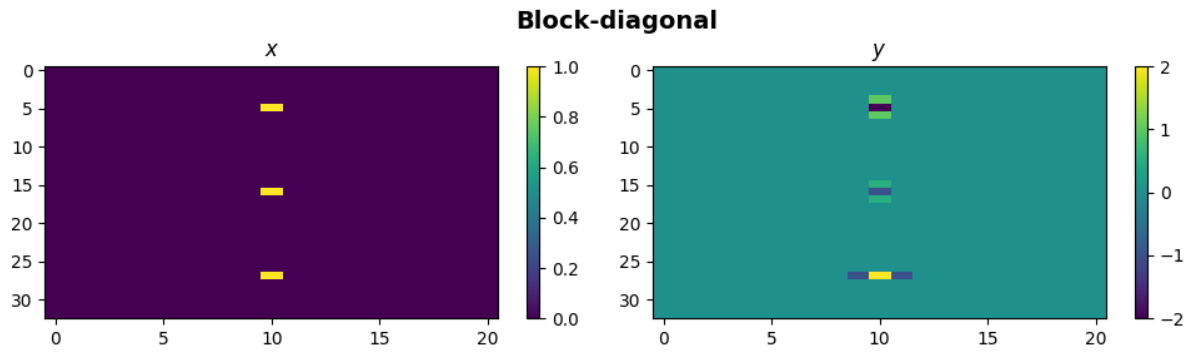


Finally we can use the *block-diagonal operator* to apply three operators to three different subset of the model and data

$$\mathbf{D_{BDiag}} = \begin{bmatrix} \mathbf{D_v} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & 0.5 * \mathbf{D_v} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & -\mathbf{D_h} \end{bmatrix}, \quad \mathbf{y} = \begin{bmatrix} \mathbf{D_v x_1} \\ 0.5 * \mathbf{D_v x_2} \\ -\mathbf{D_h x_3} \end{bmatrix}$$

```
BD = pylops.BlockDiag([D2vop, 0.5 * D2vop, -1 * D2hop])
Y = np.reshape(BD * X.ravel(), (3 * Nv, Nh))

fig, axs = plt.subplots(1, 2, figsize=(10, 3))
fig.suptitle("Block-diagonal", fontsize=14, fontweight="bold", y=0.95)
im = axs[0].imshow(X, interpolation="nearest")
axs[0].axis("tight")
axs[0].set_title(r"$x$")
plt.colorbar(im, ax=axs[0])
im = axs[1].imshow(Y, interpolation="nearest")
axs[1].axis("tight")
axs[1].set_title(r"$y$")
plt.colorbar(im, ax=axs[1])
plt.tight_layout()
plt.subplots_adjust(top=0.8)
```

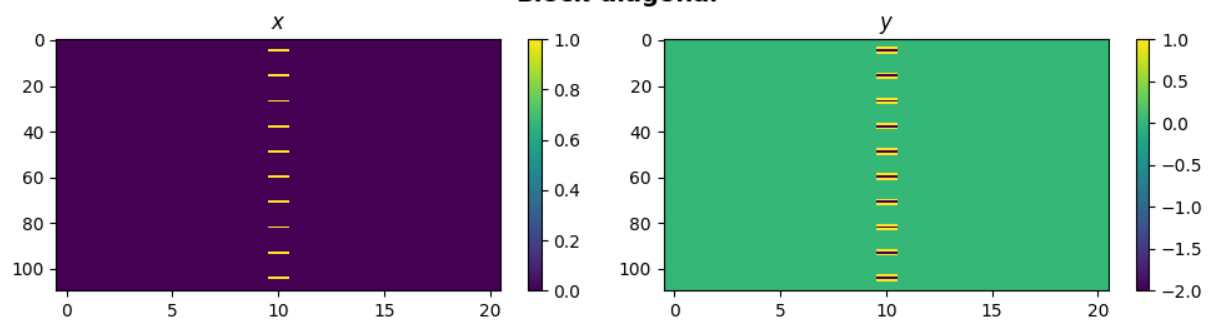


If we consider now the case of having a large number of operators inside a blockdiagonal structure, it may be convenient to span multiple processes handling subset of operators at the same time. This can be easily achieved by simply defining the number of processes we want to use via `nproc`.

```
X = np.zeros((Nv * 10, Nh))
for iv in range(10):
    X[int(Nv / 2) + iv * Nv, int(Nh / 2)] = 1

BD = pylops.BlockDiag([D2vop] * 10, nproc=2)
print("BD Operator multiprocessing pool", BD.pool)
Y = np.reshape(BD * X.ravel(), (10 * Nv, Nh))
BD.pool.close()

fig, axs = plt.subplots(1, 2, figsize=(10, 3))
fig.suptitle("Block-diagonal", fontsize=14, fontweight="bold", y=0.95)
im = axs[0].imshow(X, interpolation="nearest")
axs[0].axis("tight")
axs[0].set_title(r"$x$")
plt.colorbar(im, ax=axs[0])
im = axs[1].imshow(Y, interpolation="nearest")
axs[1].axis("tight")
axs[1].set_title(r"$y$")
plt.colorbar(im, ax=axs[1])
plt.tight_layout()
plt.subplots_adjust(top=0.8)
```




```
BD Operator multiprocessing pool <multiprocessing.pool.Pool state=RUN pool_size=2>
```

Finally we use the *Kronecker operator* and replicate this example on wiki.

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \otimes \begin{bmatrix} 0 & 5 \\ 6 & 7 \end{bmatrix} = \begin{bmatrix} 0 & 5 & 0 & 10 \\ 6 & 7 & 12 & 14 \\ 0 & 15 & 0 & 20 \\ 18 & 21 & 24 & 28 \end{bmatrix}$$

```
A = np.array([[1, 2], [3, 4]])
B = np.array([[0, 5], [6, 7]])
AB = np.kron(A, B)

n1, m1 = A.shape
n2, m2 = B.shape

Aop = pylops.MatrixMult(A)
Bop = pylops.MatrixMult(B)

ABop = pylops.Kronecker(Aop, Bop)
x = np.ones(m1 * m2)

y = AB.dot(x)
yop = ABop * x
xinv = ABop / yop

print(f"AB = \n {AB}")

print(f"x = {x}")
print(f"y = {y}")
print(f"yop = {yop}")
print(f"xinv = {xinv}")
```

```
AB =
 [[ 0  5  0 10]
 [ 6  7 12 14]
 [ 0 15  0 20]
 [18 21 24 28]]
x = [1. 1. 1. 1.]
y = [15. 39. 35. 91.]
yop = [15. 39. 35. 91.]
xinv = [1. 1. 1. 1.]
```

We can also use *pylops.Kronecker* to do something more interesting. Any operator can in fact be applied on a single direction of a multi-dimensional input array if combined with an *pylops.Identity* operator via Kronecker product. We apply here the *pylops.FirstDerivative* to the second dimension of the model.

Note that for those operators whose implementation allows their application to a single axis via the `axis` parameter, using the Kronecker product would lead to slower performance. Nevertheless, the Kronecker product allows any other operator to be applied to a single dimension.

```
Nv, Nh = 11, 21

Iop = pylops.Identity(Nv, dtype="float32")
D2hop = pylops.FirstDerivative(Nh, dtype="float32")
```

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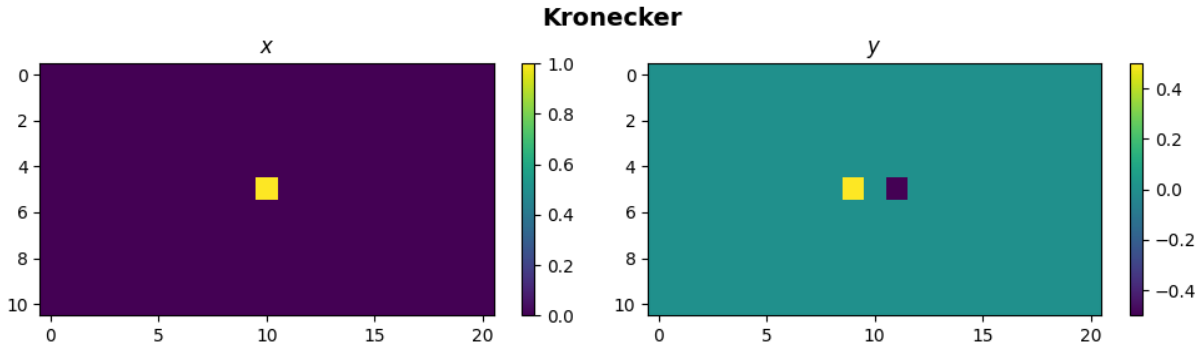
```
X = np.zeros((Nv, Nh))
X[Nv // 2, Nh // 2] = 1
D2hop = pylops.Kronecker(Iop, D2hop)

Y = D2hop * X.ravel()
Y = Y.reshape(Nv, Nh)

fig, axs = plt.subplots(1, 2, figsize=(10, 3))
fig.suptitle("Kronecker", fontsize=14, fontweight="bold", y=0.95)
im = axs[0].imshow(X, interpolation="nearest")
axs[0].axis("tight")
axs[0].set_title(r"$x$")
plt.colorbar(im, ax=axs[0])
im = axs[1].imshow(Y, interpolation="nearest")
axs[1].axis("tight")
axs[1].set_title(r"$y$")
plt.colorbar(im, ax=axs[1])
plt.tight_layout()
plt.subplots_adjust(top=0.8)
```



Total running time of the script: (0 minutes 2.387 seconds)

3.5.24 Operators with Multiprocessing

This example shows how perform a scalability test for one of PyLops operators that uses `multiprocessing` to spawn multiple processes. Operators that support such feature are `pylops.basicoperators.VStack`, `pylops.basicoperators.HStack`, and `pylops.basicoperators.BlockDiagonal`, and `pylops.basicoperators.Block`.

In this example we will consider the BlockDiagonal operator which contains `pylops.basicoperators.MatrixMult` operators along its main diagonal.

```
import matplotlib.pyplot as plt
import numpy as np

import pylops

plt.close("all")
```

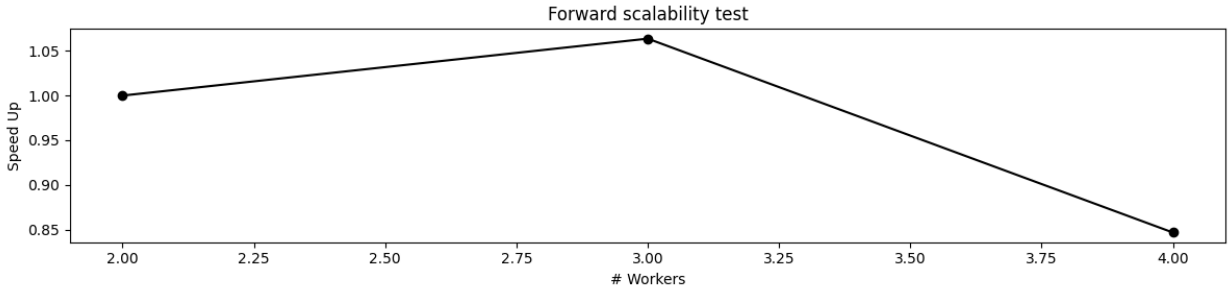
Let's start by creating N MatrixMult operators and the BlockDiag operator

```
N = 100
Nops = 32
Ops = [pylops.MatrixMult(np.random.normal(0.0, 1.0, (N, N))) for _ in range(Nops)]

Op = pylops.BlockDiag(Ops, nproc=1)
```

We can now perform a scalability test on the forward operation

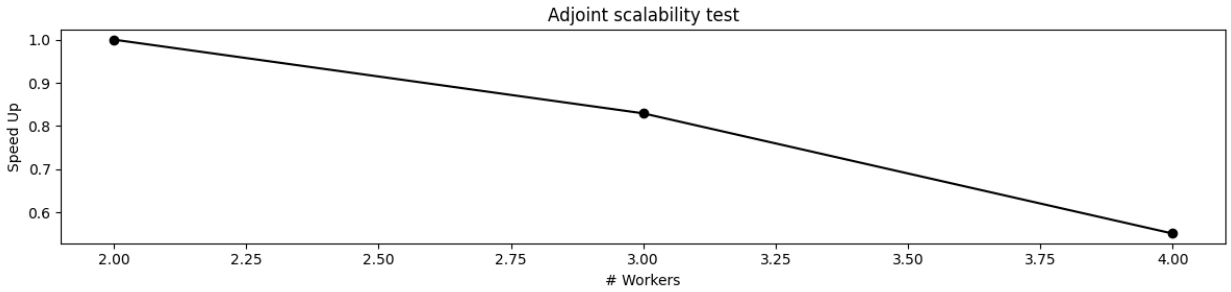
```
workers = [2, 3, 4]
compute_times, speedup = pylops.utils.multiproc.scalability_test(
    Op, np.ones(Op.shape[1]), workers=workers, forward=True
)
plt.figure(figsize=(12, 3))
plt.plot(workers, speedup, "ko-")
plt.xlabel("# Workers")
plt.ylabel("Speed Up")
plt.title("Forward scalability test")
plt.tight_layout()
```




```
Working with 2 workers...
Working with 3 workers...
Working with 4 workers...
```

And likewise on the adjoint operation

```
compute_times, speedup = pylops.utils.multiproc.scalability_test(
    Op, np.ones(Op.shape[0]), workers=workers, forward=False
)
plt.figure(figsize=(12, 3))
plt.plot(workers, speedup, "ko-")
plt.xlabel("# Workers")
plt.ylabel("Speed Up")
plt.title("Adjoint scalability test")
plt.tight_layout()
```




```
Working with 2 workers...
Working with 3 workers...
Working with 4 workers...
```

Note that we have not tested here the case with 1 worker. In this specific case, since the computations are very small, the overhead of spawning processes is actually dominating the time of computations and so computing the forward and adjoint operations with a single worker is more efficient. We hope that this example can serve as a basis to inspect the scalability of multiprocessing-enabled operators and choose the best number of processes.

Total running time of the script: (0 minutes 0.873 seconds)

3.5.25 Padding

This example shows how to use the *pylops.Pad* operator to zero-pad a model

```
import matplotlib.pyplot as plt
import numpy as np

import pylops

plt.close("all")
```

Let's define a pad operator Pop for one dimensional data

```
dims = 10
pad = (2, 3)

Pop = pylops.Pad(dims, pad)

x = np.arange(dims) + 1.0
y = Pop * x
xadj = Pop.H * y

print(f"x = {x}")
print(f"P*x = {y}")
print(f"P'*y = {xadj}")
```

```
x = [ 1.  2.  3.  4.  5.  6.  7.  8.  9. 10.]
P*x = [ 0.  0.  1.  2.  3.  4.  5.  6.  7.  8.  9. 10.  0.  0.  0.]
P'*y = [ 1.  2.  3.  4.  5.  6.  7.  8.  9. 10.]
```

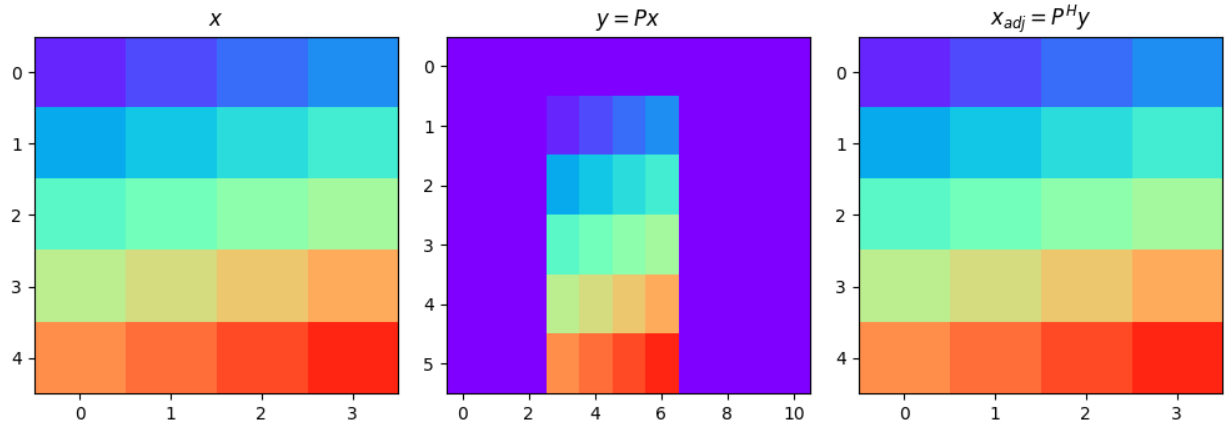
We move now to a multi-dimensional case. We pad the input model with different extents along both dimensions

```
dims = (5, 4)
pad = ((1, 0), (3, 4))

Pop = pylops.Pad(dims, pad)

x = (np.arange(np.prod(np.array(dims))) + 1.0).reshape(dims)
y = Pop * x
xadj = Pop.H * y

fig, axs = plt.subplots(1, 3, figsize=(10, 4))
fig.suptitle("Pad for 2d data", fontsize=14, fontweight="bold", y=1.15)
axs[0].imshow(x, cmap="rainbow", vmin=0, vmax=np.prod(np.array(dims)) + 1)
axs[0].set_title(r"$x$")
axs[0].axis("tight")
axs[1].imshow(y, cmap="rainbow", vmin=0, vmax=np.prod(np.array(dims)) + 1)
axs[1].set_title(r"$y = P x$")
axs[1].axis("tight")
axs[2].imshow(xadj, cmap="rainbow", vmin=0, vmax=np.prod(np.array(dims)) + 1)
axs[2].set_title(r"$x_{adj} = P^{H} y$")
axs[2].axis("tight")
plt.tight_layout()
```



Total running time of the script: (0 minutes 0.309 seconds)

3.5.26 Patching

This example shows how to use the *pylops.signalprocessing.Patch2D* and *pylops.signalprocessing.Patch3D* operators to perform repeated transforms over small patches of a 2-dimensional or 3-dimensional array. The transforms that we apply in this example are the *pylops.signalprocessing.FFT2D* and `pylops.signalprocessing.FFT3D` but this operator has been designed to allow a variety of transforms as long as they operate with signals that are 2- or 3-dimensional in nature, respectively.

```
import matplotlib.pyplot as plt
import numpy as np

import pylops
```

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```
plt.close("all")
```

Let's start by creating an 2-dimensional array of size $n_x \times n_t$ composed of 3 parabolic events

```
par = {"ox": -140, "dx": 2, "nx": 140, "ot": 0, "dt": 0.004, "nt": 200, "f0": 20}

v = 1500
t0 = [0.2, 0.4, 0.5]
px = [0, 0, 0]
pxx = [1e-5, 5e-6, 1e-20]
amp = [1.0, -2, 0.5]

# Create axis
t, t2, x, y = pylops.utils.seismicevents.makeaxis(par)

# Create wavelet
wav = pylops.utils.wavelets.ricker(t[:41], f0=par["f0"])[0]

# Generate model
_, data = pylops.utils.seismicevents.parabolic2d(x, t, t0, px, pxx, amp, wav)
```

We want to divide this 2-dimensional data into small overlapping patches in the spatial direction and apply the adjoint of the *pylops.signalprocessing.FFT2D* operator to each patch. This is done by simply using the adjoint of the *pylops.signalprocessing.Patch2D* operator. Note that for non- orthogonal operators, this must be replaced by an inverse.

```
nwin = (20, 34)  # window size in data domain
nop = (
    128,
    128 // 2 + 1,
)  # window size in model domain; we use real FFT, second axis is half
nover = (10, 4)  # overlap between windows
dimsd = data.shape

# Sliding window transform without taper
Op = pylops.signalprocessing.FFT2D(nwin, nffts=(128, 128), real=True)

nwins, dims, mwin_inends, dwin_inends = pylops.signalprocessing.patch2d_design(
    dimsd, nwin, nover, (128, 65)
)
Patch = pylops.signalprocessing.Patch2D(
    Op.H, dims, dimsd, nwin, nover, nop, tapertype=None
)
fftdata = Patch.H * data
```

We now create a similar operator but we also add a taper to the overlapping parts of the patches. We then apply the forward to restore the original signal.

```
Patch = pylops.signalprocessing.Patch2D(
    Op.H, dims, dimsd, nwin, nover, nop, tapertype="hanning"
)
```

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```
reconstructed_data = Patch * fftdata
```

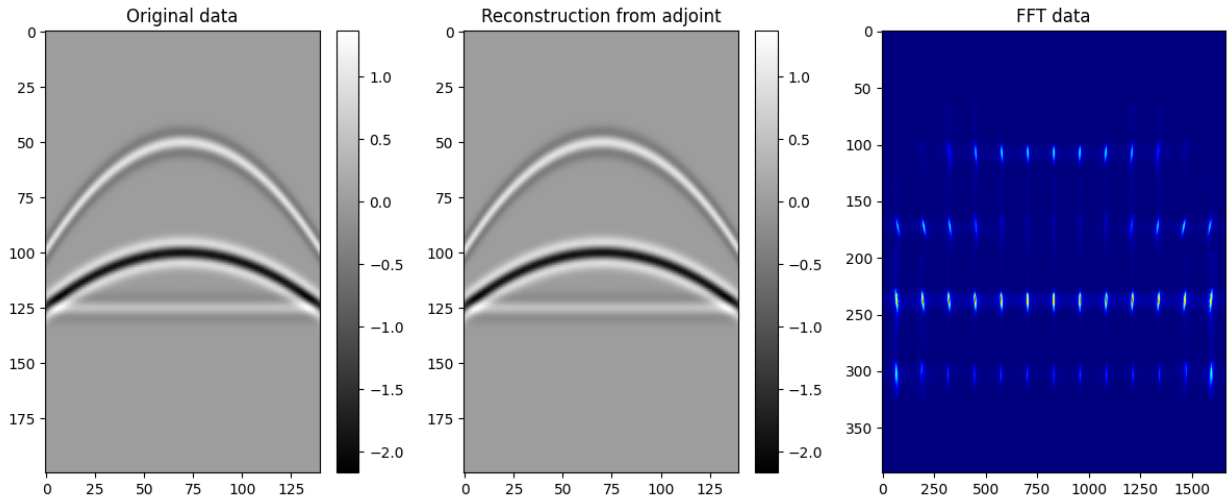
Finally we re-arrange the transformed patches so that we can also display them

```
fftdatareshaped = np.zeros((nop[0] * nwins[0], nop[1] * nwins[1]), dtype=fftdata.dtype)

iwin = 1
for ix in range(nwins[0]):
    for it in range(nwins[1]):
        fftdatareshaped[
            ix * nop[0] : (ix + 1) * nop[0], it * nop[1] : (it + 1) * nop[1]
        ] = np.fft.fftshift(fftdata[ix, it])
        iwin += 1
```

Let's finally visualize all the intermediate results as well as our final data reconstruction after inverting the *pylops.signalprocessing.Sliding2D* operator.

```
fig, axs = plt.subplots(1, 3, figsize=(12, 5))
im = axs[0].imshow(data.T, cmap="gray")
axs[0].set_title("Original data")
plt.colorbar(im, ax=axs[0])
axs[0].axis("tight")
im = axs[1].imshow(reconstructed_data.real.T, cmap="gray")
axs[1].set_title("Reconstruction from adjoint")
plt.colorbar(im, ax=axs[1])
axs[1].axis("tight")
axs[2].imshow(np.abs(fftdatareshaped).T, cmap="jet")
axs[2].set_title("FFT data")
axs[2].axis("tight")
plt.tight_layout()
```



We repeat now the same exercise in 3d

```
par = {
    "oy": -60,
```

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```
    "dy": 2,
    "ny": 60,
    "ox": -50,
    "dx": 2,
    "nx": 50,
    "ot": 0,
    "dt": 0.004,
    "nt": 100,
    "f0": 20,
}

v = 1500
t0 = [0.05, 0.2, 0.3]
vrms = [500, 700, 1700]
amp = [1.0, -2, 0.5]

# Create axis
t, t2, x, y = pylops.utils.seismicevents.makeaxis(par)

# Create wavelet
wav = pylops.utils.wavelets.ricker(t[:41], f0=par["f0"])[0]

# Generate model
_, data = pylops.utils.seismicevents.hyperbolic3d(x, y, t, t0, vrms, vrms, amp, wav)


fig, axs = plt.subplots(1, 3, figsize=(12, 5))
fig.suptitle("Original data", fontsize=12, fontweight="bold", y=0.95)
axs[0].imshow(
    data[par["ny"] // 2].T,
    aspect="auto",
    interpolation="nearest",
    vmin=-2,
    vmax=2,
    cmap="gray",
    extent=(x.min(), x.max(), t.max(), t.min()),
)
axs[0].set_xlabel(r"$x(m)$")
axs[0].set_ylabel(r"$t(s)$")
axs[1].imshow(
    data[:, par["nx"] // 2].T,
    aspect="auto",
    interpolation="nearest",
    vmin=-2,
    vmax=2,
    cmap="gray",
    extent=(y.min(), y.max(), t.max(), t.min()),
)
axs[1].set_xlabel(r"$y(m)$")
axs[1].set_ylabel(r"$t(s)$")
axs[2].imshow(
    data[:, :, par["nt"] // 2],
```

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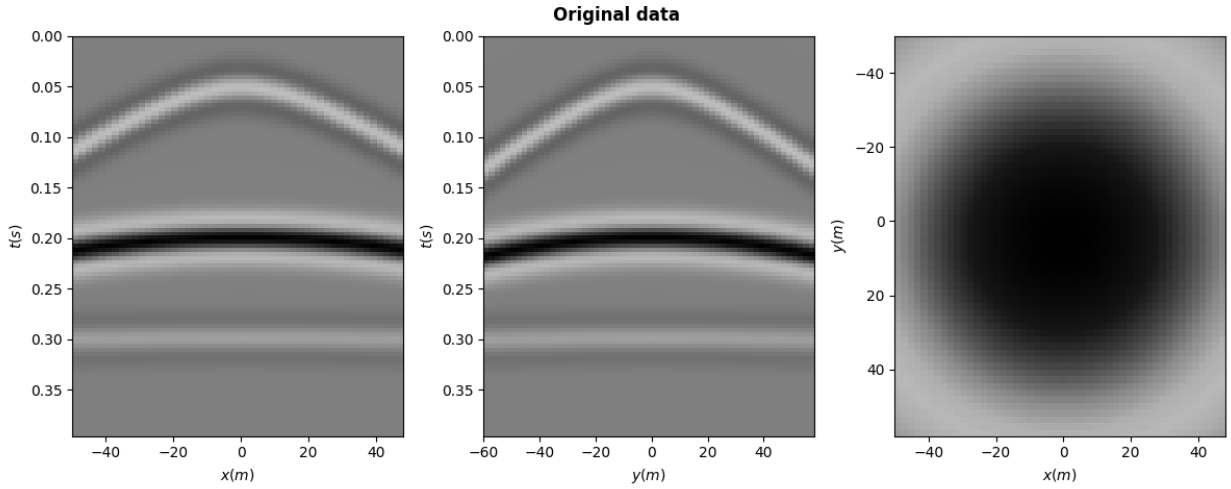
```
    aspect="auto",
    interpolation="nearest",
    vmin=-2,
    vmax=2,
    cmap="gray",
    extent=(x.min(), x.max(), y.max(), x.min()),
)
axs[2].set_xlabel(r"$x(m)$")
axs[2].set_ylabel(r"$y(m)$")
plt.tight_layout()
```



Let's create now the *pylops.signalprocessing.Patch3D* operator applying the adjoint of the `pylops.signalprocessing.FFT3D` operator to each patch.

```
nwin = (20, 20, 34)  # window size in data domain
nop = (
    128,
    128,
    128 // 2 + 1,
)  # window size in model domain; we use real FFT, third axis is half
nover = (10, 10, 4)  # overlap between windows
dimsd = data.shape

# Sliding window transform without taper
Op = pylops.signalprocessing.FFTND(nwin, nffts=(128, 128, 128), real=True)

nwins, dims, mwin_inends, dwin_inends = pylops.signalprocessing.patch3d_design(
    dimsd, nwin, nover, (128, 128, 65)
)
Patch = pylops.signalprocessing.Patch3D(
    Op.H, dims, dimsd, nwin, nover, nop, tapertype=None
)
fftdata = Patch.H * data

Patch = pylops.signalprocessing.Patch3D(
```

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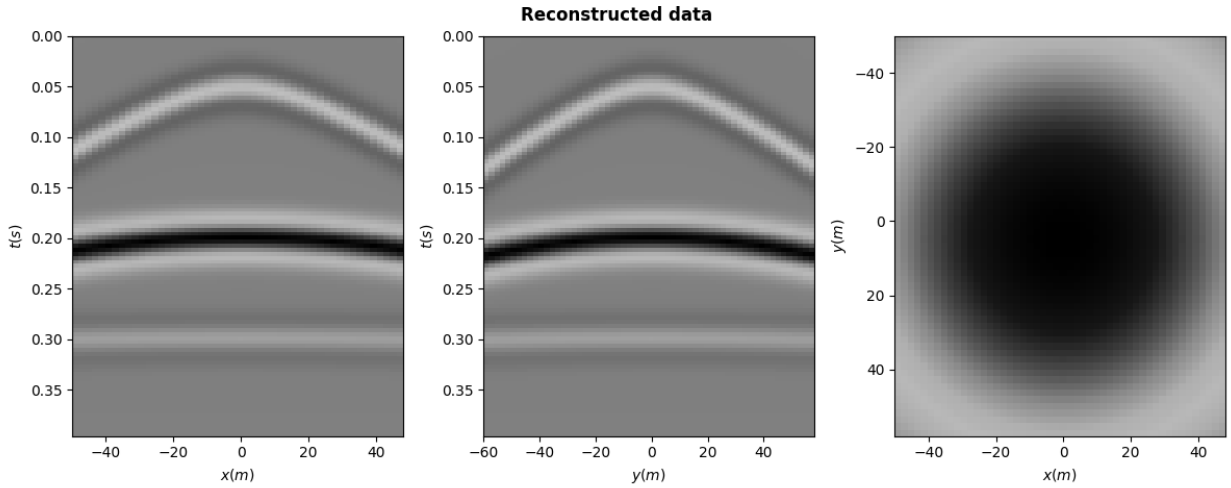
```
    Op.H, dims, dimsd, nwin, nover, nop, tapertype="hanning"
)
reconstructed_data = np.real(Patch * fftdata)

fig, axs = plt.subplots(1, 3, figsize=(12, 5))
fig.suptitle("Reconstructed data", fontsize=12, fontweight="bold", y=0.95)
axs[0].imshow(
    reconstructed_data[par["ny"] // 2].T,
    aspect="auto",
    interpolation="nearest",
    vmin=-2,
    vmax=2,
    cmap="gray",
    extent=(x.min(), x.max(), t.max(), t.min()),
)
axs[0].set_xlabel(r"$x(m)$")
axs[0].set_ylabel(r"$t(s)$")
axs[1].imshow(
    reconstructed_data[:, par["nx"] // 2].T,
    aspect="auto",
    interpolation="nearest",
    vmin=-2,
    vmax=2,
    cmap="gray",
    extent=(y.min(), y.max(), t.max(), t.min()),
)
axs[1].set_xlabel(r"$y(m)$")
axs[1].set_ylabel(r"$t(s)$")
axs[2].imshow(
    reconstructed_data[:, :, par["nt"] // 2],
    aspect="auto",
    interpolation="nearest",
    vmin=-2,
    vmax=2,
    cmap="gray",
    extent=(x.min(), x.max(), y.max(), x.min()),
)
axs[2].set_xlabel(r"$x(m)$")
axs[2].set_ylabel(r"$y(m)$")
plt.tight_layout()
```



Total running time of the script: (0 minutes 7.681 seconds)

3.5.27 PhaseShift operator

This example shows how to use the *pylops.waveeqprocessing.PhaseShift* operator to perform frequency-wavenumber shift of an input multi-dimensional signal. Such a procedure is applied in a variety of disciplines including geophysics, medical imaging and non-destructive testing.

```
import matplotlib.pyplot as plt
import numpy as np

import pylops

plt.close("all")
```

Let's first create a synthetic dataset composed of a number of hyperbolas

```
par = {
    "ox": -300,
    "dx": 20,
    "nx": 31,
    "oy": -200,
    "dy": 20,
    "ny": 21,
    "ot": 0,
    "dt": 0.004,
    "nt": 201,
    "f0": 20,
    "nfmax": 210,
}

# Create axis
t, t2, x, y = pylops.utils.seismicevents.makeaxis(par)

# Create wavelet
wav = pylops.utils.wavelets.ricker(np.arange(41) * par["dt"], f0=par["f0"])[0]
```

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```
vrms = [900, 1300, 1800]
t0 = [0.2, 0.3, 0.6]
amp = [1.0, 0.6, -2.0]

_, m = pylops.utils.seismicevents.hyperbolic2d(x, t, t0, vrms, amp, wav)
```

We can now apply a taper at the edges and also pad the input to avoid artifacts during the phase shift

```
pad = 11
taper = pylops.utils.tapers.taper2d(par["nt"], par["nx"], 5)
mpad = np.pad(m * taper, ((pad, pad), (0, 0)), mode="constant")
```

We perform now forward propagation in a constant velocity $v = 2000$ for a depth of $z_{prop} = 100m$. We should expect the hyperbolas to move forward in time and become flatter.

```
vel = 1500.0
zprop = 100
freq = np.fft.rfftfreq(par["nt"], par["dt"])
kx = np.fft.fftshift(np.fft.fftfreq(par["nx"] + 2 * pad, par["dx"]))
Pop = pylops.waveeqprocessing.PhaseShift(vel, zprop, par["nt"], freq, kx)

mdown = Pop * mpad.T.ravel()
```

We now take the forward propagated wavefield and apply backward propagation, which is in this case simply the adjoint of our operator. We should expect the hyperbolas to move backward in time and show the same traveltime as the original dataset. Of course, as we are only performing the adjoint operation we should expect some small differences between this wavefield and the input dataset.

```
mup = Pop.H * mdown.ravel()

mdown = np.real(mdown.reshape(par["nt"], par["nx"] + 2 * pad)[:, pad:-pad])
mup = np.real(mup.reshape(par["nt"], par["nx"] + 2 * pad)[:, pad:-pad])

fig, axs = plt.subplots(1, 3, figsize=(10, 6), sharey=True)
fig.suptitle("2D Phase shift", fontsize=12, fontweight="bold")
axs[0].imshow(
    m.T,
    aspect="auto",
    interpolation="nearest",
    vmin=-2,
    vmax=2,
    cmap="gray",
    extent=(x.min(), x.max(), t.max(), t.min()),
)
axs[0].set_xlabel(r"$x(m)$")
axs[0].set_ylabel(r"$t(s)$")
axs[0].set_title("Original data")
axs[1].imshow(
    mdown,
    aspect="auto",
    interpolation="nearest",
    vmin=-2,
```

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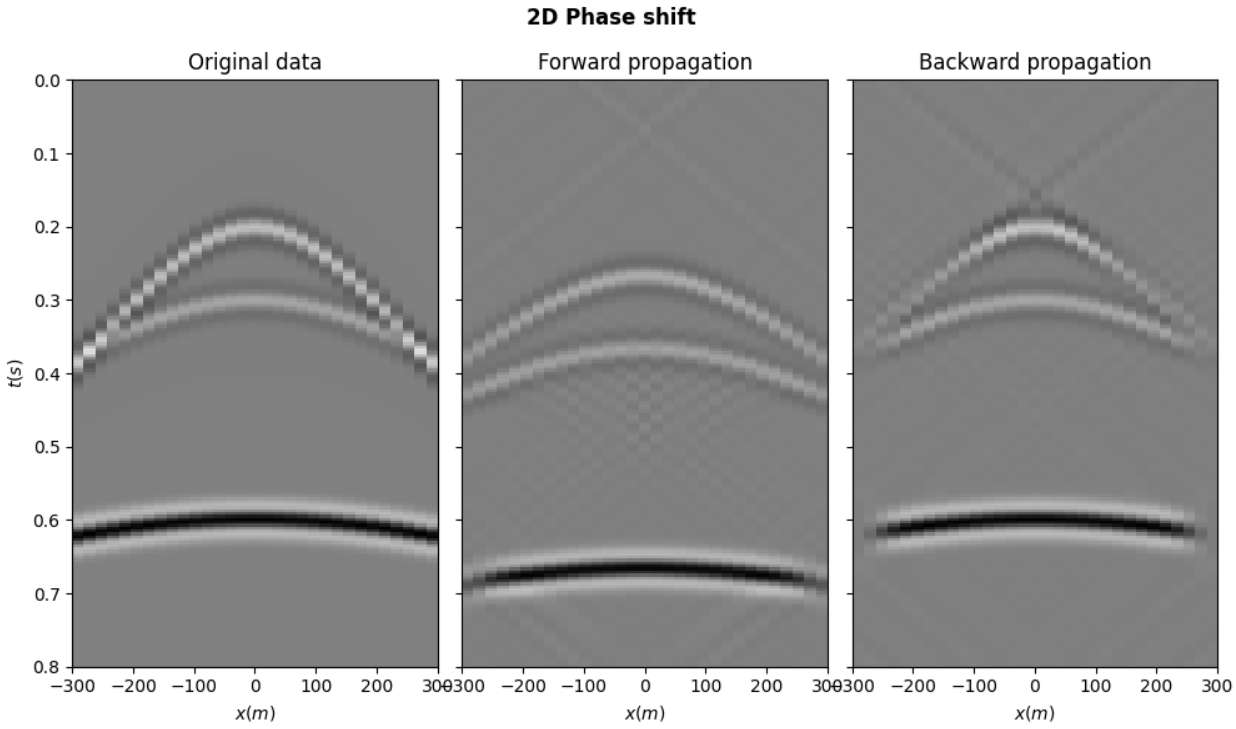
```
    vmax=2,
    cmap="gray",
    extent=(x.min(), x.max(), t.max(), t.min()),
)
axs[1].set_xlabel(r"$x(m)$")
axs[1].set_title("Forward propagation")
axs[2].imshow(
    mup,
    aspect="auto",
    interpolation="nearest",
    vmin=-2,
    vmax=2,
    cmap="gray",
    extent=(x.min(), x.max(), t.max(), t.min()),
)
axs[2].set_xlabel(r"$x(m)$")
axs[2].set_title("Backward propagation")
plt.tight_layout()
```



Finally we perform the same for a 3-dimensional signal

```
_, m = pylops.utils.seismicevents.hyperbolic3d(x, y, t, t0, vrms, vrms, amp, wav)

pad = 11
taper = pylops.utils.tapers.taper3d(par["nt"], (par["ny"], par["nx"]), (3, 3))
mpad = np.pad(m * taper, ((pad, pad), (pad, pad), (0, 0)), mode="constant")

kx = np.fft.fftshift(np.fft.fftfreq(par["nx"] + 2 * pad, par["dx"]))
```

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```
ky = np.fft.fftshift(np.fft.fftfreq(par["ny"] + 2 * pad, par["dy"]))
Pop = pylops.waveeqprocessing.PhaseShift(vel, zprop, par["nt"], freq, kx, ky)

mdown = Pop * mpad.transpose(2, 1, 0).ravel()

mup = Pop.H * mdown.ravel()

mdown = np.real(
    mdown.reshape(par["nt"], par["nx"] + 2 * pad, par["ny"] + 2 * pad)[
        :, pad:-pad, pad:-pad
    ]
)
mup = np.real(
    mup.reshape(par["nt"], par["nx"] + 2 * pad, par["ny"] + 2 * pad)[
        :, pad:-pad, pad:-pad
    ]
)

fig, axs = plt.subplots(2, 3, figsize=(10, 12), sharey=True)
fig.suptitle("3D Phase shift", fontsize=12, fontweight="bold")
axs[0][0].imshow(
    m[:, par["nx"] // 2].T,
    aspect="auto",
    interpolation="nearest",
    vmin=-2,
    vmax=2,
    cmap="gray",
    extent=(x.min(), x.max(), t.max(), t.min()),
)
axs[0][0].set_xlabel(r"$y(m)$")
axs[0][0].set_ylabel(r"$t(s)$")
axs[0][0].set_title("Original data")
axs[0][1].imshow(
    mdown[:, par["nx"] // 2],
    aspect="auto",
    interpolation="nearest",
    vmin=-2,
    vmax=2,
    cmap="gray",
    extent=(x.min(), x.max(), t.max(), t.min()),
)
axs[0][1].set_xlabel(r"$y(m)$")
axs[0][1].set_title("Forward propagation")
axs[0][2].imshow(
    mup[:, par["nx"] // 2],
    aspect="auto",
    interpolation="nearest",
    vmin=-2,
    vmax=2,
    cmap="gray",
    extent=(x.min(), x.max(), t.max(), t.min()),
)
```

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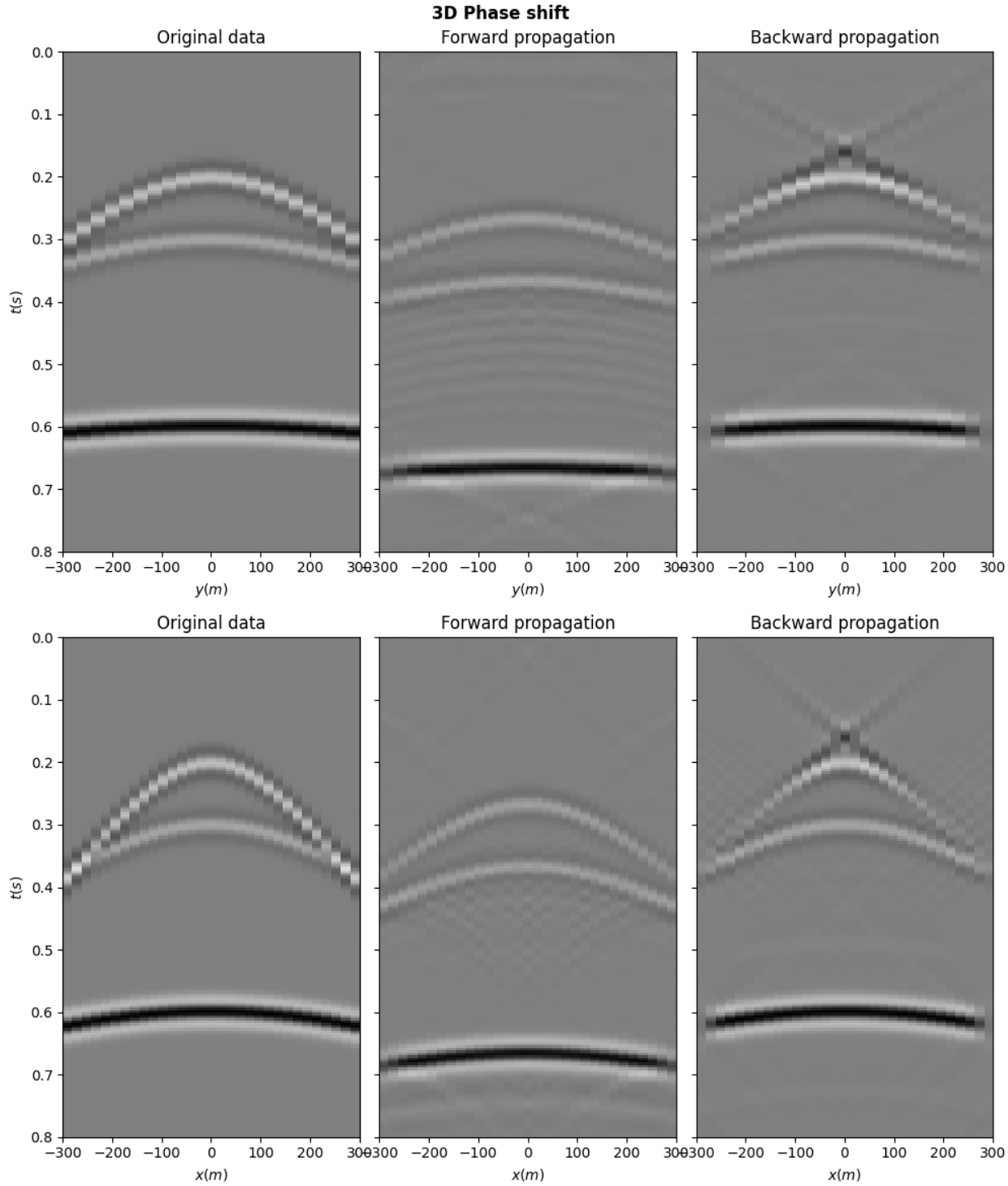
```
axs[0][2].set_xlabel(r"$y(m)$")
axs[0][2].set_title("Backward propagation")
axs[1][0].imshow(
    m[par["ny"] // 2].T,
    aspect="auto",
    interpolation="nearest",
    vmin=-2,
    vmax=2,
    cmap="gray",
    extent=(x.min(), x.max(), t.max(), t.min()),
)
axs[1][0].set_xlabel(r"$x(m)$")
axs[1][0].set_ylabel(r"$t(s)$")
axs[1][0].set_title("Original data")
axs[1][1].imshow(
    mdown[:, :, par["ny"] // 2],
    aspect="auto",
    interpolation="nearest",
    vmin=-2,
    vmax=2,
    cmap="gray",
    extent=(x.min(), x.max(), t.max(), t.min()),
)
axs[1][1].set_xlabel(r"$x(m)$")
axs[1][1].set_title("Forward propagation")
axs[1][2].imshow(
    mup[:, :, par["ny"] // 2],
    aspect="auto",
    interpolation="nearest",
    vmin=-2,
    vmax=2,
    cmap="gray",
    extent=(x.min(), x.max(), t.max(), t.min()),
)
axs[1][2].set_xlabel(r"$x(m)$")
axs[1][2].set_title("Backward propagation")
plt.tight_layout()
```



Total running time of the script: (0 minutes 1.117 seconds)

3.5.28 Polynomial Regression

This example shows how to use the *pylops.Regression* operator to perform *Polynomial regression analysis*.

In short, polynomial regression is the problem of finding the best fitting coefficients for the following equation:

$$y_i = \sum_{n=0}^{\text{order}} x_n t_i^n \qquad \forall i = 0, 1, \ldots, N-1$$

As we can express this problem in a matrix form:

$$\mathbf{y} = \mathbf{A}\mathbf{x}$$

our solution can be obtained by solving the following optimization problem:

$$J = ||\mathbf{y} - \mathbf{A}\mathbf{x}||_2$$

See documentation of *pylops.Regression* for more detailed definition of the forward problem.

```
import matplotlib.pyplot as plt
import numpy as np

import pylops

plt.close("all")
np.random.seed(10)
```

Define the input parameters: number of samples along the t-axis (`N`), order (`order`), regression coefficients (`x`), and standard deviation of noise to be added to data (`sigma`).

```
N = 30
order = 3
x = np.array([1.0, 0.05, 0.0, -0.01])
sigma = 1
```

Let's create the time axis and initialize the *pylops.Regression* operator

```
t = np.arange(N, dtype="float64") - N // 2
PRop = pylops.Regression(t, order=order, dtype="float64")
```

We can then apply the operator in forward mode to compute our data points along the x-axis (`y`). We will also generate some random gaussian noise and create a noisy version of the data (`yn`).

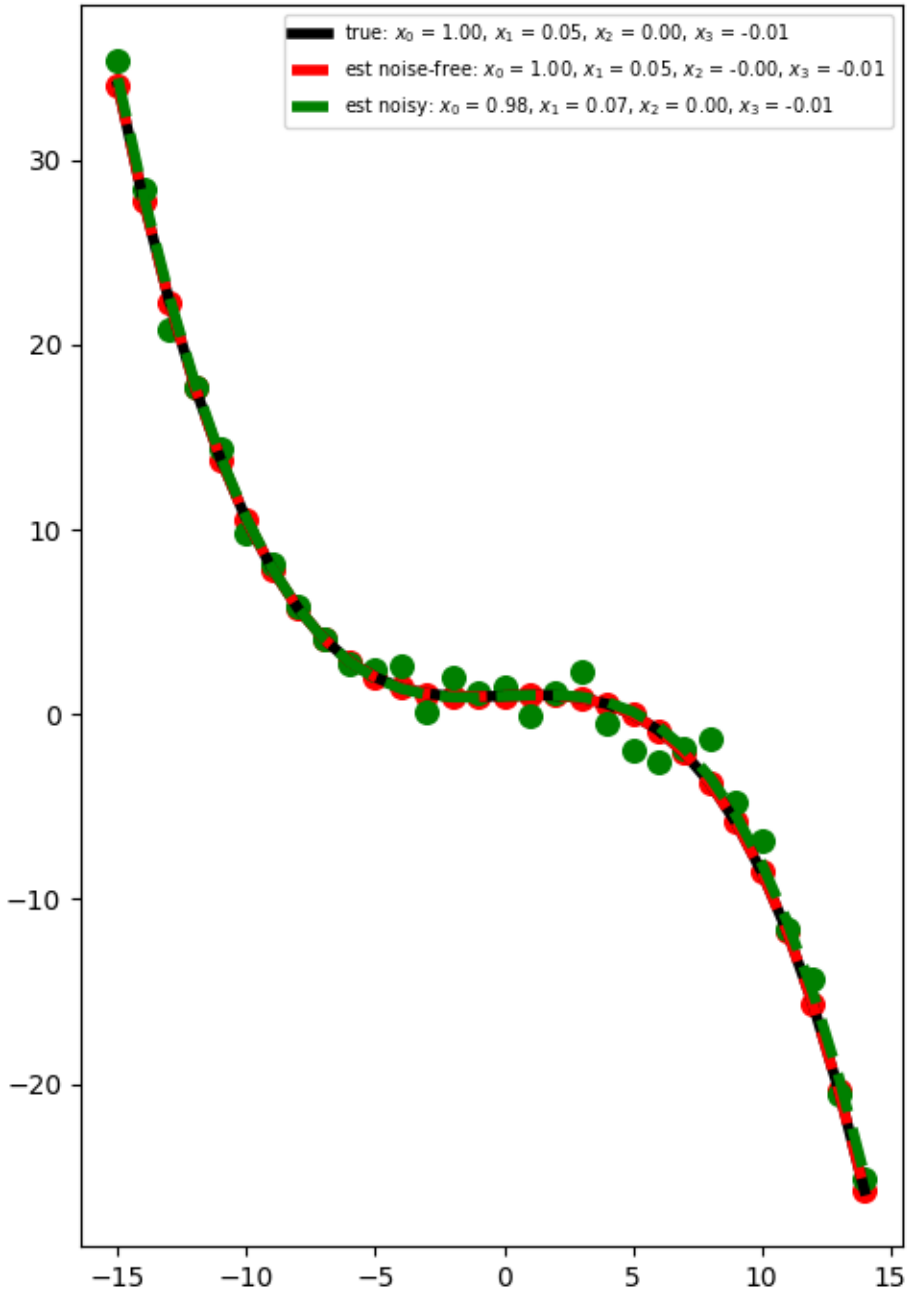
```
y = PRop * x
yn = y + np.random.normal(0, sigma, N)
```

We are now ready to solve our problem. As we are using an operator from the *pylops.LinearOperator* family, we can simply use `/`, which in this case will solve the system by means of an iterative solver (i.e., `scipy.sparse.linalg.lsqr`).

```
xest = PRop / y
xnest = PRop / yn
```

Let's plot the best fitting curve for the case of noise free and noisy data

```
plt.figure(figsize=(5, 7))
plt.plot(
    t,
    PRop * x,
    "k",
    lw=4,
    label=r"true: $x_0$ = %.2f, $x_1$ = %.2f, "
    r"$x_2$ = %.2f, $x_3$ = %.2f" % (x[0], x[1], x[2], x[3]),
)
plt.plot(
    t,
    PRop * xest,
    "--r",
    lw=4,
    label="est noise-free: $x_0$ = %.2f, $x_1$ = %.2f, "
    r"$x_2$ = %.2f, $x_3$ = %.2f" % (xest[0], xest[1], xest[2], xest[3]),
)
plt.plot(
    t,
    PRop * xnest,
    "--g",
    lw=4,
    label="est noisy: $x_0$ = %.2f, $x_1$ = %.2f, "
    r"$x_2$ = %.2f, $x_3$ = %.2f" % (xnest[0], xnest[1], xnest[2], xnest[3]),
)
plt.scatter(t, y, c="r", s=70)
plt.scatter(t, yn, c="g", s=70)
plt.legend(fontsize="x-small")
plt.tight_layout()
```



We consider now the case where some of the observations have large errors. Such elements are generally referred to as *outliers* and can affect the quality of the least-squares solution if not treated with care. In this example we will see how using a L1 solver such as `pylops.optimization.sparsity.IRLS` can drammatically improve the quality of the estimation of intercept and gradient.

```
# Add outliers
yn[1] += 40
yn[N - 2] -= 20

# IRLS
```

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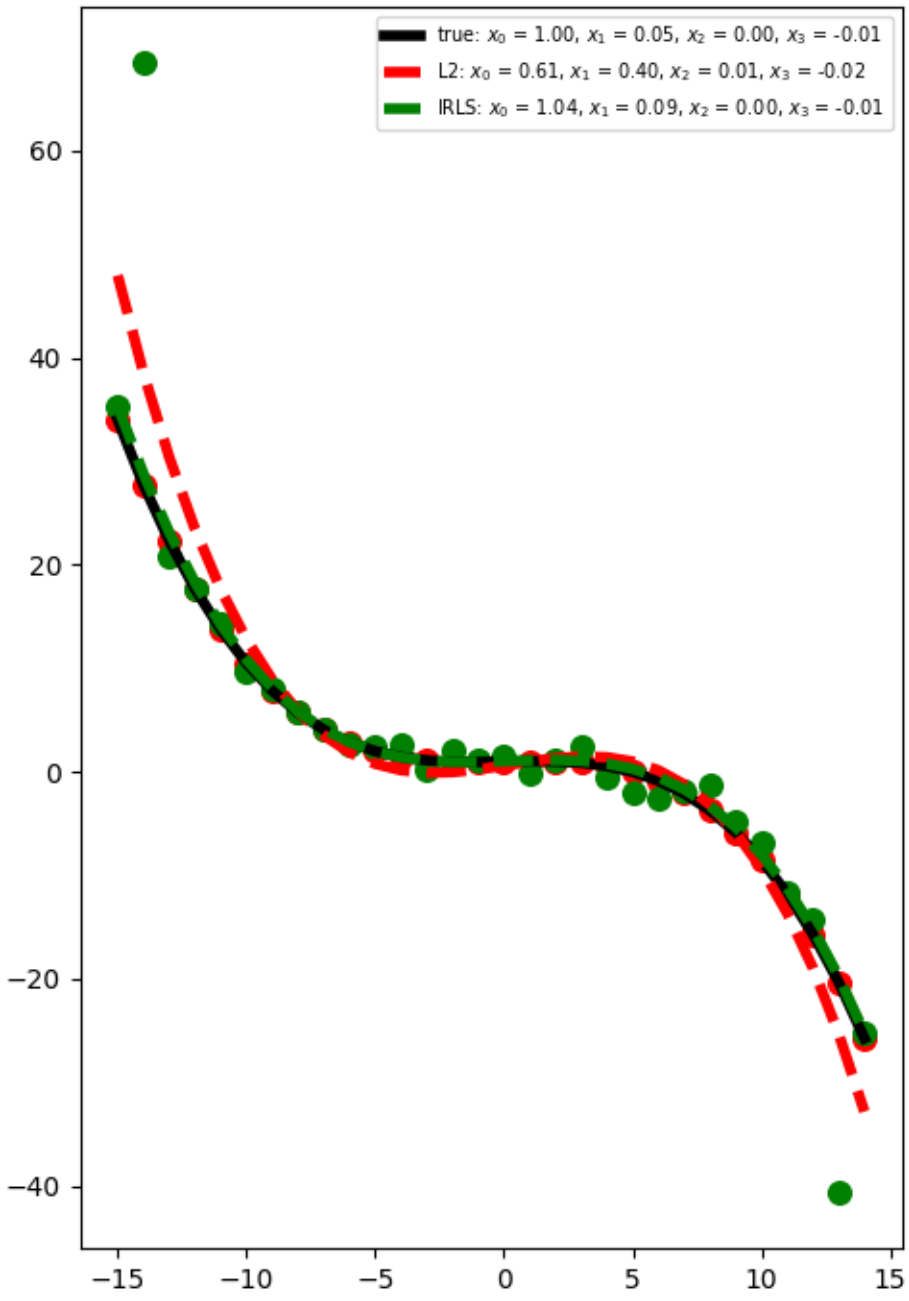
```
nouter = 20
epsR = 1e-2
epsI = 0
tolIRLS = 1e-2

xnest = PRop / yn
xirls, nouter = pylops.optimization.sparsity.irls(
    PRop,
    yn,
    nouter=nouter,
    threshR=False,
    epsR=epsR,
    epsI=epsI,
    tolIRLS=tolIRLS,
)
print(f"IRLS converged at {nouter} iterations...")

plt.figure(figsize=(5, 7))
plt.plot(
    t,
    PRop * x,
    "k",
    lw=4,
    label=r"true: $x_0$ = %.2f, $x_1$ = %.2f, "
    r"$x_2$ = %.2f, $x_3$ = %.2f" % (x[0], x[1], x[2], x[3]),
)
plt.plot(
    t,
    PRop * xnest,
    "--r",
    lw=4,
    label=r"L2: $x_0$ = %.2f, $x_1$ = %.2f, "
    r"$x_2$ = %.2f, $x_3$ = %.2f" % (xnest[0], xnest[1], xnest[2], xnest[3]),
)
plt.plot(
    t,
    PRop * xirls,
    "--g",
    lw=4,
    label=r"IRLS: $x_0$ = %.2f, $x_1$ = %.2f, "
    r"$x_2$ = %.2f, $x_3$ = %.2f" % (xirls[0], xirls[1], xirls[2], xirls[3]),
)
plt.scatter(t, y, c="r", s=70)
plt.scatter(t, yn, c="g", s=70)
plt.legend(fontsize="x-small")
plt.tight_layout()
```




```
IRLS converged at 6 iterations...
```

Total running time of the script: (0 minutes 0.473 seconds)

3.5.29 Pre-stack modelling

This example shows how to create pre-stack angle gathers using the *pylops.avo.prestack.PrestackLinearModelling* operator.

```
import matplotlib.pyplot as plt
import numpy as np
from mpl_toolkits.axes_grid1.axes_divider import make_axes_locatable
from scipy.signal import filtfilt

import pylops
from pylops.utils.wavelets import ricker

plt.close("all")
np.random.seed(0)
```

Let's start by creating the input elastic property profiles and wavelet

```
nt0 = 501
dt0 = 0.004
ntheta = 21

t0 = np.arange(nt0) * dt0
thetamin, thetamax = 0, 40
theta = np.linspace(thetamin, thetamax, ntheta)

# Elastic property profiles
vp = (
    2000
    + 5 * np.arange(nt0)
    + 2 * filtfilt(np.ones(5) / 5.0, 1, np.random.normal(0, 160, nt0))
)
vs = 600 + vp / 2 + 3 * filtfilt(np.ones(5) / 5.0, 1, np.random.normal(0, 100, nt0))
rho = 1000 + vp + filtfilt(np.ones(5) / 5.0, 1, np.random.normal(0, 120, nt0))
vp[201:] += 1500
vs[201:] += 500
rho[201:] += 100

# Wavelet
ntwav = 81
wav, twav, wavc = ricker(t0[: ntwav // 2 + 1], 5)

# vs/vp profile
vsvp = 0.5
vsvp_z = vs / vp

# Model
m = np.stack((np.log(vp), np.log(vs), np.log(rho)), axis=1)

fig, axs = plt.subplots(1, 3, figsize=(9, 7), sharey=True)
axs[0].plot(vp, t0, "k", lw=3)
axs[0].set(xlabel="[m/s]", ylabel=r"$t$ [s]", ylim=[t0[0], t0[-1]], title="Vp")
axs[0].grid()
axs[1].plot(vp / vs, t0, "k", lw=3)
```

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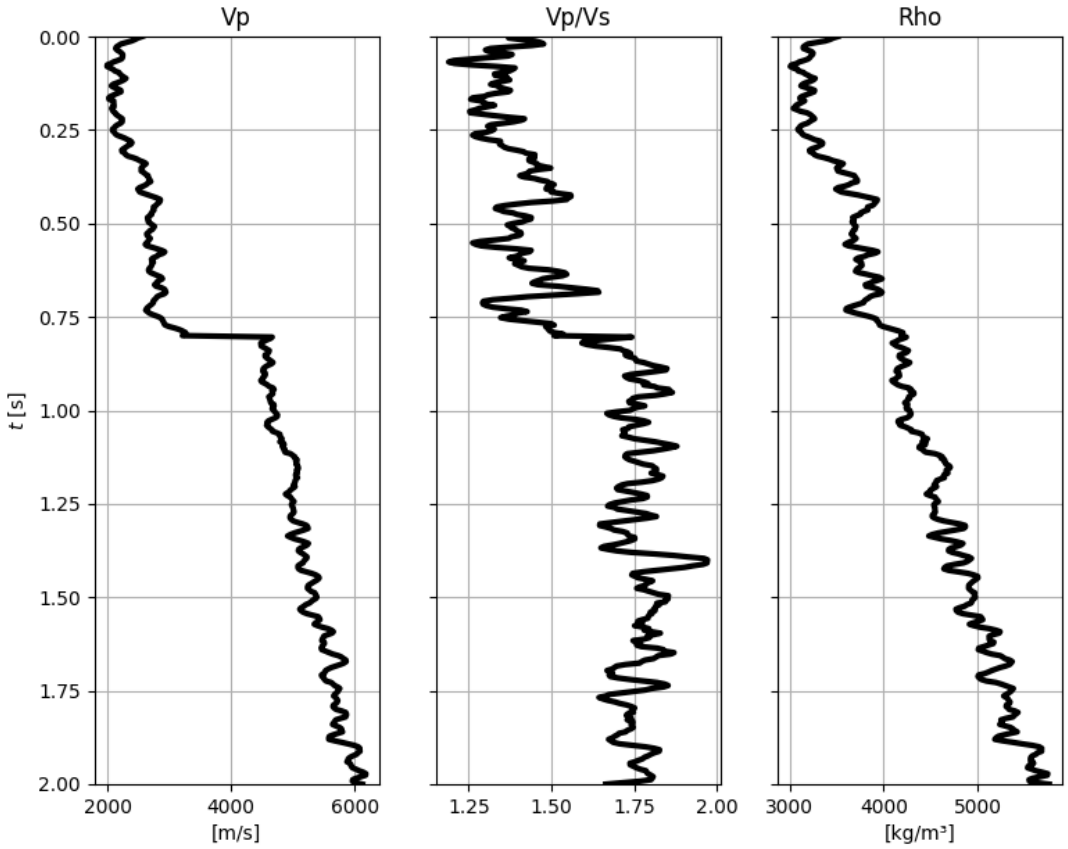
```
axs[1].set(title="Vp/Vs")
axs[1].grid()
axs[2].plot(rho, t0, "k", lw=3)
axs[2].set(xlabel="[kg/m$^3$]", title="Rho")
axs[2].invert_yaxis()
axs[2].grid()
```



We create now the operators to model a synthetic pre-stack seismic gather with a zero-phase using both a constant and a depth-variant `vsvp` profile

```
# constant vsvp
PPop_const = pylops.avo.prestack.PrestackLinearModelling(
    wav, theta, vsvp=vsvp, nt0=nt0, linearization="akirich"
)

# depth-variant vsvp
PPop_variant = pylops.avo.prestack.PrestackLinearModelling(
    wav, theta, vsvp=vsvp_z, linearization="akirich"
)
```

Let's apply those operators to the elastic model and create some synthetic data

```
dPP_const = PPop_const * m
dPP_variant = PPop_variant * m
```

Finally we visualize the two datasets

```
# sphinx_gallery_thumbnail_number = 2
fig = plt.figure(figsize=(6, 7))
ax1 = plt.subplot2grid((3, 2), (0, 0), rowspan=2)
ax2 = plt.subplot2grid((3, 2), (0, 1), rowspan=2, sharey=ax1)
ax3 = plt.subplot2grid((3, 2), (2, 0), sharex=ax1)
ax4 = plt.subplot2grid((3, 2), (2, 1), sharex=ax2)
im = ax1.imshow(
    dPP_const,
    cmap="bwr",
    extent=(theta[0], theta[-1], t0[-1], t0[0]),
    vmin=-0.2,
    vmax=0.2,
)
cax = make_axes_locatable(ax1).append_axes("bottom", size="5%", pad="3%")
cb = fig.colorbar(im, cax=cax, orientation="horizontal")
cb.ax.xaxis.set_ticks_position("bottom")
ax1.set(ylabel=r"$t$ [s]")
ax1.set_title(r"Data with constant $VP/VS$", fontsize=10)
ax1.tick_params(labelbottom=False)
ax1.axhline(t0[nt0 // 4], color="k", linestyle="--")
ax1.axhline(t0[nt0 // 2], color="k", linestyle="--")
ax1.axis("tight")

im = ax2.imshow(
    dPP_variant,
    cmap="bwr",
    extent=(theta[0], theta[-1], t0[-1], t0[0]),
    vmin=-0.2,
    vmax=0.2,
)
cax = make_axes_locatable(ax2).append_axes("bottom", size="5%", pad="3%")
cb = fig.colorbar(im, cax=cax, orientation="horizontal")
cb.ax.xaxis.set_ticks_position("bottom")
ax2.set_title(r"Data with depth-variant $VP/VS$", fontsize=10)
ax2.tick_params(labelbottom=False, labelleft=False)
ax2.axhline(t0[nt0 // 4], color="k", linestyle="--")
ax2.axhline(t0[nt0 // 2], color="k", linestyle="--")
ax2.axis("tight")

ax3.plot(theta, dPP_const[nt0 // 4], "k", lw=2)
ax3.plot(theta, dPP_variant[nt0 // 4], "--r", lw=2)
ax3.set(xlabel=r"$\theta$ [°]")
ax3.set_title("AVO curve at t=%.2f s" % t0[nt0 // 4], fontsize=10)
ax4.plot(theta, dPP_const[nt0 // 2], "k", lw=2, label=r"constant $VP/VS$")
ax4.plot(theta, dPP_variant[nt0 // 2], "--r", lw=2, label=r"variable $VP/VS$")
ax4.set(xlabel=r"$\theta$ [°]")
ax4.set_title("AVO curve at t=%.2f s" % t0[nt0 // 2], fontsize=10)
ax4.legend()
```

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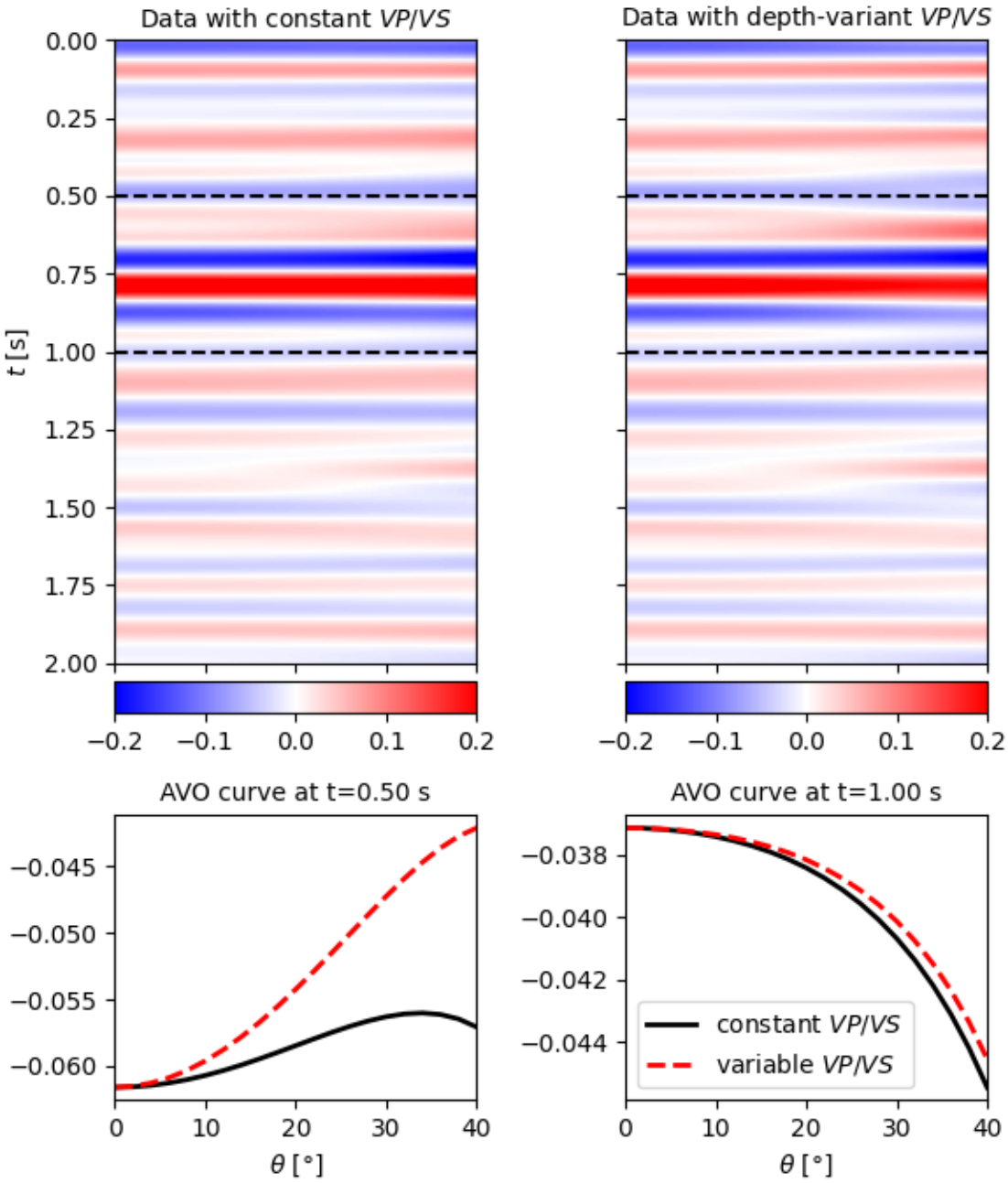
```
plt.tight_layout()
```



Total running time of the script: (0 minutes 0.761 seconds)

3.5.30 Radon Transform

This example shows how to use the *pylops.signalprocessing.Radon2D* and *pylops.signalprocessing.Radon3D* operators to apply the Radon Transform to 2-dimensional or 3-dimensional signals, respectively. In our implementation both linear, parabolic and hyperbolic parametrization can be chosen.

```
import matplotlib.pyplot as plt
import numpy as np

import pylops

plt.close("all")
```

Let's start by creating an empty 2d matrix of size $n_{p_x} \times n_t$ and add a single spike in it. We will see that applying the forward Radon operator will result in a single event (linear, parabolic or hyperbolic) in the resulting data vector.

```
nt, nh = 41, 51
npx, pxmax = 41, 1e-2

dt, dh = 0.005, 1
t = np.arange(nt) * dt
h = np.arange(nh) * dh
px = np.linspace(0, pxmax, npx)

x = np.zeros((npx, nt))
x[4, nt // 2] = 1
```

We can now define our operators for different parametric curves and apply them to the input model vector. We also apply the adjoint to the resulting data vector.

```
RLop = pylops.signalprocessing.Radon2D(
    t, h, px, centeredh=True, kind="linear", interp=False, engine="numpy"
)
RPop = pylops.signalprocessing.Radon2D(
    t, h, px, centeredh=True, kind="parabolic", interp=False, engine="numpy"
)
RHop = pylops.signalprocessing.Radon2D(
    t, h, px, centeredh=True, kind="hyperbolic", interp=False, engine="numpy"
)

# forward
yL = RLop * x
yP = RPop * x
yH = RHop * x

# adjoint
xadjL = RLop.H * yL
xadjP = RPop.H * yP
xadjH = RHop.H * yH
```

Let's now visualize the input model in the Radon domain, the data, and the adjoint model the different parametric curves.

```
fig, axs = plt.subplots(2, 4, figsize=(10, 6), sharey=True)
axs[0, 0].axis("off")
axs[1, 0].imshow(
    x.T,
    vmin=-1,
    vmax=1,
    cmap="seismic_r",
    extent=(1e3 * px[0], 1e3 * px[-1], t[-1], t[0]),
)
axs[1, 0].set(xlabel=r"$p$ [s/km]", ylabel=r"$t$ [s]", title="Input model")
axs[1, 0].axis("tight")
axs[0, 1].imshow(
    yL.T, vmin=-1, vmax=1, cmap="seismic_r", extent=(h[0], h[-1], t[-1], t[0])
)
axs[0, 1].tick_params(labelleft=True)
axs[0, 1].set(xlabel=r"$x$ [m]", ylabel=r"$t$ [s]", title="Linear data")
axs[0, 1].axis("tight")
axs[0, 2].imshow(
    yP.T, vmin=-1, vmax=1, cmap="seismic_r", extent=(h[0], h[-1], t[-1], t[0])
)
axs[0, 2].set(xlabel=r"$x$ [m]", title="Parabolic data")
axs[0, 2].axis("tight")
axs[0, 3].imshow(
    yH.T, vmin=-1, vmax=1, cmap="seismic_r", extent=(h[0], h[-1], t[-1], t[0])
)
axs[0, 3].set(xlabel=r"$x$ [m]", title="Hyperbolic data")
axs[0, 3].axis("tight")
axs[1, 1].imshow(
    xadjL.T,
    vmin=-20,
    vmax=20,
    cmap="seismic_r",
    extent=(1e3 * px[0], 1e3 * px[-1], t[-1], t[0]),
)
axs[1, 1].set(xlabel=r"$p$ [s/km]", title="Linear adjoint")
axs[1, 1].axis("tight")
axs[1, 2].imshow(
    xadjP.T,
    vmin=-20,
    vmax=20,
    cmap="seismic_r",
    extent=(1e3 * px[0], 1e3 * px[-1], t[-1], t[0]),
)
axs[1, 2].set(xlabel=r"$p$ [s/km]", title="Parabolic adjoint")
axs[1, 2].axis("tight")
axs[1, 3].imshow(
    xadjH.T,
    vmin=-20,
    vmax=20,
    cmap="seismic_r",
    extent=(1e3 * px[0], 1e3 * px[-1], t[-1], t[0]),
)
axs[1, 3].set(xlabel=r"$p$ [s/km]", title="Hyperbolic adjoint")
```

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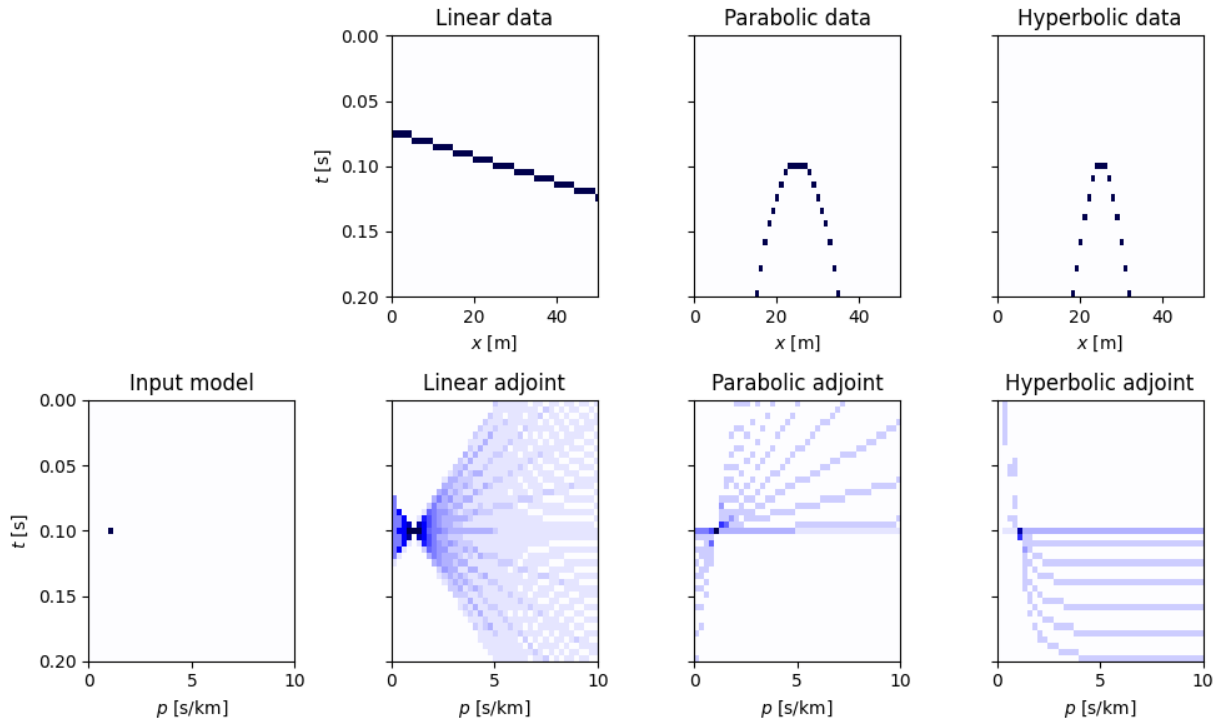
```
axs[1, 3].axis("tight")
fig.tight_layout()
```



As we can see in the bottom figures, the adjoint Radon transform is far from being close to the inverse Radon transform, i.e. $\mathbf{R^H R} \neq \mathbf{I}$ (compared to the case of FFT where the adjoint and inverse are equivalent, i.e. $\mathbf{F^H F} = \mathbf{I}$). In fact when we apply the adjoint Radon Transform we obtain a *model* that is a smoothed version of the original model polluted by smearing and artifacts. In tutorial *11. Radon filtering* we will exploit a sparsity-promiting Radon transform to perform filtering of unwanted signals from an input data.

Finally we repeat the same exercise with 3d data.

```
nt, ny, nx = 21, 21, 11
npy, pymax = 13, 5e-3
npx, pxmax = 11, 5e-3

dt, dy, dx = 0.005, 1, 1
t = np.arange(nt) * dt
hy = np.arange(ny) * dy
hx = np.arange(nx) * dx

py = np.linspace(0, pymax, npy)
px = np.linspace(0, pxmax, npx)

x = np.zeros((npy, npx, nt))
x[npy // 2, npx // 2 - 2, nt // 2] = 1

RLop = pylops.signalprocessing.Radon3D(
    t, hy, hx, py, px, centeredh=True, kind="linear", interp=False, engine="numpy"
)
```

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```
RPop = pylops.signalprocessing.Radon3D(
    t, hy, hx, py, px, centeredh=True, kind="parabolic", interp=False, engine="numpy"
)
RHop = pylops.signalprocessing.Radon3D(
    t, hy, hx, py, px, centeredh=True, kind="hyperbolic", interp=False, engine="numpy"
)

# forward
yL = RLop * x.reshape(npy * npx, nt)
yP = RPop * x.reshape(npy * npx, nt)
yH = RHop * x.reshape(npy * npx, nt)

# adjoint
xadjL = RLop.H * yL
xadjP = RPop.H * yP
xadjH = RHop.H * yH

# reshape
yL = yL.reshape(ny, nx, nt)
yP = yP.reshape(ny, nx, nt)
yH = yH.reshape(ny, nx, nt)
xadjL = xadjL.reshape(npy, npx, nt)
xadjP = xadjP.reshape(npy, npx, nt)
xadjH = xadjH.reshape(npy, npx, nt)

# plotting
fig, axs = plt.subplots(2, 4, figsize=(10, 6), sharey=True)
axs[1, 0].imshow(
    x[npy // 2].T,
    vmin=-1,
    vmax=1,
    cmap="seismic_r",
    extent=(1e3 * px[0], 1e3 * px[-1], t[-1], t[0]),
)
axs[1, 0].set(xlabel=r"$p_x$ [s/km]", ylabel=r"$t$ [s]", title="Input model")
axs[1, 0].axis("tight")
axs[0, 1].imshow(
    yL[ny // 2].T,
    vmin=-1,
    vmax=1,
    cmap="seismic_r",
    extent=(hx[0], hx[-1], t[-1], t[0]),
)
axs[0, 1].tick_params(labelleft=True)
axs[0, 1].set(xlabel=r"$x$ [m]", ylabel=r"$t$ [s]", title="Linear data")
axs[0, 1].axis("tight")
axs[0, 2].imshow(
    yP[ny // 2].T,
    vmin=-1,
    vmax=1,
    cmap="seismic_r",
    extent=(hx[0], hx[-1], t[-1], t[0]),
```

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```
)
axs[0, 2].set(xlabel=r"$x$ [m]", title="Parabolic data")
axs[0, 2].axis("tight")
axs[0, 3].imshow(
    yH[ny // 2].T,
    vmin=-1,
    vmax=1,
    cmap="seismic_r",
    extent=(hx[0], hx[-1], t[-1], t[0]),
)
axs[0, 3].set(xlabel=r"$x$ [m]", title="Hyperbolic data")
axs[0, 3].axis("tight")
axs[1, 1].imshow(
    xadjL[npy // 2].T,
    vmin=-100,
    vmax=100,
    cmap="seismic_r",
    extent=(1e3 * px[0], 1e3 * px[-1], t[-1], t[0]),
)
axs[0, 0].axis("off")
axs[1, 1].set(xlabel=r"$p_x$ [s/km]", title="Linear adjoint")
axs[1, 1].axis("tight")
axs[1, 2].imshow(
    xadjP[npy // 2].T,
    vmin=-100,
    vmax=100,
    cmap="seismic_r",
    extent=(1e3 * px[0], 1e3 * px[-1], t[-1], t[0]),
)
axs[1, 2].set(xlabel=r"$p_x$ [s/km]", title="Parabolic adjoint")
axs[1, 2].axis("tight")
axs[1, 3].imshow(
    xadjH[npy // 2].T,
    vmin=-100,
    vmax=100,
    cmap="seismic_r",
    extent=(1e3 * px[0], 1e3 * px[-1], t[-1], t[0]),
)
axs[1, 3].set(xlabel=r"$p_x$ [s/km]", title="Hyperbolic adjoint")
axs[1, 3].axis("tight")
fig.tight_layout()

fig, axs = plt.subplots(2, 4, figsize=(10, 6), sharey=True)
axs[1, 0].imshow(
    x[:, npx // 2 - 2].T,
    vmin=-1,
    vmax=1,
    cmap="seismic_r",
    extent=(1e3 * py[0], 1e3 * py[-1], t[-1], t[0]),
)
axs[1, 0].set(xlabel=r"$p_y$ [s/km]", ylabel=r"$t$ [s]", title="Input model")
axs[1, 0].axis("tight")
```

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```
axs[0, 1].imshow(
    yL[:, nx // 2].T,
    vmin=-1,
    vmax=1,
    cmap="seismic_r",
    extent=(hy[0], hy[-1], t[-1], t[0]),
)
axs[0, 1].tick_params(labelleft=True)
axs[0, 1].set(xlabel=r"$y$ [m]", ylabel=r"$t$ [s]", title="Linear data")
axs[0, 1].axis("tight")
axs[0, 2].imshow(
    yP[:, nx // 2].T,
    vmin=-1,
    vmax=1,
    cmap="seismic_r",
    extent=(hy[0], hy[-1], t[-1], t[0]),
)
axs[0, 2].set(xlabel=r"$y$ [m]", title="Parabolic data")
axs[0, 2].axis("tight")
axs[0, 3].imshow(
    yH[:, nx // 2].T,
    vmin=-1,
    vmax=1,
    cmap="seismic_r",
    extent=(hy[0], hy[-1], t[-1], t[0]),
)
axs[0, 3].set(xlabel=r"$y$ [m]", title="Hyperbolic data")
axs[0, 3].axis("tight")
axs[1, 1].imshow(
    xadjL[:, npx // 2 - 5].T,
    vmin=-100,
    vmax=100,
    cmap="seismic_r",
    extent=(1e3 * py[0], 1e3 * py[-1], t[-1], t[0]),
)
axs[0, 0].axis("off")
axs[1, 1].set(xlabel=r"$p_y$ [s/km]", title="Linear adjoint")
axs[1, 1].axis("tight")
axs[1, 2].imshow(
    xadjP[:, npx // 2 - 2].T,
    vmin=-100,
    vmax=100,
    cmap="seismic_r",
    extent=(1e3 * py[0], 1e3 * py[-1], t[-1], t[0]),
)
axs[1, 2].set(xlabel=r"$p_y$ [s/km]", title="Parabolic adjoint")
axs[1, 2].axis("tight")
axs[1, 3].imshow(
    xadjH[:, npx // 2 - 2].T,
    vmin=-100,
    vmax=100,
    cmap="seismic_r",
```

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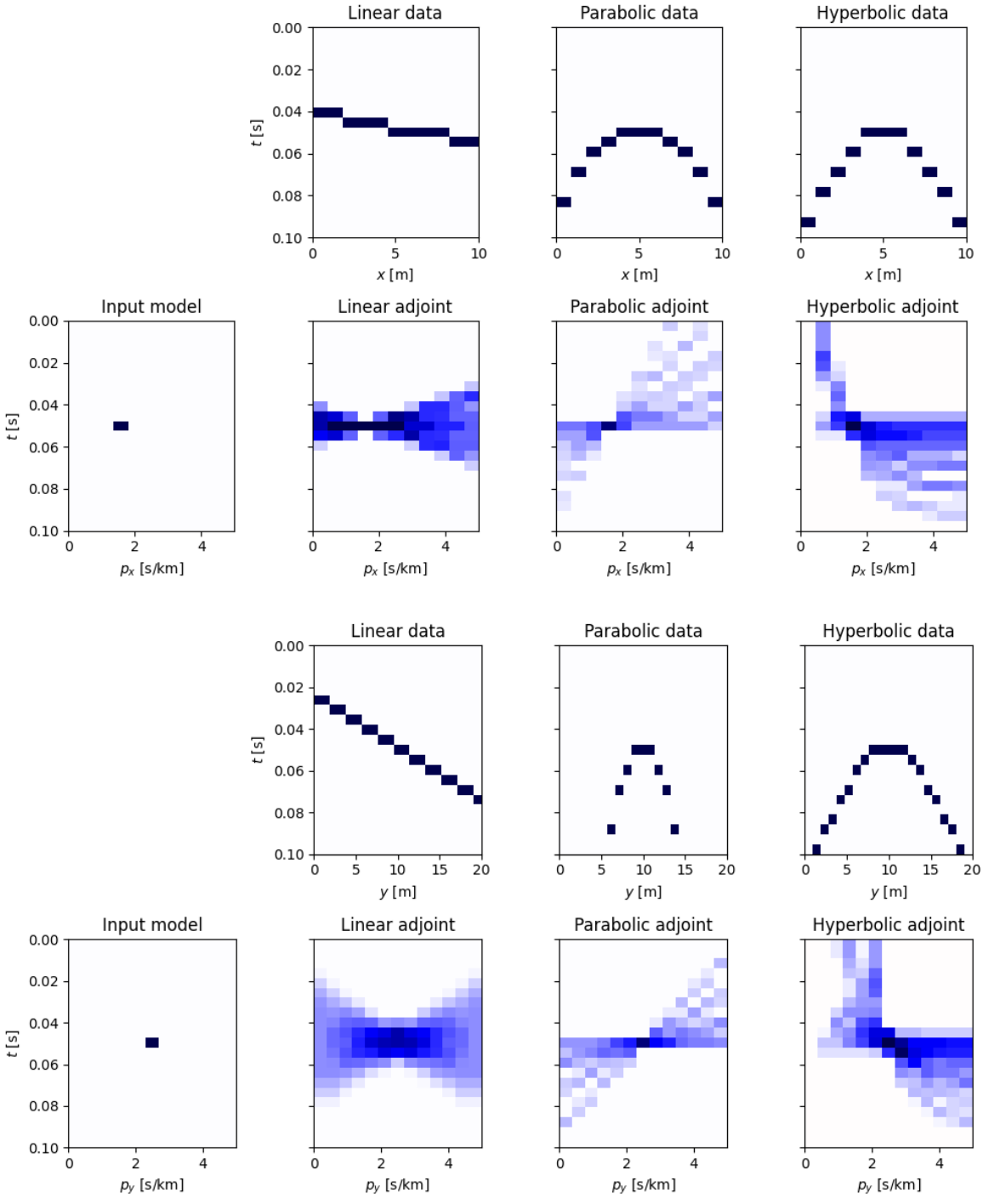
```
    extent=(1e3 * py[0], 1e3 * py[-1], t[-1], t[0]),
)
axs[1, 3].set(xlabel=r"$p_y$ [s/km]", title="Hyperbolic adjoint")
axs[1, 3].axis("tight")
fig.tight_layout()
```



Total running time of the script: (0 minutes 2.203 seconds)

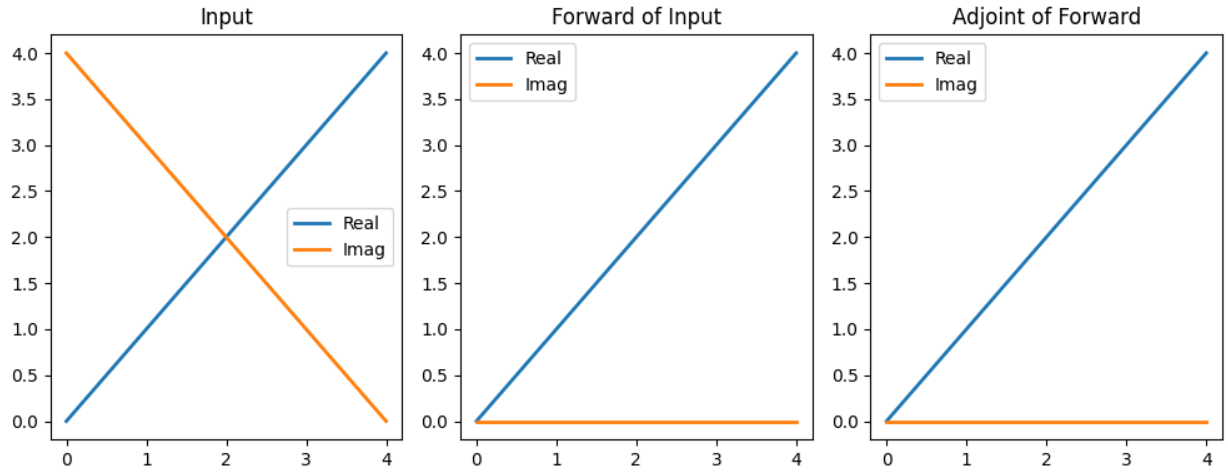
3.5.31 Real

This example shows how to use the `pylops.basicoperators.Real` operator. This operator returns the real part of the data in forward and adjoint mode, but the forward output will be a real number, while the adjoint output will be a complex number with a zero-valued imaginary part.

```
import matplotlib.pyplot as plt
import numpy as np

import pylops

plt.close("all")
```

Let's define a Real operator $\Re$ to extract the real component of the input.

```
M = 5
x = np.arange(M) + 1j * np.arange(M)[::-1]
Rop = pylops.basicoperators.Real(M, dtype="complex128")

y = Rop * x
xadj = Rop.H * y

_, axs = plt.subplots(1, 3, figsize=(10, 4))
axs[0].plot(np.real(x), lw=2, label="Real")
axs[0].plot(np.imag(x), lw=2, label="Imag")
axs[0].legend()
axs[0].set_title("Input")
axs[1].plot(np.real(y), lw=2, label="Real")
axs[1].plot(np.imag(y), lw=2, label="Imag")
axs[1].legend()
axs[1].set_title("Forward of Input")
axs[2].plot(np.real(xadj), lw=2, label="Real")
axs[2].plot(np.imag(xadj), lw=2, label="Imag")
axs[2].legend()
axs[2].set_title("Adjoint of Forward")
plt.tight_layout()
```



Total running time of the script: (0 minutes 0.358 seconds)

3.5.32 Restriction and Interpolation

This example shows how to use the *pylops.Restriction* operator to sample a certain input vector at desired locations `iava`. Moreover, we go one step further and use the *pylops.signalprocessing.Interp* operator to show how we can also sample values at locations that are not exactly on the grid of the input vector.

As explained in the *03. Solvers* tutorial, such operators can be used as forward model in an inverse problem aimed at interpolate irregularly sampled 1d or 2d signals onto a regular grid.

```
import matplotlib.pyplot as plt
import numpy as np

import pylops

plt.close("all")
np.random.seed(10)
```

Let's create a signal of size `nt` and sampling `dt` that is composed of three sinusoids at frequencies `freqs`.

```
nt = 200
dt = 0.004

freqs = [5.0, 3.0, 8.0]

t = np.arange(nt) * dt
x = np.zeros(nt)

for freq in freqs:
    x = x + np.sin(2 * np.pi * freq * t)
```

First of all, we subsample the signal at random locations and we retain 40% of the initial samples.

```
perc_subsampling = 0.4
ntsub = int(np.round(nt * perc_subsampling))

isample = np.arange(nt)
iava = np.sort(np.random.permutation(np.arange(nt))[:ntsub])
```

We then create the restriction and interpolation operators and display the original signal as well as the subsampled signal.

```
Rop = pylops.Restriction(nt, iava, dtype="float64")
NNop, iavann = pylops.signalprocessing.Interp(
    nt, iava + 0.4, kind="nearest", dtype="float64"
)
LIop, iavali = pylops.signalprocessing.Interp(
    nt, iava + 0.4, kind="linear", dtype="float64"
)
SIop, iavasi = pylops.signalprocessing.Interp(
    nt, iava + 0.4, kind="sinc", dtype="float64"
)
y = Rop * x
ynn = NNop * x
yli = LIop * x
```

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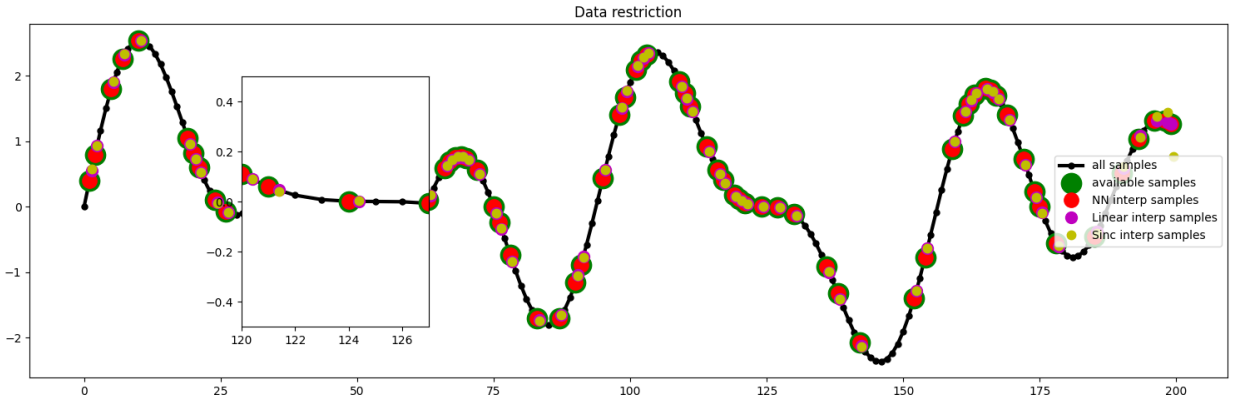
```
ysi = SIop * x
ymask = Rop.mask(x)

# Visualize data
fig = plt.figure(figsize=(15, 5))
plt.plot(isample, x, ".-k", lw=3, ms=10, label="all samples")
plt.plot(isample, ymask, ".g", ms=35, label="available samples")
plt.plot(iavann, ynn, ".r", ms=25, label="NN interp samples")
plt.plot(iavali, yli, ".m", ms=20, label="Linear interp samples")
plt.plot(iavasi, ysi, ".y", ms=15, label="Sinc interp samples")
plt.legend(loc="right")
plt.title("Data restriction")

subax = fig.add_axes([0.2, 0.2, 0.15, 0.6])
subax.plot(isample, x, ".-k", lw=3, ms=10)
subax.plot(isample, ymask, ".g", ms=35)
subax.plot(iavann, ynn, ".r", ms=25)
subax.plot(iavali, yli, ".m", ms=20)
subax.plot(iavasi, ysi, ".y", ms=15)
subax.set_xlim([120, 127])
subax.set_ylim([-0.5, 0.5])
plt.tight_layout()
```



Finally we show how the *pylops.Restriction* is not limited to one dimensional signals but can be applied to sample locations of a specific axis of a multi-dimensional array. subsampling locations

```
nx, nt = 100, 50

x = np.random.normal(0, 1, (nx, nt))

perc_subsampling = 0.4
nxsub = int(np.round(nx * perc_subsampling))
iava = np.sort(np.random.permutation(np.arange(nx))[:nxsub])

Rop = pylops.Restriction((nx, nt), iava, axis=0, dtype="float64")
y = Rop * x
ymask = Rop.mask(x)
```

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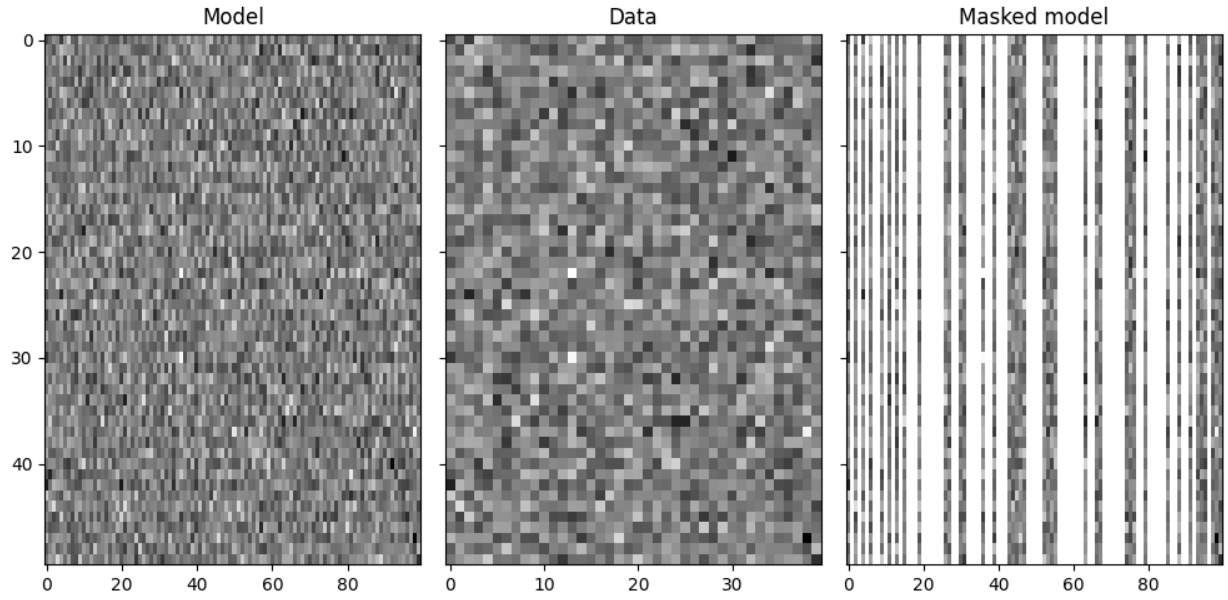
```
fig, axs = plt.subplots(1, 3, figsize=(10, 5), sharey=True)
axs[0].imshow(x.T, cmap="gray")
axs[0].set_title("Model")
axs[0].axis("tight")
axs[1].imshow(y.T, cmap="gray")
axs[1].set_title("Data")
axs[1].axis("tight")
axs[2].imshow(ymask.T, cmap="gray")
axs[2].set_title("Masked model")
axs[2].axis("tight")
plt.tight_layout()
```



Total running time of the script: (0 minutes 0.581 seconds)

3.5.33 Roll

This example shows how to use the *pylops.Roll* operator.

This operator simply shifts elements of multi-dimensional array along a specified direction a chosen number of samples.

```
import matplotlib.pyplot as plt
import numpy as np

import pylops

plt.close("all")
```

Let's start with a 1d example. We make a signal, shift it by two samples and then shift it back using its adjoint. We can immediately see how the adjoint of this operator is equivalent to its inverse.

```
nx = 10
x = np.arange(nx)
```

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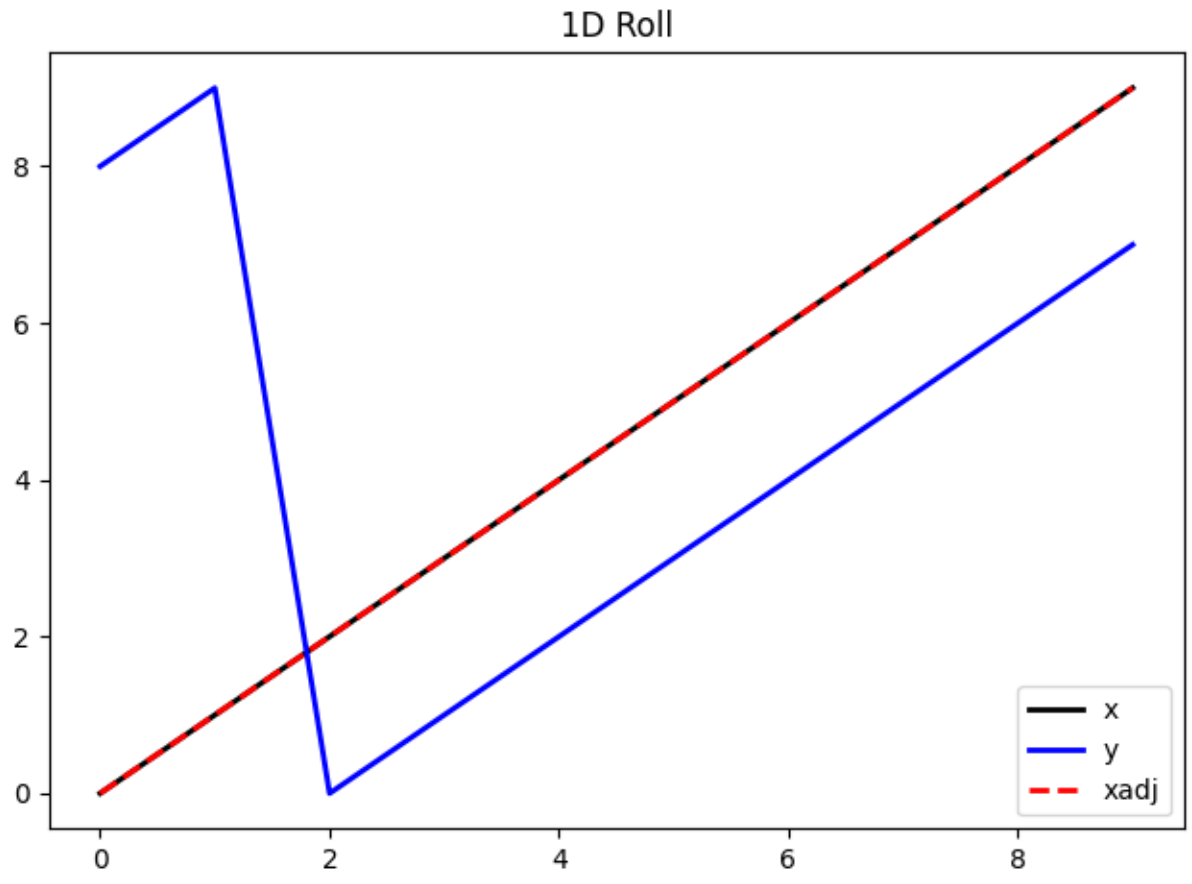
```
Rop = pylops.Roll(nx, shift=2)

y = Rop * x
xadj = Rop.H * y

plt.figure()
plt.plot(x, "k", lw=2, label="x")
plt.plot(y, "b", lw=2, label="y")
plt.plot(xadj, "--r", lw=2, label="xadj")
plt.title("1D Roll")
plt.legend()
plt.tight_layout()
```



We can now do the same with a 2d array.

```
ny, nx = 10, 5
x = np.arange(ny * nx).reshape(ny, nx)

Rop = pylops.Roll(dims=(ny, nx), axis=1, shift=-2)

y = Rop * x
xadj = Rop.H * y
```

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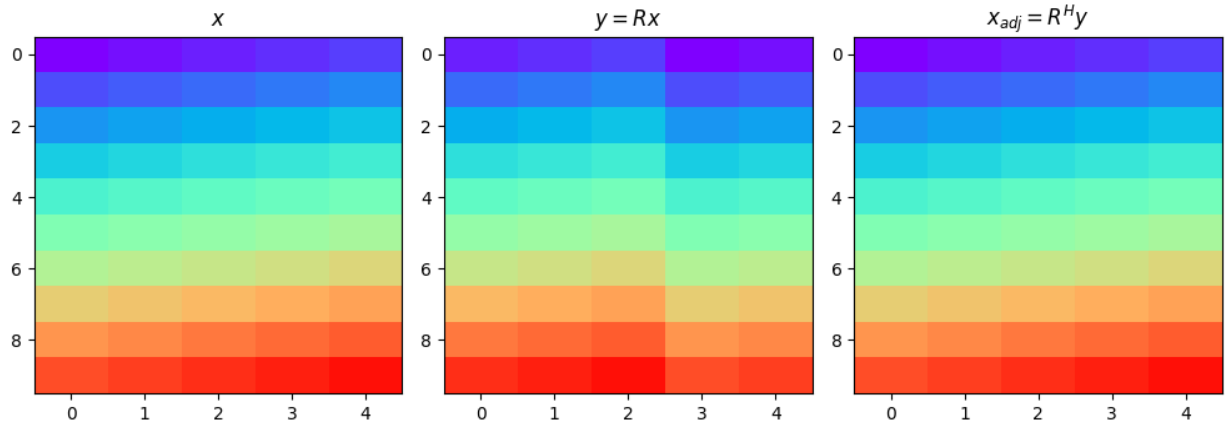
```
fig, axs = plt.subplots(1, 3, figsize=(10, 4))
fig.suptitle("Roll for 2d data", fontsize=14, fontweight="bold", y=1.15)
axs[0].imshow(x, cmap="rainbow", vmin=0, vmax=50)
axs[0].set_title(r"$x$")
axs[0].axis("tight")
axs[1].imshow(y, cmap="rainbow", vmin=0, vmax=50)
axs[1].set_title(r"$y = R x$")
axs[1].axis("tight")
axs[2].imshow(xadj, cmap="rainbow", vmin=0, vmax=50)
axs[2].set_title(r"$x_{adj} = R^H y$")
axs[2].axis("tight")
plt.tight_layout()
```



Total running time of the script: (0 minutes 0.471 seconds)

3.5.34 Seislet transform

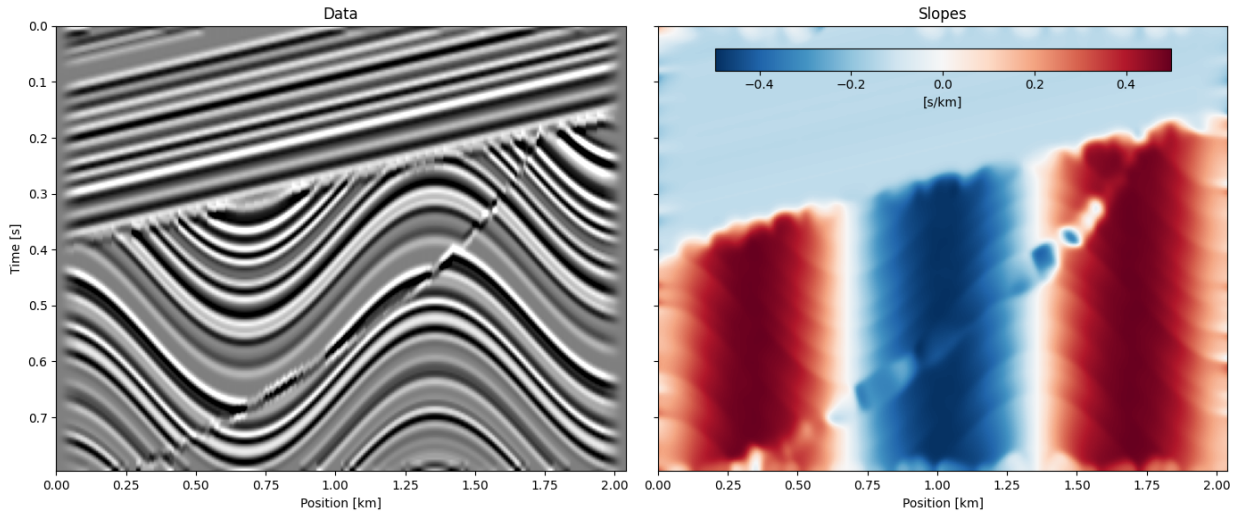
This example shows how to use the *pylops.signalprocessing.Seislet* operator. This operator the forward, adjoint and inverse Seislet transform that is a modification of the well-know Wavelet transform where local slopes are used in the prediction and update steps to further improve the prediction of a trace from its previous (or subsequent) one and reduce the amount of information passed to the subsequent scale. While this transform was initially developed in the context of processing and compression of seismic data, it is also suitable to any other oscillatory dataset such as GPR or Acoustic recordings.

```
import matplotlib.pyplot as plt
import numpy as np
from matplotlib.ticker import MaxNLocator
from mpl_toolkits.axes_grid1 import make_axes_locatable

import pylops

plt.close("all")
```

In this example we use the same benchmark dataset that was used in the original paper describing the Seislet transform. First, local slopes are estimated using *pylops.utils.signalprocessing.slope_estimate*.

```
inputfile = "../testdata/sigmoid.npz"

d = np.load(inputfile)
d = d["sigmoid"]
nx, nt = d.shape
dx, dt = 0.008, 0.004
x, t = np.arange(nx) * dx, np.arange(nt) * dt

# slope estimation
slope, _ = pylops.utils.signalprocessing.slope_estimate(d.T, dt, dx, smooth=2.5)
slope = -slope.T  # t-axis points down, reshape
# clip slopes above 80°
pmax = np.arctan(80 * np.pi / 180)
slope[slope > pmax] = pmax
slope[slope < -pmax] = -pmax
```

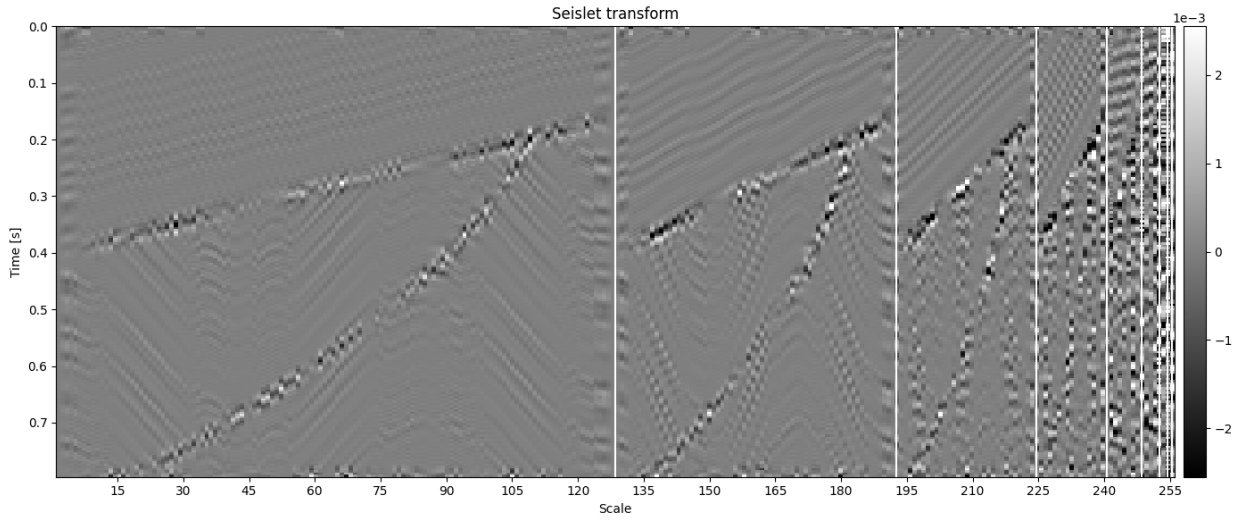
```
clip = 0.5 * np.max(np.abs(d))
clip_s = min(pmax, np.max(np.abs(slope)))
opts = dict(aspect=2, extent=(x[0], x[-1], t[-1], t[0]))

fig, axs = plt.subplots(1, 2, figsize=(14, 7), sharey=True, sharex=True)
axs[0].imshow(d.T, cmap="gray", vmin=-clip, vmax=clip, **opts)
axs[0].set(xlabel="Position [km]", ylabel="Time [s]", title="Data")

im = axs[1].imshow(slope.T, cmap="RdBu_r", vmin=-clip_s, vmax=clip_s, **opts)
axs[1].set(xlabel="Position [km]", title="Slopes")
fig.tight_layout()

pos = axs[1].get_position()
cbpos = [
    pos.x0 + 0.1 * pos.width,
    pos.y0 + 0.9 * pos.height,
    0.8 * pos.width,
    0.05 * pos.height,
]
cax = fig.add_axes(cbpos)
cb = fig.colorbar(im, cax=cax, orientation="horizontal")
cb.set_label("[s/km]")
```



Next the Seislet transform is computed.

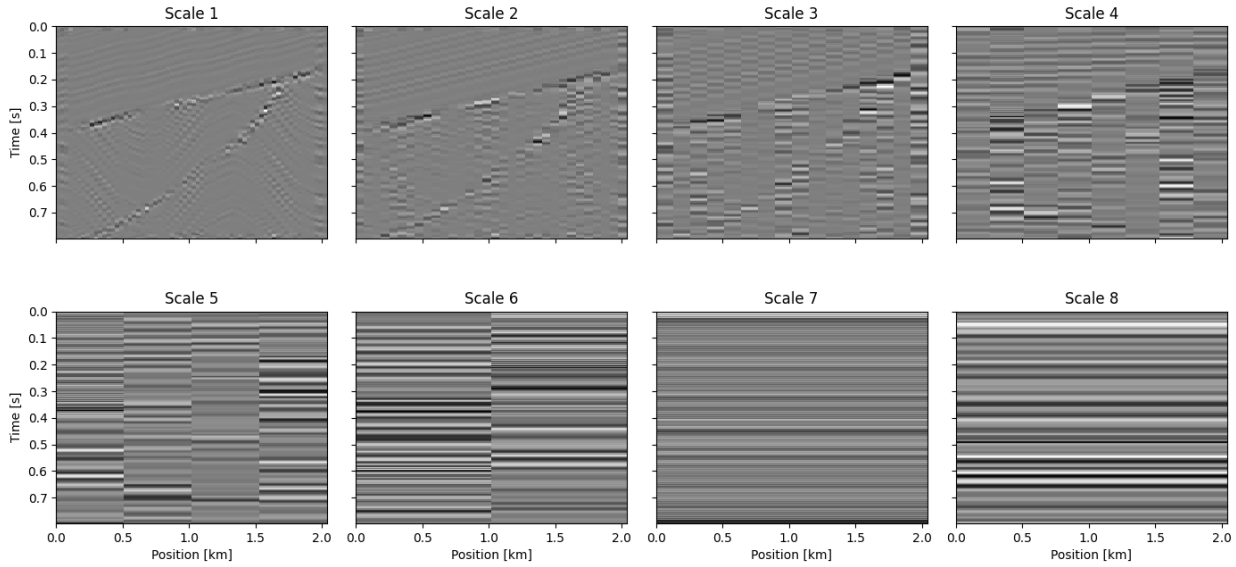
```
Sop = pylops.signalprocessing.Seislet(slope, sampling=(dx, dt))

seis = Sop * d

nlevels_max = int(np.log2(nx))
levels_size = np.flip(np.array([2**i for i in range(nlevels_max)]))
levels_cum = np.cumsum(levels_size)
```

```
fig, ax = plt.subplots(figsize=(14, 6))
im = ax.imshow(
    seis.T,
    cmap="gray",
    vmin=-clip,
    vmax=clip,
    aspect="auto",
    interpolation="none",
    extent=(1, seis.shape[0], t[-1], t[0]),
)
ax.xaxis.set_major_locator(MaxNLocator(nbins=20, integer=True))
for level in levels_cum:
    ax.axvline(level + 0.5, color="w")
ax.set(xlabel="Scale", ylabel="Time [s]", title="Seislet transform")
cax = make_axes_locatable(ax).append_axes("right", size="2%", pad=0.1)
cb = fig.colorbar(im, cax=cax, orientation="vertical")
cb.formatter.set_powerlimits((0, 0))
fig.tight_layout()
```



We may also stretch the finer scales to be the width of the image

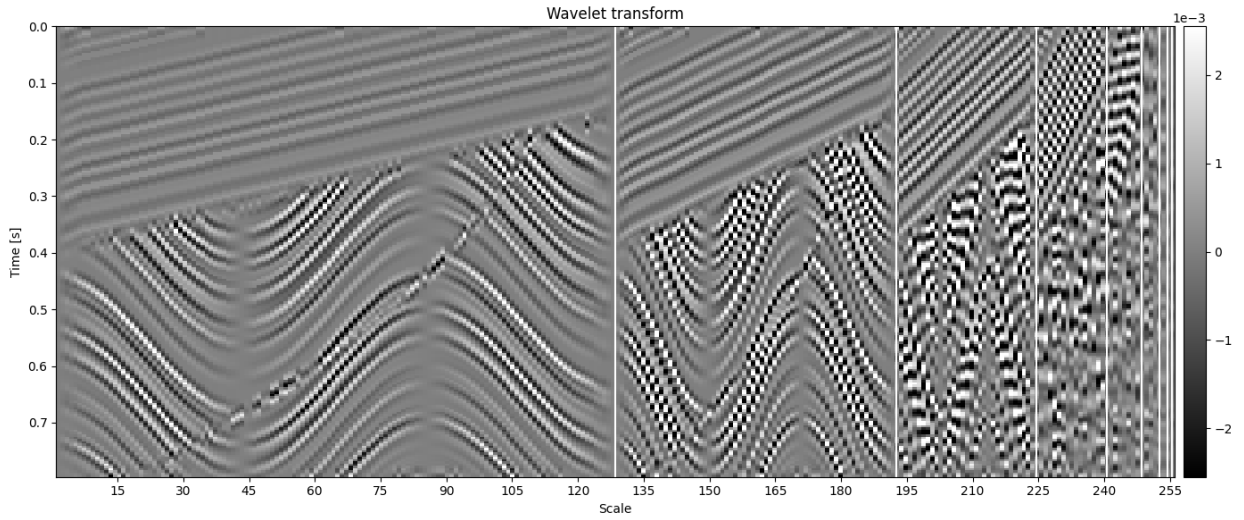
```
fig, axs = plt.subplots(2, nlevels_max // 2, figsize=(14, 7), sharex=True, sharey=True)
for i, ax in enumerate(axs.ravel()[:-1]):
    curdata = seis[levels_cum[i] : levels_cum[i + 1], :].T
    vmax = np.max(np.abs(curdata))
    ax.imshow(curdata, vmin=-vmax, vmax=vmax, cmap="gray", interpolation="none", **opts)
    ax.set(title=f"Scale {i+1}")
    if i + 1 > nlevels_max // 2:
        ax.set(xlabel="Position [km]")
curdata = seis[levels_cum[-1] :, :].T
vmax = np.max(np.abs(curdata))
axs[-1, -1].imshow(
    curdata, vmin=-vmax, vmax=vmax, cmap="gray", interpolation="none", **opts
)
axs[0, 0].set(ylabel="Time [s]")
axs[1, 0].set(ylabel="Time [s]")
axs[-1, -1].set(xlabel="Position [km]", title=f"Scale {nlevels_max}")
fig.tight_layout()
```



As a comparison we also compute the Seislet transform fixing slopes to zero. This way we turn the Seislet tranform into a basic 1D Wavelet transform performed over the spatial axis.

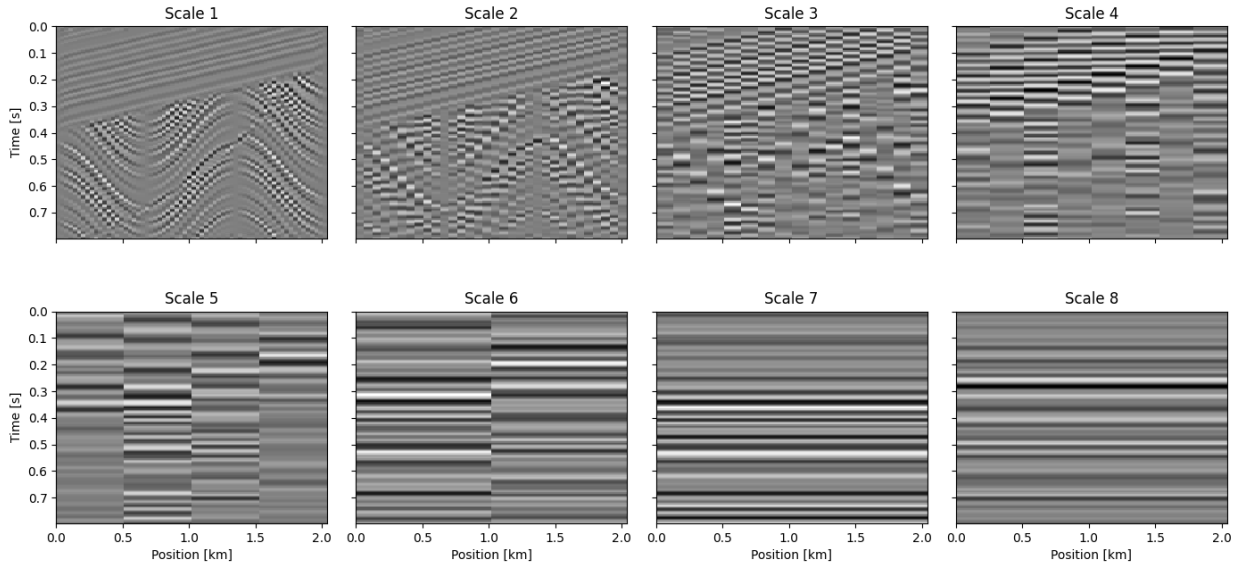
```
Wop = pylops.signalprocessing.Seislet(np.zeros_like(slope), sampling=(dx, dt))
dwt = Wop * d
```

```
fig, ax = plt.subplots(figsize=(14, 6))
im = ax.imshow(
    dwt.T,
    cmap="gray",
    vmin=-clip,
    vmax=clip,
    aspect="auto",
    interpolation="none",
    extent=(1, dwt.shape[0], t[-1], t[0]),
)
ax.xaxis.set_major_locator(MaxNLocator(nbins=20, integer=True))
for level in levels_cum:
    ax.axvline(level + 0.5, color="w")
ax.set(xlabel="Scale", ylabel="Time [s]", title="Wavelet transform")
cax = make_axes_locatable(ax).append_axes("right", size="2%", pad=0.1)
cb = fig.colorbar(im, cax=cax, orientation="vertical")
cb.formatter.set_powerlimits((0, 0))
fig.tight_layout()
```



Again, we may decompress the finer scales

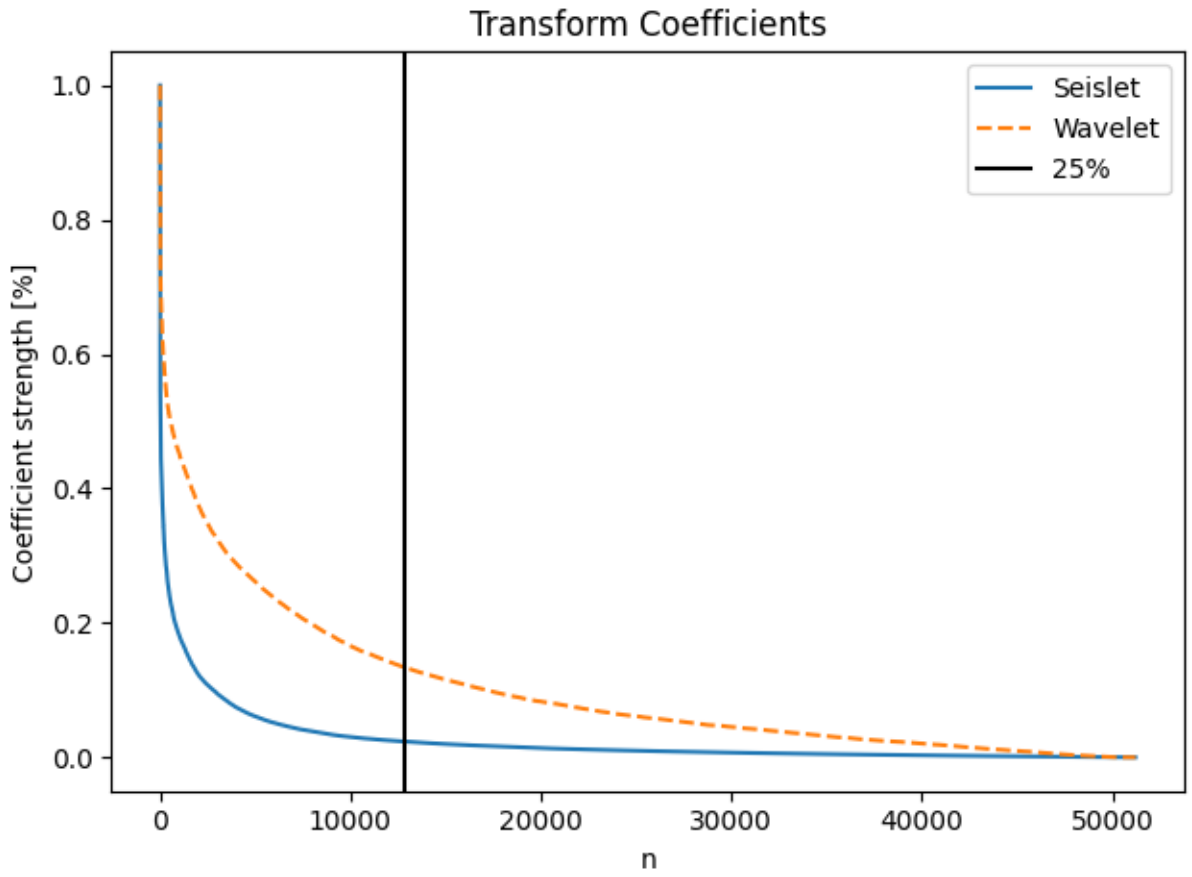
```
fig, axs = plt.subplots(2, nlevels_max // 2, figsize=(14, 7), sharex=True, sharey=True)
for i, ax in enumerate(axs.ravel()[:-1]):
    curdata = dwt[levels_cum[i] : levels_cum[i + 1], :].T
    vmax = np.max(np.abs(curdata))
    ax.imshow(curdata, vmin=-vmax, vmax=vmax, cmap="gray", interpolation="none", **opts)
    ax.set(title=f"Scale {i+1}")
    if i + 1 > nlevels_max // 2:
        ax.set(xlabel="Position [km]")
curdata = dwt[levels_cum[-1] :, :].T
vmax = np.max(np.abs(curdata))
axs[-1, -1].imshow(
    curdata, vmin=-vmax, vmax=vmax, cmap="gray", interpolation="none", **opts
)
axs[0, 0].set(ylabel="Time [s]")
axs[1, 0].set(ylabel="Time [s]")
axs[-1, -1].set(xlabel="Position [km]", title=f"Scale {nlevels_max}")
fig.tight_layout()
```



Finally we evaluate the compression capabilities of the Seislet transform compared to the 1D Wavelet transform. We zero-out all but the strongest 25% of the components. We perform the inverse transforms and assess the compression error.

```
perc = 0.25
seis_strong_idx = np.argsort(-np.abs(seis.ravel()))
dwt_strong_idx = np.argsort(-np.abs(dwt.ravel()))
seis_strong = np.abs(seis.ravel())[seis_strong_idx]
dwt_strong = np.abs(dwt.ravel())[dwt_strong_idx]
```

```
fig, ax = plt.subplots()
ax.plot(range(1, len(seis_strong) + 1), seis_strong / seis_strong[0], label="Seislet")
ax.plot(
    range(1, len(dwt_strong) + 1), dwt_strong / dwt_strong[0], "--", label="Wavelet"
)
ax.set(xlabel="n", ylabel="Coefficient strength [%]", title="Transform Coefficients")
ax.axvline(np.rint(len(seis_strong) * perc), color="k", label=f"{100*perc:.0f}%")
ax.legend()
fig.tight_layout()
```




```
seis1 = np.zeros_like(seis.ravel())
seis_strong_idx = seis_strong_idx[: int(np.rint(len(seis_strong) * perc))]
seis1[seis_strong_idx] = seis.ravel()[seis_strong_idx]
d_seis = Sop.inverse(seis1).reshape(Sop.dims)

dwt1 = np.zeros_like(dwt.ravel())
dwt_strong_idx = dwt_strong_idx[: int(np.rint(len(dwt_strong) * perc))]
dwt1[dwt_strong_idx] = dwt.ravel()[dwt_strong_idx]
d_dwt = Wop.inverse(dwt1).reshape(Wop.dims)
```

```
opts.update(dict(cmap="gray", vmin=-clip, vmax=clip))
fig, axs = plt.subplots(2, 3, figsize=(14, 7), sharex=True, sharey=True)
axs[0, 0].imshow(d.T, **opts)
axs[0, 0].set(title="Data")
axs[0, 1].imshow(d_seis.T, **opts)
axs[0, 1].set(title=f"Rec. from Seislet ({100*perc:.0f}% of coeffs.)")
axs[0, 2].imshow((d - d_seis).T, **opts)
axs[0, 2].set(title="Error from Seislet Rec.")
axs[1, 0].imshow(d.T, **opts)
axs[1, 0].set(ylabel="Time [s]", title="Data [Repeat]")
axs[1, 1].imshow(d_dwt.T, **opts)
axs[1, 1].set(title=f"Rec. from Wavelet ({100*perc:.0f}% of coeffs.)")
axs[1, 2].imshow((d - d_dwt).T, **opts)
```

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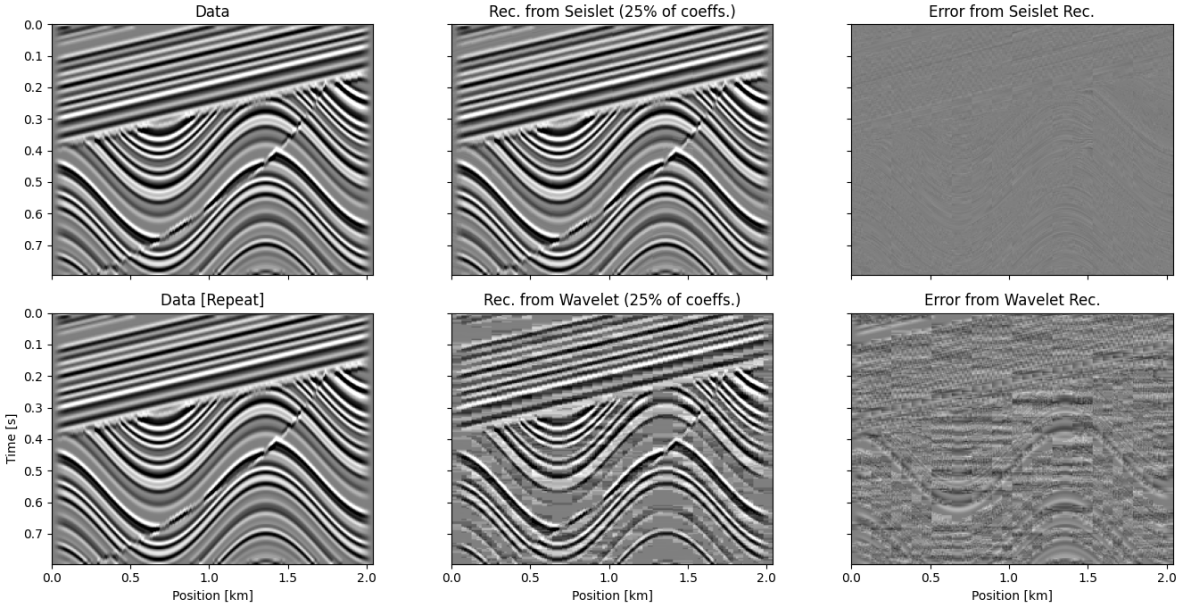
```
axs[1, 2].set(title="Error from Wavelet Rec.")
for i in range(3):
    axs[1, i].set(xlabel="Position [km]")
plt.tight_layout()
```



To conclude it is worth noting that the Seislet transform, differently to the Wavelet transform, is not orthogonal: in other words, its adjoint and inverse are not equivalent. While we have used the forward and inverse transformations, when used as linear operator in composition with other operators, the Seislet transform requires the adjoint be defined and that it also passes the dot-test pair that is. As shown below, this is the case when using the implementation in the PyLops package.

```
pylops.utils.dottest(Sop, verb=True)
```

```
Dot test passed, v^H(Opu)=-98.52334183020213 - u^H(Op^Hv)=-98.52334183020218

True
```

Total running time of the script: (0 minutes 12.160 seconds)

3.5.35 Shift

This example shows how to use the *pylops.signalprocessing.Shift* operator to apply fractional delay to an input signal. Whilst this operator acts on 1D signals it can also be applied on any multi-dimensional signal on a specific direction of it.

```
import matplotlib.pyplot as plt
import numpy as np

import pylops

plt.close("all")
```

Let's start with a 1D example. Define the input parameters: number of samples of input signal (`nt`), sampling step (`dt`) as well as the input signal which will be equal to a ricker wavelet:

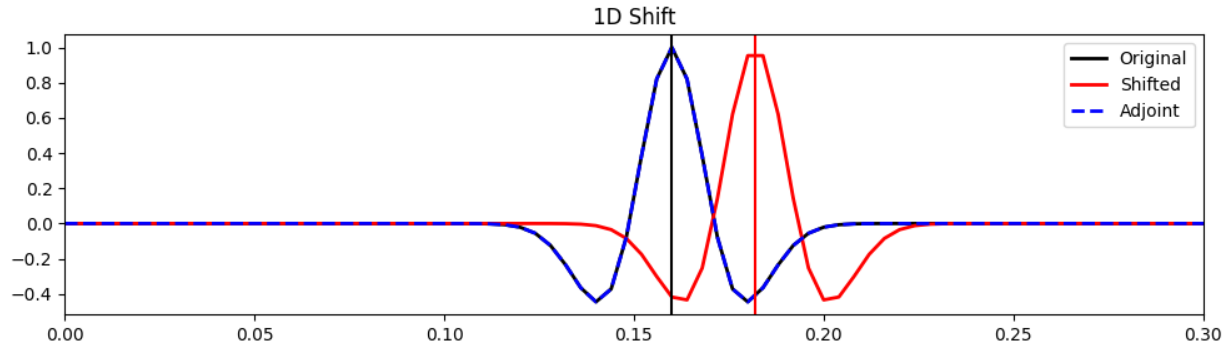
```
nt = 127
dt = 0.004
t = np.arange(nt) * dt
ntwav = 41

wav = pylops.utils.wavelets.ricker(t[:ntwav], f0=20)[0]
wav = np.pad(wav, [0, nt - len(wav)])
WAV = np.fft.rfft(wav, n=nt)
```

We can shift this wavelet by 5.5dt:

```
shift = 5.5 * dt
Op = pylops.signalprocessing.Shift(nt, shift, sampling=dt, real=True, dtype=np.float64)
wavshift = Op * wav
wavshiftback = Op.H * wavshift

plt.figure(figsize=(10, 3))
plt.plot(t, wav, "k", lw=2, label="Original")
plt.plot(t, wavshift, "r", lw=2, label="Shifted")
plt.plot(t, wavshiftback, "--b", lw=2, label="Adjoint")
plt.axvline(t[ntwav - 1], color="k")
plt.axvline(t[ntwav - 1] + shift, color="r")
plt.xlim(0, 0.3)
plt.legend()
plt.title("1D Shift")
plt.tight_layout()
```



We can repeat the same exercise for a 2D signal and perform the shift along the first and second dimensions.

```
shift = 10.5 * dt

# 1st axis
wav2d = np.outer(wav, np.ones(10))
Op = pylops.signalprocessing.Shift(
    (nt, 10), shift, axis=0, sampling=dt, real=True, dtype=np.float64
)
wav2dshift = Op * wav2d
wav2dshiftback = Op.H * wav2dshift
```

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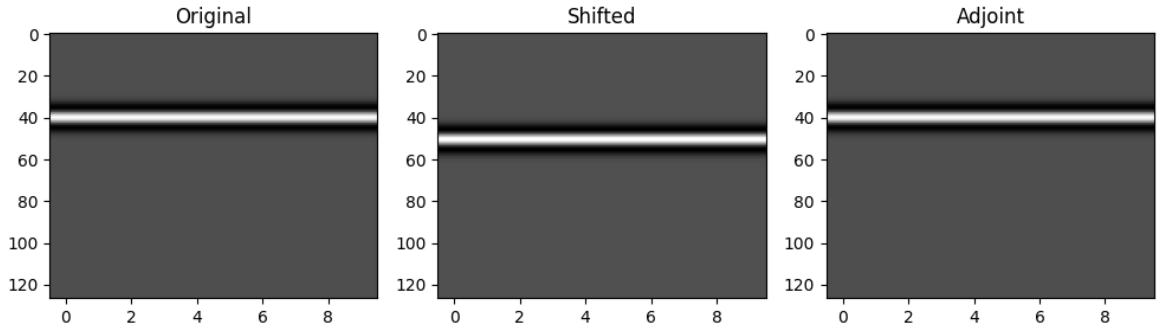
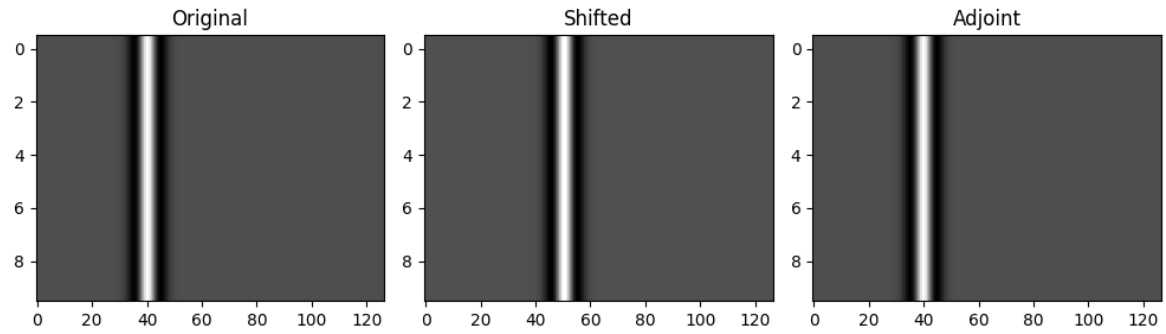
```
fig, axs = plt.subplots(1, 3, figsize=(10, 3))
axs[0].imshow(wav2d, cmap="gray")
axs[0].axis("tight")
axs[0].set_title("Original")
axs[1].imshow(wav2dshift, cmap="gray")
axs[1].set_title("Shifted")
axs[1].axis("tight")
axs[2].imshow(wav2dshiftback, cmap="gray")
axs[2].set_title("Adjoint")
axs[2].axis("tight")
fig.tight_layout()

# 2nd axis
wav2d = np.outer(wav, np.ones(10)).T
Op = pylops.signalprocessing.Shift(
    (10, nt), shift, axis=1, sampling=dt, real=True, dtype=np.float64
)
wav2dshift = Op * wav2d
wav2dshiftback = Op.H * wav2dshift

fig, axs = plt.subplots(1, 3, figsize=(10, 3))
axs[0].imshow(wav2d, cmap="gray")
axs[0].axis("tight")
axs[0].set_title("Original")
axs[1].imshow(wav2dshift, cmap="gray")
axs[1].set_title("Shifted")
axs[1].axis("tight")
axs[2].imshow(wav2dshiftback, cmap="gray")
axs[2].set_title("Adjoint")
axs[2].axis("tight")
fig.tight_layout()
```



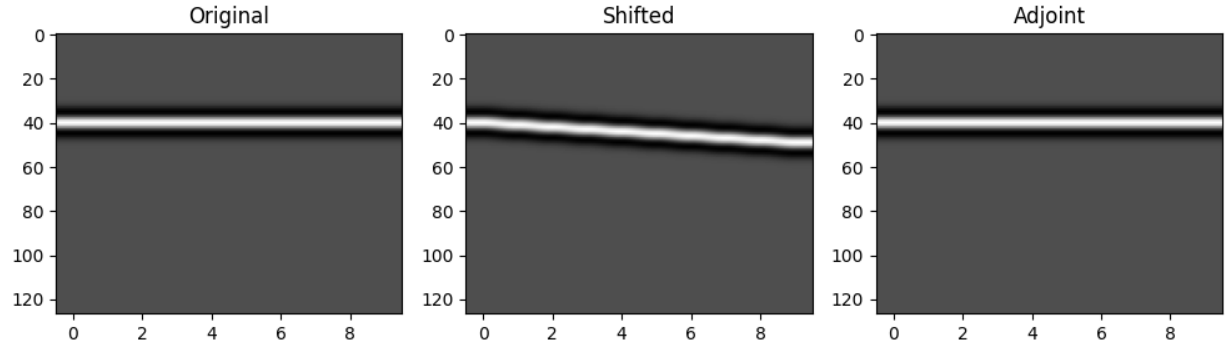


Finally we consider a more generic case where we apply a trace varying shift

```
shift = dt * np.arange(10)

wav2d = np.outer(wav, np.ones(10))
Op = pylops.signalprocessing.Shift(
    (nt, 10), shift, axis=0, sampling=dt, real=True, dtype=np.float64
)
wav2dshift = Op * wav2d
wav2dshiftback = Op.H * wav2dshift

fig, axs = plt.subplots(1, 3, figsize=(10, 3))
axs[0].imshow(wav2d, cmap="gray")
axs[0].axis("tight")
axs[0].set_title("Original")
axs[1].imshow(wav2dshift, cmap="gray")
axs[1].set_title("Shifted")
axs[1].axis("tight")
axs[2].imshow(wav2dshiftback, cmap="gray")
axs[2].set_title("Adjoint")
axs[2].axis("tight")
fig.tight_layout()
```



Total running time of the script: (0 minutes 1.116 seconds)

3.5.36 Slope estimation via Structure Tensor algorithm

This example shows how to estimate local slopes or local dips of a two-dimensional array using *pylops.utils.signalprocessing.slope_estimate* and *pylops.utils.signalprocessing.dip_estimate*.

Knowing the local slopes of an image (or a seismic data) can be useful for a variety of tasks in image (or geophysical) processing such as denoising, smoothing, or interpolation. When slopes are used with the *pylops.signalprocessing.Seislet* operator, the input dataset can be compressed and the sparse nature of the Seislet transform can also be used to precondition sparsity-promoting inverse problems.

We will show examples of a variety of different settings, including a comparison with the original implementation in [1].

```
import matplotlib.pyplot as plt
import numpy as np
from matplotlib.image import imread
from matplotlib.ticker import FuncFormatter, MultipleLocator
from mpl_toolkits.axes_grid1 import make_axes_locatable

import pylops
from pylops.signalprocessing.seislet import _predict_trace
from pylops.utils.signalprocessing import dip_estimate, slope_estimate

plt.close("all")
np.random.seed(10)
```

Python logo

To start we import a 2d image and estimate the local dips of the image.

```
im = np.load("../testdata/python.npy")[..., 0]
im = im / 255.0 - 0.5

angles, anisotropy = dip_estimate(im, smooth=7)
angles = -np.rad2deg(angles)
```

```
fig, axs = plt.subplots(1, 3, figsize=(12, 4), sharex=True, sharey=True)
iax = axs[0].imshow(im, cmap="viridis", origin="lower")
axs[0].set_title("Data")
cax = make_axes_locatable(axs[0]).append_axes("right", size="5%", pad=0.05)
cax.axis("off")

iax = axs[1].imshow(angles, cmap="twilight_shifted", origin="lower", vmin=-90, vmax=90)
axs[1].set_title("Angle of incline")
cax = make_axes_locatable(axs[1]).append_axes("right", size="5%", pad=0.05)
cb = fig.colorbar(
    iax,
    ticks=MultipleLocator(30),
    format=FuncFormatter(lambda x, pos: "{:.0f}°".format(x)),
    cax=cax,
    orientation="vertical",
)
```

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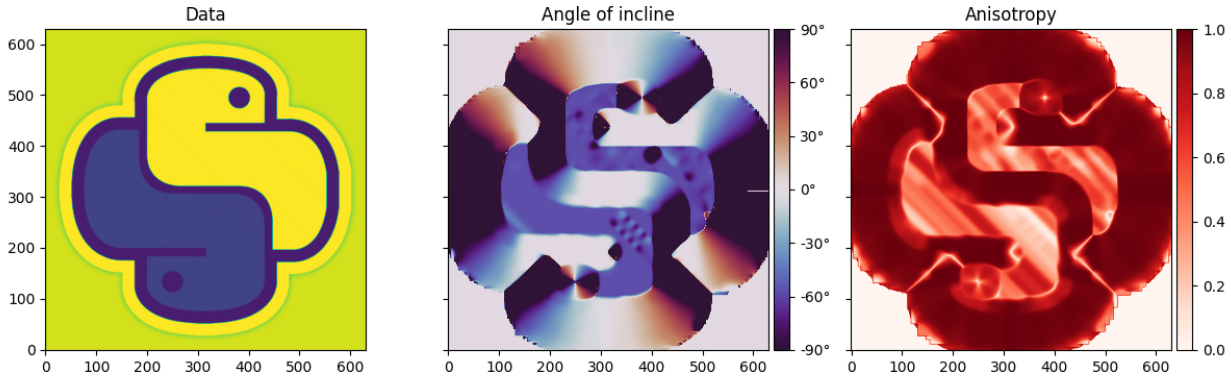
```
iax = axs[2].imshow(anisotropy, cmap="Reds", origin="lower", vmin=0, vmax=1)
axs[2].set_title("Anisotropy")
cax = make_axes_locatable(axs[2]).append_axes("right", size="5%", pad=0.05)
cb = fig.colorbar(iax, cax=cax, orientation="vertical")
fig.tight_layout()
```



Seismic data

We can now repeat the same using some seismic data. We will first define a single trace and a slope field, apply such slope field to the trace recursively to create the other traces of the data and finally try to recover the underlying slope field from the data alone.

```
# Reflectivity model
nx, nt = 2**7, 121
dx, dt = 0.01, 0.004
x, t = np.arange(nx) * dx, np.arange(nt) * dt

nspike = nt // 8
refl = np.zeros(nt)
it = np.sort(np.random.permutation(range(10, nt - 20))[:nspike])
refl[it] = np.random.normal(0.0, 1.0, nspike)

# Wavelet
ntwav = 41
f0 = 30
twav = np.arange(ntwav) * dt
wav, *_ = pylops.utils.wavelets.ricker(twav, f0)

# Input trace
trace = np.convolve(refl, wav, mode="same")

# Slopes
theta = np.deg2rad(np.linspace(0, 30, nx))
slope = np.outer(np.ones(nt), np.tan(theta) * dt / dx)

# Model data
d = np.zeros((nt, nx))
```

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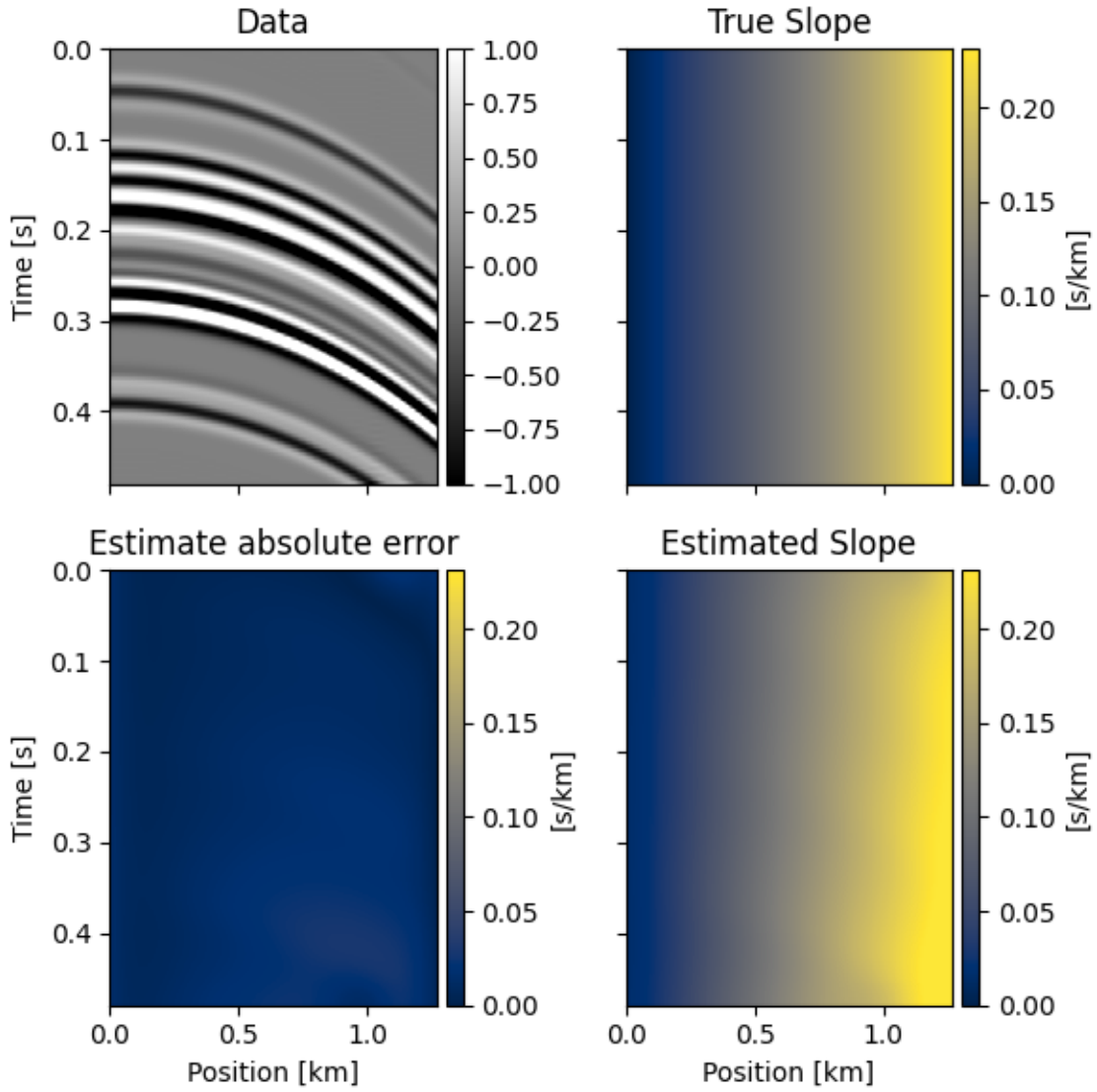
```
tr = trace.copy()
for ix in range(nx):
    tr = _predict_trace(tr, t, dt, dx, slope[:, ix])
    d[:, ix] = tr

# Estimate slopes
slope_est, _ = slope_estimate(d, dt, dx, smooth=10)
slope_est *= -1
```

```
fig, axs = plt.subplots(2, 2, figsize=(6, 6), sharex=True, sharey=True)

opts = dict(aspect="auto", extent=(x[0], x[-1], t[-1], t[0]))
iax = axs[0, 0].imshow(d, cmap="gray", vmin=-1, vmax=1, **opts)
axs[0, 0].set(title="Data", ylabel="Time [s]")
cax = make_axes_locatable(axs[0, 0]).append_axes("right", size="5%", pad=0.05)
fig.colorbar(iax, cax=cax, orientation="vertical")

opts.update(dict(cmap="cividis", vmin=np.min(slope), vmax=np.max(slope)))
iax = axs[0, 1].imshow(slope, **opts)
axs[0, 1].set(title="True Slope")
cax = make_axes_locatable(axs[0, 1]).append_axes("right", size="5%", pad=0.05)
fig.colorbar(iax, cax=cax, orientation="vertical")
cax.set_ylabel("[s/km]")

iax = axs[1, 0].imshow(np.abs(slope - slope_est), **opts)
axs[1, 0].set(
    title="Estimate absolute error", ylabel="Time [s]", xlabel="Position [km]"
)
cax = make_axes_locatable(axs[1, 0]).append_axes("right", size="5%", pad=0.05)
fig.colorbar(iax, cax=cax, orientation="vertical")
cax.set_ylabel("[s/km]")

iax = axs[1, 1].imshow(slope_est, **opts)
axs[1, 1].set(title="Estimated Slope", xlabel="Position [km]")
cax = make_axes_locatable(axs[1, 1]).append_axes("right", size="5%", pad=0.05)
fig.colorbar(iax, cax=cax, orientation="vertical")
cax.set_ylabel("[s/km]")

fig.tight_layout()
```



Concentric circles

The original paper by van Vliet and Verbeek [1] has an example with concentric circles. We recover their original images and compare our implementation with theirs.

```
def rgb2gray(rgb):
    return np.dot(rgb[..., :3], [0.2989, 0.5870, 0.1140])


circles_input = rgb2gray(imread("../testdata/slope_estimate/concentric.png"))
circles_angles = rgb2gray(imread("../testdata/slope_estimate/concentric_angles.png"))

angles, anisos_sm0 = dip_estimate(circles_input, smooth=0)
angles_sm0 = np.rad2deg(angles)
```

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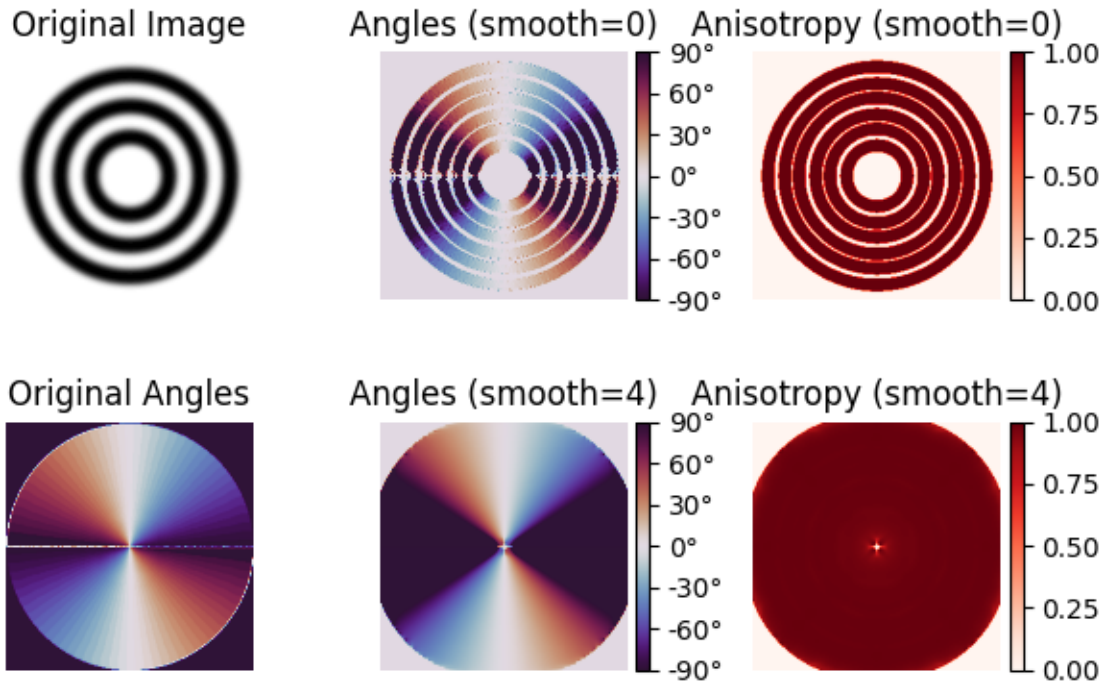
```
angles, anisos_sm4 = dip_estimate(circles_input, smooth=4)
angles_sm4 = np.rad2deg(angles)
```

```
fig, axs = plt.subplots(2, 3, figsize=(6, 4), sharex=True, sharey=True)
axs[0, 0].imshow(circles_input, cmap="gray", aspect="equal")
axs[0, 0].set(title="Original Image")
cax = make_axes_locatable(axs[0, 0]).append_axes("right", size="5%", pad=0.05)
cax.axis("off")

axs[1, 0].imshow(-circles_angles, cmap="twilight_shifted")
axs[1, 0].set(title="Original Angles")
cax = make_axes_locatable(axs[1, 0]).append_axes("right", size="5%", pad=0.05)
cax.axis("off")

im = axs[0, 1].imshow(angles_sm0, cmap="twilight_shifted", vmin=-90, vmax=90)
cax = make_axes_locatable(axs[0, 1]).append_axes("right", size="5%", pad=0.05)
cb = fig.colorbar(
    im,
    ticks=MultipleLocator(30),
    format=FuncFormatter(lambda x, pos: "{:.0f}°".format(x)),
    cax=cax,
    orientation="vertical",
)
axs[0, 1].set(title="Angles (smooth=0)")

im = axs[1, 1].imshow(angles_sm4, cmap="twilight_shifted", vmin=-90, vmax=90)
cax = make_axes_locatable(axs[1, 1]).append_axes("right", size="5%", pad=0.05)
cb = fig.colorbar(
    im,
    ticks=MultipleLocator(30),
    format=FuncFormatter(lambda x, pos: "{:.0f}°".format(x)),
    cax=cax,
    orientation="vertical",
)
axs[1, 1].set(title="Angles (smooth=4)")

im = axs[0, 2].imshow(anisos_sm0, cmap="Reds", vmin=0, vmax=1)
cax = make_axes_locatable(axs[0, 2]).append_axes("right", size="5%", pad=0.05)
cb = fig.colorbar(im, cax=cax, orientation="vertical")
axs[0, 2].set(title="Anisotropy (smooth=0)")

im = axs[1, 2].imshow(anisos_sm4, cmap="Reds", vmin=0, vmax=1)
cax = make_axes_locatable(axs[1, 2]).append_axes("right", size="5%", pad=0.05)
cb = fig.colorbar(im, cax=cax, orientation="vertical")
axs[1, 2].set(title="Anisotropy (smooth=4)")

for ax in axs.ravel():
    ax.axis("off")
fig.tight_layout()
```



Core samples

The original paper by van Vliet and Verbeek [1] also has an example with images of core samples. Since the original paper does not have a scale with which to plot the angles, we have chosen ours it to match their image as closely as possible.

```
core_input = rgb2gray(imread("../testdata/slope_estimate/core_sample.png"))
core_angles = rgb2gray(imread("../testdata/slope_estimate/core_sample_orientation.png"))
core_aniso = rgb2gray(imread("../testdata/slope_estimate/core_sample_anisotropy.png"))


angles, anisos_sm4 = dip_estimate(core_input, smooth=4)
angles_sm4 = np.rad2deg(angles)

angles, anisos_sm8 = dip_estimate(core_input, smooth=8)
angles_sm8 = np.rad2deg(angles)
```

```
fig, axs = plt.subplots(1, 6, figsize=(10, 6))

axs[0].imshow(core_input, cmap="gray_r", aspect="equal")
axs[0].set(title="Original\nImage")
cax = make_axes_locatable(axs[0]).append_axes("right", size="20%", pad=0.05)
cax.axis("off")

axs[1].imshow(-core_angles, cmap="YlGnBu_r")
axs[1].set(title="Original\nAngles")
cax = make_axes_locatable(axs[1]).append_axes("right", size="20%", pad=0.05)
```

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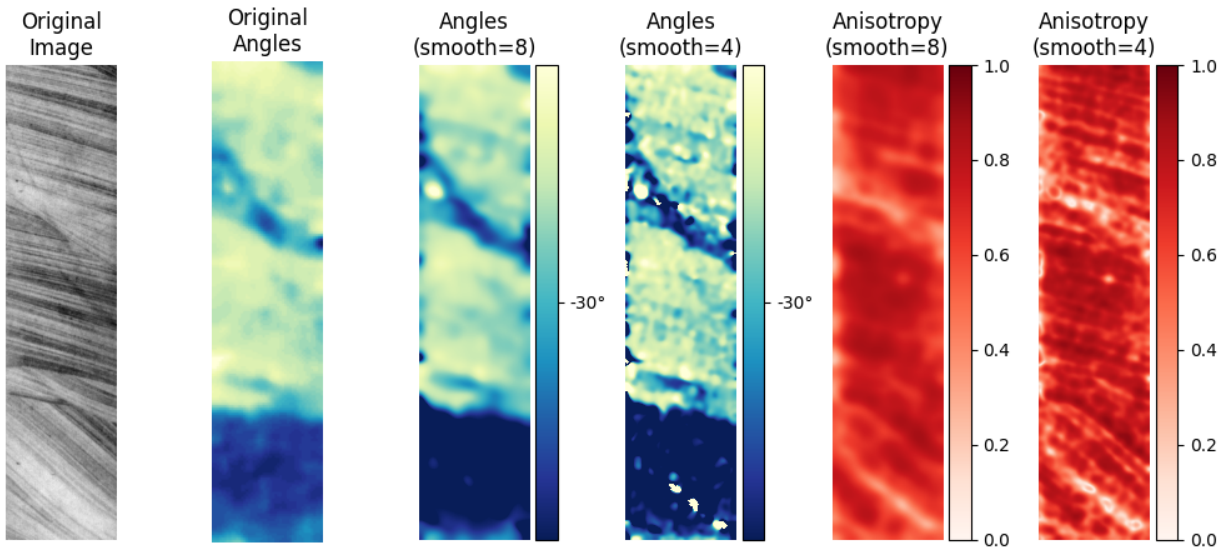
```
cax.axis("off")

im = axs[2].imshow(angles_sm8, cmap="YlGnBu_r", vmin=-49, vmax=-11)
cax = make_axes_locatable(axs[2]).append_axes("right", size="20%", pad=0.05)
cb = fig.colorbar(
    im,
    ticks=MultipleLocator(30),
    format=FuncFormatter(lambda x, pos: "{:.0f}°".format(x)),
    cax=cax,
    orientation="vertical",
)
axs[2].set(title="Angles\n(smooth=8)")

im = axs[3].imshow(angles_sm4, cmap="YlGnBu_r", vmin=-49, vmax=-11)
cax = make_axes_locatable(axs[3]).append_axes("right", size="20%", pad=0.05)
cb = fig.colorbar(
    im,
    ticks=MultipleLocator(30),
    format=FuncFormatter(lambda x, pos: "{:.0f}°".format(x)),
    cax=cax,
    orientation="vertical",
)
axs[3].set(title="Angles\n(smooth=4)")

im = axs[4].imshow(anisos_sm8, cmap="Reds", vmin=0, vmax=1)
cax = make_axes_locatable(axs[4]).append_axes("right", size="20%", pad=0.05)
cb = fig.colorbar(im, cax=cax, orientation="vertical")
axs[4].set(title="Anisotropy\n(smooth=8)")

im = axs[5].imshow(anisos_sm4, cmap="Reds", vmin=0, vmax=1)
cax = make_axes_locatable(axs[5]).append_axes("right", size="20%", pad=0.05)
cb = fig.colorbar(im, cax=cax, orientation="vertical")
axs[5].set(title="Anisotropy\n(smooth=4)")

for ax in axs.ravel():
    ax.axis("off")
fig.tight_layout()
```



Final considerations

As you can see the Structure Tensor algorithm is a very fast, general purpose algorithm that can be used to estimate local slopes to input datasets of very different natures.

Total running time of the script: (0 minutes 2.322 seconds)

3.5.37 Spread How-to

This example focuses on the `pylops.basicoperators.Spread` operator, which is a highly versatile operator in PyLops to perform spreading/stacking operations in a vectorized manner (or efficiently via Numba-jitted `for` loops).

The `pylops.basicoperators.Spread` is powerful in its generality, but it may not be obvious for at first how to structure your code to leverage it properly. While it is highly recommended for advanced users to inspect the *`pylops.signalprocessing.Radon2D`* and *`pylops.signalprocessing.Radon3D`* operators since they are built using the `pylops.basicoperators.Spread` class, here we provide a simple example on how to get started.

In this example we will recreate a simplified version of the famous linear Radon operator, which stacks data along straight lines with a given intercept and slope.

```
import matplotlib.pyplot as plt
import numpy as np

import pylops

plt.close("all")
```

Let's first define the time and space axes as well as some auxiliary input parameters that we will use to create a Ricker wavelet

```
par = {
    "ox": -200,
    "dx": 2,
    "nx": 201,
    "ot": 0,
    "dt": 0.004,
    "nt": 501,
    "f0": 20,
    "nfmax": 210,
}

# Create axis
t, _, x, _ = pylops.utils.seismicevents.makeaxis(par)

# Create centered Ricker wavelet
t_wav = np.arange(41) * par["dt"]
wav, _, _ = pylops.utils.wavelets.ricker(t_wav, f0=par["f0"])
```

We will create a 2d data with a number of crossing linear events, to which we will later apply our Radon transforms. We use the convenience function *pylops.utils.seismicevents.linear2d*.

```
v = 1500  # m/s
t0 = [0.2, 0.7, 1.6]  # seconds
theta = [40, 0, -60]  # degrees
amp = [1.0, 0.6, -2.0]

mlin, mlinwav = pylops.utils.seismicevents.linear2d(x, t, v, t0, theta, amp, wav)
```

Let's now define the slowness axis and use *pylops.signalprocessing.Radon2D* to implement our benchmark linear Radon. Refer to the documentation of the operator for a more detailed mathematical description of linear Radon. Note that `pxmax` is in s/m, which explains the small value. Its highest value corresponds to the lowest value of velocity in the transform. In this case we choose that to be 1000 m/s.

```
npx, pxmax = 41, 1e-3
px = np.linspace(-pxmax, pxmax, npx)

RLop = pylops.signalprocessing.Radon2D(
    t, x, px, centeredh=False, kind="linear", interp=False, engine="numpy"
)

# Compute adjoint = Radon transform
mlinwavR = RLop.H * mlinwav
```

Now, let's try to reimplement this operator from scratch using `pylops.basicoperators.Spread`. Using the on-the-fly approach, and we need to create a function which takes indices of the model domain, here (p_x, t_0) where p_x is the slope and t_0 is the intercept of the parametric curve $t(x) = t_0 + p_x x$ we wish to spread the model over in the data domain. The function must return an array of size `nx`, containing the indices corresponding to $t(x)$.

The on-the-fly approach is useful when storing the indices in RAM may exhaust resources, especially when computing the indices is fast. When there is enough memory to store the full table of indices (an array of size $n_x \times n_t \times n_{p_x}$) the `pylops.basicoperators.Spread` operator can be used with tables instead. We will see an example of this later.

Returning to our on-the-fly example, we need to create a function which only depends on `ipx` and `it0`, so we create a closure around it with all our other auxiliary variables.

```
def create_radon_fh(xaxis, taxis, pxaxis):
    ot = taxis[0]
    dt = taxis[1] - taxis[0]
    nt = len(taxis)

    def fh(ipx, it0):
        tx = t[it0] + xaxis * pxaxis[ipx]
        it0_frac = (tx - ot) / dt
        itx = np.rint(it0_frac)
        # Indices outside time axis set to nan
        itx[np.isin(itx, range(nt), invert=True)] = np.nan
        return itx

    return fh


fRad = create_radon_fh(x, t, px)
ROTFOp = pylops.Spread((npx, par["nt"]), (par["nx"], par["nt"]), fh=fRad)

mlinwavROTF = ROTFOp.H * mlinwav
```

Compare the results between the native Radon transform and the one using our on-the-fly `pylops.basicoperators.Spread`.

```
fig, axs = plt.subplots(1, 3, figsize=(9, 5), sharey=True)
axs[0].imshow(
    mlinwav.T,
    aspect="auto",
    interpolation="nearest",
    vmin=-1,
    vmax=1,
    cmap="gray",
    extent=(x.min(), x.max(), t.max(), t.min()),
)
axs[0].set_title("Linear events", fontsize=12, fontweight="bold")
axs[0].set_xlabel(r"$x$ [m]")
axs[0].set_ylabel(r"$t$ [s]")
axs[1].imshow(
    mlinwavR.T,
    aspect="auto",
    interpolation="nearest",
    vmin=-10,
    vmax=10,
    cmap="gray",
    extent=(px.min(), px.max(), t.max(), t.min()),
)
axs[1].set_title("Native Linear Radon", fontsize=12, fontweight="bold")
axs[1].set_xlabel(r"$p_x$ [s/m]")
axs[1].ticklabel_format(style="sci", axis="x", scilimits=(0, 0))

axs[2].imshow(
    mlinwavROTF.T,
    aspect="auto",
```

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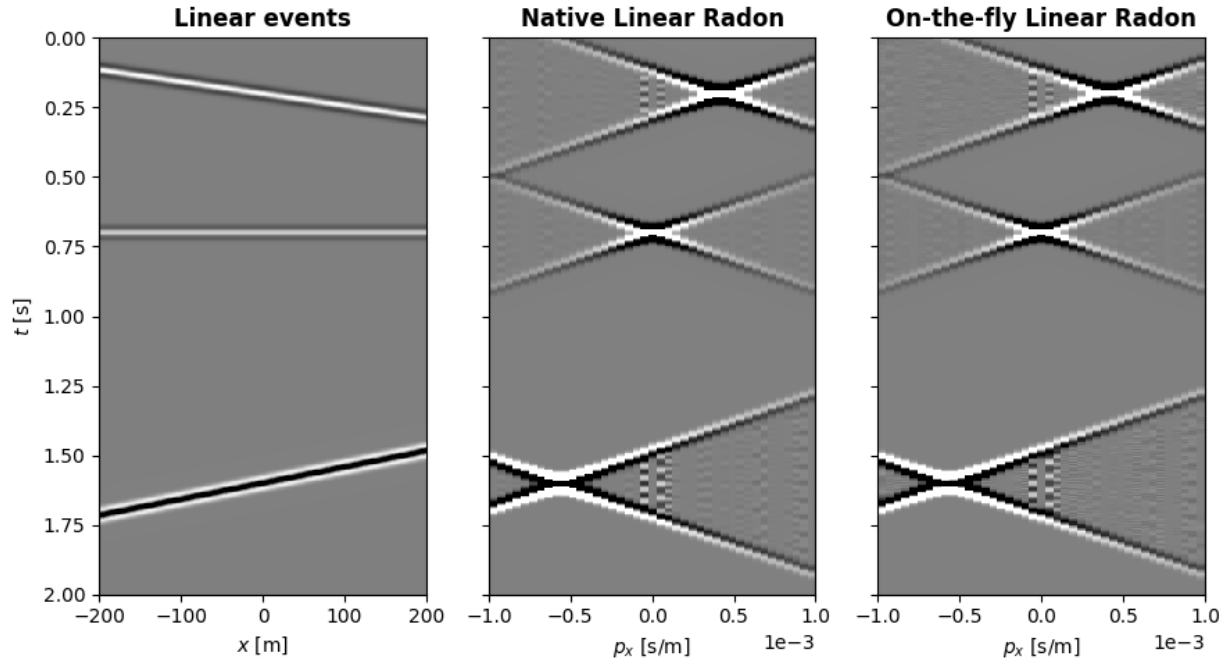
```
    interpolation="nearest",
    vmin=-10,
    vmax=10,
    cmap="gray",
    extent=(px.min(), px.max(), t.max(), t.min()),
)
axs[2].set_title("On-the-fly Linear Radon", fontsize=12, fontweight="bold")
axs[2].set_xlabel(r"$p_x$ [s/m]")
axs[2].ticklabel_format(style="sci", axis="x", scilimits=(0, 0))
fig.tight_layout()
```



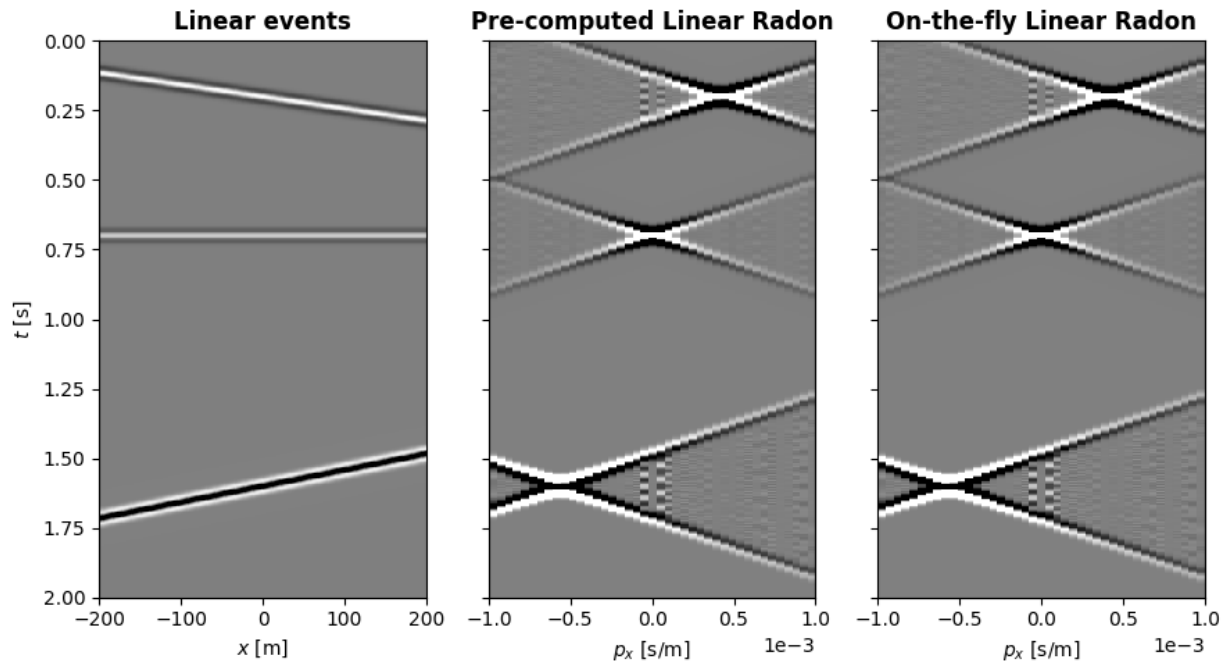
Finally, we will re-implement the example above using pre-computed tables. This is useful when `fh` is expensive to compute, or requires manual edition prior to usage.

Using a table instead of a function is simple, we just need to apply `fh` to all our points and store the results.

```
def create_table(npx, nt, nx):
    table = np.full((npx, nt, nx), fill_value=np.nan)
    for ipx in range(npx):
        for it0 in range(nt):
            table[ipx, it0, :] = fRad(ipx, it0)
    return table


table = create_table(npx, par["nt"], par["nx"])
RPCOp = pylops.Spread((npx, par["nt"]), (par["nx"], par["nt"]), table=table)

mlinwavRPC = RPCOp.H * mlinwav
```

Compare the results between the pre-computed or on-the-fly Radon transforms

```
fig, axs = plt.subplots(1, 3, figsize=(9, 5), sharey=True)
axs[0].imshow(
    mlinwav.T,
    aspect="auto",
    interpolation="nearest",
    vmin=-1,
    vmax=1,
    cmap="gray",
    extent=(x.min(), x.max(), t.max(), t.min()),
)
axs[0].set_title("Linear events", fontsize=12, fontweight="bold")
axs[0].set_xlabel(r"$x$ [m]")
axs[0].set_ylabel(r"$t$ [s]")
axs[1].imshow(
    mlinwavRPC.T,
    aspect="auto",
    interpolation="nearest",
    vmin=-10,
    vmax=10,
    cmap="gray",
    extent=(px.min(), px.max(), t.max(), t.min()),
)
axs[1].set_title("Pre-computed Linear Radon", fontsize=12, fontweight="bold")
axs[1].set_xlabel(r"$p_x$ [s/m]")
axs[1].ticklabel_format(style="sci", axis="x", scilimits=(0, 0))

axs[2].imshow(
    mlinwavROTF.T,
    aspect="auto",
    interpolation="nearest",
    vmin=-10,
    vmax=10,
    cmap="gray",
    extent=(px.min(), px.max(), t.max(), t.min()),
)
axs[2].set_title("On-the-fly Linear Radon", fontsize=12, fontweight="bold")
axs[2].set_xlabel(r"$p_x$ [s/m]")
axs[2].ticklabel_format(style="sci", axis="x", scilimits=(0, 0))
fig.tight_layout()
```



Total running time of the script: (0 minutes 7.838 seconds)

3.5.38 Sum

This example shows how to use the *pylops.Sum* operator to stack values along an axis of a multi-dimensional array

```
import matplotlib.gridspec as pltgs
import matplotlib.pyplot as plt
import numpy as np

import pylops

plt.close("all")
```

Let's start by defining a 2-dimensional data

```
ny, nx = 5, 7
x = (np.arange(ny * nx)).reshape(ny, nx)
```

We can now create the operator and peform forward and adjoint

```
Sop = pylops.Sum(dims=(ny, nx), axis=0)

y = Sop * x
xadj = Sop.H * y

gs = pltgs.GridSpec(1, 7)
fig = plt.figure(figsize=(7, 4))
ax = plt.subplot(gs[0, 0:3])
im = ax.imshow(x, cmap="rainbow", vmin=0, vmax=ny * nx)
```

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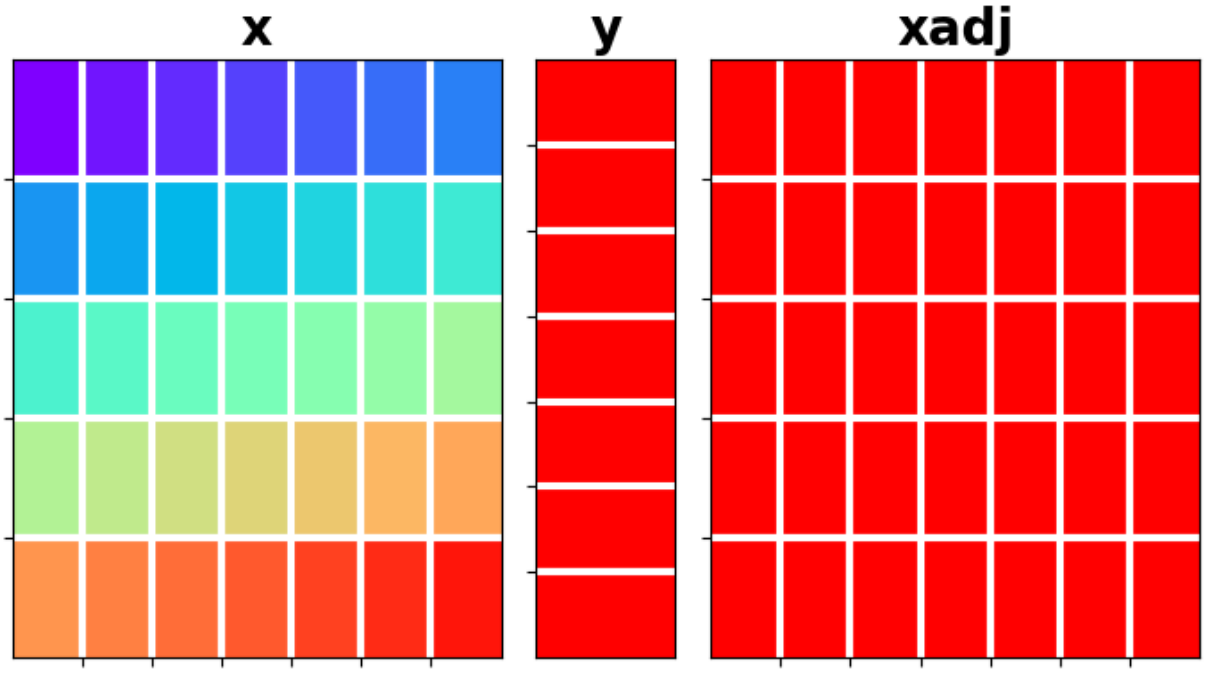
```
ax.set_title("x", size=20, fontweight="bold")
ax.set_xticks(np.arange(nx - 1) + 0.5)
ax.set_yticks(np.arange(ny - 1) + 0.5)
ax.grid(linewidth=3, color="white")
ax.xaxis.set_ticklabels([])
ax.yaxis.set_ticklabels([])
ax.axis("tight")
ax = plt.subplot(gs[0, 3])
ax.imshow(y[:, np.newaxis], cmap="rainbow", vmin=0, vmax=ny * nx)
ax.set_title("y", size=20, fontweight="bold")
ax.set_xticks([])
ax.set_yticks(np.arange(nx - 1) + 0.5)
ax.grid(linewidth=3, color="white")
ax.xaxis.set_ticklabels([])
ax.yaxis.set_ticklabels([])
ax.axis("tight")
ax = plt.subplot(gs[0, 4:])
ax.imshow(xadj, cmap="rainbow", vmin=0, vmax=ny * nx)
ax.set_title("xadj", size=20, fontweight="bold")
ax.set_xticks(np.arange(nx - 1) + 0.5)
ax.set_yticks(np.arange(ny - 1) + 0.5)
ax.grid(linewidth=3, color="white")
ax.xaxis.set_ticklabels([])
ax.yaxis.set_ticklabels([])
ax.axis("tight")
plt.tight_layout()
```



Note that since the Sum operator creates and under-determined system of equations (data has always lower dimensionality than the model), an exact inverse is not possible for this operator.

Total running time of the script: (0 minutes 0.185 seconds)

3.5.39 Symmetrize

This example shows how to use the *pylops.Symmetrize* operator which takes an input signal and returns a symmetric signal by pre-pending the input signal in reversed order. Such an operation can be inverted as we will see in this example.

Moreover the *pylops.Symmetrize* can be used as *preconditioning* to any inverse problem where we are after inverting for a signal that we want to ensure is symmetric. Refer to *Wavelet estimation* for an example of such a type.

```
import matplotlib.pyplot as plt
import numpy as np

import pylops

plt.close("all")
```

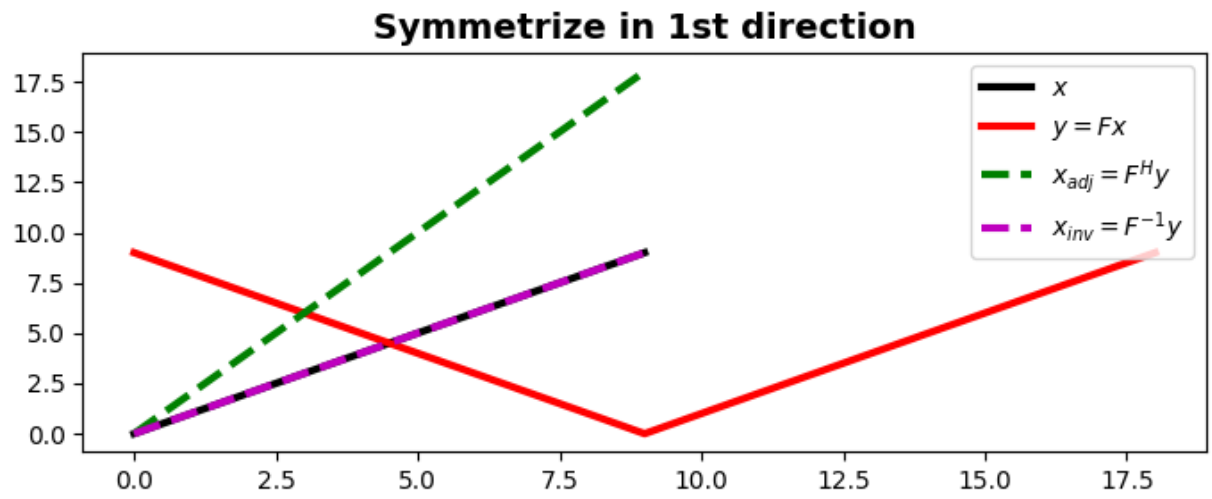
Let's start with a 1D example. Define an input signal composed of `nt` samples

```
nt = 10
x = np.arange(nt)
```

We can now create our flip operator and apply it to the input signal. We can also apply the adjoint to the flipped signal and we can see how for this operator the adjoint is effectively equivalent to the inverse.

```
Sop = pylops.Symmetrize(nt)
y = Sop * x
xadj = Sop.H * y
xinv = Sop / y

plt.figure(figsize=(7, 3))
plt.plot(x, "k", lw=3, label=r"$x$")
plt.plot(y, "r", lw=3, label=r"$y=Fx$")
plt.plot(xadj, "--g", lw=3, label=r"$x_{adj} = F^H y$")
plt.plot(xinv, "--m", lw=3, label=r"$x_{inv} = F^{-1} y$")
plt.title("Symmetrize in 1st direction", fontsize=14, fontweight="bold")
plt.legend()
plt.tight_layout()
```



Let's now repeat the same exercise on a two dimensional signal. We will first flip the model along the first axis and then along the second axis

```
nt, nx = 10, 6
x = np.outer(np.arange(nt), np.ones(nx))

Sop = pylops.Symmetrize((nt, nx), axis=0)
y = Sop * x
xadj = Sop.H * y
xinv = Sop / y.ravel()
xinv = xinv.reshape(Sop.dims)

fig, axs = plt.subplots(1, 3, figsize=(7, 3))
fig.suptitle(
    "Symmetrize in 2nd direction for 2d data", fontsize=14, fontweight="bold", y=0.95
)
axs[0].imshow(x, cmap="rainbow", vmin=0, vmax=9)
axs[0].set_title(r"$x$")
axs[0].axis("tight")
axs[1].imshow(y, cmap="rainbow", vmin=0, vmax=9)
axs[1].set_title(r"$y=Fx$")
axs[1].axis("tight")
axs[2].imshow(xinv, cmap="rainbow", vmin=0, vmax=9)
axs[2].set_title(r"$x_{adj}=F^{-1}y$")
axs[2].axis("tight")
plt.tight_layout()
plt.subplots_adjust(top=0.8)


x = np.outer(np.ones(nt), np.arange(nx))
Sop = pylops.Symmetrize((nt, nx), axis=1)

y = Sop * x
xadj = Sop.H * y
xinv = Sop / y.ravel()
xinv = xinv.reshape(Sop.dims)
```

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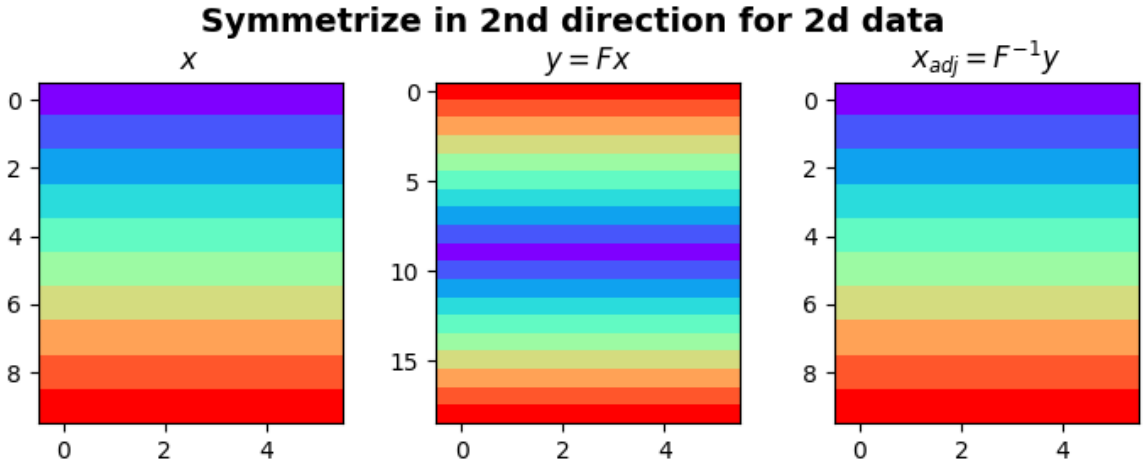
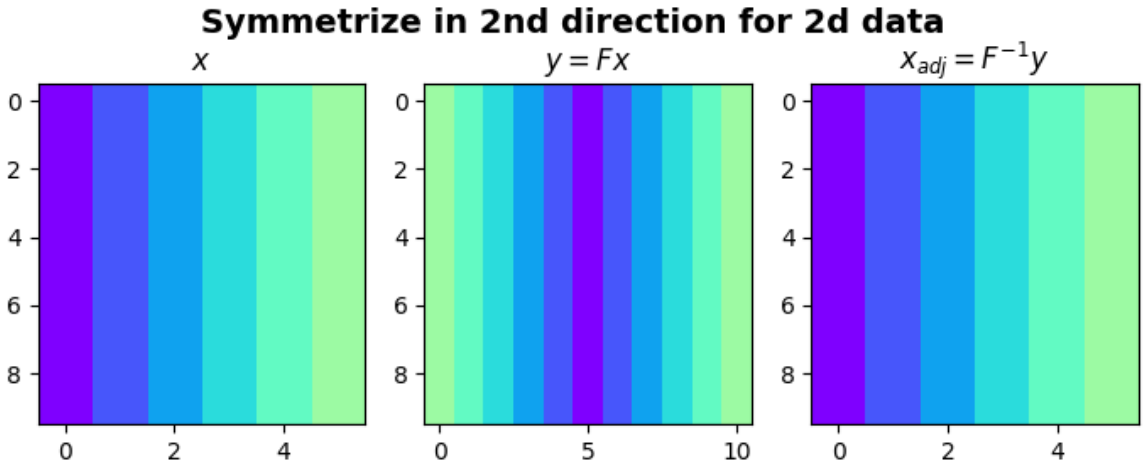
```
# sphinx_gallery_thumbnail_number = 3
fig, axs = plt.subplots(1, 3, figsize=(7, 3))
fig.suptitle(
    "Symmetrize in 2nd direction for 2d data", fontsize=14, fontweight="bold", y=0.95
)
axs[0].imshow(x, cmap="rainbow", vmin=0, vmax=9)
axs[0].set_title(r"$x$")
axs[0].axis("tight")
axs[1].imshow(y, cmap="rainbow", vmin=0, vmax=9)
axs[1].set_title(r"$y=Fx$")
axs[1].axis("tight")
axs[2].imshow(xinv, cmap="rainbow", vmin=0, vmax=9)
axs[2].set_title(r"$x_{adj}=F^{-1}y$")
axs[2].axis("tight")
plt.tight_layout()
plt.subplots_adjust(top=0.8)
```

- 

- 

Total running time of the script: (0 minutes 0.777 seconds)

3.5.40 Synthetic seismic

This example shows how to use the `pylops.utils.seismicevents` module to quickly create synthetic seismic data to be used for toy examples and tests.

```
import matplotlib.pyplot as plt
import numpy as np

import pylops

plt.close("all")
```

Let's first define the time and space axes as well as some auxiliary input parameters that we will use to create a Ricker wavelet

```
par = {
    "ox": -200,
    "dx": 2,
    "nx": 201,
    "oy": -100,
    "dy": 2,
    "ny": 101,
    "ot": 0,
    "dt": 0.004,
    "nt": 501,
    "f0": 20,
    "nfmax": 210,
}

# Create axis
t, t2, x, y = pylops.utils.seismicevents.makeaxis(par)

# Create wavelet
wav = pylops.utils.wavelets.ricker(np.arange(41) * par["dt"], f0=par["f0"])[0]
```

We want to create a 2d data with a number of crossing linear events using the *pylops.utils.seismicevents.linear2d* routine.

```
v = 1500
t0 = [0.2, 0.7, 1.6]
theta = [40, 0, -60]
amp = [1.0, 0.6, -2.0]

mlin, mlinwav = pylops.utils.seismicevents.linear2d(x, t, v, t0, theta, amp, wav)
```

We can also create a 2d data with a number of crossing parabolic events using the *pylops.utils.seismicevents.parabolic2d* routine.

```
px = [0, 0, 0]
pxx = [1e-5, 5e-6, 1e-6]

mpar, mparwav = pylops.utils.seismicevents.parabolic2d(x, t, t0, px, pxx, amp, wav)
```

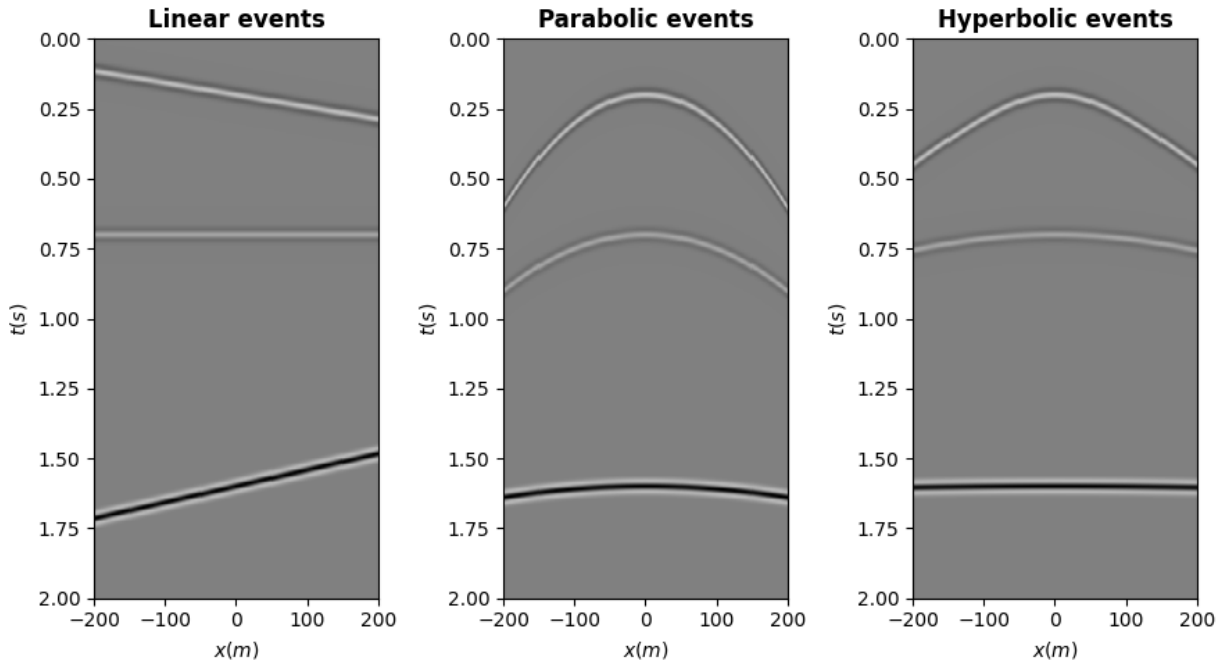
And similarly we can create a 2d data with a number of crossing hyperbolic events using the *pylops.utils.seismicevents.hyperbolic2d* routine.

```
vrms = [500, 700, 1700]

mhyp, mhypwav = pylops.utils.seismicevents.hyperbolic2d(x, t, t0, vrms, amp, wav)
```

We can now visualize the different events

```
# sphinx_gallery_thumbnail_number = 2
fig, axs = plt.subplots(1, 3, figsize=(9, 5))
axs[0].imshow(
    mlinwav.T,
    aspect="auto",
    interpolation="nearest",
    vmin=-2,
    vmax=2,
    cmap="gray",
    extent=(x.min(), x.max(), t.max(), t.min()),
)
axs[0].set_title("Linear events", fontsize=12, fontweight="bold")
axs[0].set_xlabel(r"$x(m)$")
axs[0].set_ylabel(r"$t(s)$")
axs[1].imshow(
    mparwav.T,
    aspect="auto",
    interpolation="nearest",
    vmin=-2,
    vmax=2,
    cmap="gray",
    extent=(x.min(), x.max(), t.max(), t.min()),
)
axs[1].set_title("Parabolic events", fontsize=12, fontweight="bold")
axs[1].set_xlabel(r"$x(m)$")
axs[1].set_ylabel(r"$t(s)$")
axs[2].imshow(
    mhypwav.T,
    aspect="auto",
    interpolation="nearest",
    vmin=-2,
    vmax=2,
    cmap="gray",
    extent=(x.min(), x.max(), t.max(), t.min()),
)
axs[2].set_title("Hyperbolic events", fontsize=12, fontweight="bold")
axs[2].set_xlabel(r"$x(m)$")
axs[2].set_ylabel(r"$t(s)$")
plt.tight_layout()
```



Let's finally repeat the same exercise in 3d

```
phi = [20, 0, -10]

mlin, mlinwav = pylops.utils.seismicevents.linear3d(
    x, y, t, v, t0, theta, phi, amp, wav
)

fig, axs = plt.subplots(1, 2, figsize=(7, 5), sharey=True)
fig.suptitle("Linear events in 3d", fontsize=12, fontweight="bold", y=0.95)
axs[0].imshow(
    mlinwav[par["ny"] // 2].T,
    aspect="auto",
    interpolation="nearest",
    vmin=-2,
    vmax=2,
    cmap="gray",
    extent=(x.min(), x.max(), t.max(), t.min()),
)
axs[0].set_xlabel(r"$x(m)$")
axs[0].set_ylabel(r"$t(s)$")
axs[1].imshow(
    mlinwav[:, par["nx"] // 2].T,
    aspect="auto",
    interpolation="nearest",
    vmin=-2,
    vmax=2,
    cmap="gray",
    extent=(y.min(), y.max(), t.max(), t.min()),
)
axs[1].set_xlabel(r"$y(m)$")
```

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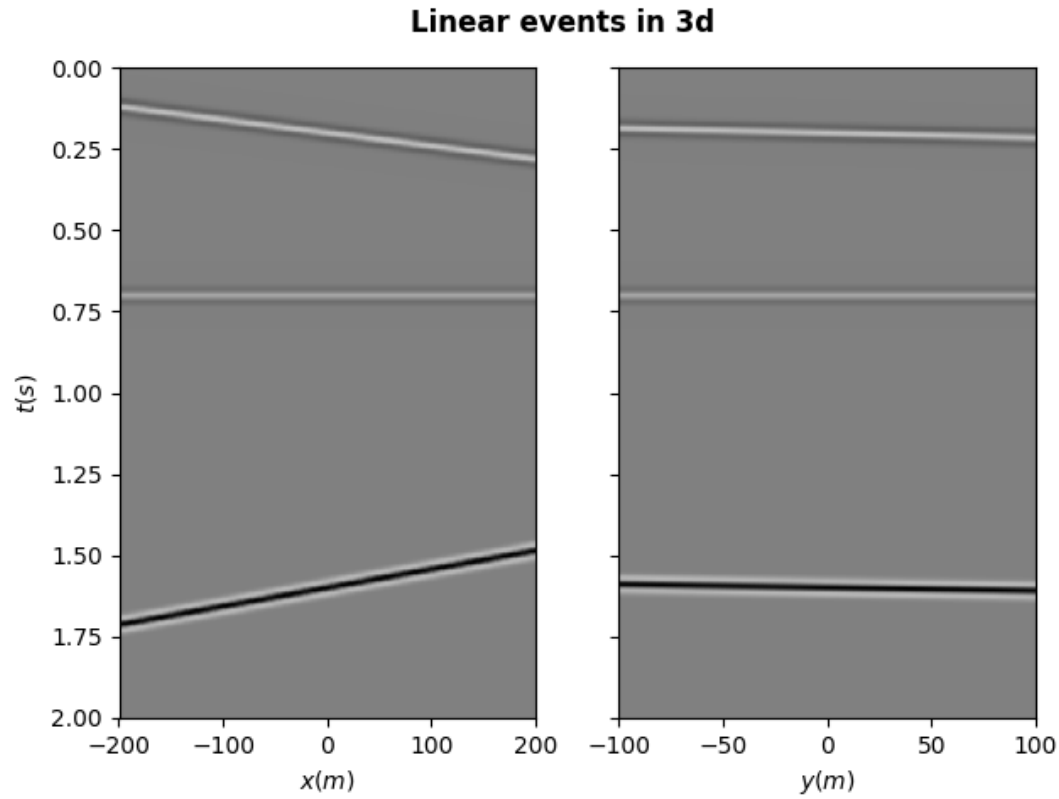
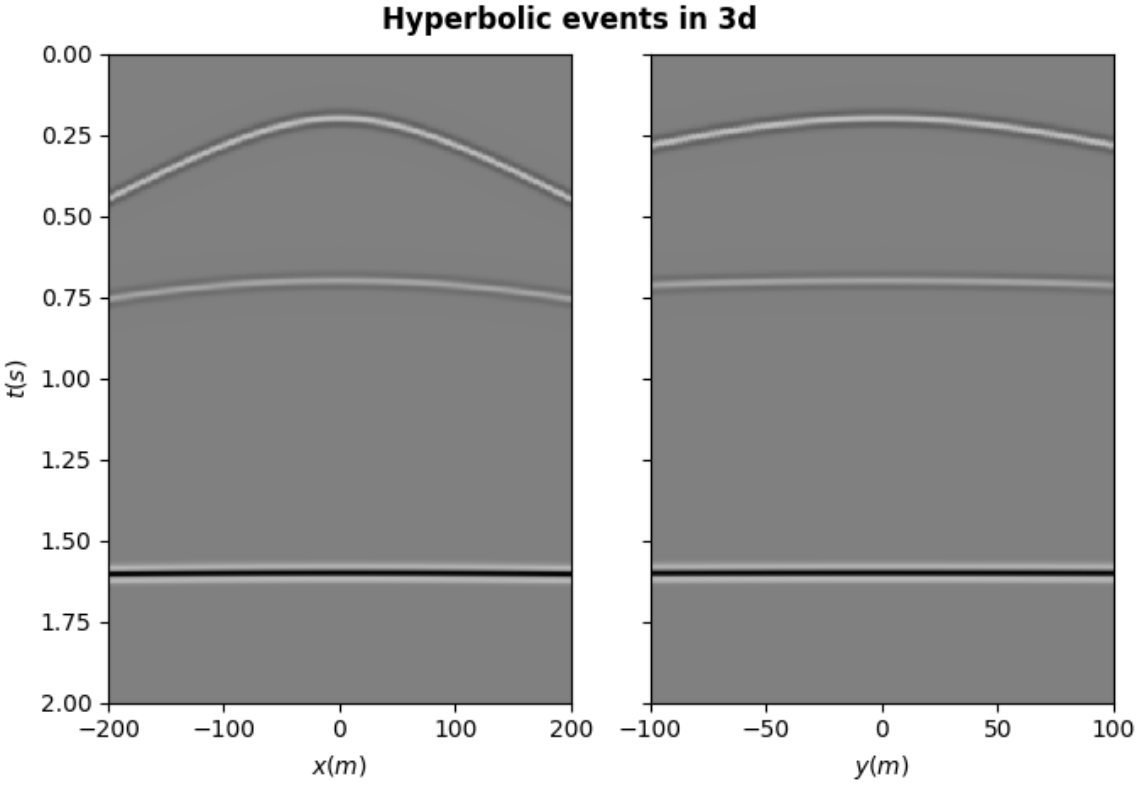
```
mhyp, mhypwav = pylops.utils.seismicevents.hyperbolic3d(
    x, y, t, t0, vrms, vrms, amp, wav
)

fig, axs = plt.subplots(1, 2, figsize=(7, 5), sharey=True)
fig.suptitle("Hyperbolic events in 3d", fontsize=12, fontweight="bold", y=0.95)
axs[0].imshow(
    mhypwav[par["ny"] // 2].T,
    aspect="auto",
    interpolation="nearest",
    vmin=-2,
    vmax=2,
    cmap="gray",
    extent=(x.min(), x.max(), t.max(), t.min()),
)
axs[0].set_xlabel(r"$x(m)$")
axs[0].set_ylabel(r"$t(s)$")
axs[1].imshow(
    mhypwav[:, par["nx"] // 2].T,
    aspect="auto",
    interpolation="nearest",
    vmin=-2,
    vmax=2,
    cmap="gray",
    extent=(y.min(), y.max(), t.max(), t.min()),
)
axs[1].set_xlabel(r"$y(m)$")
plt.tight_layout()
```


Linear events in 3d
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00
t(s)
−200
−100
0
100
200
x(m)
−100
−50
0
50
100
y(m)


Hyperbolic events in 3d
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00
t(s)
−200
−100
0
100
200
x(m)
−100
−50
0
50
100
y(m)

Total running time of the script: (0 minutes 2.052 seconds)

3.5.41 Tapers

This example shows how to create some basic tapers in 1d, 2d, and 3d using the `pylops.utils.tapers` module.

```
import matplotlib.pyplot as plt

import pylops

plt.close("all")
```

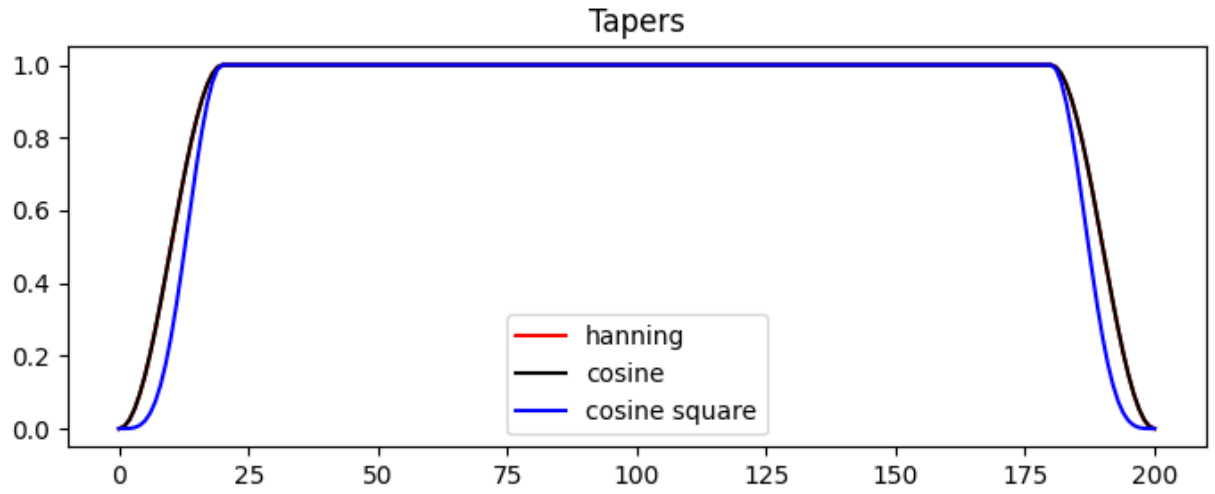
Let's first define the time and space axes

```
par = {
    "ox": -200,
    "dx": 2,
    "nx": 201,
    "oy": -100,
    "dy": 2,
    "ny": 101,
    "ot": 0,
    "dt": 0.004,
    "nt": 501,
    "ntapx": 21,
    "ntapy": 31,
}
```

We can now create tapers in 1d

```
tap_han = pylops.utils.tapers.hanningtaper(par["nx"], par["ntapx"])
tap_cos = pylops.utils.tapers.cosinetaper(par["nx"], par["ntapx"], False)
tap_cos2 = pylops.utils.tapers.cosinetaper(par["nx"], par["ntapx"], True)

plt.figure(figsize=(7, 3))
plt.plot(tap_han, "r", label="hanning")
plt.plot(tap_cos, "k", label="cosine")
plt.plot(tap_cos2, "b", label="cosine square")
plt.title("Tapers")
plt.legend()
plt.tight_layout()
```



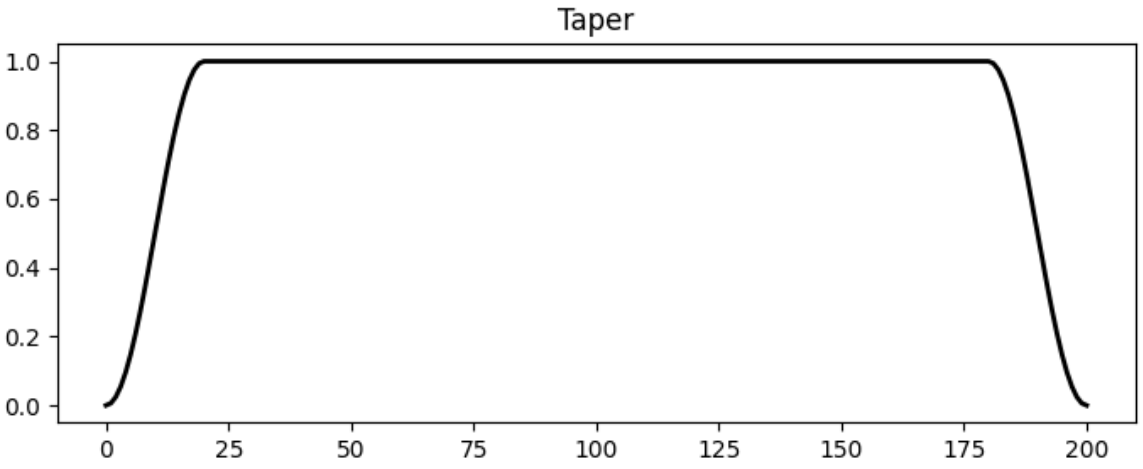
Similarly we can create 2d and 3d tapers with any of the tapers above

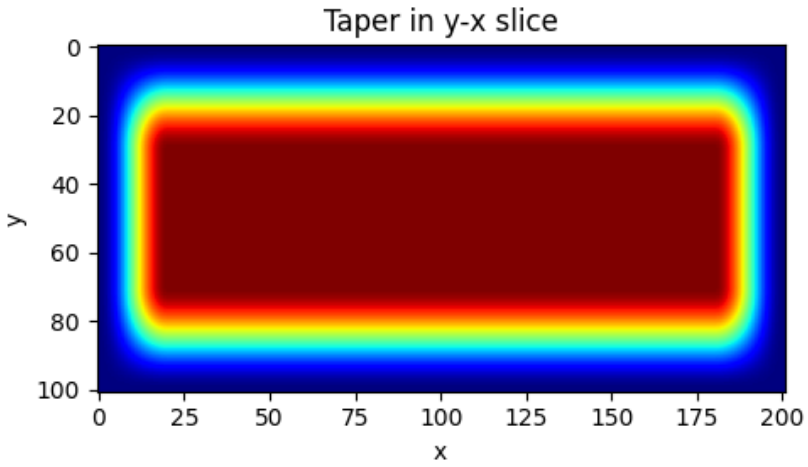
```
tap2d = pylops.utils.tapers.taper2d(par["nt"], par["nx"], par["ntapx"])

plt.figure(figsize=(7, 3))
plt.plot(tap2d[:, par["nt"] // 2], "k", lw=2)
plt.title("Taper")
plt.tight_layout()

tap3d = pylops.utils.tapers.taper3d(
    par["nt"], (par["ny"], par["nx"]), (par["ntapy"], par["ntapx"])
)

plt.figure(figsize=(7, 3))
plt.imshow(tap3d[:, :, par["nt"] // 2], "jet")
plt.title("Taper in y-x slice")
plt.xlabel("x")
plt.ylabel("y")
plt.tight_layout()
```





-

Total running time of the script: (0 minutes 0.584 seconds)

3.5.42 Total Variation (TV) Regularization

This set of examples shows how to add Total Variation (TV) regularization to an inverse problem in order to enforce blockiness in the reconstructed model.

To do so we will use the generalizated Split Bregman iterations by means of `pylops.optimization.sparsity.SplitBregman` solver.

The first example is concerned with denoising of a piece-wise step function which has been contaminated by noise. The forward model is:

$$\mathbf{y} = \mathbf{x} + \mathbf{n}$$

meaning that we have an identity operator ($\mathbf{I}$) and inverting for $\mathbf{x}$ from $\mathbf{y}$ is impossible without adding prior information. We will enforce blockiness in the solution by adding a regularization term that enforces sparsity in the first derivative of the solution:

$$J = \mu/2||\mathbf{y} - \mathbf{I}\mathbf{x}||_2 + ||\nabla \mathbf{x}||_1$$

```
import matplotlib.pyplot as plt

# sphinx_gallery_thumbnail_number = 5
import numpy as np

import pylops

plt.close("all")
np.random.seed(1)
```

Let's start by creating the model and data

```
nx = 101
x = np.zeros(nx)
x[: nx // 2] = 10
x[nx // 2 : 3 * nx // 4] = -5
```

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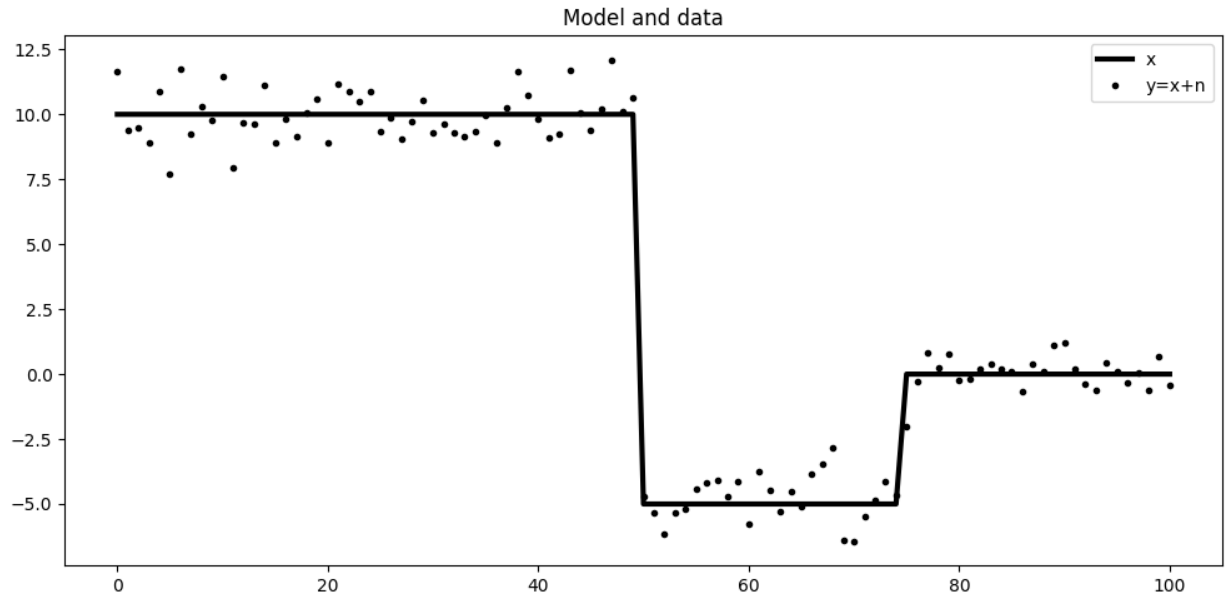
```
Iop = pylops.Identity(nx)

n = np.random.normal(0, 1, nx)
y = Iop * (x + n)

plt.figure(figsize=(10, 5))
plt.plot(x, "k", lw=3, label="x")
plt.plot(y, ".k", label="y=x+n")
plt.legend()
plt.title("Model and data")
plt.tight_layout()
```

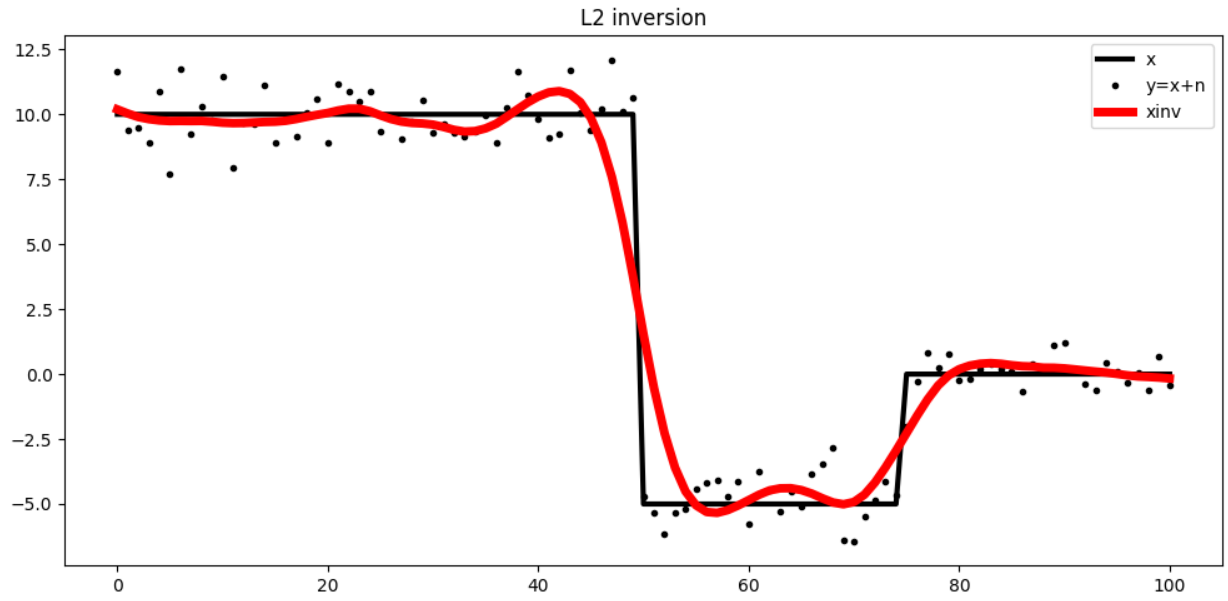


To start we will try to use a simple L2 regularization that enforces smoothness in the solution. We can see how denoising is succesfully achieved but the solution is much smoother than we wish for.

```
D2op = pylops.SecondDerivative(nx, edge=True)
lamda = 1e2

xinv = pylops.optimization.leastsquares.regularized_inversion(
    Iop, y, [D2op], epsRs=[np.sqrt(lamda / 2)], **dict(iter_lim=30)
)[0]

plt.figure(figsize=(10, 5))
plt.plot(x, "k", lw=3, label="x")
plt.plot(y, ".k", label="y=x+n")
plt.plot(xinv, "r", lw=5, label="xinv")
plt.legend()
plt.title("L2 inversion")
plt.tight_layout()
```

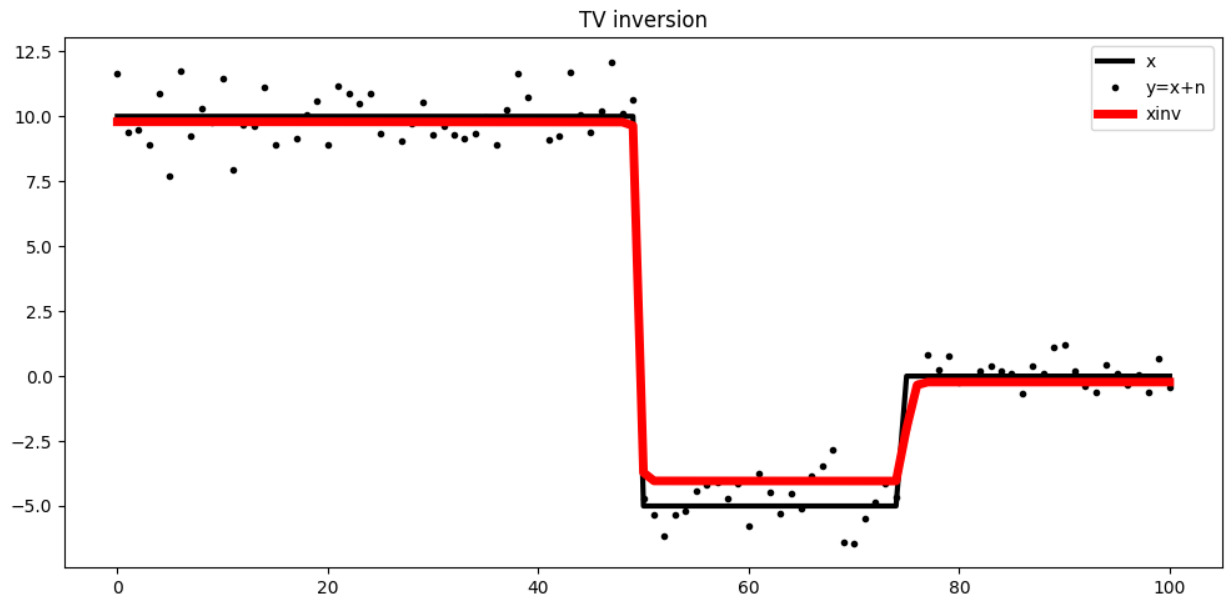


Now we impose blockiness in the solution using the Split Bregman solver

```
Dop = pylops.FirstDerivative(nx, edge=True, kind="backward")
mu = 0.01
lamda = 0.3
niter_out = 50
niter_in = 3

xinv = pylops.optimization.sparsity.splitbregman(
    Iop,
    y,
    [Dop],
    niter_outer=niter_out,
    niter_inner=niter_in,
    mu=mu,
    epsRL1s=[lamda],
    tol=1e-4,
    tau=1.0,
    **dict(iter_lim=30, damp=1e-10)
)[0]

plt.figure(figsize=(10, 5))
plt.plot(x, "k", lw=3, label="x")
plt.plot(y, ".k", label="y=x+n")
plt.plot(xinv, "r", lw=5, label="xinv")
plt.legend()
plt.title("TV inversion")
plt.tight_layout()
```



Finally, we repeat the same exercise on a 2-dimensional image. In this case we mock a medical imaging problem: the data is created by appling a 2D Fourier Transform to the input model and by randomly sampling 60% of its values.

```
x = np.load("../testdata/optimization/shepp_logan_phantom.npy")
x = x / x.max()
ny, nx = x.shape

perc_subsampling = 0.6
nxsub = int(np.round(ny * nx * perc_subsampling))
iava = np.sort(np.random.permutation(np.arange(ny * nx))[:nxsub])
Rop = pylops.Restriction(ny * nx, iava, axis=0, dtype=np.complex128)
Fop = pylops.signalprocessing.FFT2D(dims=(ny, nx))

n = np.random.normal(0, 0.0, (ny, nx))
y = Rop * Fop * (x.ravel() + n.ravel())
yfft = Fop * (x.ravel() + n.ravel())
yfft = np.fft.fftshift(yfft.reshape(ny, nx))

ymask = Rop.mask(Fop * (x.ravel()) + n.ravel())
ymask = ymask.reshape(ny, nx)
ymask.data[:] = np.fft.fftshift(ymask.data)
ymask.mask[:] = np.fft.fftshift(ymask.mask)

fig, axs = plt.subplots(1, 3, figsize=(14, 5))
axs[0].imshow(x, vmin=0, vmax=1, cmap="gray")
axs[0].set_title("Model")
axs[0].axis("tight")
axs[1].imshow(np.abs(yfft), vmin=0, vmax=1, cmap="rainbow")
axs[1].set_title("Full data")
axs[1].axis("tight")
axs[2].imshow(np.abs(ymask), vmin=0, vmax=1, cmap="rainbow")
axs[2].set_title("Sampled data")
axs[2].axis("tight")
```

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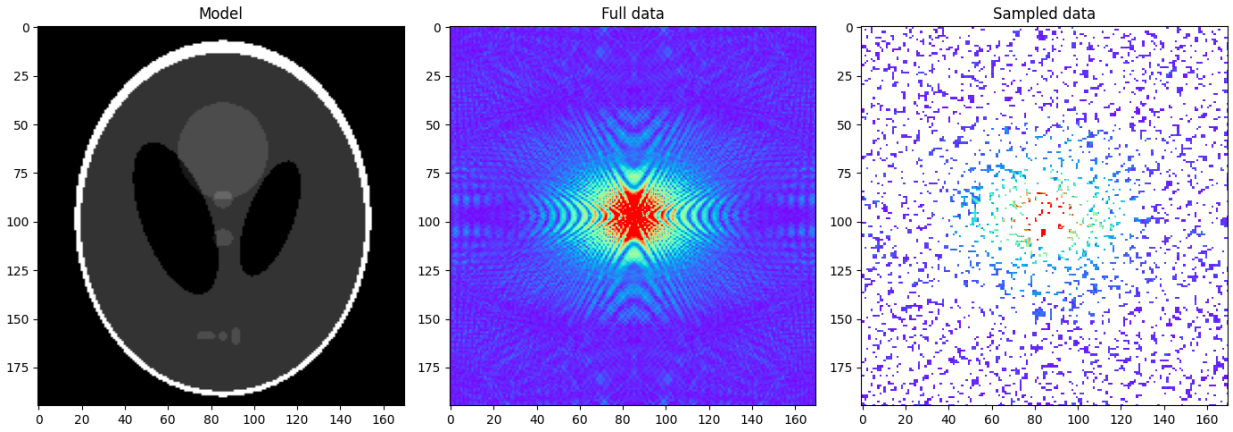
```
plt.tight_layout()
```



Let's attempt now to reconstruct the model using the Split Bregman with anisotropic TV regularization (aka sum of L1 norms of the first derivatives over x and y):

$$J = \mu/2||\mathbf{y} - \mathbf{R}\mathbf{F}\mathbf{x}||_2 + ||\nabla_x \mathbf{x}||_1 + ||\nabla_y \mathbf{x}||_1$$

```
Dop = [
    pylops.FirstDerivative(
        (ny, nx), axis=0, edge=False, kind="backward", dtype=np.complex128
    ),
    pylops.FirstDerivative(
        (ny, nx), axis=1, edge=False, kind="backward", dtype=np.complex128
    ),
]

# TV
mu = 1.5
lamda = [0.1, 0.1]
niter_out = 20
niter_in = 10

xinv = pylops.optimization.sparsity.splitbregman(
    Rop * Fop,
    y.ravel(),
    Dop,
    niter_outer=niter_out,
    niter_inner=niter_in,
    mu=mu,
    epsRL1s=lamda,
    tol=1e-4,
    tau=1.0,
    show=False,
    **dict(iter_lim=5, damp=1e-4)
)[0]
xinv = np.real(xinv.reshape(ny, nx))
```

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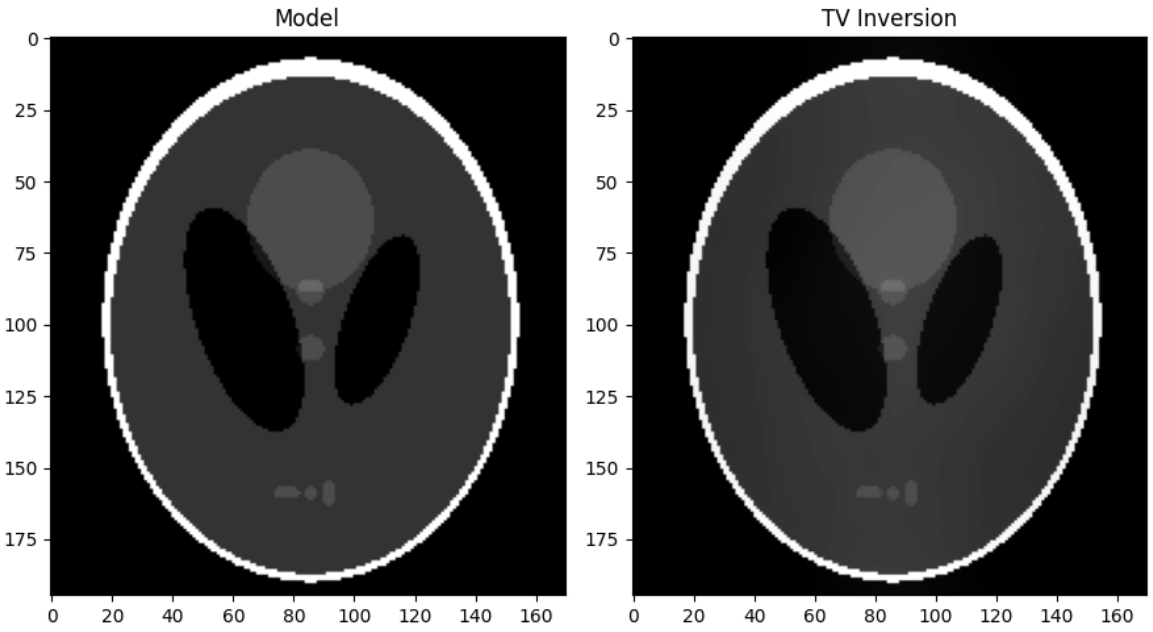
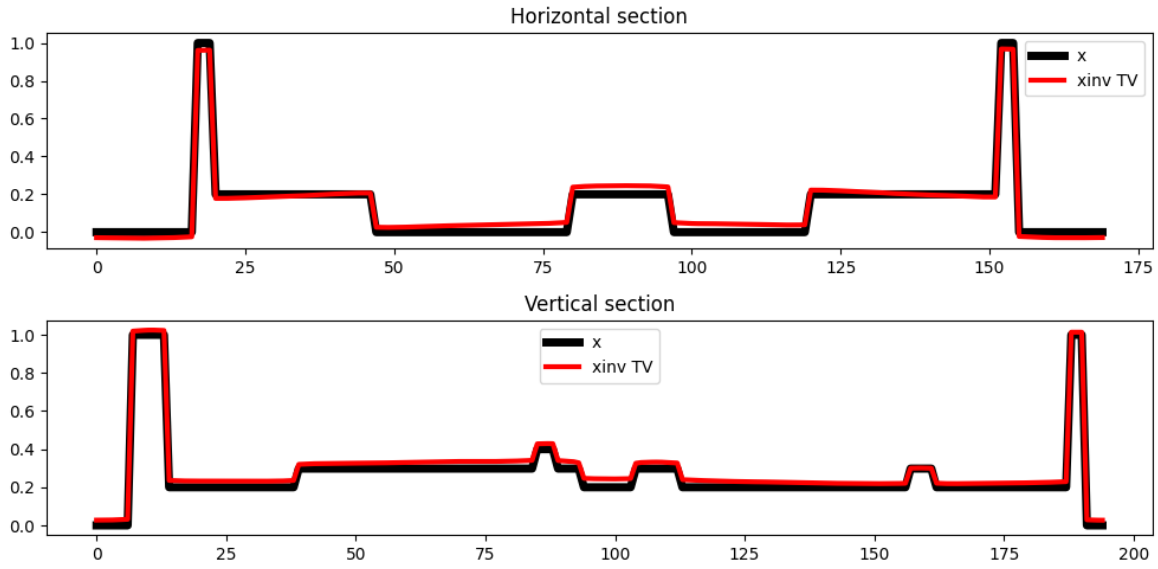
```
fig, axs = plt.subplots(1, 2, figsize=(9, 5))
axs[0].imshow(x, vmin=0, vmax=1, cmap="gray")
axs[0].set_title("Model")
axs[0].axis("tight")
axs[1].imshow(xinv, vmin=0, vmax=1, cmap="gray")
axs[1].set_title("TV Inversion")
axs[1].axis("tight")
plt.tight_layout()

fig, axs = plt.subplots(2, 1, figsize=(10, 5))
axs[0].plot(x[ny // 2], "k", lw=5, label="x")
axs[0].plot(xinv[ny // 2], "r", lw=3, label="xinv TV")
axs[0].set_title("Horizontal section")
axs[0].legend()
axs[1].plot(x[:, nx // 2], "k", lw=5, label="x")
axs[1].plot(xinv[:, nx // 2], "r", lw=3, label="xinv TV")
axs[1].set_title("Vertical section")
axs[1].legend()
plt.tight_layout()
```





Note that more optimized variations of the Split Bregman algorithm have been proposed in the literature for this specific problem, both improving the overall quality of the inversion and the speed of convergence.

In PyLops we however prefer to implement the generalized Split Bergman algorithm as this can used for any sort of problem where we wish to add any number of L1 and/or L2 regularization terms to the cost function to minimize.

Total running time of the script: (0 minutes 17.360 seconds)

3.5.43 Transpose

This example shows how to use the *pylops.Transpose* operator. For arrays that are 2-dimensional in nature this operator simply transposes rows and columns. For multi-dimensional arrays, this operator can be used to permute dimensions

```
import matplotlib.pyplot as plt
import numpy as np

import pylops

plt.close("all")
np.random.seed(0)
```

Let's start by creating a 2-dimensional array

```
dims = (20, 40)
x = np.arange(800).reshape(dims)
```

We use now the *pylops.Transpose* operator to swap the two dimensions. As you will see the adjoint of this operator brings the data back to its original model, or in other words the adjoint operator is equal in this case to the inverse operator.

```
Top = pylops.Transpose(dims=dims, axes=(1, 0))

y = Top * x
xadj = Top.H * y
```

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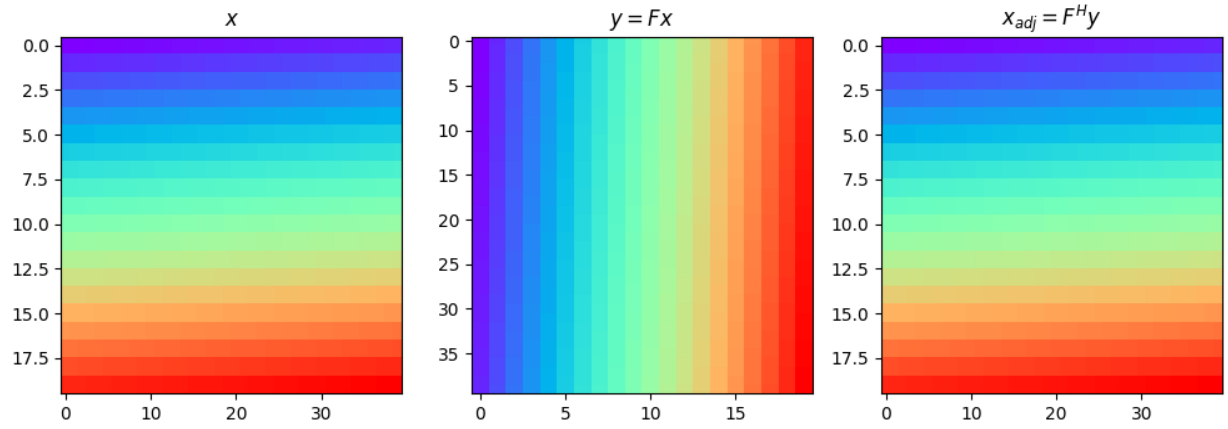
```
fig, axs = plt.subplots(1, 3, figsize=(10, 4))
fig.suptitle("Transpose for 2d data", fontsize=14, fontweight="bold", y=1.15)
axs[0].imshow(x, cmap="rainbow", vmin=0, vmax=800)
axs[0].set_title(r"$x$")
axs[0].axis("tight")
axs[1].imshow(y, cmap="rainbow", vmin=0, vmax=800)
axs[1].set_title(r"$y = F x$")
axs[1].axis("tight")
axs[2].imshow(xadj, cmap="rainbow", vmin=0, vmax=800)
axs[2].set_title(r"$x_{adj} = F^H y$")
axs[2].axis("tight")
plt.tight_layout()
```



A similar approach can of course be taken two swap multiple axes of multi-dimensional arrays for any number of dimensions.

Total running time of the script: (0 minutes 0.327 seconds)

3.5.44 Wavelet estimation

This example shows how to use the *pylops.avo.prestack.PrestackWaveletModelling* to estimate a wavelet from pre-stack seismic data. This problem can be written in mathematical form as:

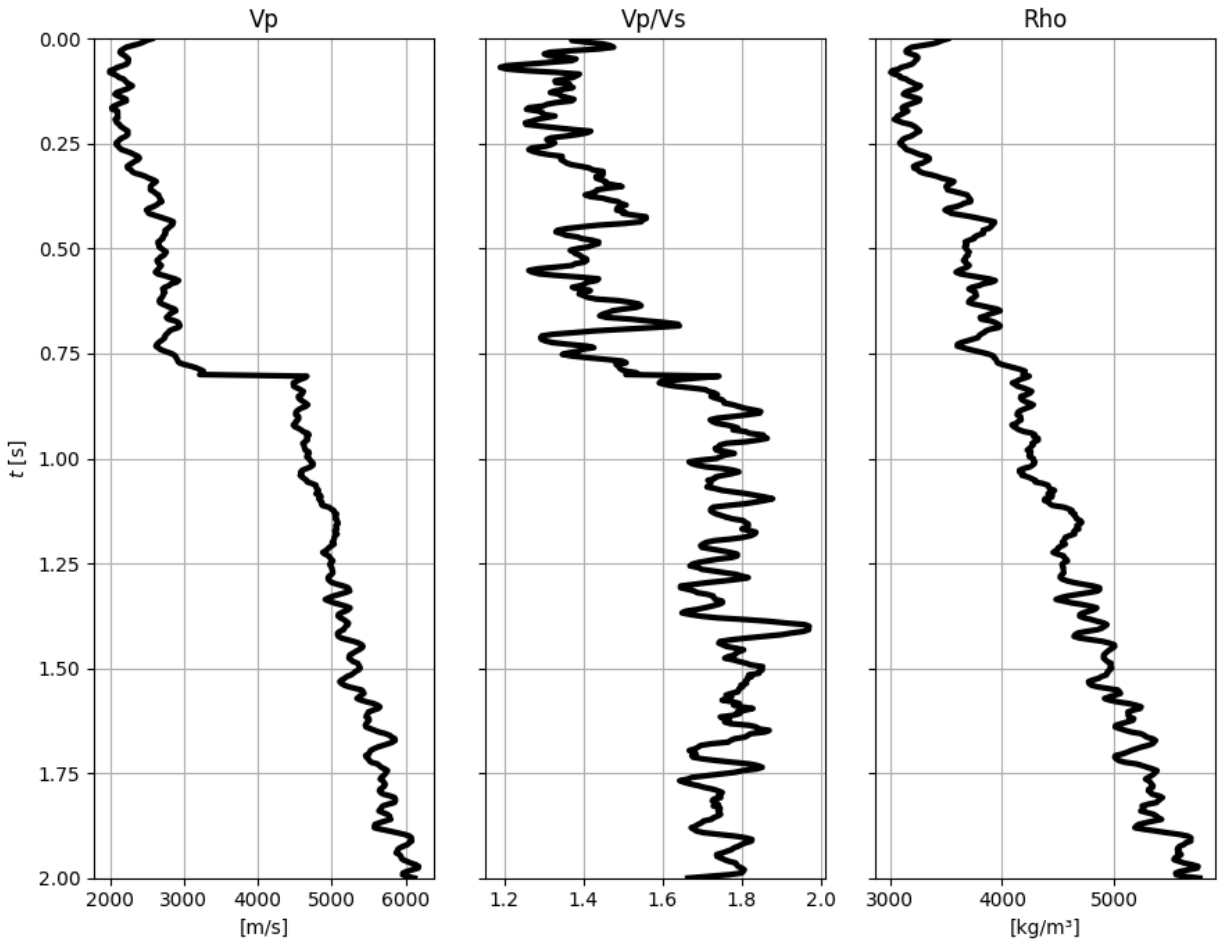
$$\mathbf{d} = \mathbf{G}\mathbf{w}$$

where $\mathbf{G}$ is an operator that convolves an angle-variant reflectivity series with the wavelet $\mathbf{w}$ that we aim to retrieve.

```
import matplotlib.pyplot as plt
import numpy as np
from scipy.signal import filtfilt

import pylops
from pylops.utils.wavelets import ricker

plt.close("all")
np.random.seed(0)
```

Let's start by creating the input elastic property profiles and wavelet

```
nt0 = 501
dt0 = 0.004
ntheta = 21

t0 = np.arange(nt0) * dt0
thetamin, thetamax = 0, 40
theta = np.linspace(thetamin, thetamax, ntheta)

# Elastic property profiles
vp = (
    2000
    + 5 * np.arange(nt0)
    + 2 * filtfilt(np.ones(5) / 5.0, 1, np.random.normal(0, 160, nt0))
)
vs = 600 + vp / 2 + 3 * filtfilt(np.ones(5) / 5.0, 1, np.random.normal(0, 100, nt0))
rho = 1000 + vp + filtfilt(np.ones(5) / 5.0, 1, np.random.normal(0, 120, nt0))
vp[201:] += 1500
vs[201:] += 500
rho[201:] += 100

# Wavelet
ntwav = 41
wavoff = 10
wav, twav, wavc = ricker(t0[: ntwav // 2 + 1], 20)
wav_phase = np.hstack((wav[wavoff:], np.zeros(wavoff)))

# vs/vp profile
vsvp = vs / vp

# Model
m = np.stack((np.log(vp), np.log(vs), np.log(rho)), axis=1)

fig, axs = plt.subplots(1, 3, figsize=(9, 7), sharey=True)
axs[0].plot(vp, t0, "k", lw=3)
axs[0].set(xlabel="[m/s]", ylabel=r"$t$ [s]", ylim=[t0[0], t0[-1]], title="Vp")
axs[0].grid()
axs[1].plot(vp / vs, t0, "k", lw=3)
axs[1].set(title="Vp/Vs")
axs[1].grid()
axs[2].plot(rho, t0, "k", lw=3)
axs[2].set(xlabel="[kg/m$^3$]", title="Rho")
axs[2].invert_yaxis()
axs[2].grid()
plt.tight_layout()
```



We create now the operators to model a synthetic pre-stack seismic gather with a zero-phase as well as a mixed phase wavelet.

```
# Create operators
Wavesop = pylops.avo.prestack.PrestackWaveletModelling(
    m, theta, nwav=ntwav, wavc=wavc, vsvp=vsvp, linearization="akirich"
)
Wavesop_phase = pylops.avo.prestack.PrestackWaveletModelling(
    m, theta, nwav=ntwav, wavc=wavc, vsvp=vsvp, linearization="akirich"
)
```

Let's apply those operators to the elastic model and create some synthetic data

```
d = (Wavesop * wav).reshape(ntheta, nt0).T
d_phase = (Wavesop_phase * wav_phase).reshape(ntheta, nt0).T

# add noise
dn = d + np.random.normal(0, 3e-2, d.shape)

fig, axs = plt.subplots(1, 3, figsize=(13, 7), sharey=True)
axs[0].imshow(
    d, cmap="gray", extent=(theta[0], theta[-1], t0[-1], t0[0]), vmin=-0.1, vmax=0.1
```

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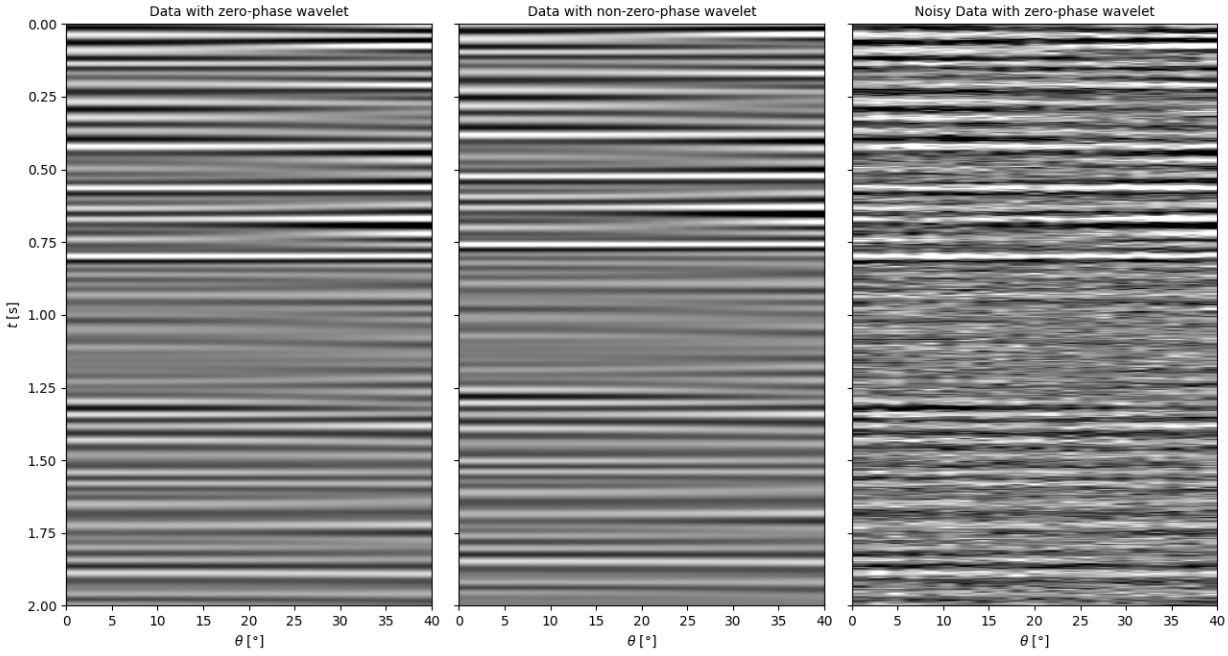
```
)
axs[0].axis("tight")
axs[0].set(xlabel=r"$\theta$ [°]", ylabel=r"$t$ [s]")
axs[0].set_title("Data with zero-phase wavelet", fontsize=10)
axs[1].imshow(
    d_phase,
    cmap="gray",
    extent=(theta[0], theta[-1], t0[-1], t0[0]),
    vmin=-0.1,
    vmax=0.1,
)
axs[1].axis("tight")
axs[1].set_title("Data with non-zero-phase wavelet", fontsize=10)
axs[1].set_xlabel(r"$\theta$ [°]")
axs[2].imshow(
    dn, cmap="gray", extent=(theta[0], theta[-1], t0[-1], t0[0]), vmin=-0.1, vmax=0.1
)
axs[2].axis("tight")
axs[2].set_title("Noisy Data with zero-phase wavelet", fontsize=10)
axs[2].set_xlabel(r"$\theta$ [°]")
plt.tight_layout()
```



We can invert the data. First we will consider noise-free data, subsequently we will add some noise and add a regularization terms in the inversion process to obtain a well-behaved wavelet also under noise conditions.

```
wav_est = Wavesop / d.T.ravel()
wav_phase_est = Wavesop_phase / d_phase.T.ravel()
wavn_est = Wavesop / dn.T.ravel()

# Create regularization operator
```

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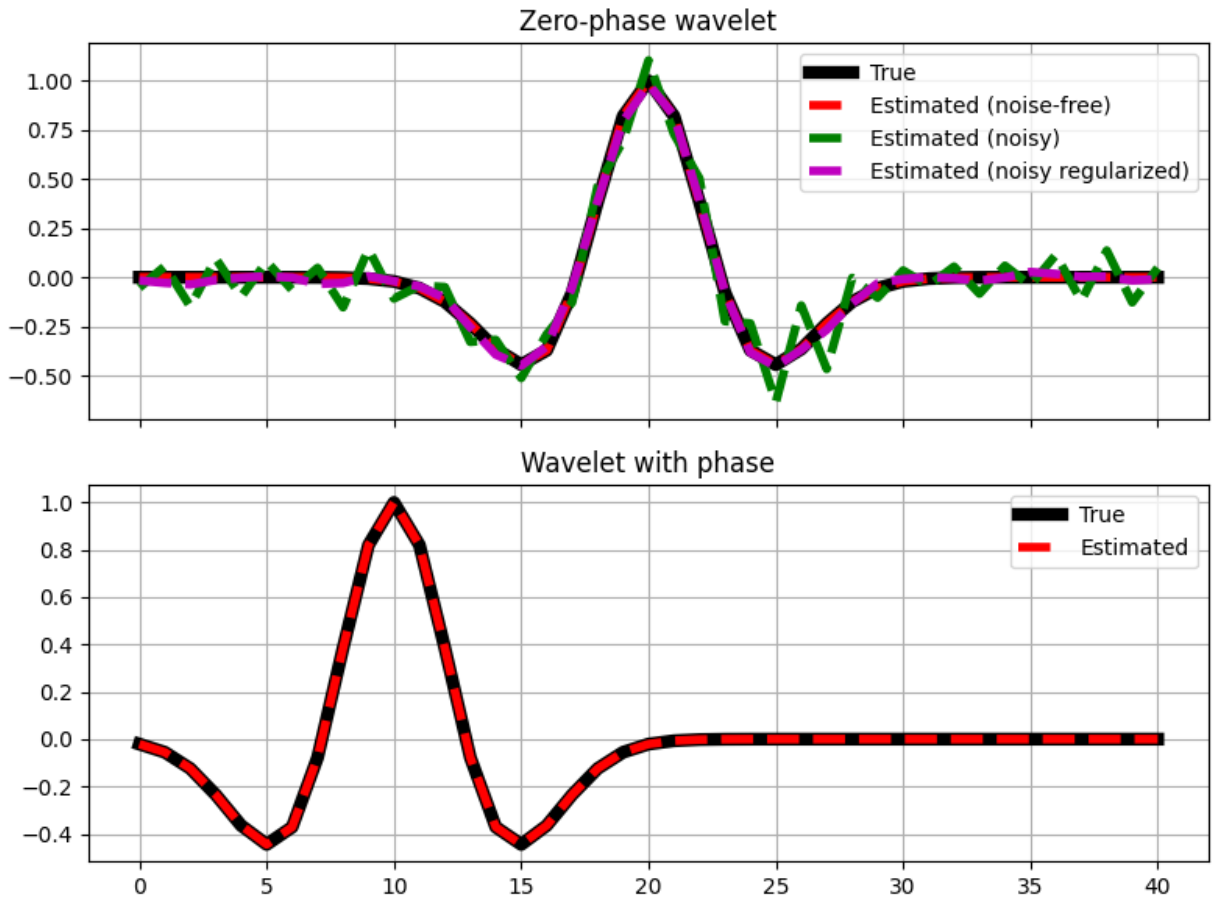
```
D2op = pylops.SecondDerivative(ntwav, dtype="float64")

# Invert for wavelet
(
    wavn_reg_est,
    istop,
    itn,
    r1norm,
    r2norm,
) = pylops.optimization.leastsquares.regularized_inversion(
    Wavesop,
    dn.T.ravel(),
    [D2op],
    epsRs=[np.sqrt(0.1)],
    **dict(damp=np.sqrt(1e-4), iter_lim=200, show=0)
)
```

As expected, the regularization helps to retrieve a smooth wavelet even under noisy conditions.

```
# sphinx_gallery_thumbnail_number = 3
fig, axs = plt.subplots(2, 1, sharex=True, figsize=(8, 6))
axs[0].plot(wav, "k", lw=6, label="True")
axs[0].plot(wav_est, "--r", lw=4, label="Estimated (noise-free)")
axs[0].plot(wavn_est, "--g", lw=4, label="Estimated (noisy)")
axs[0].plot(wavn_reg_est, "--m", lw=4, label="Estimated (noisy regularized)")
axs[0].set_title("Zero-phase wavelet")
axs[0].grid()
axs[0].legend(loc="upper right")
axs[0].axis("tight")
axs[1].plot(wav_phase, "k", lw=6, label="True")
axs[1].plot(wav_phase_est, "--r", lw=4, label="Estimated")
axs[1].set_title("Wavelet with phase")
axs[1].grid()
axs[1].legend(loc="upper right")
axs[1].axis("tight")
plt.tight_layout()
```



Finally we repeat the same exercise, but this time we use a *preconditioner*. Initially, our preconditioner is a *pylops.Symmetrize* operator to ensure that our estimated wavelet is zero-phase. After we chain the *pylops.Symmetrize* and the *pylops.Smoothing1D* operators to also guarantee a smooth wavelet.

```
# Create symmetrize operator
Sop = pylops.Symmetrize((ntwav + 1) // 2)

# Create smoothing operator
Smop = pylops.Smoothing1D(5, dims=((ntwav + 1) // 2,), dtype="float64")

# Invert for wavelet
wavn_prec_est = pylops.optimization.leastsquares.preconditioned_inversion(
    Wavesop, dn.T.ravel(), Sop, **dict(damp=np.sqrt(1e-4), iter_lim=200, show=0)
)[0]

wavn_smooth_est = pylops.optimization.leastsquares.preconditioned_inversion(
    Wavesop, dn.T.ravel(), Sop * Smop, **dict(damp=np.sqrt(1e-4), iter_lim=200, show=0)
)[0]

fig, ax = plt.subplots(1, 1, sharex=True, figsize=(8, 3))
ax.plot(wav, "k", lw=6, label="True")
ax.plot(wav_est, "--r", lw=4, label="Estimated (noise-free)")
ax.plot(wavn_prec_est, "--g", lw=4, label="Estimated (noisy symmetric)")
```

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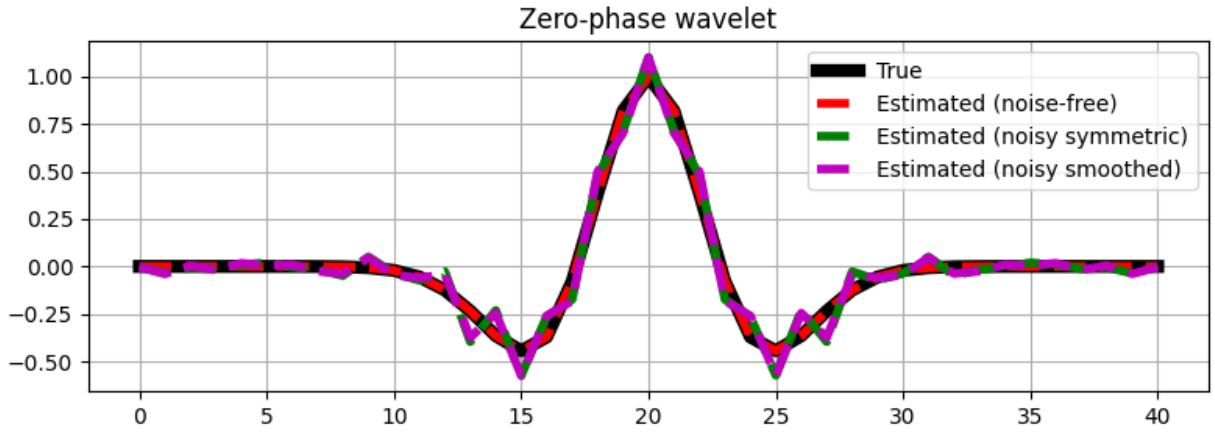
```
ax.plot(wavn_smooth_est, "--m", lw=4, label="Estimated (noisy smoothed)")
ax.set_title("Zero-phase wavelet")
ax.grid()
ax.legend(loc="upper right")
plt.tight_layout()
```



Total running time of the script: (0 minutes 1.937 seconds)

3.5.45 Wavelet transform

This example shows how to use the `pylops.DWT` and `pylops.DWT2D` operators to perform 1- and 2-dimensional DWT.

```
import matplotlib.pyplot as plt
import numpy as np

import pylops

plt.close("all")
```

Let's start with a 1-dimensional signal. We apply the 1-dimensional wavelet transform, keep only the first 30 coefficients and perform the inverse transform.

```
nt = 200
dt = 0.004
t = np.arange(nt) * dt
freqs = [10, 7, 9]
amps = [1, -2, 0.5]
x = np.sum([amp * np.sin(2 * np.pi * f * t) for (f, amp) in zip(freqs, amps)], axis=0)

Wop = pylops.signalprocessing.DWT(nt, wavelet="dmey", level=5)
y = Wop * x
yf = y.copy()
yf[25:] = 0
xinv = Wop.H * yf

plt.figure(figsize=(8, 2))
```

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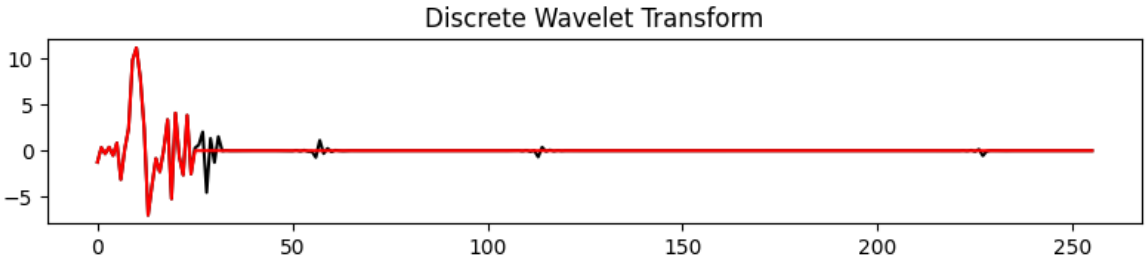
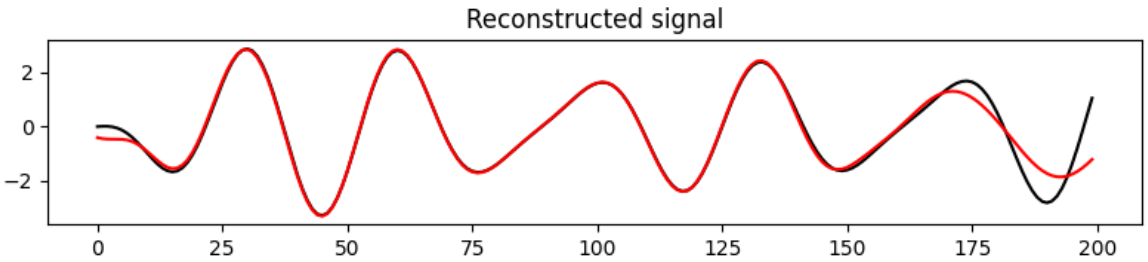
```
plt.plot(y, "k", label="Full")
plt.plot(yf, "r", label="Extracted")
plt.title("Discrete Wavelet Transform")
plt.tight_layout()

plt.figure(figsize=(8, 2))
plt.plot(x, "k", label="Original")
plt.plot(xinv, "r", label="Reconstructed")
plt.title("Reconstructed signal")
plt.tight_layout()
```





We repeat the same procedure with an image. In this case the 2-dimensional DWT will be applied instead. Only a quarter of the coefficients of the DWT will be retained in this case.

```
im = np.load("../testdata/python.npy")[::5, ::5, 0]

Nz, Nx = im.shape
Wop = pylops.signalprocessing.DWT2D((Nz, Nx), wavelet="haar", level=5)
y = Wop * im
yf = y.copy()
yf.flat[y.size // 4 :] = 0
iminv = Wop.H * yf

fig, axs = plt.subplots(2, 2, figsize=(6, 6))
axs[0, 0].imshow(im, cmap="gray")
axs[0, 0].set_title("Image")
axs[0, 0].axis("tight")
axs[0, 1].imshow(y, cmap="gray_r", vmin=-1e2, vmax=1e2)
axs[0, 1].set_title("DWT2 coefficients")
axs[0, 1].axis("tight")
axs[1, 0].imshow(iminv, cmap="gray")
axs[1, 0].set_title("Reconstructed image")
```

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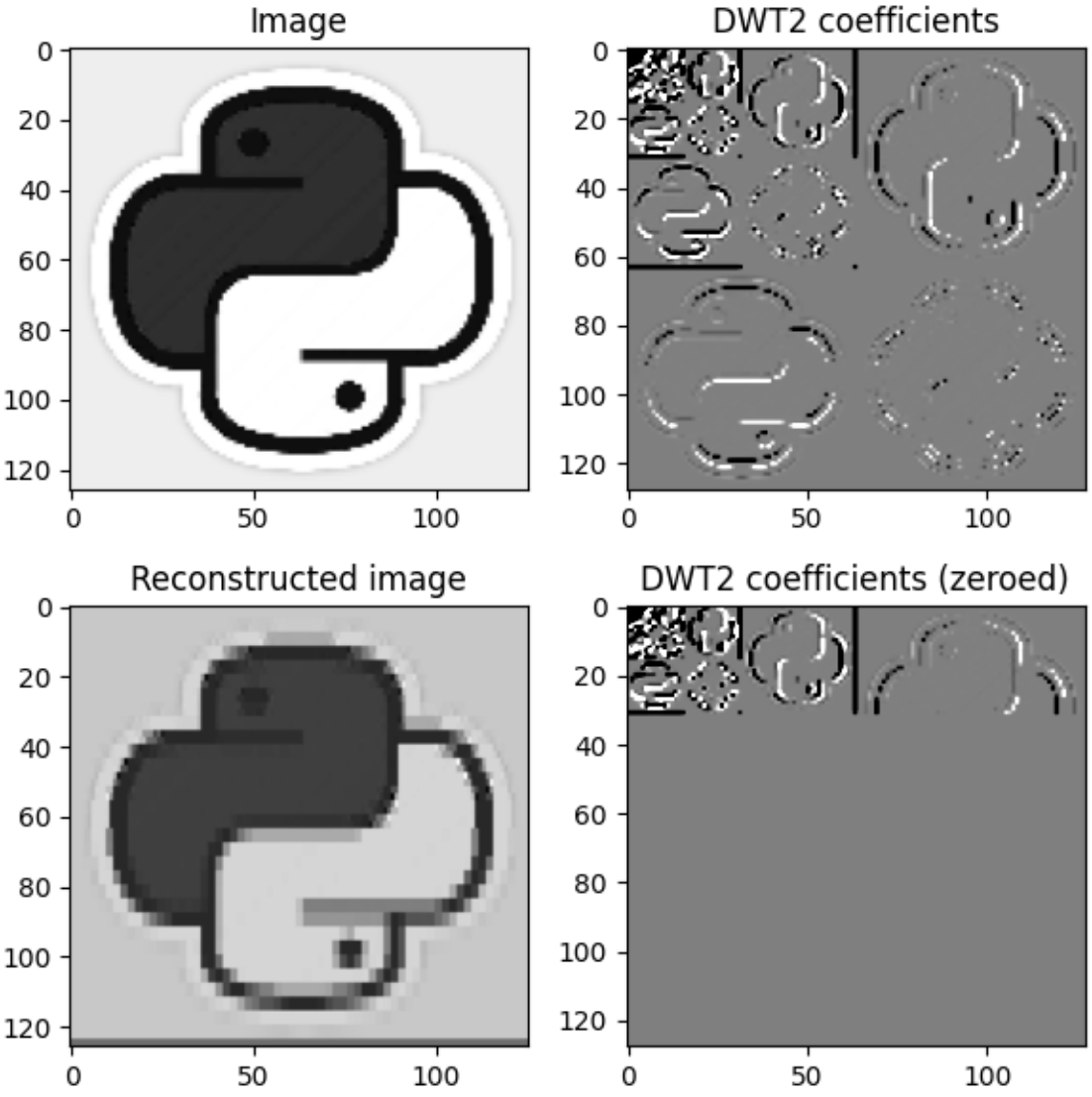
```
axs[1, 0].axis("tight")
axs[1, 1].imshow(yf, cmap="gray_r", vmin=-1e2, vmax=1e2)
axs[1, 1].set_title("DWT2 coefficients (zeroed)")
axs[1, 1].axis("tight")
plt.tight_layout()
```



Total running time of the script: (0 minutes 0.656 seconds)

3.5.46 Wavelets

This example shows how to use the different wavelets available PyLops.

```
import matplotlib.pyplot as plt
import numpy as np

import pylops

plt.close("all")
```

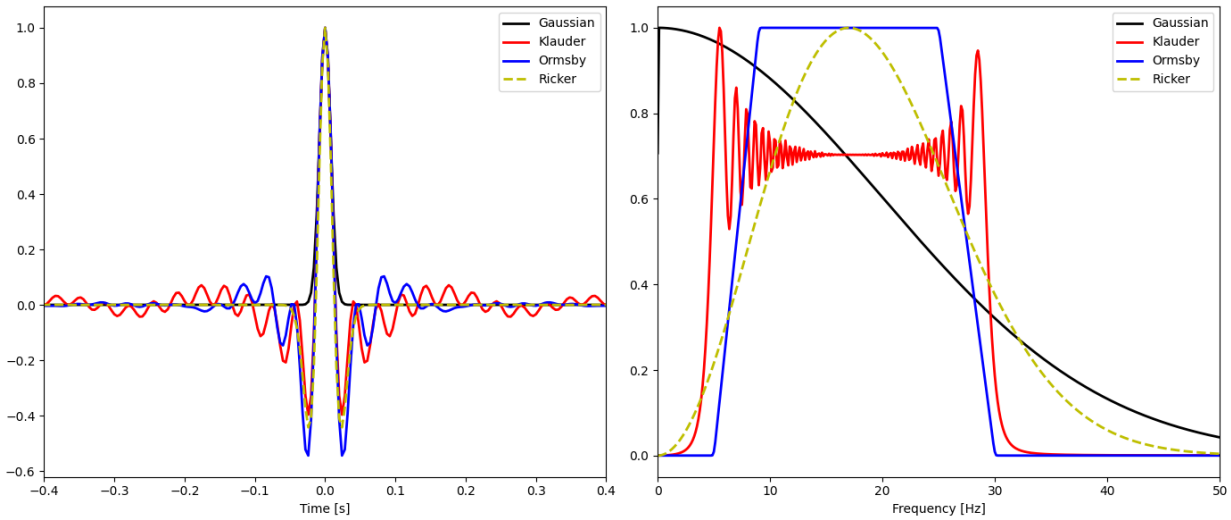
Let's start with defining a time axis and creating the FFT operator

```
dt = 0.004
nt = 1001
t = np.arange(nt) * dt

Fop = pylops.signalprocessing.FFT(2 * nt - 1, sampling=dt, real=True)
f = Fop.f
```

We can now create the different wavelets and display them

```
# Gaussian
wg, twg, wgc = pylops.utils.wavelets.gaussian(t, std=2)

# Gaussian
wk, twk, wgk = pylops.utils.wavelets.klauder(t, f=[4, 30], taper=np.hanning)

# Ormsby
wo, two, woc = pylops.utils.wavelets.ormsby(t, f=[5, 9, 25, 30], taper=np.hanning)

# Ricker
wr, twr, wrc = pylops.utils.wavelets.ricker(t, f0=17)

# Frequency domain
wgf = Fop @ wg
wkf = Fop @ wk
wof = Fop @ wo
wrf = Fop @ wr
```

```
fig, axs = plt.subplots(1, 2, figsize=(14, 6))
axs[0].plot(twg, wg, "k", lw=2, label="Gaussian")
axs[0].plot(twk, wk, "r", lw=2, label="Klauder")
axs[0].plot(two, wo, "b", lw=2, label="Ormsby")
axs[0].plot(twr, wr, "y--", lw=2, label="Ricker")
axs[0].set(xlim=(-0.4, 0.4), xlabel="Time [s]")
axs[0].legend()
axs[1].plot(f, np.abs(wgf) / np.abs(wgf).max(), "k", lw=2, label="Gaussian")
axs[1].plot(f, np.abs(wkf) / np.abs(wkf).max(), "r", lw=2, label="Klauder")
axs[1].plot(f, np.abs(wof) / np.abs(wof).max(), "b", lw=2, label="Ormsby")
axs[1].plot(f, np.abs(wrf) / np.abs(wrf).max(), "y--", lw=2, label="Ricker")
axs[1].set(xlim=(0, 50), xlabel="Frequency [Hz]")
axs[1].legend()
plt.tight_layout()
```



Total running time of the script: (0 minutes 0.307 seconds)

3.5.47 Zero

This example shows how to use the `pylops.basicoperators.Zero` operator. This operators simply zeroes the data in forward mode and the model in adjoint mode.

```
import matplotlib.gridspec as pltgs
import matplotlib.pyplot as plt
import numpy as np

import pylops

plt.close("all")
```

Let's define an zero operator $\mathbf{0}$ with same number of elements for data N and model M.

```
N, M = 5, 5
x = np.arange(M)
Zop = pylops.basicoperators.Zero(M, dtype="int")

y = Zop * x
xadj = Zop.H * y

gs = pltgs.GridSpec(1, 6)
fig = plt.figure(figsize=(7, 4))
ax = plt.subplot(gs[0, 0:3])
ax.imshow(np.zeros((N, N)), cmap="rainbow", vmin=-M, vmax=M)
ax.set_title("A", size=20, fontweight="bold")
ax.set_xticks(np.arange(N - 1) + 0.5)
ax.set_yticks(np.arange(M - 1) + 0.5)
ax.grid(linewidth=3, color="white")
ax.xaxis.set_ticklabels([])
ax.yaxis.set_ticklabels([])
ax = plt.subplot(gs[0, 3])
```

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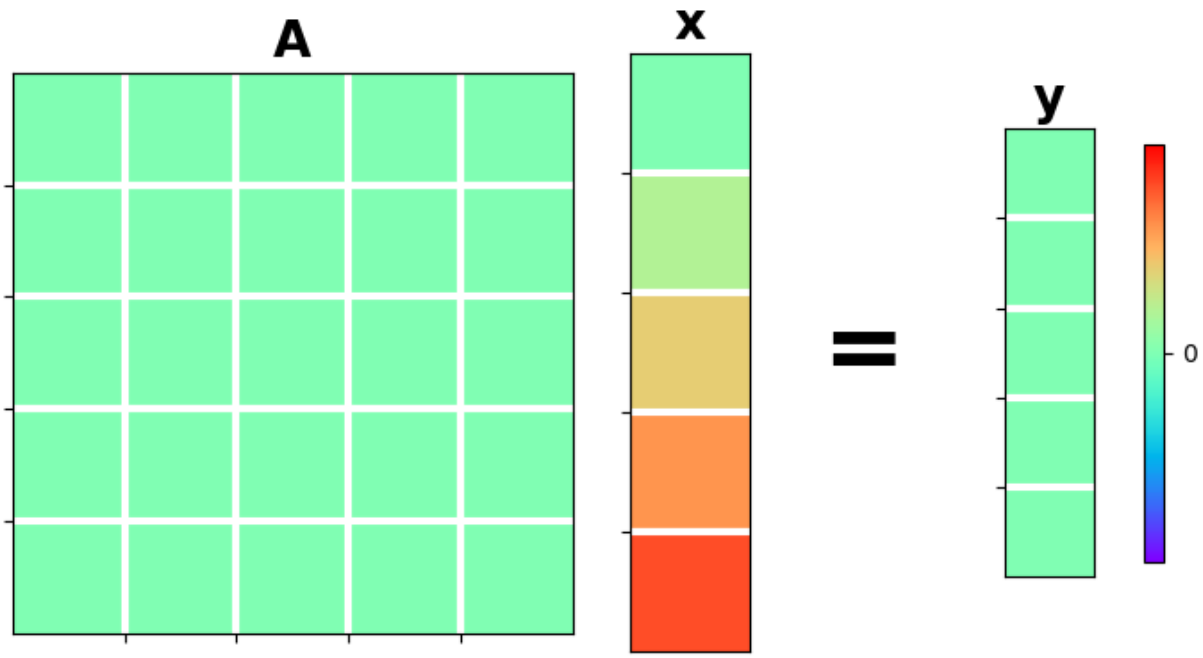
```
im = ax.imshow(x[:, np.newaxis], cmap="rainbow", vmin=-M, vmax=M)
ax.set_title("x", size=20, fontweight="bold")
ax.set_xticks([])
ax.set_yticks(np.arange(M - 1) + 0.5)
ax.grid(linewidth=3, color="white")
ax.xaxis.set_ticklabels([])
ax.yaxis.set_ticklabels([])
ax = plt.subplot(gs[0, 4])
ax.text(
    0.35,
    0.5,
    "=",
    horizontalalignment="center",
    verticalalignment="center",
    size=40,
    fontweight="bold",
)
ax.axis("off")
ax = plt.subplot(gs[0, 5])
ax.imshow(y[:, np.newaxis], cmap="rainbow", vmin=-M, vmax=M)
ax.set_title("y", size=20, fontweight="bold")
ax.set_xticks([])
ax.set_yticks(np.arange(N - 1) + 0.5)
ax.grid(linewidth=3, color="white")
ax.xaxis.set_ticklabels([])
ax.yaxis.set_ticklabels([])
fig.colorbar(im, ax=ax, ticks=[0], pad=0.3, shrink=0.7)
plt.tight_layout()
```



Similarly we can consider the case with data bigger than model

```
N, M = 10, 5
x = np.arange(M)
Zop = pylops.Zero(N, M, dtype="int")

y = Zop * x
xadj = Zop.H * y

print(f"x = {x}")
print(f"0*x = {y}")
print(f"0'*y = {xadj}")
```

```
x = [0 1 2 3 4]
0*x = [0 0 0 0 0 0 0 0 0 0]
0'*y = [0 0 0 0 0]
```

and model bigger than data

```
N, M = 5, 10
x = np.arange(M)
Zop = pylops.Zero(N, M, dtype="int")

y = Zop * x
xadj = Zop.H * y

print(f"x = {x}")
print(f"0*x = {y}")
print(f"0'*y = {xadj}")
```

```
x = [0 1 2 3 4 5 6 7 8 9]
0*x = [0 0 0 0 0]
0'*y = [0 0 0 0 0 0 0 0 0 0]
```

Note that this operator can be useful in many real-life applications when for example we want to manipulate a subset of the model array and keep intact the rest of the array. For example:

$$\begin{bmatrix} \mathbf{A} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{x_1} \\ \mathbf{x_2} \end{bmatrix} = \mathbf{A}\mathbf{x_1}$$

Refer to the tutorial on *Optimization* for more details on this.

Total running time of the script: (0 minutes 0.209 seconds)

3.6 Frequenty Asked Questions

1. Can I visualize my operator?

Yes, you can. Every operator has a method called `todense` that will return the dense matrix equivalent of the operator. Note, however, that in order to do so we need to allocate a `numpy` array of the size of your operator and apply the operator `N` times, where `N` is the number of columns of the operator. The allocation can be very heavy on your memory and the computation may take long time, so use it with care only for small toy examples to understand what your operator

looks like. This method should however not be abused, as the reason of working with linear operators is indeed that you don't really need to access the explicit matrix representation of an operator.

2. Can I have an older version of `cupy` or `cusignal` installed in my system (`cupy-cudaXX<8.1.0` or `cusignal>=0.16.0`)?

Yes. Nevertheless you need to tell PyLops that you don't want to use its `cupy` backend by setting the environment variable `CUPY_PYLOPS=0` or `CUPY_SIGNAL=0`. Failing to do so will lead to an error when you import `pylops` because some of the `cupyx` routines that we use are not available in earlier version of `cupy`.

3.7 PyLops API

The Application Programming Interface (API) of PyLops can be loosely seen as composed of a stack of three main layers:

- *Linear operators*: building blocks for the setting up of inverse problems
- *Solvers*: interfaces to a variety of solvers, providing an easy way to augment an inverse problem with additional regularization and/or preconditioning term
- *Applications*: high-level interfaces allowing users to easily setup and solve specific problems (while hiding the non-needed details - i.e., creation and setup of linear operators and solvers).

3.7.1 Linear operators

Templates

LinearOperator(*args, **kwargs)	Common interface for performing matrix-vector products.
FunctionOperator(*args, **kwargs)	Function Operator.
MemoizeOperator(*args, **kwargs)	Memoize Operator.
TorchOperator(*args, **kwargs)	Wrap a PyLops operator into a Torch function.

pylops.LinearOperator

class pylops.**LinearOperator**(**args*, ***kwargs*)

Common interface for performing matrix-vector products.

This class is an overload of the `scipy.sparse.linalg.LinearOperator` class. It adds functionalities by overloading standard operators such as `__truediv__` as well as creating convenience methods such as `eigs`, `cond`, and `conj`.

Note: End users of PyLops should not use this class directly but simply use operators that are already implemented. This class is meant for developers and it has to be used as the parent class of any new operator developed within PyLops. Find more details regarding implementation of new operators at *Implementing new operators*.

Parameters

Op
[`scipy.sparse.linalg.LinearOperator` or `scipy.sparse.linalg._ProductLinearOperator` or `scipy.sparse.linalg._SumLinearOperator`]

Operator. If other arguments are provided, they will overwrite those obtained from `Op`.

dtype
[`str`, optional] Type of elements in input array.

shape
[`tuple(int, int)`, optional] Shape of operator. If not provided, obtained from `dims` and `dimsd`.

dims
[`tuple(int, ..., int)`, optional] New in version 2.0.0.

Dimensions of model. If not provided, `(self.shape[1],)` is used.

dimsd
[`tuple(int, ..., int)`, optional] New in version 2.0.0.

Dimensions of data. If not provided, `(self.shape[0],)` is used.

explicit
[`bool`, optional] Operator contains a matrix that can be solved explicitly (`True`) or not (`False`). Defaults to `False`.

clinear
[`bool`, optional] New in version 1.17.0.

Operator is complex-linear. Defaults to `True`.

name
[`str`, optional] New in version 2.0.0.

Name of operator (to be used by *pylops.utils.describe.describe*)

Methods

`__init__`([Op, dtype, shape, dims, dimsd, ...])	Initialize this LinearOperator.
`adjoint`()	Hermitian adjoint.
`apply_columns`(cols)	Apply subset of columns of operator
`cond`([uselobpcg])	Condition number of linear operator.
`conj`()	Complex conjugate operator
`div`(y[, niter, densesolver])	Solve the linear problem $\mathbf{y} = \mathbf{A}\mathbf{x}$.
`dot`(x)	Matrix-matrix or matrix-vector multiplication.
`eigs`([neigs, symmetric, niter, uselobpcg])	Most significant eigenvalues of linear operator.
`matmat`(X)	Matrix-matrix multiplication.
`matvec`(x)	Matrix-vector multiplication.
`reset_count`()	Reset counters
`rmatmat`(X)	Matrix-matrix multiplication.
`rmatvec`(x)	Adjoint matrix-vector multiplication.
`todense`([backend])	Return dense matrix.
`toimag`([forw, adj])	Imag operator
`toreal`([forw, adj])	Real operator
`tosparse`()	Return sparse matrix.
`trace`([neval, method, backend])	Trace of linear operator.
`transpose`()	Transpose this linear operator.

Examples using `pylops.LinearOperator`

- *1D, 2D and 3D Sliding*
- *AVO modelling*
- *Bilinear Interpolation*
- *CGLS and LSQR Solvers*
- *Causal Integration*
- *Chirp Radon Transform*
- *Conj*
- *Convolution*
- *Derivatives*
- *Describe*
- *Diagonal*
- *Flip along an axis*
- *Identity*
- *Imag*
- *Linear Regression*
- *MP, OMP, ISTA and FISTA*
- *Matrix Multiplication*
- *Multi-Dimensional Convolution*
- *Normal Moveout (NMO) Correction*
- *Operators concatenation*
- *Operators with Multiprocessing*
- *Padding*
- *Patching*
- *PhaseShift operator*
- *Polynomial Regression*
- *Pre-stack modelling*
- *Real*
- *Restriction and Interpolation*
- *Roll*
- *Seislet transform*
- *Shift*
- *Spread How-to*
- *Sum*
- *Symmetrize*
- *Total Variation (TV) Regularization*

- *Transpose*
- *Wavelet estimation*
- *Wavelet transform*
- *Zero*
- *01. The LinearOpeator*
- *02. The Dot-Test*
- *03. Solvers*
- *03. Solvers (Advanced)*
- *04. Bayesian Inversion*
- *05. Image deblurring*
- *06. 2D Interpolation*
- *07. Post-stack inversion*
- *08. Pre-stack (AVO) inversion*
- *09. Multi-Dimensional Deconvolution*
- *12. Seismic regularization*
- *14. Seismic wavefield decomposition*
- *16. CT Scan Imaging*
- *17. Real/Complex Inversion*
- *18. Deblending*
- *19. Automatic Differentiation*

pylops.FunctionOperator

class pylops.**FunctionOperator**(**args*, ***kwargs*)

Function Operator.

Simple wrapper to functions for forward *f* and adjoint *f_c* multiplication.

Functions f and f_c are such that $f : \mathbb{F}^m \to \mathbb{F}_c^n$ and $f_c : \mathbb{F}_c^n \to \mathbb{F}^m$ where $\mathbb{F}$ and $\mathbb{F}_c$ are the underlying fields (e.g., $\mathbb{R}$ for real or $\mathbb{C}$ for complex)

FunctionOperator can be called in the following ways: `FunctionOperator(f, n)`, `FunctionOperator(f, n, m)`, `FunctionOperator(f, fc, n)`, and `FunctionOperator(f, fc, n, m)`.

The first two methods can only be used for forward modelling and will return `NotImplementedError` if the adjoint is called. The first and third method assume the matrix (or matrices) to be square. All methods can be called with the `dtype` keyword argument.

Parameters

f
[`callable`] Function for forward multiplication.

fc
[`callable`, optional] Function for adjoint multiplication.

n
[int, optional] Number of rows (length of data vector).

m
[int, optional] Number of columns (length of model vector).

dtype
[str, optional] Type of elements in input array.

name
[str, optional] New in version 2.0.0.

Name of operator (to be used by *pylops.utils.describe.describe*)

Examples

```
>>> from pylops.basicoperators import FunctionOperator
>>> def forward(v):
...     return np.array([2*v[0], 3*v[1]])
...
>>> A = FunctionOperator(forward, 2)
>>> A
<2x2 FunctionOperator with dtype=float64>
>>> A.matvec(np.ones(2))
array([2.,  3.])
>>> A @ np.ones(2)
array([2.,  3.])
```

Attributes

shape
[tuple] Operator shape $[n \times m]$

explicit
[bool] Operator contains a matrix that can be solved explicitly (`True`) or not (`False`)

Methods

`__init__`(f, *args, **kwargs)	Initialize this LinearOperator.
`adjoint`()	Hermitian adjoint.
`apply_columns`(cols)	Apply subset of columns of operator
`cond`([uselobpcg])	Condition number of linear operator.
`conj`()	Complex conjugate operator
`div`(y[, niter, densesolver])	Solve the linear problem $\mathbf{y} = \mathbf{A}\mathbf{x}$.
`dot`(x)	Matrix-matrix or matrix-vector multiplication.
`eigs`([neigs, symmetric, niter, uselobpcg])	Most significant eigenvalues of linear operator.
`matmat`(X)	Matrix-matrix multiplication.
`matvec`(x)	Matrix-vector multiplication.
`reset_count`()	Reset counters
`rmatmat`(X)	Matrix-matrix multiplication.
`rmatvec`(x)	Adjoint matrix-vector multiplication.
`todense`([backend])	Return dense matrix.
`toimag`([forw, adj])	Imag operator
`toreal`([forw, adj])	Real operator
`tosparse`()	Return sparse matrix.
`trace`([neval, method, backend])	Trace of linear operator.
`transpose`()	Transpose this linear operator.

pylops.MemoizeOperator

class pylops.**MemoizeOperator**(**args*, ***kwargs*)

Memoize Operator.

This operator can be used to wrap any PyLops operator and add a memoize functionality and stores the last `max_neval` model/data vector pairs

Parameters

Op
[*pylops.LinearOperator*] PyLops linear operator

max_neval
[`int`, optional] Maximum number of previous evaluations stored, use `np.inf` for infinite memory

Attributes

shape
[`tuple`] Operator shape $[n \times m]$

explicit
[`bool`] Operator contains a matrix that can be solved explicitly (`True`) or not (`False`)

Methods

`__init__`(Op[, max_neval])	Initialize this LinearOperator.
`adjoint`()	Hermitian adjoint.
`apply_columns`(cols)	Apply subset of columns of operator
`cond`([uselobpcg])	Condition number of linear operator.
`conj`()	Complex conjugate operator
`div`(y[, niter, densesolver])	Solve the linear problem $\mathbf{y} = \mathbf{A}\mathbf{x}$.
`dot`(x)	Matrix-matrix or matrix-vector multiplication.
`eigs`([neigs, symmetric, niter, uselobpcg])	Most significant eigenvalues of linear operator.
`matmat`(X)	Matrix-matrix multiplication.
`matvec`(x)	Matrix-vector multiplication.
`reset_count`()	Reset counters
`rmatmat`(X)	Matrix-matrix multiplication.
`rmatvec`(x)	Adjoint matrix-vector multiplication.
`todense`([backend])	Return dense matrix.
`toimag`([forw, adj])	Imag operator
`toreal`([forw, adj])	Real operator
`tosparse`()	Return sparse matrix.
`trace`([neval, method, backend])	Trace of linear operator.
`transpose`()	Transpose this linear operator.

Examples using `pylops.MemoizeOperator`

- *17. Real/Complex Inversion*

pylops.TorchOperator

class pylops.**TorchOperator**(**args*, ***kwargs*)

Wrap a PyLops operator into a Torch function.

This class can be used to wrap a pylops operator into a torch function. Doing so, users can mix native torch functions (e.g. basic linear algebra operations, neural networks, etc.) and pylops operators.

Since all operators in PyLops are linear operators, a Torch function is simply implemented by using the forward operator for its forward pass and the adjoint operator for its backward (gradient) pass.

Parameters

Op
[*pylops.LinearOperator*] PyLops operator

batch
[bool, optional] Input has single sample (`False`) or batch of samples (`True`). If `batch==False` the input must be a 1-d Torch tensor, if *batch==False* ` the input must be a 2-d Torch tensor with batches along the first dimension

device
[str, optional] Device to be used when applying operator (`cpu` or `gpu`)

devicetorch
[str, optional] Device to be assigned the output of the operator to (any Torch-compatible device)

Methods

`__init__`(Op[, batch, device, devicetorch])	Initialize this LinearOperator.
`adjoint`()	Hermitian adjoint.
`apply`(x)	Apply forward pass to input vector
`apply_columns`(cols)	Apply subset of columns of operator
`cond`([uselobpcg])	Condition number of linear operator.
`conj`()	Complex conjugate operator
`div`(y[, niter, densesolver])	Solve the linear problem $\mathbf{y} = \mathbf{A}\mathbf{x}$.
`dot`(x)	Matrix-matrix or matrix-vector multiplication.
`eigs`([neigs, symmetric, niter, uselobpcg])	Most significant eigenvalues of linear operator.
`matmat`(X)	Matrix-matrix multiplication.
`matvec`(x)	Matrix-vector multiplication.
`reset_count`()	Reset counters
`rmatmat`(X)	Matrix-matrix multiplication.
`rmatvec`(x)	Adjoint matrix-vector multiplication.
`todense`([backend])	Return dense matrix.
`toimag`([forw, adj])	Imag operator
`toreal`([forw, adj])	Real operator
`tosparse`()	Return sparse matrix.
`trace`([neval, method, backend])	Trace of linear operator.
`transpose`()	Transpose this linear operator.

Examples using `pylops.TorchOperator`

- *19. Automatic Differentiation*

Basic operators

MatrixMult(*args, **kwargs)	Matrix multiplication.
Identity(*args, **kwargs)	Identity operator.
Zero(*args, **kwargs)	Zero operator.
Diagonal(*args, **kwargs)	Diagonal operator.
Transpose(*args, **kwargs)	Transpose operator.
Flip(*args, **kwargs)	Flip along an axis.
Roll(*args, **kwargs)	Roll along an axis.
Pad(*args, **kwargs)	Pad operator.
Sum(*args, **kwargs)	Sum operator.
Symmetrize(*args, **kwargs)	Symmetrize along an axis.
Restriction(*args, **kwargs)	Restriction (or sampling) operator.
Regression(*args, **kwargs)	Polynomial regression.
LinearRegression(taxis[, dtype])	Linear regression.
CausalIntegration(*args, **kwargs)	Causal integration.
Spread(*args, **kwargs)	Spread operator.
VStack(*args, **kwargs)	Vertical stacking.
HStack(*args, **kwargs)	Horizontal stacking.
Block(ops[, nproc, dtype])	Block operator.
BlockDiag(*args, **kwargs)	Block-diagonal operator.
Kronecker(*args, **kwargs)	Kronecker operator.
Real(*args, **kwargs)	Real operator.
Imag(*args, **kwargs)	Imag operator.
Conj(*args, **kwargs)	Complex conjugate operator.

pylops.MatrixMult

class pylops.**MatrixMult**(**args*, ***kwargs*)

Matrix multiplication.

Simple wrapper to `numpy.dot` and `numpy.vdot` for an input matrix $\mathbf{A}$.

Parameters

A
[`numpy.ndarray` or `scipy.sparse` matrix] Matrix.

otherdims
[`tuple`, optional] Number of samples for each other dimension of model (model/data will be reshaped and `A` applied multiple times to each column of the model/data).

dtype
[`str`, optional] Type of elements in input array.

name
[`str`, optional] New in version 2.0.0.

Name of operator (to be used by *pylops.utils.describe.describe*)

Attributes

dimsd
[`tuple`] Shape of the array after the forward, but before linearization.

For example, `y_reshaped = (Op * x.ravel()).reshape(Op.dimsd)`.

shape
[tuple] Operator shape

explicit
[bool] Operator contains a matrix that can be solved explicitly (True) or not (False)

complex
[bool] Matrix has complex numbers (True) or not (False)

Methods

__init__(A[, otherdims, dtype, name])	Initialize this LinearOperator.
adjoint()	Hermitian adjoint.
apply_columns(cols)	Apply subset of columns of operator
cond([uselobpcg])	Condition number of linear operator.
conj()	Complex conjugate operator
div(y[, niter, densesolver])	Solve the linear problem $\mathbf{y} = \mathbf{A}\mathbf{x}$.
dot(x)	Matrix-matrix or matrix-vector multiplication.
eigs([neigs, symmetric, niter, uselobpcg])	Most significant eigenvalues of linear operator.
inv()	Return the inverse of $\mathbf{A}$.
matmat(X)	Matrix-matrix multiplication.
matvec(x)	Matrix-vector multiplication.
reset_count()	Reset counters
rmatmat(X)	Matrix-matrix multiplication.
rmatvec(x)	Adjoint matrix-vector multiplication.
todense([backend])	Return dense matrix.
toimag([forw, adj])	Imag operator
toreal([forw, adj])	Real operator
tosparse()	Return sparse matrix.
trace([neval, method, backend])	Trace of linear operator.
transpose()	Transpose this linear operator.

Examples using pylops.MatrixMult

- *CGLS and LSQR Solvers*
- *Describe*
- *MP, OMP, ISTA and FISTA*
- *Matrix Multiplication*
- *Operators concatenation*
- *Operators with Multiprocessing*
- *Restriction and Interpolation*
- *02. The Dot-Test*
- *03. Solvers*
- *07. Post-stack inversion*
- *08. Pre-stack (AVO) inversion*
- *17. Real/Complex Inversion*

- *19. Automatic Differentiation*

pylops.Identity

class pylops.**Identity**(**args*, ***kwargs*)

Identity operator.

Simply move model to data in forward model and viceversa in adjoint mode if $M = N$. If $M > N$ removes last $M - N$ elements from model in forward and pads with 0 in adjoint. If $N > M$ removes last $N - M$ elements from data in adjoint and pads with 0 in forward.

Parameters

N
[int] Number of samples in data (and model, if `M` is not provided).

M
[int, optional] Number of samples in model.

inplace
[bool, optional] Work inplace (`True`) or make a new copy (`False`). By default, data is a reference to the model (in forward) and model is a reference to the data (in adjoint).

dtype
[str, optional] Type of elements in input array.

name
[str, optional] New in version 2.0.0.

Name of operator (to be used by *pylops.utils.describe.describe*)

Notes

For $M = N$, an *Identity* operator simply moves the model $\mathbf{x}$ to the data $\mathbf{y}$ in forward mode and viceversa in adjoint mode:

$$y_i = x_i \quad \forall i = 1, 2, \ldots, N$$

or in matrix form:

$$\mathbf{y} = \mathbf{I}\mathbf{x} = \mathbf{x}$$

and

$$\mathbf{x} = \mathbf{I}\mathbf{y} = \mathbf{y}$$

For $M > N$, the *Identity* operator takes the first M elements of the model $\mathbf{x}$ into the data $\mathbf{y}$ in forward mode

$$y_i = x_i \quad \forall i = 1, 2, \ldots, N$$

and all the elements of the data $\mathbf{y}$ into the first M elements of model in adjoint mode (other elements are 0):

$$x_i = y_i \quad \forall i = 1, 2, \ldots, M$$
$$x_i = 0 \quad \forall i = M + 1, \ldots, N$$

Attributes

shape
[tuple] Operator shape

explicit
[bool] Operator contains a matrix that can be solved explicitly (True) or not (False)

Methods

__init__(N[, M, inplace, dtype, name])	Initialize this LinearOperator.
adjoint()	Hermitian adjoint.
apply_columns(cols)	Apply subset of columns of operator
cond([uselobpcg])	Condition number of linear operator.
conj()	Complex conjugate operator
div(y[, niter, densesolver])	Solve the linear problem $\mathbf{y} = \mathbf{A}\mathbf{x}$.
dot(x)	Matrix-matrix or matrix-vector multiplication.
eigs([neigs, symmetric, niter, uselobpcg])	Most significant eigenvalues of linear operator.
matmat(X)	Matrix-matrix multiplication.
matvec(x)	Matrix-vector multiplication.
reset_count()	Reset counters
rmatmat(X)	Matrix-matrix multiplication.
rmatvec(x)	Adjoint matrix-vector multiplication.
todense([backend])	Return dense matrix.
toimag([forw, adj])	Imag operator
toreal([forw, adj])	Real operator
tosparse()	Return sparse matrix.
trace([neval, method, backend])	Trace of linear operator.
transpose()	Transpose this linear operator.

Examples using pylops.Identity

- *Identity*
- *Operators concatenation*
- *Total Variation (TV) Regularization*
- *04. Bayesian Inversion*

pylops.Zero

class pylops.**Zero**(**args*, ***kwargs*)

Zero operator.

Transform model into array of zeros of size N in forward and transform data into array of zeros of size N in adjoint.

Parameters

N
[int] Number of samples in data (and model in M is not provided).

M
[int, optional] Number of samples in model.

dtype
[str, optional] Type of elements in input array.

name
[str, optional] New in version 2.0.0.

Name of operator (to be used by *pylops.utils.describe.describe*)

Notes

An *Zero* operator simply creates a null data vector $\mathbf{y}$ in forward mode:

$$\mathbf{0x} = \mathbf{0}_N$$

and a null model vector $\mathbf{x}$ in forward mode:

$$\mathbf{0y} = \mathbf{0}_M$$

Attributes

shape
[tuple] Operator shape

explicit
[bool] Operator contains a matrix that can be solved explicitly (`True`) or not (`False`)

Methods

`__init__`(N[, M, dtype, name])	Initialize this LinearOperator.
`adjoint`()	Hermitian adjoint.
`apply_columns`(cols)	Apply subset of columns of operator
`cond`([uselobpcg])	Condition number of linear operator.
`conj`()	Complex conjugate operator
`div`(y[, niter, densesolver])	Solve the linear problem $\mathbf{y} = \mathbf{Ax}$.
`dot`(x)	Matrix-matrix or matrix-vector multiplication.
`eigs`([neigs, symmetric, niter, uselobpcg])	Most significant eigenvalues of linear operator.
`matmat`(X)	Matrix-matrix multiplication.
`matvec`(x)	Matrix-vector multiplication.
`reset_count`()	Reset counters
`rmatmat`(X)	Matrix-matrix multiplication.
`rmatvec`(x)	Adjoint matrix-vector multiplication.
`todense`([backend])	Return dense matrix.
`toimag`([forw, adj])	Imag operator
`toreal`([forw, adj])	Real operator
`tosparse`()	Return sparse matrix.
`trace`([neval, method, backend])	Trace of linear operator.
`transpose`()	Transpose this linear operator.

Examples using `pylops.Zero`

- *Zero*

pylops.Diagonal

class pylops.**Diagonal**(**args*, ***kwargs*)

Diagonal operator.

Applies element-wise multiplication of the input vector with the vector `diag` in forward and with its complex conjugate in adjoint mode.

This operator can also broadcast; in this case the input vector is reshaped into its dimensions `dims` and the element-wise multiplication with `diag` is perfomed along `axis`. Note that the vector `diag` will need to have size equal to `dims[axis]`.

Parameters

diag
[numpy.ndarray] Vector to be used for element-wise multiplication.

dims
[list, optional] Number of samples for each dimension (`None` if only one dimension is available)

axis
[int, optional] New in version 2.0.0.

Axis along which multiplication is applied.

dtype
[str, optional] Type of elements in input array.

name
[str, optional] New in version 2.0.0.

Name of operator (to be used by *pylops.utils.describe.describe*)

Notes

Element-wise multiplication between the model $\mathbf{x}$ and/or data $\mathbf{y}$ vectors and the array $\mathbf{d}$ can be expressed as

$$y_i = d_i x_i \quad \forall i = 1, 2, \ldots, N$$

This is equivalent to a matrix-vector multiplication with a matrix containing the vector $\mathbf{d}$ along its main diagonal.

For real-valued `diag`, the Diagonal operator is self-adjoint as the adjoint of a diagonal matrix is the diagonal matrix itself. For complex-valued `diag`, the adjoint is equivalent to the element-wise multiplication with the complex conjugate elements of `diag`.

Attributes

shape
[tuple] Operator shape

explicit
[bool] Operator contains a matrix that can be solved explicitly (`True`) or not (`False`)

Methods

`__init__`(diag[, dims, axis, dtype, name])	Initialize this LinearOperator.
`adjoint`()	Hermitian adjoint.
`apply_columns`(cols)	Apply subset of columns of operator
`cond`([uselobpcg])	Condition number of linear operator.
`conj`()	Complex conjugate operator
`div`(y[, niter, densesolver])	Solve the linear problem $\mathbf{y} = \mathbf{A}\mathbf{x}$.
`dot`(x)	Matrix-matrix or matrix-vector multiplication.
`eigs`([neigs, symmetric, niter, uselobpcg])	Most significant eigenvalues of linear operator.
`matmat`(X)	Matrix-matrix multiplication.
`matrix`()	Return diagonal matrix as dense `numpy.ndarray`
`matvec`(x)	Matrix-vector multiplication.
`reset_count`()	Reset counters
`rmatmat`(X)	Matrix-matrix multiplication.
`rmatvec`(x)	Adjoint matrix-vector multiplication.
`todense`()	Fast implementation of todense based on known structure of the operator
`toimag`([forw, adj])	Imag operator
`toreal`([forw, adj])	Real operator
`tosparse`()	Return sparse matrix.
`trace`([neval, method, backend])	Trace of linear operator.
`transpose`()	Transpose this linear operator.

Examples using `pylops.Diagonal`

- *Describe*
- *Diagonal*
- *01. The LinearOpeator*
- *02. The Dot-Test*

pylops.Transpose

class pylops.**Transpose**(**args*, ***kwargs*)

Transpose operator.

Transpose axes of a multi-dimensional array. This operator works with flattened input model (or data), which are however multi-dimensional in nature and will be reshaped and treated as such in both forward and adjoint modes.

Parameters

dims
[tuple, optional] Number of samples for each dimension

axes
[tuple, optional] Direction along which transposition is applied

dtype
[str, optional] Type of elements in input array

name
[str, optional] New in version 2.0.0.

Name of operator (to be used by *pylops.utils.describe.describe*)

Raises

ValueError
If `axes` contains repeated dimensions (or a dimension is missing)

Notes

The Transpose operator reshapes the input model into a multi-dimensional array of size `dims` and transposes (or swaps) its axes as defined in `axes`.

Similarly, in adjoint mode the data is reshaped into a multi-dimensional array whose size is a permuted version of `dims` defined by `axes`. The array is then rearragned into the original model dimensions `dims`.

Attributes

shape
[tuple] Operator shape

explicit
[bool] Operator contains a matrix that can be solved explicitly (`True`) or not (`False`)

Methods

`__init__`(dims, axes[, dtype, name])	Initialize this LinearOperator.
`adjoint`()	Hermitian adjoint.
`apply_columns`(cols)	Apply subset of columns of operator
`cond`([uselobpcg])	Condition number of linear operator.
`conj`()	Complex conjugate operator
`div`(y[, niter, densesolver])	Solve the linear problem $\mathbf{y} = \mathbf{A}\mathbf{x}$.
`dot`(x)	Matrix-matrix or matrix-vector multiplication.
`eigs`([neigs, symmetric, niter, uselobpcg])	Most significant eigenvalues of linear operator.
`matmat`(X)	Matrix-matrix multiplication.
`matvec`(x)	Matrix-vector multiplication.
`reset_count`()	Reset counters
`rmatmat`(X)	Matrix-matrix multiplication.
`rmatvec`(x)	Adjoint matrix-vector multiplication.
`todense`([backend])	Return dense matrix.
`toimag`([forw, adj])	Imag operator
`toreal`([forw, adj])	Real operator
`tosparse`()	Return sparse matrix.
`trace`([neval, method, backend])	Trace of linear operator.
`transpose`()	Transpose this linear operator.

Examples using `pylops.Transpose`

- *Transpose*

pylops.Flip

class pylops.**Flip**(**args*, ***kwargs*)

Flip along an axis.

Flip a multi-dimensional array along `axis`.

Parameters

dims
[`list` or `int`] Number of samples for each dimension

axis
[`int`, optional] New in version 2.0.0.

Axis along which model is flipped.

dtype
[`str`, optional] Type of elements in input array.

name
[`str`, optional] New in version 2.0.0.

Name of operator (to be used by *`pylops.utils.describe.describe`*)

Notes

The Flip operator flips the input model (and data) along any chosen direction. For simplicity, given a one dimensional array, in forward mode this is equivalent to:

$$y[i] = x[N - 1 - i] \quad \forall i = 0, 1, 2, \ldots, N - 1$$

where N is the dimension of the input model along `axis`. As this operator is self-adjoint, x and y in the equation above are simply swapped in adjoint mode.

Attributes

shape
[`tuple`] Operator shape

explicit
[`bool`] Operator contains a matrix that can be solved explicitly (`True`) or not (`False`)

Methods

`__init__`(dims[, axis, dtype, name])	Initialize this LinearOperator.
`adjoint`()	Hermitian adjoint.
`apply_columns`(cols)	Apply subset of columns of operator
`cond`([uselobpcg])	Condition number of linear operator.
`conj`()	Complex conjugate operator
`div`(y[, niter, densesolver])	Solve the linear problem $\mathbf{y} = \mathbf{A}\mathbf{x}$.
`dot`(x)	Matrix-matrix or matrix-vector multiplication.
`eigs`([neigs, symmetric, niter, uselobpcg])	Most significant eigenvalues of linear operator.
`matmat`(X)	Matrix-matrix multiplication.
`matvec`(x)	Matrix-vector multiplication.
`reset_count`()	Reset counters
`rmatmat`(X)	Matrix-matrix multiplication.
`rmatvec`(x)	Adjoint matrix-vector multiplication.
`todense`([backend])	Return dense matrix.
`toimag`([forw, adj])	Imag operator
`toreal`([forw, adj])	Real operator
`tosparse`()	Return sparse matrix.
`trace`([neval, method, backend])	Trace of linear operator.
`transpose`()	Transpose this linear operator.

Examples using `pylops.Flip`

- *Flip along an axis*

pylops.Roll

class pylops.**Roll**(**args*, ***kwargs*)

Roll along an axis.

Roll a multi-dimensional array along `axis` for a chosen number of samples (`shift`).

Parameters

dims
[`list` or `int`] Number of samples for each dimension

axis
[`int`, optional] New in version 2.0.0.

Axis along which model is rolled.

shift
[`int`, optional] Number of samples by which elements are shifted

dtype
[`str`, optional] Type of elements in input array.

name
[`str`, optional] New in version 2.0.0.

Name of operator (to be used by *pylops.utils.describe.describe*)

Notes

The Roll operator is a thin wrapper around `numpy.roll` and shifts elements in a multi-dimensional array along a specified direction for a chosen number of samples.

Attributes

shape
[`tuple`] Operator shape

explicit
[`bool`] Operator contains a matrix that can be solved explicitly (`True`) or not (`False`)

Methods

`__init__`(dims[, axis, shift, dtype, name])	Initialize this LinearOperator.
`adjoint`()	Hermitian adjoint.
`apply_columns`(cols)	Apply subset of columns of operator
`cond`([uselobpcg])	Condition number of linear operator.
`conj`()	Complex conjugate operator
`div`(y[, niter, densesolver])	Solve the linear problem $\mathbf{y} = \mathbf{A}\mathbf{x}$.
`dot`(x)	Matrix-matrix or matrix-vector multiplication.
`eigs`([neigs, symmetric, niter, uselobpcg])	Most significant eigenvalues of linear operator.
`matmat`(X)	Matrix-matrix multiplication.
`matvec`(x)	Matrix-vector multiplication.
`reset_count`()	Reset counters
`rmatmat`(X)	Matrix-matrix multiplication.
`rmatvec`(x)	Adjoint matrix-vector multiplication.
`todense`([backend])	Return dense matrix.
`toimag`([forw, adj])	Imag operator
`toreal`([forw, adj])	Real operator
`tosparse`()	Return sparse matrix.
`trace`([neval, method, backend])	Trace of linear operator.
`transpose`()	Transpose this linear operator.

Examples using `pylops.Roll`

- *Roll*

pylops.Pad

class pylops.**Pad**(**args*, ***kwargs*)

Pad operator.

Zero-pad model in forward model and extract non-zero subsequence in adjoint. Padding can be performed in one or multiple directions to any multi-dimensional input arrays.

Parameters

dims
[`int` or `tuple`] Number of samples for each dimension

pad
[tuple] Number of samples to pad. If `dims` is a scalar, `pad` is a single tuple `(pad_in, pad_end)`. If `dims` is a tuple, `pad` is a tuple of tuples where each inner tuple contains the number of samples to pad in each dimension

dtype
[str, optional] Type of elements in input array.

name
[str, optional] New in version 2.0.0.

Name of operator (to be used by *pylops.utils.describe.describe*)

Raises

ValueError
If any element of `pad` is negative.

Notes

Given an array of size N, the *Pad* operator simply adds pad_{in} at the start and pad_{end} at the end in forward mode:

$$y_i = x_{i-\text{pad}_{\text{in}}} \quad \forall i = \text{pad}_{\text{in}}, \dots, \text{pad}_{\text{in}} + N - 1$$

and $y_i = 0 \quad \forall i = 0, \dots, \text{pad}_{\text{in}} - 1, \text{pad}_{\text{in}} + N - 1, \dots, N + \text{pad}_{\text{in}} + \text{pad}_{\text{end}}$

In adjoint mode, values from pad_{in} to $N - \text{pad}_{\text{end}}$ are extracted from the data:

$$x_i = y_{\text{pad}_{\text{in}}+i} \quad \forall i = 0, N - 1$$

Attributes

shape
[tuple] Operator shape

explicit
[bool] Operator contains a matrix that can be solved explicitly (`True`) or not (`False`)

Methods

`__init__`(dims, pad[, dtype, name])	Initialize this LinearOperator.
`adjoint`()	Hermitian adjoint.
`apply_columns`(cols)	Apply subset of columns of operator
`cond`([uselobpcg])	Condition number of linear operator.
`conj`()	Complex conjugate operator
`div`(y[, niter, densesolver])	Solve the linear problem $\mathbf{y} = \mathbf{A}\mathbf{x}$.
`dot`(x)	Matrix-matrix or matrix-vector multiplication.
`eigs`([neigs, symmetric, niter, uselobpcg])	Most significant eigenvalues of linear operator.
`matmat`(X)	Matrix-matrix multiplication.
`matvec`(x)	Matrix-vector multiplication.
`reset_count`()	Reset counters
`rmatmat`(X)	Matrix-matrix multiplication.
`rmatvec`(x)	Adjoint matrix-vector multiplication.
`todense`([backend])	Return dense matrix.
`toimag`([forw, adj])	Imag operator
`toreal`([forw, adj])	Real operator
`tosparse`()	Return sparse matrix.
`trace`([neval, method, backend])	Trace of linear operator.
`transpose`()	Transpose this linear operator.

Examples using `pylops.Pad`

- *Padding*
- *18. Deblending*

pylops.Sum

class pylops.**Sum**(**args*, ***kwargs*)

Sum operator.

Sum along `axis` of a multi-dimensional array (at least 2 dimensions are required) in forward model, and spread along the same axis in adjoint mode.

Parameters

dims
[`tuple`] Number of samples for each dimension

axis
[`int`, optional] New in version 2.0.0.

Axis along which model is summed.

dtype
[`str`, optional] Type of elements in input array.

name
[`str`, optional] New in version 2.0.0.

Name of operator (to be used by *`pylops.utils.describe.describe`*)

Notes

Given a two dimensional array, the *Sum* operator re-arranges the input model into a multi-dimensional array of size `dims` and sums values along `axis`:

$$y_j = \sum_i x_{i,j}$$

In adjoint mode, the data is spread along the same direction:

$$x_{i,j} = y_j \quad \forall i = 0, N-1$$

Attributes

shape
[`tuple`] Operator shape

explicit
[`bool`] Operator contains a matrix that can be solved explicitly (`True`) or not (`False`)

Methods

`__init__`(dims[, axis, dtype, name])	Initialize this LinearOperator.
`adjoint`()	Hermitian adjoint.
`apply_columns`(cols)	Apply subset of columns of operator
`cond`([uselobpcg])	Condition number of linear operator.
`conj`()	Complex conjugate operator
`div`(y[, niter, densesolver])	Solve the linear problem $\mathbf{y} = \mathbf{A}\mathbf{x}$.
`dot`(x)	Matrix-matrix or matrix-vector multiplication.
`eigs`([neigs, symmetric, niter, uselobpcg])	Most significant eigenvalues of linear operator.
`matmat`(X)	Matrix-matrix multiplication.
`matvec`(x)	Matrix-vector multiplication.
`reset_count`()	Reset counters
`rmatmat`(X)	Matrix-matrix multiplication.
`rmatvec`(x)	Adjoint matrix-vector multiplication.
`todense`([backend])	Return dense matrix.
`toimag`([forw, adj])	Imag operator
`toreal`([forw, adj])	Real operator
`tosparse`()	Return sparse matrix.
`trace`([neval, method, backend])	Trace of linear operator.
`transpose`()	Transpose this linear operator.

Examples using `pylops.Sum`

- *Sum*

pylops.Symmetrize

class pylops.**Symmetrize**(**args*, ***kwargs*)

Symmetrize along an axis.

Symmetrize a multi-dimensional array along `axis`.

Parameters

dims
[`list` or `int`] Number of samples for each dimension (`None` if only one dimension is available)

axis
[`int`, optional] New in version 2.0.0.

Axis along which model is symmetrized.

dtype
[`str`, optional] Type of elements in input array

name
[`str`, optional] New in version 2.0.0.

Name of operator (to be used by *pylops.utils.describe.describe*)

Notes

The Symmetrize operator constructs a symmetric array given an input model in forward mode, by pre-pending the input model in reversed order.

For simplicity, given a one dimensional array, the forward operation can be expressed as:

$$y[i] = \begin{cases} x[i - N + 1], & i \geq N \\ x[N - 1 - i], & \text{otherwise} \end{cases}$$

for $i = 0, 1, 2, \ldots, 2N - 2$, where N is the dimension of the input model.

In adjoint mode, the Symmetrize operator assigns the sums of the elements in position $N - 1 - i$ and $N - 1 + i$ to position i as follows:

apart from the central sample where $x[0] = y[N - 1]$.

Attributes

shape
[`tuple`] Operator shape

explicit
[`bool`] Operator contains a matrix that can be solved explicitly (`True`) or not (`False`)

Methods

`__init__`(dims[, axis, dtype, name])	Initialize this LinearOperator.
`adjoint`()	Hermitian adjoint.
`apply_columns`(cols)	Apply subset of columns of operator
`cond`([uselobpcg])	Condition number of linear operator.
`conj`()	Complex conjugate operator
`div`(y[, niter, densesolver])	Solve the linear problem $\mathbf{y} = \mathbf{A}\mathbf{x}$.
`dot`(x)	Matrix-matrix or matrix-vector multiplication.
`eigs`([neigs, symmetric, niter, uselobpcg])	Most significant eigenvalues of linear operator.
`matmat`(X)	Matrix-matrix multiplication.
`matvec`(x)	Matrix-vector multiplication.
`reset_count`()	Reset counters
`rmatmat`(X)	Matrix-matrix multiplication.
`rmatvec`(x)	Adjoint matrix-vector multiplication.
`todense`([backend])	Return dense matrix.
`toimag`([forw, adj])	Imag operator
`toreal`([forw, adj])	Real operator
`tosparse`()	Return sparse matrix.
`trace`([neval, method, backend])	Trace of linear operator.
`transpose`()	Transpose this linear operator.

Examples using `pylops.Symmetrize`

- *Symmetrize*
- *Wavelet estimation*

pylops.Restriction

class pylops.**Restriction**(**args*, ***kwargs*)

Restriction (or sampling) operator.

Extract subset of values from input vector at locations `iava` in forward mode and place those values at locations `iava` in an otherwise zero vector in adjoint mode.

Parameters

dims
[`list` or `int`] Number of samples for each dimension

iava
[`list` or `numpy.ndarray`] Integer indices of available samples for data selection.

axis
[`int`, optional] New in version 2.0.0.

Axis along which restriction is applied to model.

inplace
[`bool`, optional] Work inplace (`True`) or make a new copy (`False`). By default, data is a reference to the model (in forward) and model is a reference to the data (in adjoint).

dtype
[`str`, optional] Type of elements in input array.

name
[str, optional] New in version 2.0.0.

Name of operator (to be used by *pylops.utils.describe.describe*)

See also:

pylops.signalprocessing.Interp
Interpolation operator

Notes

Extraction (or *sampling*) of a subset of N values at locations `iava` from an input (or model) vector $\mathbf{x}$ of size M can be expressed as:

$$y_i = x_{l_i} \quad \forall i = 0, 1, \ldots, N-1$$

where $\mathbf{l} = [l_0, l_1, \ldots, l_{N-1}]$ is a vector containing the indices of the original array at which samples are taken.

Conversely, in adjoint mode the available values in the data vector $\mathbf{y}$ are placed at locations $\mathbf{l} = [l_0, l_1, \ldots, l_{M-1}]$ in the model vector:

$$x_{l_i} = y_i \quad \forall i = 0, 1, \ldots, N-1$$

and $x_j = 0$ for $j \neq l_i$ (i.e., at all other locations in input vector).

Attributes

shape
[tuple] Operator shape

explicit
[bool] Operator contains a matrix that can be solved explicitly (`True`) or not (`False`)

Methods

`__init__`(dims, iava[, axis, inplace, dtype, ...])	Initialize this LinearOperator.
`adjoint`()	Hermitian adjoint.
`apply_columns`(cols)	Apply subset of columns of operator
`cond`([uselobpcg])	Condition number of linear operator.
`conj`()	Complex conjugate operator
`div`(y[, niter, densesolver])	Solve the linear problem $\mathbf{y} = \mathbf{A}\mathbf{x}$.
`dot`(x)	Matrix-matrix or matrix-vector multiplication.
`eigs`([neigs, symmetric, niter, uselobpcg])	Most significant eigenvalues of linear operator.
`mask`(x)	Apply mask to input signal returning a signal of same size with values at `iava` locations and `0` at other locations
`matmat`(X)	Matrix-matrix multiplication.
`matvec`(x)	Matrix-vector multiplication.
`reset_count`()	Reset counters
`rmatmat`(X)	Matrix-matrix multiplication.
`rmatvec`(x)	Adjoint matrix-vector multiplication.
`todense`([backend])	Return dense matrix.
`toimag`([forw, adj])	Imag operator
`toreal`([forw, adj])	Real operator
`tosparse`()	Return sparse matrix.
`trace`([neval, method, backend])	Trace of linear operator.
`transpose`()	Transpose this linear operator.

Examples using `pylops.Restriction`

- *Restriction and Interpolation*
- *Total Variation (TV) Regularization*
- *03. Solvers*
- *03. Solvers (Advanced)*
- *04. Bayesian Inversion*
- *06. 2D Interpolation*
- *12. Seismic regularization*

pylops.Regression

class pylops.**Regression**(**args*, ***kwargs*)

Polynomial regression.

Creates an operator that applies polynomial regression to a set of points. Values along the t-axis must be provided while initializing the operator. The coefficients of the polynomial regression form the model vector to be provided in forward mode, while the values of the regression curve shall be provided in adjoint mode.

Parameters

taxis
[numpy.ndarray] Elements along the t-axis.

order
[int] Order of the regressed polynomial.

dtype
[str, optional] Type of elements in input array.

name
[str, optional] New in version 2.0.0.

Name of operator (to be used by *pylops.utils.describe.describe*)

Raises

TypeError
If `taxis` is not numpy.ndarray.

See also:

LinearRegression
Linear regression

Notes

The Regression operator solves the following problem:

$$y_i = \sum_{n=0}^{\text{order}} x_n t_i^n \qquad \forall i = 0, 1, \ldots, N-1$$

where N represents the number of points in `taxis`. We can express this problem in a matrix form

$$\mathbf{y} = \mathbf{A}\mathbf{x}$$

where

$$\mathbf{y} = [y_0, y_1, \ldots, y_{N-1}]^T, \qquad \mathbf{x} = [x_0, x_1, \ldots, x_{\text{order}}]^T$$

and

$$\mathbf{A} = \begin{bmatrix} 1 & t_0 & t_0^2 & \ldots & t_0^{\text{order}} \\ 1 & t_1 & t_1^2 & \ldots & t_1^{\text{order}} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & t_{N-1} & t_{N-1}^2 & \ldots & t_{N-1}^{\text{order}} \end{bmatrix}_{N \times \text{order}+1}$$

Attributes

shape
[tuple] Operator shape

explicit
[bool] Operator contains a matrix that can be solved explicitly (`True`) or not (`False`)

Methods

`__init__`(taxis, order[, dtype, name])	Initialize this LinearOperator.
`adjoint`()	Hermitian adjoint.
`apply`(t, x)	Return values along y-axis given certain `t` location(s) along t-axis and regression coefficients `x`
`apply_columns`(cols)	Apply subset of columns of operator
`cond`([uselobpcg])	Condition number of linear operator.
`conj`()	Complex conjugate operator
`div`(y[, niter, densesolver])	Solve the linear problem $\mathbf{y} = \mathbf{A}\mathbf{x}$.
`dot`(x)	Matrix-matrix or matrix-vector multiplication.
`eigs`([neigs, symmetric, niter, uselobpcg])	Most significant eigenvalues of linear operator.
`matmat`(X)	Matrix-matrix multiplication.
`matvec`(x)	Matrix-vector multiplication.
`reset_count`()	Reset counters
`rmatmat`(X)	Matrix-matrix multiplication.
`rmatvec`(x)	Adjoint matrix-vector multiplication.
`todense`([backend])	Return dense matrix.
`toimag`([forw, adj])	Imag operator
`toreal`([forw, adj])	Real operator
`tosparse`()	Return sparse matrix.
`trace`([neval, method, backend])	Trace of linear operator.
`transpose`()	Transpose this linear operator.

Examples using `pylops.Regression`

- *Linear Regression*
- *Polynomial Regression*

pylops.LinearRegression

`pylops.LinearRegression`(*taxis*, *dtype='float64'*)

Linear regression.

Creates an operator that applies linear regression to a set of points. Values along the t-axis must be provided while initializing the operator. Intercept and gradient form the model vector to be provided in forward mode, while the values of the regression line curve shall be provided in adjoint mode.

Parameters

taxis
[`numpy.ndarray`] Elements along the t-axis.

dtype
[`str`, optional] Type of elements in input array.

Raises

TypeError
If `taxis` is not `numpy.ndarray`.

See also:

Regression
Polynomial regression

Notes

The LinearRegression operator solves the following problem:

$$y_i = x_0 + x_1 t_i \qquad \forall i = 0, 1, \ldots, N-1$$

We can express this problem in a matrix form

$$\mathbf{y} = \mathbf{A}\mathbf{x}$$

where

$$\mathbf{y} = [y_0, y_1, \ldots, y_{N-1}]^T, \qquad \mathbf{x} = [x_0, x_1]^T$$

and

$$\mathbf{A} = \begin{bmatrix} 1 & t_0 \\ 1 & t_1 \\ \vdots & \vdots \\ 1 & t_{N-1} \end{bmatrix}$$

Note that this is a particular case of the *pylops.Regression* operator and it is in fact just a lazy call of that operator with `order=1`.

Attributes

shape
[`tuple`] Operator shape

explicit
[`bool`] Operator contains a matrix that can be solved explicitly (`True`) or not (`False`)

Examples using `pylops.LinearRegression`

- *Linear Regression*

pylops.CausalIntegration

class pylops.**CausalIntegration**(**args*, ***kwargs*)

Causal integration.

Apply causal integration to a multi-dimensional array along `axis`.

Parameters

dims
[`list` or `int`] Number of samples for each dimension

axis
[`int`, optional] New in version 2.0.0.

Axis along which the model is integrated.

sampling
[float, optional] Sampling step `dx`.

kind
[str, optional] Integration kind (`full`, `half`, or `trapezoidal`).

removefirst
[bool, optional] Remove first sample (`True`) or not (`False`).

dtype
[str, optional] Type of elements in input array.

name
[str, optional] New in version 2.0.0.

Name of operator (to be used by *pylops.utils.describe.describe*)

Notes

The CausalIntegration operator applies a causal integration to any chosen direction of a multi-dimensional array.

For simplicity, given a one dimensional array, the causal integration is:

$$y(t) = \int_{-\infty}^{t} x(\tau)\,\mathrm{d}\tau$$

which can be discretised as :

$$y[i] = \sum_{j=0}^{i} x[j]\,\Delta t$$

or

$$y[i] = \left(\sum_{j=0}^{i-1} x[j] + 0.5x[i]\right)\,\Delta t$$

or

$$y[i] = \left(\sum_{j=1}^{i-1} x[j] + 0.5x[0] + 0.5x[i]\right)\,\Delta t$$

where Δt is the `sampling` interval, and assuming the signal is zero before sample $j = 0$. In our implementation, the choice to add $x[i]$ or $0.5x[i]$ is made by selecting `kind=full` or `kind=half`, respectively. The choice to add $0.5x[i]$ and $0.5x[0]$ instead of made by selecting the `kind=trapezoidal`.

Note that the causal integral of a signal will depend, up to a constant, on causal start of the signal. For example if $x(\tau) = t^2$ the resulting indefinite integration is:

$$y(t) = \int \tau^2\,\mathrm{d}\tau = \frac{t^3}{3} + C$$

However, if we apply a first derivative to y always obtain:

$$x(t) = \frac{\mathrm{d}y}{\mathrm{d}t} = t^2$$

no matter the choice of C.

Attributes

shape
[tuple] Operator shape

explicit
[bool] Operator contains a matrix that can be solved explicitly (True) or not (False)

Methods

__init__(dims[, axis, sampling, kind, ...])	Initialize this LinearOperator.
adjoint()	Hermitian adjoint.
apply_columns(cols)	Apply subset of columns of operator
cond([uselobpcg])	Condition number of linear operator.
conj()	Complex conjugate operator
div(y[, niter, densesolver])	Solve the linear problem $\mathbf{y} = \mathbf{A}\mathbf{x}$.
dot(x)	Matrix-matrix or matrix-vector multiplication.
eigs([neigs, symmetric, niter, uselobpcg])	Most significant eigenvalues of linear operator.
matmat(X)	Matrix-matrix multiplication.
matvec(x)	Matrix-vector multiplication.
reset_count()	Reset counters
rmatmat(X)	Matrix-matrix multiplication.
rmatvec(x)	Adjoint matrix-vector multiplication.
todense([backend])	Return dense matrix.
toimag([forw, adj])	Imag operator
toreal([forw, adj])	Real operator
tosparse()	Return sparse matrix.
trace([neval, method, backend])	Trace of linear operator.
transpose()	Transpose this linear operator.

Examples using pylops.CausalIntegration

- *Causal Integration*

pylops.Spread

class pylops.**Spread**(**args*, ***kwargs*)

Spread operator.

Spread values from the input model vector arranged as a 2-dimensional array of size $[n_{x_0} \times n_{t_0}]$ into the data vector of size $[n_x \times n_t]$. Note that the value at each single pair (x_0, t_0) in the input is spread over the entire x axis in the output.

Spreading is performed along parametric curves provided as look-up table of pre-computed indices (table) or computed on-the-fly using a function handle (fh).

In adjont mode, values from the data vector are instead stacked along the same parametric curves.

Parameters

dims
[tuple] Dimensions of model vector (vector will be reshaped internally into a two-dimensional array of size $[n_{x_0} \times n_{t_0}]$, where the first dimension is the spreading direction)

dimsd
[tuple] Dimensions of data vector (vector will be reshaped internal into a two-dimensional array of size $[n_x \times n_t]$, where the first dimension is the stacking direction)

table
[np.ndarray, optional] Look-up table of indices of size $[n_{x_0} \times n_{t_0} \times n_x]$ (if None use function handle fh). When dtable is not provided, the data will be created as follows

```
data[ix, table[ix0, it0, ix]] += model[ix0, it0]
```

Note: When using table without dtable, its elements must be between 0 and $n_{t_0} - 1$ (or numpy.nan).

---

dtable
[np.ndarray, optional] Look-up table of decimals remainders for linear interpolation of size $[n_{x_0} \times n_{t_0} \times n_x]$ (if None use function handle fh). When provided, the data will be created as follows

```
data[ix, table[ix0, it0, ix]]     += (1 - dtable[ix0, it0, ix]) *␣
↪model[ix0, it0]
data[ix, table[ix0, it0, ix] + 1] +=      dtable[ix0, it0, ix]  *␣
↪model[ix0, it0]
```

Note: When using table and dtable, the elements of table indices must be between 0 and $n_{t_0} - 2$ (or numpy.nan).

---

fh
[callable, optional] If None will use look-up table table. When provided, should be a function which takes indices ix0 and it0 and returns an array of size n_x containing each respective time index. Alternatively, if linear interpolation is required, it should output in addition to the time indices, a weight for interpolation with linear interpolation, to be used as follows

```
data[ix, index]     += (1 - dindices[ix]) * model[ix0, it0]
data[ix, index + 1] +=      dindices[ix]  * model[ix0, it0]
```

where index refers to a time index in the first array returned by fh and dindices refers to the weight in the second array returned by fh.

Note: When using fh with one output (time indices), the time indices must be between 0 and $n_{t_0} - 1$ (or numpy.nan). When using fh with two outputs (time indices and weights), they must be within the between 0 and $n_{t_0} - 2$ (or numpy.nan).

---

interp
[bool, optional] Use only if engine engine='numba'. Apply linear interpolation (True) or nearest interpolation (False) during stacking/spreading along parametric curve. When using engine="numpy", it will be inferred directly from fh or the presence of dtable.

engine
[str, optional] Engine used for spread computation (numpy or numba). Note that numba can only be used when providing a look-up table

dtype
[str, optional] Type of elements in input array.

name
[str, optional] New in version 2.0.0.

Name of operator (to be used by *pylops.utils.describe.describe*)

Raises

KeyError
If `engine` is neither `numpy` nor `numba`

NotImplementedError
If both `table` and `fh` are not provided

ValueError
If `table` has shape different from $[n_{x_0} \times n_{t_0} \times n_x]$

Notes

The Spread operator applies the following linear transform in forward mode to the model vector after reshaping it into a 2-dimensional array of size $[n_x \times n_t]$:

$$m(x_0, t_0) \rightarrow d(x, t = f(x_0, x, t_0)) \quad \forall x$$

where $f(x_0, x, t)$ is a mapping function that returns a value t given values x_0, x, and t_0. Note that for each (x_0, t_0) pair, spreading is done over the entire x axis in the data domain.

In adjoint mode, the model is reconstructed by means of the following stacking operation:

$$m(x_0, t_0) = \int d(x, t = f(x_0, x, t_0)) \, \mathrm{d}x$$

Note that `table` (or `fh`) must return integer numbers representing indices in the axis t. However it also possible to perform linear interpolation as part of the spreading/stacking process by providing the decimal part of the mapping function ($t - \lfloor t \rfloor$) either in `dtable` input parameter or as second value in the return of `fh` function.

Attributes

shape
[tuple] Operator shape

explicit
[bool] Operator contains a matrix that can be solved explicitly (`True`) or not (`False`)

Methods

`__init__`(dims, dimsd[, table, dtable, fh, ...])	Initialize this LinearOperator.
`adjoint`()	Hermitian adjoint.
`apply_columns`(cols)	Apply subset of columns of operator
`cond`([uselobpcg])	Condition number of linear operator.
`conj`()	Complex conjugate operator
`div`(y[, niter, densesolver])	Solve the linear problem $\mathbf{y} = \mathbf{A}\mathbf{x}$.
`dot`(x)	Matrix-matrix or matrix-vector multiplication.
`eigs`([neigs, symmetric, niter, uselobpcg])	Most significant eigenvalues of linear operator.
`matmat`(X)	Matrix-matrix multiplication.
`matvec`(x)	Matrix-vector multiplication.
`reset_count`()	Reset counters
`rmatmat`(X)	Matrix-matrix multiplication.
`rmatvec`(x)	Adjoint matrix-vector multiplication.
`todense`([backend])	Return dense matrix.
`toimag`([forw, adj])	Imag operator
`toreal`([forw, adj])	Real operator
`tosparse`()	Return sparse matrix.
`trace`([neval, method, backend])	Trace of linear operator.
`transpose`()	Transpose this linear operator.

Examples using `pylops.Spread`

- *1D, 2D and 3D Sliding*
- *Normal Moveout (NMO) Correction*
- *Radon Transform*
- *Spread How-to*
- *11. Radon filtering*
- *12. Seismic regularization*
- *16. CT Scan Imaging*

pylops.VStack

class pylops.**VStack**(**args*, ***kwargs*)

Vertical stacking.

Stack a set of N linear operators vertically.

Parameters

ops
[`list`] Linear operators to be stacked. Alternatively, `numpy.ndarray` or `scipy.sparse` matrices can be passed in place of one or more operators.

nproc
[`int`, optional] Number of processes used to evaluate the N operators in parallel using `multiprocessing`. If `nproc=1`, work in serial mode.

dtype
[str, optional] Type of elements in input array.

Raises

ValueError
If ops have different number of rows

Notes

A vertical stack of N linear operators is created such as its application in forward mode leads to

$$\begin{bmatrix} \mathbf{L}_1 \\ \mathbf{L}_2 \\ \vdots \\ \mathbf{L}_N \end{bmatrix} \mathbf{x} = \begin{bmatrix} \mathbf{L}_1\mathbf{x} \\ \mathbf{L}_2\mathbf{x} \\ \vdots \\ \mathbf{L}_N\mathbf{x} \end{bmatrix} = \begin{bmatrix} \mathbf{y}_1 \\ \mathbf{y}_2 \\ \vdots \\ \mathbf{y}_N \end{bmatrix}$$

while its application in adjoint mode leads to

$$\begin{bmatrix} \mathbf{L}_1^H & \mathbf{L}_2^H & \dots & \mathbf{L}_N^H \end{bmatrix} \begin{bmatrix} \mathbf{y}_1 \\ \mathbf{y}_2 \\ \vdots \\ \mathbf{y}_N \end{bmatrix} = \mathbf{L}_1^H\mathbf{y}_1 + \mathbf{L}_2^H\mathbf{y}_2 + \ldots + \mathbf{L}_N^H\mathbf{y}_N$$

Attributes

shape
[tuple] Operator shape

explicit
[bool] Operator contains a matrix that can be solved explicitly (True) or not (False)

Methods

__init__(ops[, nproc, dtype])	Initialize this LinearOperator.
adjoint()	Hermitian adjoint.
apply_columns(cols)	Apply subset of columns of operator
cond([uselobpcg])	Condition number of linear operator.
conj()	Complex conjugate operator
div(y[, niter, densesolver])	Solve the linear problem $\mathbf{y} = \mathbf{A}\mathbf{x}$.
dot(x)	Matrix-matrix or matrix-vector multiplication.
eigs([neigs, symmetric, niter, uselobpcg])	Most significant eigenvalues of linear operator.
matmat(X)	Matrix-matrix multiplication.
matvec(x)	Matrix-vector multiplication.
reset_count()	Reset counters
rmatmat(X)	Matrix-matrix multiplication.
rmatvec(x)	Adjoint matrix-vector multiplication.
todense([backend])	Return dense matrix.
toimag([forw, adj])	Imag operator
toreal([forw, adj])	Real operator
tosparse()	Return sparse matrix.
trace([neval, method, backend])	Trace of linear operator.
transpose()	Transpose this linear operator.

Examples using `pylops.VStack`

- *Derivatives*
- *Describe*
- *Operators concatenation*
- *Operators with Multiprocessing*
- *Wavelet estimation*
- *08. Pre-stack (AVO) inversion*
- *17. Real/Complex Inversion*

pylops.HStack

class pylops.**HStack**(**args*, ***kwargs*)

Horizontal stacking.

Stack a set of N linear operators horizontally.

Parameters

ops
[list] Linear operators to be stacked. Alternatively, numpy.ndarray or scipy.sparse matrices can be passed in place of one or more operators.

nproc
[int, optional] Number of processes used to evaluate the N operators in parallel using multiprocessing. If nproc=1, work in serial mode.

dtype
[str, optional] Type of elements in input array.

Raises

ValueError
If ops have different number of columns

Notes

An horizontal stack of N linear operators is created such as its application in forward mode leads to

$$\begin{bmatrix} \mathbf{L}_1 & \mathbf{L}_2 & \dots & \mathbf{L}_N \end{bmatrix} \begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \\ \vdots \\ \mathbf{x}_N \end{bmatrix} = \mathbf{L}_1\mathbf{x}_1 + \mathbf{L}_2\mathbf{x}_2 + \ldots + \mathbf{L}_N\mathbf{x}_N$$

while its application in adjoint mode leads to

$$\begin{bmatrix} \mathbf{L}_1^H \\ \mathbf{L}_2^H \\ \vdots \\ \mathbf{L}_N^H \end{bmatrix} \mathbf{y} = \begin{bmatrix} \mathbf{L}_1^H\mathbf{y} \\ \mathbf{L}_2^H\mathbf{y} \\ \vdots \\ \mathbf{L}_N^H\mathbf{y} \end{bmatrix} = \begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \\ \vdots \\ \mathbf{x}_N \end{bmatrix}$$

Attributes

shape
[tuple] Operator shape

explicit
[bool] Operator contains a matrix that can be solved explicitly (True) or not (False)

Methods

__init__(ops[, nproc, dtype])	Initialize this LinearOperator.
adjoint()	Hermitian adjoint.
apply_columns(cols)	Apply subset of columns of operator
cond([uselobpcg])	Condition number of linear operator.
conj()	Complex conjugate operator
div(y[, niter, densesolver])	Solve the linear problem $\mathbf{y} = \mathbf{A}\mathbf{x}$.
dot(x)	Matrix-matrix or matrix-vector multiplication.
eigs([neigs, symmetric, niter, uselobpcg])	Most significant eigenvalues of linear operator.
matmat(X)	Matrix-matrix multiplication.
matvec(x)	Matrix-vector multiplication.
reset_count()	Reset counters
rmatmat(X)	Matrix-matrix multiplication.
rmatvec(x)	Adjoint matrix-vector multiplication.
todense([backend])	Return dense matrix.
toimag([forw, adj])	Imag operator
toreal([forw, adj])	Real operator
tosparse()	Return sparse matrix.
trace([neval, method, backend])	Trace of linear operator.
transpose()	Transpose this linear operator.

Examples using pylops.HStack

- *Describe*
- *Operators concatenation*
- *Operators with Multiprocessing*
- *18. Deblending*

pylops.Block

pylops.**Block**(*ops*, *nproc=1*, *dtype=None*)
Block operator.

Create a block operator from N lists of M linear operators each.

Parameters

ops
[list] List of lists of operators to be combined in block fashion. Alternatively, numpy.ndarray or scipy.sparse matrices can be passed in place of one or more operators.

nproc
[int, optional] Number of processes used to evaluate the N operators in parallel using multiprocessing. If nproc=1, work in serial mode.

dtype
[str, optional] Type of elements in input array.

Notes

In mathematics, a block or a partitioned matrix is a matrix that is interpreted as being broken into sections called blocks or submatrices. Similarly a block operator is composed of N sets of M linear operators each such that its application in forward mode leads to

$$\begin{bmatrix} \mathbf{L}_{1,1} & \mathbf{L}_{1,2} & \dots & \mathbf{L}_{1,M} \\ \mathbf{L}_{2,1} & \mathbf{L}_{2,2} & \dots & \mathbf{L}_{2,M} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{L}_{N,1} & \mathbf{L}_{N,2} & \dots & \mathbf{L}_{N,M} \end{bmatrix} \begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \\ \vdots \\ \mathbf{x}_M \end{bmatrix} = \begin{bmatrix} \mathbf{L}_{1,1}\mathbf{x}_1 + \mathbf{L}_{1,2}\mathbf{x}_2 + \mathbf{L}_{1,M}\mathbf{x}_M \\ \mathbf{L}_{2,1}\mathbf{x}_1 + \mathbf{L}_{2,2}\mathbf{x}_2 + \mathbf{L}_{2,M}\mathbf{x}_M \\ \vdots \\ \mathbf{L}_{N,1}\mathbf{x}_1 + \mathbf{L}_{N,2}\mathbf{x}_2 + \mathbf{L}_{N,M}\mathbf{x}_M \end{bmatrix}$$

while its application in adjoint mode leads to

$$\begin{bmatrix} \mathbf{L}_{1,1}^H & \mathbf{L}_{2,1}^H & \dots & \mathbf{L}_{N,1}^H \\ \mathbf{L}_{1,2}^H & \mathbf{L}_{2,2}^H & \dots & \mathbf{L}_{N,2}^H \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{L}_{1,M}^H & \mathbf{L}_{2,M}^H & \dots & \mathbf{L}_{N,M}^H \end{bmatrix} \begin{bmatrix} \mathbf{y}_1 \\ \mathbf{y}_2 \\ \vdots \\ \mathbf{y}_N \end{bmatrix} = \begin{bmatrix} \mathbf{L}_{1,1}^H\mathbf{y}_1 + \mathbf{L}_{2,1}^H\mathbf{y}_2 + \mathbf{L}_{N,1}^H\mathbf{y}_N \\ \mathbf{L}_{1,2}^H\mathbf{y}_1 + \mathbf{L}_{2,2}^H\mathbf{y}_2 + \mathbf{L}_{N,2}^H\mathbf{y}_N \\ \vdots \\ \mathbf{L}_{1,M}^H\mathbf{y}_1 + \mathbf{L}_{2,M}^H\mathbf{y}_2 + \mathbf{L}_{N,M}^H\mathbf{y}_N \end{bmatrix}$$

Attributes

shape
[tuple] Operator shape

explicit
[bool] Operator contains a matrix that can be solved explicitly (`True`) or not (`False`)

Examples using `pylops.Block`

- *Describe*
- *Operators concatenation*
- *Operators with Multiprocessing*

pylops.BlockDiag

class pylops.**BlockDiag**(**args*, ***kwargs*)

Block-diagonal operator.

Create a block-diagonal operator from N linear operators.

Parameters

ops
[list] Linear operators to be stacked. Alternatively, numpy.ndarray or scipy.sparse matrices can be passed in place of one or more operators.

nproc
[int, optional] Number of processes used to evaluate the N operators in parallel using `multiprocessing`. If `nproc=1`, work in serial mode.

dtype
[str, optional] Type of elements in input array.

Notes

A block-diagonal operator composed of N linear operators is created such as its application in forward mode leads to

$$\begin{bmatrix} \mathbf{L}_1 & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & \mathbf{L}_2 & \dots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \dots & \mathbf{L}_N \end{bmatrix} \begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \\ \vdots \\ \mathbf{x}_N \end{bmatrix} = \begin{bmatrix} \mathbf{L}_1\mathbf{x}_1 \\ \mathbf{L}_2\mathbf{x}_2 \\ \vdots \\ \mathbf{L}_N\mathbf{x}_N \end{bmatrix}$$

while its application in adjoint mode leads to

$$\begin{bmatrix} \mathbf{L}_1^H & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & \mathbf{L}_2^H & \dots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \dots & \mathbf{L}_N^H \end{bmatrix} \begin{bmatrix} \mathbf{y}_1 \\ \mathbf{y}_2 \\ \vdots \\ \mathbf{y}_N \end{bmatrix} = \begin{bmatrix} \mathbf{L}_1^H\mathbf{y}_1 \\ \mathbf{L}_2^H\mathbf{y}_2 \\ \vdots \\ \mathbf{L}_N^H\mathbf{y}_N \end{bmatrix}$$

Attributes

shape
[tuple] Operator shape

explicit
[bool] Operator contains a matrix that can be solved explicitly (`True`) or not (`False`)

Methods

`__init__`(ops[, nproc, dtype])	Initialize this LinearOperator.
`adjoint`()	Hermitian adjoint.
`apply_columns`(cols)	Apply subset of columns of operator
`cond`([uselobpcg])	Condition number of linear operator.
`conj`()	Complex conjugate operator
`div`(y[, niter, densesolver])	Solve the linear problem $\mathbf{y} = \mathbf{A}\mathbf{x}$.
`dot`(x)	Matrix-matrix or matrix-vector multiplication.
`eigs`([neigs, symmetric, niter, uselobpcg])	Most significant eigenvalues of linear operator.
`matmat`(X)	Matrix-matrix multiplication.
`matvec`(x)	Matrix-vector multiplication.
`reset_count`()	Reset counters
`rmatmat`(X)	Matrix-matrix multiplication.
`rmatvec`(x)	Adjoint matrix-vector multiplication.
`todense`([backend])	Return dense matrix.
`toimag`([forw, adj])	Imag operator
`toreal`([forw, adj])	Real operator
`tosparse`()	Return sparse matrix.
`trace`([neval, method, backend])	Trace of linear operator.
`transpose`()	Transpose this linear operator.

Examples using `pylops.BlockDiag`

- *Operators concatenation*
- *Operators with Multiprocessing*

pylops.Kronecker

class pylops.**Kronecker**(**args*, ***kwargs*)

Kronecker operator.

Perform Kronecker product of two operators. Note that the combined operator is never created explicitly, rather the product of this operator with the model vector is performed in forward mode, or the product of the adjoint of this operator and the data vector in adjoint mode.

Parameters

Op1
[*pylops.LinearOperator*] First operator

Op2
[*pylops.LinearOperator*] Second operator

dtype
[str, optional] Type of elements in input array.

name
[str, optional] New in version 2.0.0.

Name of operator (to be used by *pylops.utils.describe.describe*)

Notes

The Kronecker product (denoted with $\otimes$) is an operation on two operators $\mathbf{Op}_1$ and $\mathbf{Op}_2$ of sizes $[n_1 \times m_1]$ and $[n_2 \times m_2]$ respectively, resulting in a block matrix of size $[n_1 n_2 \times m_1 m_2]$.

$$\mathbf{Op}_1 \otimes \mathbf{Op}_2 = \begin{bmatrix} \text{Op}_1^{1,1}\mathbf{Op}_2 & \dots & \text{Op}_1^{1,m_1}\mathbf{Op}_2 \\ \vdots & \ddots & \vdots \\ \text{Op}_1^{n_1,1}\mathbf{Op}_2 & \dots & \text{Op}_1^{n_1,m_1}\mathbf{Op}_2 \end{bmatrix}$$

The application of the resulting matrix to a vector $\mathbf{x}$ of size $[m_1 m_2 \times 1]$ is equivalent to the application of the second operator $\mathbf{Op}_2$ to the rows of a matrix of size $[m_2 \times m_1]$ obtained by reshaping the input vector $\mathbf{x}$, followed by the application of the first operator to the transposed matrix produced by the first operator. In adjoint mode the same procedure is followed but the adjoint of each operator is used.

Attributes

shape
[tuple] Operator shape

explicit
[bool] Operator contains a matrix that can be solved explicitly (`True`) or not (`False`)

Methods

`__init__`(Op1, Op2[, dtype, name])	Initialize this LinearOperator.
`adjoint`()	Hermitian adjoint.
`apply_columns`(cols)	Apply subset of columns of operator
`cond`([uselobpcg])	Condition number of linear operator.
`conj`()	Complex conjugate operator
`div`(y[, niter, densesolver])	Solve the linear problem $\mathbf{y} = \mathbf{A}\mathbf{x}$.
`dot`(x)	Matrix-matrix or matrix-vector multiplication.
`eigs`([neigs, symmetric, niter, uselobpcg])	Most significant eigenvalues of linear operator.
`matmat`(X)	Matrix-matrix multiplication.
`matvec`(x)	Matrix-vector multiplication.
`reset_count`()	Reset counters
`rmatmat`(X)	Matrix-matrix multiplication.
`rmatvec`(x)	Adjoint matrix-vector multiplication.
`todense`([backend])	Return dense matrix.
`toimag`([forw, adj])	Imag operator
`toreal`([forw, adj])	Real operator
`tosparse`()	Return sparse matrix.
`trace`([neval, method, backend])	Trace of linear operator.
`transpose`()	Transpose this linear operator.

Examples using `pylops.Kronecker`

- *Operators concatenation*

pylops.Real

class pylops.**Real**(**args*, ***kwargs*)

Real operator.

Return the real component of the input. The adjoint returns a complex number with the same real component as the input and zero imaginary component.

Parameters

dims
[`int` or `tuple`] Number of samples for each dimension

dtype
[`str`, optional] Type of elements in input array.

name
[`str`, optional] New in version 2.0.0.

Name of operator (to be used by *`pylops.utils.describe.describe`*)

Notes

In forward mode:

$$y_i = \Re\{x_i\} \quad \forall i = 0, \ldots, N-1$$

In adjoint mode:

$$x_i = \Re\{y_i\} + 0i \quad \forall i = 0, \ldots, N-1$$

Attributes

shape
[tuple] Operator shape

explicit
[bool] Operator contains a matrix that can be solved explicitly (True) or not (False)

Methods

__init__(dims[, dtype, name])	Initialize this LinearOperator.
adjoint()	Hermitian adjoint.
apply_columns(cols)	Apply subset of columns of operator
cond([uselobpcg])	Condition number of linear operator.
conj()	Complex conjugate operator
div(y[, niter, densesolver])	Solve the linear problem $\mathbf{y} = \mathbf{A}\mathbf{x}$.
dot(x)	Matrix-matrix or matrix-vector multiplication.
eigs([neigs, symmetric, niter, uselobpcg])	Most significant eigenvalues of linear operator.
matmat(X)	Matrix-matrix multiplication.
matvec(x)	Matrix-vector multiplication.
reset_count()	Reset counters
rmatmat(X)	Matrix-matrix multiplication.
rmatvec(x)	Adjoint matrix-vector multiplication.
todense([backend])	Return dense matrix.
toimag([forw, adj])	Imag operator
toreal([forw, adj])	Real operator
tosparse()	Return sparse matrix.
trace([neval, method, backend])	Trace of linear operator.
transpose()	Transpose this linear operator.

Examples using pylops.Real

- *Real*

pylops.Imag

class pylops.**Imag**(**args*, ***kwargs*)

Imag operator.

Return the imaginary component of the input as a real value. The adjoint returns a complex number with zero real component and the imaginary component set to the real component of the input.

Parameters

dims
[int or tuple] Number of samples for each dimension

dtype
[str, optional] Type of elements in input array.

name
[str, optional] New in version 2.0.0.

Name of operator (to be used by *pylops.utils.describe.describe*)

Notes

In forward mode:

$$y_i = \Im\{x_i\} \quad \forall i = 0, \ldots, N-1$$

In adjoint mode:

$$x_i = 0 + i\Re\{y_i\} \quad \forall i = 0, \ldots, N-1$$

Attributes

shape
[tuple] Operator shape

explicit
[bool] Operator contains a matrix that can be solved explicitly (`True`) or not (`False`)

Methods

`__init__`(dims[, dtype, name])	Initialize this LinearOperator.
`adjoint`()	Hermitian adjoint.
`apply_columns`(cols)	Apply subset of columns of operator
`cond`([uselobpcg])	Condition number of linear operator.
`conj`()	Complex conjugate operator
`div`(y[, niter, densesolver])	Solve the linear problem $\mathbf{y} = \mathbf{A}\mathbf{x}$.
`dot`(x)	Matrix-matrix or matrix-vector multiplication.
`eigs`([neigs, symmetric, niter, uselobpcg])	Most significant eigenvalues of linear operator.
`matmat`(X)	Matrix-matrix multiplication.
`matvec`(x)	Matrix-vector multiplication.
`reset_count`()	Reset counters
`rmatmat`(X)	Matrix-matrix multiplication.
`rmatvec`(x)	Adjoint matrix-vector multiplication.
`todense`([backend])	Return dense matrix.
`toimag`([forw, adj])	Imag operator
`toreal`([forw, adj])	Real operator
`tosparse`()	Return sparse matrix.
`trace`([neval, method, backend])	Trace of linear operator.
`transpose`()	Transpose this linear operator.

Examples using `pylops.Imag`

- *Imag*

pylops.Conj

class pylops.**Conj**(**args*, ***kwargs*)

Complex conjugate operator.

Return the complex conjugate of the input. It is self-adjoint.

Parameters

dims
[int or tuple] Number of samples for each dimension

dtype
[str, optional] Type of elements in input array.

name
[str, optional] New in version 2.0.0.

Name of operator (to be used by *pylops.utils.describe.describe*)

Notes

In forward mode:

$$y_i = \Re\{x_i\} - i\Im\{x_i\} \quad \forall i = 0, \ldots, N-1$$

In adjoint mode:

$$x_i = \Re\{y_i\} - i\Im\{y_i\} \quad \forall i = 0, \ldots, N-1$$

Attributes

shape
[tuple] Operator shape

explicit
[bool] Operator contains a matrix that can be solved explicitly (True) or not (False)

Methods

__init__(dims[, dtype, name])	Initialize this LinearOperator.
adjoint()	Hermitian adjoint.
apply_columns(cols)	Apply subset of columns of operator
cond([uselobpcg])	Condition number of linear operator.
conj()	Complex conjugate operator
div(y[, niter, densesolver])	Solve the linear problem $\mathbf{y} = \mathbf{A}\mathbf{x}$.
dot(x)	Matrix-matrix or matrix-vector multiplication.
eigs([neigs, symmetric, niter, uselobpcg])	Most significant eigenvalues of linear operator.
matmat(X)	Matrix-matrix multiplication.
matvec(x)	Matrix-vector multiplication.
reset_count()	Reset counters
rmatmat(X)	Matrix-matrix multiplication.
rmatvec(x)	Adjoint matrix-vector multiplication.
todense([backend])	Return dense matrix.
toimag([forw, adj])	Imag operator
toreal([forw, adj])	Real operator
tosparse()	Return sparse matrix.
trace([neval, method, backend])	Trace of linear operator.
transpose()	Transpose this linear operator.

Examples using pylops.Conj

- *Conj*

Smoothing and derivatives

Smoothing1D(nsmooth, dims[, axis, dtype])	1D Smoothing.
Smoothing2D(nsmooth, dims[, axes, dtype])	2D Smoothing.
FirstDerivative(*args, **kwargs)	First derivative.
SecondDerivative(*args, **kwargs)	Second derivative.
Laplacian(dims[, axes, weights, sampling, ...])	Laplacian.
Gradient(dims[, sampling, edge, kind, dtype])	Gradient.
FirstDirectionalDerivative(dims, v[, ...])	First Directional derivative.
SecondDirectionalDerivative(dims, v[, ...])	Second Directional derivative.

pylops.Smoothing1D

pylops.**Smoothing1D**(*nsmooth*, *dims*, *axis=-1*, *dtype='float64'*)

1D Smoothing.

Apply smoothing to model (and data) to a multi-dimensional array along `axis`.

Parameters

nsmooth
[int] Length of smoothing operator (must be odd)

dims
[tuple or int] Number of samples for each dimension

axis
[int, optional] New in version 2.0.0.

Axis along which model (and data) are smoothed.

dtype
[str, optional] Type of elements in input array.

Notes

The Smoothing1D operator is a special type of convolutional operator that convolves the input model (or data) with a constant filter of size n_{smooth}:

$$\mathbf{f} = [1/n_{\text{smooth}}, 1/n_{\text{smooth}}, ..., 1/n_{\text{smooth}}]$$

When applied to the first direction:

$$y[i,j,k] = 1/n_{\text{smooth}} \sum_{l=-(n_{\text{smooth}}-1)/2}^{(n_{\text{smooth}}-1)/2} x[l,j,k]$$

Similarly when applied to the second direction:

$$y[i,j,k] = 1/n_{\text{smooth}} \sum_{l=-(n_{\text{smooth}}-1)/2}^{(n_{\text{smooth}}-1)/2} x[i,l,k]$$

and the third direction:

$$y[i,j,k] = 1/n_{\text{smooth}} \sum_{l=-(n_{\text{smooth}}-1)/2}^{(n_{\text{smooth}}-1)/2} x[i,j,l]$$

Note that since the filter is symmetrical, the *Smoothing1D* operator is self-adjoint.

Attributes

shape
[tuple] Operator shape

explicit
[bool] Operator contains a matrix that can be solved explicitly (True) or not (False)

Examples using pylops.Smoothing1D

- *1D Smoothing*
- *Causal Integration*
- *Wavelet estimation*
- *03. Solvers*

pylops.Smoothing2D

pylops.**Smoothing2D**(*nsmooth*, *dims*, *axes=(-2, -1)*, *dtype='float64'*)

2D Smoothing.

Apply smoothing to model (and data) along two axes of a multi-dimensional array.

Parameters

nsmooth
[tuple or list] Lenght of smoothing operator in 1st and 2nd dimensions (must be odd)

dims
[tuple] Number of samples for each dimension

axes
[int, optional] New in version 2.0.0.

Axes along which model (and data) are smoothed.

dtype
[str, optional] Type of elements in input array.

See also:

pylops.signalprocessing.Convolve2D
2D convolution

Notes

The 2D Smoothing operator is a special type of convolutional operator that convolves the input model (or data) with a constant 2d filter of size $n_{\text{smooth},1} \times n_{\text{smooth},2}$:

Its application to a two dimensional input signal is:

$$y[i,j] = 1/(n_{\text{smooth},1} \cdot n_{\text{smooth},2}) \sum_{l=-(n_{\text{smooth},1}-1)/2}^{(n_{\text{smooth},1}-1)/2} \sum_{m=-(n_{\text{smooth},2}-1)/2}^{(n_{\text{smooth},2}-1)/2} x[l,m]$$

Note that since the filter is symmetrical, the *Smoothing2D* operator is self-adjoint.

Attributes

shape
[tuple] Operator shape

explicit
[bool] Operator contains a matrix that can be solved explicitly (True) or not (False)

Examples using pylops.Smoothing2D

- *2D Smoothing*
- *Causal Integration*
- *19. Automatic Differentiation*

pylops.FirstDerivative

class pylops.**FirstDerivative**(**args*, ***kwargs*)

First derivative.

Apply a first derivative using a multiple-point stencil finite-difference approximation along axis.

Parameters

dims
[list or int] Number of samples for each dimension

axis
[int, optional] New in version 2.0.0.

Axis along which derivative is applied.

sampling
[float, optional] Sampling step Δx.

kind
[str, optional] Derivative kind (forward, centered, or backward).

edge
[bool, optional] Use reduced order derivative at edges (True) or ignore them (False). This is currently only available

for centered derivative

order
[int, optional] New in version 2.0.0.

Derivative order (3 or 5). This is currently only available for centered derivative

dtype
[str, optional] Type of elements in input array.

name
[str, optional] New in version 2.0.0.

Name of operator (to be used by *pylops.utils.describe.describe*)

Notes

The FirstDerivative operator applies a first derivative to any chosen direction of a multi-dimensional array using either a second- or third-order centered stencil or first-order forward/backward stencils.

For simplicity, given a one dimensional array, the second-order centered first derivative is:

$$y[i] = (0.5x[i+1] - 0.5x[i-1])/\Delta x$$

while the first-order forward stencil is:

$$y[i] = (x[i+1] - x[i])/\Delta x$$

and the first-order backward stencil is:

$$y[i] = (x[i] - x[i-1])/\Delta x$$

Formulas for the third-order centered stencil can be found at this link.

Attributes

shape
[tuple] Operator shape

explicit
[bool] Operator contains a matrix that can be solved explicitly (True) or not (False)

Methods

__init__(dims[, axis, sampling, kind, edge, ...])	Initialize this LinearOperator.
adjoint()	Hermitian adjoint.
apply_columns(cols)	Apply subset of columns of operator
cond([uselobpcg])	Condition number of linear operator.
conj()	Complex conjugate operator
div(y[, niter, densesolver])	Solve the linear problem $\mathbf{y} = \mathbf{A}\mathbf{x}$.
dot(x)	Matrix-matrix or matrix-vector multiplication.
eigs([neigs, symmetric, niter, uselobpcg])	Most significant eigenvalues of linear operator.
matmat(X)	Matrix-matrix multiplication.
matvec(x)	Matrix-vector multiplication.
reset_count()	Reset counters
rmatmat(X)	Matrix-matrix multiplication.
rmatvec(x)	Adjoint matrix-vector multiplication.
todense([backend])	Return dense matrix.
toimag([forw, adj])	Imag operator
toreal([forw, adj])	Real operator
tosparse()	Return sparse matrix.
trace([neval, method, backend])	Trace of linear operator.
transpose()	Transpose this linear operator.

Examples using `pylops.FirstDerivative`

- *Causal Integration*
- *Derivatives*
- *Operators concatenation*
- *Total Variation (TV) Regularization*
- *05. Image deblurring*
- *16. CT Scan Imaging*

pylops.SecondDerivative

class pylops.**SecondDerivative**(**args*, ***kwargs*)

Second derivative.

Apply a second derivative using a three-point stencil finite-difference approximation along `axis`.

Parameters

dims
[`list` or `int`] Number of samples for each dimension (`None` if only one dimension is available)

axis
[`int`, optional] New in version 2.0.0.

Axis along which derivative is applied.

sampling
[`float`, optional] Sampling step Δx.

kind
[`str`, optional] Derivative kind (`forward`, `centered`, or `backward`).

edge
[`bool`, optional] Use shifted derivatives at edges (`True`) or ignore them (`False`). This is currently only available

for centered derivative

dtype
[`str`, optional] Type of elements in input array.

name
[`str`, optional] New in version 2.0.0.

Name of operator (to be used by *`pylops.utils.describe.describe`*)

Notes

The SecondDerivative operator applies a second derivative to any chosen direction of a multi-dimensional array. For simplicity, given a one dimensional array, the second-order centered first derivative is:

$$y[i] = (x[i+1] - 2x[i] + x[i-1])/\Delta x^2$$

while the second-order forward stencil is:

$$y[i] = (x[i+2] - 2x[i+1] + x[i])/\Delta x^2$$

and the second-order backward stencil is:

$$y[i] = (x[i] - 2x[i-1] + x[i-2])/\Delta x^2$$

Attributes

shape
[tuple] Operator shape

explicit
[bool] Operator contains a matrix that can be solved explicitly (True) or not (False)

Methods

__init__(dims[, axis, sampling, kind, edge, ...])	Initialize this LinearOperator.
adjoint()	Hermitian adjoint.
apply_columns(cols)	Apply subset of columns of operator
cond([uselobpcg])	Condition number of linear operator.
conj()	Complex conjugate operator
div(y[, niter, densesolver])	Solve the linear problem $\mathbf{y} = \mathbf{A}\mathbf{x}$.
dot(x)	Matrix-matrix or matrix-vector multiplication.
eigs([neigs, symmetric, niter, uselobpcg])	Most significant eigenvalues of linear operator.
matmat(X)	Matrix-matrix multiplication.
matvec(x)	Matrix-vector multiplication.
reset_count()	Reset counters
rmatmat(X)	Matrix-matrix multiplication.
rmatvec(x)	Adjoint matrix-vector multiplication.
todense([backend])	Return dense matrix.
toimag([forw, adj])	Imag operator
toreal([forw, adj])	Real operator
tosparse()	Return sparse matrix.
trace([neval, method, backend])	Trace of linear operator.
transpose()	Transpose this linear operator.

Examples using `pylops.SecondDerivative`

- *Causal Integration*
- *Derivatives*
- *Operators concatenation*
- *Total Variation (TV) Regularization*
- *Wavelet estimation*
- *03. Solvers*
- *12. Seismic regularization*

pylops.Laplacian

`pylops.Laplacian`(*dims*, *axes=(-2, -1)*, *weights=(1, 1)*, *sampling=(1, 1)*, *edge=False*, *kind='centered'*, *dtype='float64'*)

Laplacian.

Apply second-order centered Laplacian operator to a multi-dimensional array.

Note: At least 2 dimensions are required, use *pylops.SecondDerivative* for 1d arrays.

Parameters

dims
[tuple] Number of samples for each dimension.

axes
[int, optional] New in version 2.0.0.

Axes along which the Laplacian is applied.

weights
[tuple, optional] Weight to apply to each direction (real laplacian operator if `weights=(1, 1)`)

sampling
[tuple, optional] Sampling steps for each direction

edge
[bool, optional] Use reduced order derivative at edges (`True`) or ignore them (`False`) for centered derivative

kind
[str, optional] Derivative kind (`forward`, `centered`, or `backward`)

dtype
[str, optional] Type of elements in input array.

Returns

l2op
[*pylops.LinearOperator*] Laplacian linear operator

Raises

ValueError
If `axes`. `weights`, and `sampling` do not have the same size

Notes

The Laplacian operator applies a second derivative along two directions of a multi-dimensional array.

For simplicity, given a two dimensional array, the Laplacian is:

$$y[i,j] = (x[i+1,j] + x[i-1,j] + x[i,j-1] + x[i,j+1] - 4x[i,j])/(\Delta x \Delta y)$$

Examples using `pylops.Laplacian`

- *Bilinear Interpolation*
- *Causal Integration*
- *Derivatives*
- *06. 2D Interpolation*
- *16. CT Scan Imaging*

pylops.Gradient

`pylops.Gradient`(*dims*, *sampling=1*, *edge=False*, *kind='centered'*, *dtype='float64'*)

Gradient.

Apply gradient operator to a multi-dimensional array.

Note: At least 2 dimensions are required, use *pylops.FirstDerivative* for 1d arrays.

Parameters

dims
[tuple] Number of samples for each dimension.

sampling
[tuple, optional] Sampling steps for each direction.

edge
[bool, optional] Use reduced order derivative at edges (`True`) or ignore them (`False`).

kind
[str, optional] Derivative kind (`forward`, `centered`, or `backward`).

dtype
[str, optional] Type of elements in input array.

Returns

l2op
[*pylops.LinearOperator*] Gradient linear operator

Notes

The Gradient operator applies a first-order derivative to each dimension of a multi-dimensional array in forward mode.

For simplicity, given a three dimensional array, the Gradient in forward mode using a centered stencil can be expressed as:

$$\mathbf{g}_{i,j,k} = (f_{i+1,j,k} - f_{i-1,j,k})/d_1\mathbf{i_1} + (f_{i,j+1,k} - f_{i,j-1,k})/d_2\mathbf{i_2} + (f_{i,j,k+1} - f_{i,j,k-1})/d_3\mathbf{i_3}$$

which is discretized as follows:

$$\mathbf{g} = \begin{bmatrix} \mathbf{df_1} \\ \mathbf{df_2} \\ \mathbf{df_3} \end{bmatrix}$$

In adjoint mode, the adjoints of the first derivatives along different axes are instead summed together.

Examples using `pylops.Gradient`

- *Derivatives*

pylops.FirstDirectionalDerivative

pylops.**FirstDirectionalDerivative**(*dims*, *v*, *sampling=1*, *edge=False*, *kind='centered'*, *dtype='float64'*)

First Directional derivative.

Apply a directional derivative operator to a multi-dimensional array along either a single common axis or different axes for each point of the array.

Note: At least 2 dimensions are required, consider using *pylops.FirstDerivative* for 1d arrays.

Parameters

dims
[tuple] Number of samples for each dimension.

v
[np.ndarray, optional] Single direction (array of size n_{dims}) or group of directions (array of size $[n_{\text{dims}} \times n_{d_0} \times ... \times n_{d_{n_{\text{dims}}}}]$)

sampling
[tuple, optional] Sampling steps for each direction.

edge
[bool, optional] Use reduced order derivative at edges (`True`) or ignore them (`False`).

kind
[str, optional] Derivative kind (`forward`, `centered`, or `backward`).

dtype
[str, optional] Type of elements in input array.

Returns

ddop
[*pylops.LinearOperator*] First directional derivative linear operator

Notes

The FirstDirectionalDerivative applies a first-order derivative to a multi-dimensional array along the direction defined by the unitary vector $\mathbf{v}$:

$$df_{\mathbf{v}} = \nabla f \mathbf{v}$$

or along the directions defined by the unitary vectors $\mathbf{v}(x, y)$:

$$df_{\mathbf{v}}(x, y) = \nabla f(x, y) \mathbf{v}(x, y)$$

where we have here considered the 2-dimensional case.

This operator can be easily implemented as the concatenation of the *pylops.Gradient* operator and the *pylops.Diagonal* operator with $\mathbf{v}$ along the main diagonal.

Examples using `pylops.FirstDirectionalDerivative`

- *Derivatives*

pylops.SecondDirectionalDerivative

`pylops.`**`SecondDirectionalDerivative`**(*dims*, *v*, *sampling=1*, *edge=False*, *dtype='float64'*)

Second Directional derivative.

Apply a second directional derivative operator to a multi-dimensional array along either a single common axis or different axes for each point of the array.

Note: At least 2 dimensions are required, consider using *pylops.SecondDerivative* for 1d arrays.

Parameters

dims
[tuple] Number of samples for each dimension.

v
[np.ndarray, optional] Single direction (array of size n_{dims}) or group of directions (array of size $[n_{\text{dims}} \times n_{d_0} \times ... \times n_{d_{n_{\text{dims}}}}]$)

sampling
[tuple, optional] Sampling steps for each direction.

edge
[bool, optional] Use reduced order derivative at edges (`True`) or ignore them (`False`).

dtype
[str, optional] Type of elements in input array.

Returns

ddop
[*pylops.LinearOperator*] Second directional derivative linear operator

Notes

The SecondDirectionalDerivative applies a second-order derivative to a multi-dimensional array along the direction defined by the unitary vector $\mathbf{v}$:

$$d^2 f_{\mathbf{v}} = -D_{\mathbf{v}}^T [D_{\mathbf{v}} f]$$

where $D_{\mathbf{v}}$ is the first-order directional derivative implemented by *pylops.SecondDirectionalDerivative*.

This operator is sometimes also referred to as directional Laplacian in the literature.

Examples using `pylops.SecondDirectionalDerivative`

- *Derivatives*

Signal processing

Convolve1D(*args, **kwargs)	1D convolution operator.
Convolve2D(dims, h[, offset, axes, method, ...])	2D convolution operator.
ConvolveND(*args, **kwargs)	ND convolution operator.
Interp(dims, iava[, axis, kind, dtype, name])	Interpolation operator.
Bilinear(*args, **kwargs)	Bilinear interpolation operator.
FFT(dims[, axis, nfft, sampling, norm, ...])	One dimensional Fast-Fourier Transform.
FFT2D(dims[, axes, nffts, sampling, norm, ...])	Two dimensional Fast-Fourier Transform.
FFTND(dims[, axes, nffts, sampling, norm, ...])	N-dimensional Fast-Fourier Transform.
Shift(dims, shift[, axis, nfft, sampling, ...])	Shift operator
DWT(*args, **kwargs)	One dimensional Wavelet operator.
DWT2D(*args, **kwargs)	Two dimensional Wavelet operator.
Seislet(*args, **kwargs)	Two dimensional Seislet operator.
Radon2D(taxis, haxis, pxaxis[, kind, ...])	Two dimensional Radon transform.
Radon3D(taxis, hyaxis, hxaxis, pyaxis, pxaxis)	Three dimensional Radon transform.
ChirpRadon2D(*args, **kwargs)	2D Chirp Radon transform
ChirpRadon3D(*args, **kwargs)	3D Chirp Radon transform
Sliding1D(Op, dim, dimd, nwin, nover[, ...])	1D Sliding transform operator.
Sliding2D(Op, dims, dimsd, nwin, nover[, ...])	2D Sliding transform operator.
Sliding3D(Op, dims, dimsd, nwin, nover, nop)	3D Sliding transform operator.w
Patch2D(Op, dims, dimsd, nwin, nover, nop[, ...])	2D Patch transform operator.
Patch3D(Op, dims, dimsd, nwin, nover, nop[, ...])	3D Patch transform operator.
Fredholm1(*args, **kwargs)	Fredholm integral of first kind.

pylops.signalprocessing.Convolve1D

class pylops.signalprocessing.**Convolve1D**(**args*, ***kwargs*)

1D convolution operator.

Apply one-dimensional convolution with a compact filter to model (and data) along an `axis` of a multi-dimensional array.

Parameters

dims
[`list` or `int`] Number of samples for each dimension

h
[numpy.ndarray] 1d compact filter to be convolved to input signal

offset
[int] Index of the center of the compact filter

axis
[int, optional] New in version 2.0.0.

Axis along which convolution is applied

method
[str, optional] Method used to calculate the convolution (`direct`, `fft`, or `overlapadd`). Note that only `direct` and `fft` are allowed when `dims=None`, whilst `fft` and `overlapadd` are allowed when `dims` is provided.

dtype
[str, optional] Type of elements in input array.

name
[str, optional] New in version 2.0.0.

Name of operator (to be used by *pylops.utils.describe.describe*)

Raises

ValueError
If `offset` is bigger than `len(h) - 1`

NotImplementedError
If `method` provided is not allowed

Notes

The Convolve1D operator applies convolution between the input signal $x(t)$ and a compact filter kernel $h(t)$ in forward model:

$$y(t) = \int_{-\infty}^{\infty} h(t-\tau)x(\tau)\,\mathrm{d}\tau$$

This operation can be discretized as follows

$$y[n] = \sum_{m=-\infty}^{\infty} h[n-m]x[m]$$

as well as performed in the frequency domain.

$$Y(f) = F(h(t)) * F(x(t))$$

Convolve1D operator uses `scipy.signal.convolve` that automatically chooses the best domain for the operation to be carried out for one dimensional inputs. The fft implementation `scipy.signal.fftconvolve` is however enforced for signals in 2 or more dimensions as this routine efficently operates on multi-dimensional arrays.

As the adjoint of convolution is correlation, Convolve1D operator applies correlation in the adjoint mode.

In time domain:

$$x(t) = \int_{-\infty}^{\infty} h(t+\tau)x(\tau)\,\mathrm{d}\tau$$

or in frequency domain:

$$y(t) = F^{-1}(H(f)^* * X(f))$$

Attributes

shape
[tuple] Operator shape

explicit
[bool] Operator contains a matrix that can be solved explicitly (True) or not (False)

Methods

__init__(dims, h[, offset, axis, method, ...])	Initialize this LinearOperator.
adjoint()	Hermitian adjoint.
apply_columns(cols)	Apply subset of columns of operator
cond([uselobpcg])	Condition number of linear operator.
conj()	Complex conjugate operator
div(y[, niter, densesolver])	Solve the linear problem $\mathbf{y} = \mathbf{A}\mathbf{x}$.
dot(x)	Matrix-matrix or matrix-vector multiplication.
eigs([neigs, symmetric, niter, uselobpcg])	Most significant eigenvalues of linear operator.
matmat(X)	Matrix-matrix multiplication.
matvec(x)	Matrix-vector multiplication.
reset_count()	Reset counters
rmatmat(X)	Matrix-matrix multiplication.
rmatvec(x)	Adjoint matrix-vector multiplication.
todense([backend])	Return dense matrix.
toimag([forw, adj])	Imag operator
toreal([forw, adj])	Real operator
tosparse()	Return sparse matrix.
trace([neval, method, backend])	Trace of linear operator.
transpose()	Transpose this linear operator.

Examples using pylops.signalprocessing.Convolve1D

- *1D Smoothing*
- *Convolution*
- *MP, OMP, ISTA and FISTA*
- *Wavelet estimation*
- *03. Solvers*
- *04. Bayesian Inversion*

pylops.signalprocessing.Convolve2D

pylops.signalprocessing.**Convolve2D**(*dims*, *h*, *offset=(0, 0)*, *axes=(-2, -1)*, *method='fft'*, *dtype='float64'*, *name='C'*)

2D convolution operator.

Apply two-dimensional convolution with a compact filter to model (and data) along a pair of `axes` of a two or three-dimensional array.

Parameters

dims
[`list` or `int`] Number of samples for each dimension

h
[`numpy.ndarray`] 2d compact filter to be convolved to input signal

offset
[`tuple`, optional] Indices of the center of the compact filter

axes
[`int`, optional] New in version 2.0.0.

Axes along which convolution is applied

method
[`str`, optional] Method used to calculate the convolution (`direct` or `fft`).

dtype
[`str`, optional] Type of elements in input array.

name
[`str`, optional] New in version 2.0.0.

Name of operator (to be used by *pylops.utils.describe.describe*)

Returns

cop
[*pylops.LinearOperator*] Convolve2D linear operator

Notes

The Convolve2D operator applies two-dimensional convolution between the input signal $d(t, x)$ and a compact filter kernel $h(t, x)$ in forward model:

$$y(t, x) = \int_{-\infty}^{\infty}\int h(t-\tau, x-\chi) d(\tau, \chi)\, \mathrm{d}\tau\, \mathrm{d}\chi$$

This operation can be discretized as follows

$$y[i, n] = \sum_{j=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} h[i-j, n-m] d[j, m]$$

as well as performed in the frequency domain.

$$Y(f, k_x) = F(h(t, x)) * F(d(t, x))$$

Convolve2D operator uses `scipy.signal.convolve2d` that automatically chooses the best domain for the operation to be carried out.

As the adjoint of convolution is correlation, Convolve2D operator applies correlation in the adjoint mode.

In time domain:

$$y(t,x) = \int\int_{-\infty}^{\infty} h(t+\tau, x+\chi) d(\tau,\chi)\, \mathrm{d}\tau\, \mathrm{d}\chi$$

or in frequency domain:

$$y(t,x) = F^{-1}(H(f,k_x)^* * X(f,k_x))$$

Examples using `pylops.signalprocessing.Convolve2D`

- *Convolution*
- *05. Image deblurring*

pylops.signalprocessing.ConvolveND

class pylops.signalprocessing.**ConvolveND**(**args*, ***kwargs*)

ND convolution operator.

Apply n-dimensional convolution with a compact filter to model (and data) along the `axes` of a n-dimensional array.

Parameters

dims
[`list` or `int`] Number of samples for each dimension

h
[`numpy.ndarray`] nd compact filter to be convolved to input signal

offset
[`tuple`, optional] Indices of the center of the compact filter

axes
[`int`, optional] New in version 2.0.0.

Axes along which convolution is applied

method
[`str`, optional] Method used to calculate the convolution (`direct` or `fft`).

dtype
[`str`, optional] Type of elements in input array.

name
[`str`, optional] New in version 2.0.0.

Name of operator (to be used by *`pylops.utils.describe.describe`*)

Notes

The ConvolveND operator applies n-dimensional convolution between the input signal $d(x_1, x_2, ..., x_N)$ and a compact filter kernel $h(x_1, x_2, ..., x_N)$ in forward model. This is a straighforward extension to multiple dimensions of *pylops.signalprocessing.Convolve2D* operator.

Attributes

shape
[tuple] Operator shape

explicit
[bool] Operator contains a matrix that can be solved explicitly (True) or not (False)

Methods

__init__(dims, h[, offset, axes, method, ...])	Initialize this LinearOperator.
adjoint()	Hermitian adjoint.
apply_columns(cols)	Apply subset of columns of operator
cond([uselobpcg])	Condition number of linear operator.
conj()	Complex conjugate operator
div(y[, niter, densesolver])	Solve the linear problem $\mathbf{y} = \mathbf{A}\mathbf{x}$.
dot(x)	Matrix-matrix or matrix-vector multiplication.
eigs([neigs, symmetric, niter, uselobpcg])	Most significant eigenvalues of linear operator.
matmat(X)	Matrix-matrix multiplication.
matvec(x)	Matrix-vector multiplication.
reset_count()	Reset counters
rmatmat(X)	Matrix-matrix multiplication.
rmatvec(x)	Adjoint matrix-vector multiplication.
todense([backend])	Return dense matrix.
toimag([forw, adj])	Imag operator
toreal([forw, adj])	Real operator
tosparse()	Return sparse matrix.
trace([neval, method, backend])	Trace of linear operator.
transpose()	Transpose this linear operator.

Examples using pylops.signalprocessing.ConvolveND

- *2D Smoothing*
- *Causal Integration*
- *Convolution*
- *05. Image deblurring*

pylops.signalprocessing.Interp

pylops.signalprocessing.**Interp**(*dims*, *iava*, *axis=-1*, *kind='linear'*, *dtype='float64'*, *name='I'*)

Interpolation operator.

Apply interpolation along `axis` from regularly sampled input vector into fractionary positions `iava` using one of the following algorithms:

- *Nearest neighbour* interpolation is a thin wrapper around *pylops.Restriction* at `np.round(iava)` locations.
- *Linear interpolation* extracts values from input vector at locations `np.floor(iava)` and `np.floor(iava)+1` and linearly combines them in forward mode, places weighted versions of the interpolated values at locations `np.floor(iava)` and `np.floor(iava)+1` in an otherwise zero vector in adjoint mode.
- *Sinc interpolation* performs sinc interpolation at locations `iava`. Note that this is the most accurate method but it has higher computational cost as it involves multiplying the input data by a matrix of size $N \times M$.

Note: The vector `iava` should contain unique values. If the same index is repeated twice an error will be raised. This also applies when values beyond the last element of the input array for *linear interpolation* as those values are forced to be just before this element.

Parameters

dims
[`list` or `int`] Number of samples for each dimension

iava
[`list` or `numpy.ndarray`] Floating indices of locations of available samples for interpolation.

axis
[`int`, optional] New in version 2.0.0.

Axis along which interpolation is applied.

kind
[`str`, optional] Kind of interpolation (`nearest`, `linear`, and `sinc` are currently supported)

dtype
[`str`, optional] Type of elements in input array.

name
[`str`, optional] New in version 2.0.0.

Name of operator (to be used by *pylops.utils.describe.describe*)

Returns

op
[*pylops.LinearOperator*] Linear intepolation operator

iava
[`list` or `numpy.ndarray`] Corrected indices of locations of available samples (samples at `M-1` or beyond are forced to be at `M-1-eps`)

Raises

ValueError
If the vector `iava` contains repeated values.

NotImplementedError
If `kind` is not `nearest`, `linear` or `sinc`

See also:

pylops.Restriction
Restriction operator

Notes

Linear interpolation of a subset of N values at locations `iava` from an input (or model) vector $\mathbf{x}$ of size M can be expressed as:

$$y_i = (1 - w_i) x_{l_i^l} + w_i x_{l_i^r} \quad \forall i = 1, 2, \ldots, N$$

where $\mathbf{l^l} = [\lfloor l_1 \rfloor, \lfloor l_2 \rfloor, \ldots, \lfloor l_N \rfloor]$ and $\mathbf{l^r} = [\lfloor l_1 \rfloor + 1, \lfloor l_2 \rfloor + 1, \ldots, \lfloor l_N \rfloor + 1]$ are vectors containing the indeces of the original array at which samples are taken, and $\mathbf{w} = [l_1 - \lfloor l_1 \rfloor, l_2 - \lfloor l_2 \rfloor, ..., l_N - \lfloor l_N \rfloor]$ are the linear interpolation weights. This operator can be implemented by simply summing two *pylops.Restriction* operators which are weighted using `pylops.basicoperators.Diagonal` operators.

Sinc interpolation of a subset of N values at locations `iava` from an input (or model) vector $\mathbf{x}$ of size M can be expressed as:

$$sincy_i = \sum_{j=0}^{M} x_j (i - j) \quad \forall i = 1, 2, \ldots, N$$

This operator can be implemented using the *pylops.MatrixMult* operator with a matrix containing the values of the sinc function at all i, j possible combinations.

Examples using `pylops.signalprocessing.Interp`

- *Restriction and Interpolation*

pylops.signalprocessing.Bilinear

class pylops.signalprocessing.**Bilinear**(**args*, ***kwargs*)

Bilinear interpolation operator.

Apply bilinear interpolation onto fractionary positions `iava` along the first two axes of a n-dimensional array.

Note: The vector `iava` should contain unique pais. If the same pair is repeated twice an error will be raised.

Parameters

iava
[`list` or `numpy.ndarray`] Array of size $[2 \times n_{\text{ava}}]$ containing pairs of floating indices of locations of available samples for interpolation.

dims
[list] Number of samples for each dimension

dtype
[str, optional] Type of elements in input array.

name
[str, optional] New in version 2.0.0.

Name of operator (to be used by *pylops.utils.describe.describe*)

Raises

ValueError
If the vector `iava` contains repeated values.

Notes

Bilinear interpolation of a subset of N values at locations `iava` from an input n-dimensional vector $\mathbf{x}$ of size $[m_1 \times m_2 \times ... \times m_{ndim}]$ can be expressed as:

$$y_\mathbf{i} = (1-w_i^0)(1-w_i^1)x_{l_i^{l,0},l_i^{l,1}} + w_i^0(1-w_i^1)x_{l_i^{r,0},l_i^{l,1}} + (1-w_i^0)w_i^1 x_{l_i^{l,0},l_i^{r,1}} + w_i^0 w_i^1 x_{l_i^{r,0},l_i^{r,1}} \quad \forall i = 1, 2, \ldots, M$$

where $\mathbf{l^{l,0}} = [\lfloor l_1^0 \rfloor, \lfloor l_2^0 \rfloor, ..., \lfloor l_N^0 \rfloor]$, $\mathbf{l^{l,1}} = [\lfloor l_1^1 \rfloor, \lfloor l_2^1 \rfloor, ..., \lfloor l_N^1 \rfloor]$, $\mathbf{l^{r,0}} = [\lfloor l_1^0 \rfloor + 1, \lfloor l_2^0 \rfloor + 1, ..., \lfloor l_N^0 \rfloor + 1]$, $\mathbf{l^{r,1}} = [\lfloor l_1^1 \rfloor + 1, \lfloor l_2^1 \rfloor + 1, ..., \lfloor l_N^1 \rfloor + 1]$, are vectors containing the indices of the original array at which samples are taken, and $\mathbf{w^j} = [l_1^i - \lfloor l_1^i \rfloor, l_2^i - \lfloor l_2^i \rfloor, ..., l_N^i - \lfloor l_N^i \rfloor]$ $(\forall j = 0, 1)$ are the bilinear interpolation weights.

Attributes

shape
[tuple] Operator shape

explicit
[bool] Operator contains a matrix that can be solved explicitly (`True`) or not (`False`)

Methods

`__init__`(iava, dims[, dtype, name])	Initialize this LinearOperator.
`adjoint`()	Hermitian adjoint.
`apply_columns`(cols)	Apply subset of columns of operator
`cond`([uselobpcg])	Condition number of linear operator.
`conj`()	Complex conjugate operator
`div`(y[, niter, densesolver])	Solve the linear problem $\mathbf{y} = \mathbf{Ax}$.
`dot`(x)	Matrix-matrix or matrix-vector multiplication.
`eigs`([neigs, symmetric, niter, uselobpcg])	Most significant eigenvalues of linear operator.
`matmat`(X)	Matrix-matrix multiplication.
`matvec`(x)	Matrix-vector multiplication.
`reset_count`()	Reset counters
`rmatmat`(X)	Matrix-matrix multiplication.
`rmatvec`(x)	Adjoint matrix-vector multiplication.
`todense`([backend])	Return dense matrix.
`toimag`([forw, adj])	Imag operator
`toreal`([forw, adj])	Real operator
`tosparse`()	Return sparse matrix.
`trace`([neval, method, backend])	Trace of linear operator.
`transpose`()	Transpose this linear operator.

Examples using `pylops.signalprocessing.Bilinear`

- *Bilinear Interpolation*

pylops.signalprocessing.FFT

pylops.signalprocessing.**FFT**(*dims*, *axis=-1*, *nfft=None*, *sampling=1.0*, *norm='ortho'*, *real=False*, *ifftshift_before=False*, *fftshift_after=False*, *engine='numpy'*, *dtype='complex128'*, *name='F'*, ***kwargs_fftw*)

One dimensional Fast-Fourier Transform.

Apply Fast-Fourier Transform (FFT) along an `axis` of a multi-dimensional array of size `dim`.

Using the default NumPy engine, the FFT operator is an overload to either the NumPy `numpy.fft.fft` (or `numpy.fft.rfft` for real models) in forward mode, and to `numpy.fft.ifft` (or `numpy.fft.irfft` for real models) in adjoint mode, or their CuPy equivalents. When `engine='fftw'` is chosen, the `pyfftw.FFTW` class is used instead. Alternatively, when the SciPy engine is chosen, the overloads are of `scipy.fft.fft` (or `scipy.fft.rfft` for real models) in forward mode, and to `scipy.fft.ifft` (or `scipy.fft.irfft` for real models) in adjoint mode.

When using `real=True`, the result of the forward is also multiplied by $\sqrt{2}$ for all frequency bins except zero and Nyquist, and the input of the adjoint is multiplied by $1/\sqrt{2}$ for the same frequencies.

For a real valued input signal, it is advised to use the flag `real=True` as it stores the values of the Fourier transform at positive frequencies only as values at negative frequencies are simply their complex conjugates.

Parameters

dims
[`tuple`] Number of samples for each dimension

axis
[`int`, optional] New in version 2.0.0.

Axis along which FFT is applied

nfft
[`int`, optional] Number of samples in Fourier Transform (same as input if `nfft=None`)

sampling
[`float`, optional] Sampling step `dt`.

norm
[*{"ortho", "none", "1/n"}*, optional] New in version 1.17.0.

- "ortho": Scales forward and adjoint FFT transforms with $1/\sqrt{N_F}$, where N_F is the number of samples in the Fourier domain given by `nfft`.
- "none": Does not scale the forward or the adjoint FFT transforms.
- "1/n": Scales both the forward and adjoint FFT transforms by $1/N_F$.

Note: For "none" and "1/n", the operator is not unitary, that is, the adjoint is not the inverse. To invert the operator, simply use `Op \ y`.

---

real
[`bool`, optional] Model to which fft is applied has real numbers (`True`) or not (`False`). Used to enforce that the output of adjoint of a real model is real.

ifftshift_before
[bool, optional] New in version 1.17.0.

Apply ifftshift (`True`) or not (`False`) to model vector (before FFT). Consider using this option when the model vector's respective axis is symmetric with respect to the zero value sample. This will shift the zero value sample to coincide with the zero index sample. With such an arrangement, FFT will not introduce a sample-dependent phase-shift when compared to the continuous Fourier Transform. Defaults to not applying ifftshift.

fftshift_after
[bool, optional] New in version 1.17.0.

Apply fftshift (`True`) or not (`False`) to data vector (after FFT). Consider using this option when you require frequencies to be arranged naturally, from negative to positive. When not applying fftshift after FFT, frequencies are arranged from zero to largest positive, and then from negative Nyquist to the frequency bin before zero.

engine
[str, optional] Engine used for fft computation (`numpy`, `fftw`, or `scipy`). Choose `numpy` when working with cupy arrays.

Note: Since version 1.17.0, accepts "scipy".

---

dtype
[str, optional] Type of elements in input array. Note that the `dtype` of the operator is the corresponding complex type even when a real type is provided. In addition, note that neither the NumPy nor the FFTW backends supports returning `dtype` different than `complex128`. As such, when using either backend, arrays will be force-casted to types corresponding to the supplied `dtype`. The SciPy backend supports all precisions natively. Under all backends, when a real `dtype` is supplied, a real result will be enforced on the result of the `rmatvec` and the input of the `matvec`.

name
[str, optional] New in version 2.0.0.

Name of operator (to be used by *pylops.utils.describe.describe*)

****kwargs_fftw**
Arbitrary keyword arguments for `pyfftw.FTTW`

Raises

ValueError

- If `dims` is provided and `axis` is bigger than `len(dims)`.
- If `norm` is not one of "ortho", "none", or "1/n".

NotImplementedError
If `engine` is neither `numpy`, `fftw`, nor `scipy`.

See also:

FFT2D
Two-dimensional FFT

FFTND
N-dimensional FFT

Notes

The FFT operator (using `norm="ortho"`) applies the forward Fourier transform to a signal $d(t)$ in forward mode:

$$D(f) = F(d) = \frac{1}{\sqrt{N_F}} \int_{-\infty}^{\infty} d(t) e^{-j2\pi ft} \, \mathrm{d}t$$

Similarly, the inverse Fourier transform is applied to the Fourier spectrum $D(f)$ in adjoint mode:

$$d(t) = F^{-1}(D) = \frac{1}{\sqrt{N_F}} \int_{-\infty}^{\infty} D(f) e^{j2\pi ft} \, \mathrm{d}f$$

where N_F is the number of samples in the Fourier domain `nfft`. Both operators are effectively discretized and solved by a fast iterative algorithm known as Fast Fourier Transform. Note that the FFT operator (using `norm="ortho"`) is a special operator in that the adjoint is also the inverse of the forward mode. For other norms, this does not hold (see `norm` help). However, for any norm, the Fourier transform is Hermitian for real input signals.

Attributes

dimsd
[`tuple`] Shape of the array after the forward, but before linearization.

For example, `y_reshaped = (Op * x.ravel()).reshape(Op.dimsd)`.

f
[`numpy.ndarray`] Discrete Fourier Transform sample frequencies

real
[`bool`] When `True`, uses `rfft/irfft`

rdtype
[`bool`] Expected input type to the forward

cdtype
[`bool`] Output type of the forward. Complex equivalent to `rdtype`.

shape
[`tuple`] Operator shape

clinear
[`bool`] New in version 1.17.0.

Operator is complex-linear. Is false when either `real=True` or when `dtype` is not a complex type.

explicit
[`bool`] Operator contains a matrix that can be solved explicitly (`True`) or not (`False`)

Examples using `pylops.signalprocessing.FFT`

- *1D, 2D and 3D Sliding*
- *Fourier Transform*
- *Wavelets*
- *02. The Dot-Test*
- *03. Solvers*

- *03. Solvers (Advanced)*
- *04. Bayesian Inversion*

pylops.signalprocessing.FFT2D

pylops.signalprocessing.**FFT2D**(*dims*, *axes=(-2, -1)*, *nffts=None*, *sampling=1.0*, *norm='ortho'*, *real=False*, *ifftshift_before=False*, *fftshift_after=False*, *engine='numpy'*, *dtype='complex128'*, *name='F'*)

Two dimensional Fast-Fourier Transform.

Apply two dimensional Fast-Fourier Transform (FFT) to any pair of `axes` of a multi-dimensional array.

Using the default NumPy engine, the FFT operator is an overload to either the NumPy `numpy.fft.fft2` (or `numpy.fft.rfft2` for real models) in forward mode, and to `numpy.fft.ifft2` (or `numpy.fft.irfft2` for real models) in adjoint mode, or their CuPy equivalents. Alternatively, when the SciPy engine is chosen, the overloads are of `scipy.fft.fft2` (or `scipy.fft.rfft2` for real models) in forward mode, and to `scipy.fft.ifft2` (or `scipy.fft.irfft2` for real models) in adjoint mode.

When using `real=True`, the result of the forward is also multiplied by $\sqrt{2}$ for all frequency bins except zero and Nyquist, and the input of the adjoint is multiplied by $1/\sqrt{2}$ for the same frequencies.

For a real valued input signal, it is advised to use the flag `real=True` as it stores the values of the Fourier transform of the last axis in `axes` at positive frequencies only as values at negative frequencies are simply their complex conjugates.

Parameters

dims
[`tuple`] Number of samples for each dimension

axes
[`tuple`, optional] New in version 2.0.0.

Pair of axes along which FFT2D is applied

nffts
[`tuple` or `int`, optional] Number of samples in Fourier Transform for each axis in `axes`. In case only one dimension needs to be specified, use `None` for the other dimension in the tuple. An axis with `None` will use `dims[axis]` as `nfft`. When supplying a tuple, the length must be 2. When a single value is passed, it will be used for both axes. As such the default is equivalent to `nffts=(None, None)`.

sampling
[`tuple` or `float`, optional] Sampling steps for each axis in `axes`. When supplied a single value, it is used for both axes. Unlike `nffts`, any `None` will not be converted to the default value.

norm
[*{"ortho", "none", "1/n"}*, optional] New in version 1.17.0.

- "ortho": Scales forward and adjoint FFT transforms with $1/\sqrt{N_F}$, where N_F is the number of samples in the Fourier domain given by product of all elements of `nffts`.
- "none": Does not scale the forward or the adjoint FFT transforms.
- "1/n": Scales both the forward and adjoint FFT transforms by $1/N_F$.

Note: For "none" and "1/n", the operator is not unitary, that is, the adjoint is not the inverse. To invert the operator, simply use `Op \ y`.

real
[bool, optional] Model to which fft is applied has real numbers (`True`) or not (`False`). Used to enforce that the output of adjoint of a real model is real. Note that the real FFT is applied only to the first dimension to which the FFT2D operator is applied (last element of `axes`)

ifftshift_before
[tuple or bool, optional] Apply ifftshift (`True`) or not (`False`) to model vector (before FFT). Consider using this option when the model vector's respective axis is symmetric with respect to the zero value sample. This will shift the zero value sample to coincide with the zero index sample. With such an arrangement, FFT will not introduce a sample-dependent phase-shift when compared to the continuous Fourier Transform. When passing a single value, the shift will the same for every axis in `axes`. Pass a tuple to specify which dimensions are shifted.

fftshift_after
[tuple or bool, optional] Apply fftshift (`True`) or not (`False`) to data vector (after FFT). Consider using this option when you require frequencies to be arranged naturally, from negative to positive. When not applying fftshift after FFT, frequencies are arranged from zero to largest positive, and then from negative Nyquist to the frequency bin before zero. When passing a single value, the shift will the same for every axis in `axes`. Pass a tuple to specify which dimensions are shifted.

engine
[str, optional] New in version 1.17.0.

Engine used for fft computation (`numpy` or `scipy`).

dtype
[str, optional] Type of elements in input array. Note that the `dtype` of the operator is the corresponding complex type even when a real type is provided. In addition, note that the NumPy backend does not support returning `dtype` different than `complex128`. As such, when using the NumPy backend, arrays will be force-casted to types corresponding to the supplied `dtype`. The SciPy backend supports all precisions natively. Under both backends, when a real `dtype` is supplied, a real result will be enforced on the result of the `rmatvec` and the input of the `matvec`.

name
[str, optional] New in version 2.0.0.

Name of operator (to be used by *pylops.utils.describe.describe*)

Raises

ValueError

- If `dims` has less than two elements.
- If `axes` does not have exactly two elements.
- If `nffts` or `sampling` are not either a single value or a tuple with two elements.
- If `norm` is not one of "ortho", "none", or "1/n".

NotImplementedError
If `engine` is neither `numpy`, nor `scipy`.

See also:

FFT
One-dimensional FFT

FFTND
N-dimensional FFT

Notes

The FFT2D operator (using `norm="ortho"`) applies the two-dimensional forward Fourier transform to a signal $d(y, x)$ in forward mode:

$$D(k_y, k_x) = F(d) = \frac{1}{\sqrt{N_F}} \int\int_{-\infty}^{\infty} d(y, x) e^{-j2\pi k_y y} e^{-j2\pi k_x x} \, \mathrm{d}y \, \mathrm{d}x$$

Similarly, the two-dimensional inverse Fourier transform is applied to the Fourier spectrum $D(k_y, k_x)$ in adjoint mode:

$$d(y, x) = F^{-1}(D) = \frac{1}{\sqrt{N_F}} \int\int_{-\infty}^{\infty} D(k_y, k_x) e^{j2\pi k_y y} e^{j2\pi k_x x} \, \mathrm{d}k_y \, \mathrm{d}k_x$$

where N_F is the number of samples in the Fourier domain given by the product of the element of `nffts`. Both operators are effectively discretized and solved by a fast iterative algorithm known as Fast Fourier Transform. Note that the FFT2D operator (using `norm="ortho"`) is a special operator in that the adjoint is also the inverse of the forward mode. For other norms, this does not hold (see `norm` help). However, for any norm, the 2D Fourier transform is Hermitian for real input signals.

Attributes

dimsd
[`tuple`] Shape of the array after the forward, but before linearization.

For example, `y_reshaped = (Op * x.ravel()).reshape(Op.dimsd)`.

f1
[`numpy.ndarray`] Discrete Fourier Transform sample frequencies along `axes[0]`

f2
[`numpy.ndarray`] Discrete Fourier Transform sample frequencies along `axes[1]`

real
[`bool`] When `True`, uses `rfft2/irfft2`

rdtype
[`bool`] Expected input type to the forward

cdtype
[`bool`] Output type of the forward. Complex equivalent to `rdtype`.

shape
[`tuple`] Operator shape

clinear
[`bool`] New in version 1.17.0.

Operator is complex-linear. Is false when either `real=True` or when `dtype` is not a complex type.

explicit
[`bool`] Operator contains a matrix that can be solved explicitly (`True`) or not (`False`)

Examples using `pylops.signalprocessing.FFT2D`

- *Fourier Transform*
- *Patching*
- *Total Variation (TV) Regularization*
- *12. Seismic regularization*
- *14. Seismic wavefield decomposition*
- *18. Deblending*

pylops.signalprocessing.FFTND

`pylops.signalprocessing.FFTND`(*dims*, *axes=(-3, -2, -1)*, *nffts=None*, *sampling=1.0*, *norm='ortho'*, *real=False*, *ifftshift_before=False*, *fftshift_after=False*, *engine='scipy'*, *dtype='complex128'*, *name='F'*)

N-dimensional Fast-Fourier Transform.

Apply N-dimensional Fast-Fourier Transform (FFT) to any n `axes` of a multi-dimensional array.

Using the default NumPy engine, the FFT operator is an overload to either the NumPy `numpy.fft.fftn` (or `numpy.fft.rfftn` for real models) in forward mode, and to `numpy.fft.ifftn` (or `numpy.fft.irfftn` for real models) in adjoint mode, or their CuPy equivalents. Alternatively, when the SciPy engine is chosen, the overloads are of `scipy.fft.fftn` (or `scipy.fft.rfftn` for real models) in forward mode, and to `scipy.fft.ifftn` (or `scipy.fft.irfftn` for real models) in adjoint mode.

When using `real=True`, the result of the forward is also multiplied by $\sqrt{2}$ for all frequency bins except zero and Nyquist along the last `axes`, and the input of the adjoint is multiplied by $1/\sqrt{2}$ for the same frequencies.

For a real valued input signal, it is advised to use the flag `real=True` as it stores the values of the Fourier transform of the last axis in `axes` at positive frequencies only as values at negative frequencies are simply their complex conjugates.

Parameters

dims
[`tuple`] Number of samples for each dimension

axes
[`int`, optional] New in version 2.0.0.

Axes (or axis) along which FFTND is applied

nffts
[`tuple` or `int`, optional] Number of samples in Fourier Transform for each axis in `axes`. In case only one dimension needs to be specified, use `None` for the other dimension in the tuple. An axis with `None` will use `dims[axis]` as `nfft`. When supplying a tuple, the length must agree with that of `axes`. When a single value is passed, it will be used for all `axes`. As such the default is equivalent to ``nffts=(None, ..., None)``.

sampling
[`tuple` or `float`, optional] Sampling steps for each direction. When supplied a single value, it is used for all directions. Unlike `nffts`, any `None` will not be converted to the default value.

norm
[*{"ortho", "none", "1/n"}*, optional] New in version 1.17.0.

- "ortho": Scales forward and adjoint FFT transforms with $1/\sqrt{N_F}$, where N_F is the number of samples in the Fourier domain given by product of all elements of `nffts`.
- "none": Does not scale the forward or the adjoint FFT transforms.
- "1/n": Scales both the forward and adjoint FFT transforms by $1/N_F$.

Note: For "none" and "1/n", the operator is not unitary, that is, the adjoint is not the inverse. To invert the operator, simply use `Op \ y`.

---

real
[bool, optional] Model to which fft is applied has real numbers (`True`) or not (`False`). Used to enforce that the output of adjoint of a real model is real. Note that the real FFT is applied only to the first dimension to which the FFTND operator is applied (last element of `axes`)

ifftshift_before
[tuple or bool, optional] New in version 1.17.0.

Apply ifftshift (`True`) or not (`False`) to model vector (before FFT). Consider using this option when the model vector's respective axis is symmetric with respect to the zero value sample. This will shift the zero value sample to coincide with the zero index sample. With such an arrangement, FFT will not introduce a sample-dependent phase-shift when compared to the continuous Fourier Transform. When passing a single value, the shift will the same for every direction. Pass a tuple to specify which dimensions are shifted.

fftshift_after
[tuple or bool, optional] New in version 1.17.0.

Apply fftshift (`True`) or not (`False`) to data vector (after FFT). Consider using this option when you require frequencies to be arranged naturally, from negative to positive. When not applying fftshift after FFT, frequencies are arranged from zero to largest positive, and then from negative Nyquist to the frequency bin before zero. When passing a single value, the shift will the same for every direction. Pass a tuple to specify which dimensions are shifted.

engine
[str, optional] New in version 1.17.0.

Engine used for fft computation (`numpy` or `scipy`).

dtype
[str, optional] Type of elements in input array. Note that the `dtype` of the operator is the corresponding complex type even when a real type is provided. In addition, note that the NumPy backend does not support returning `dtype` different than `complex128`. As such, when using the NumPy backend, arrays will be force-cast to types corresponding to the supplied `dtype`. The SciPy backend supports all precisions natively. Under both backends, when a real `dtype` is supplied, a real result will be enforced on the result of the `rmatvec` and the input of the `matvec`.

name
[str, optional] New in version 2.0.0.

Name of operator (to be used by *pylops.utils.describe.describe*)

Raises

ValueError

- If `nffts` or `sampling` are not either a single value or tuple with the same dimension `axes`.

- If `norm` is not one of "ortho", "none", or "1/n".

NotImplementedError
If `engine` is neither `numpy`, nor `scipy`.

See also:

FFT
One-dimensional FFT

FFT2D
Two-dimensional FFT

Notes

The FFTND operator (using `norm="ortho"`) applies the N-dimensional forward Fourier transform to a multi-dimensional array. Considering an N-dimensional signal $d(x_1, \ldots, x_N)$. The FFTND in forward mode is:

$$D(k_1, \ldots, k_N) = F(d) = \frac{1}{\sqrt{N_F}} \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} d(x_1, \ldots, x_N) e^{-j2\pi k_1 x_1} \cdots e^{-j2\pi k_N x_N} \, \mathrm{d}x_1 \cdots \mathrm{d}x_N$$

Similarly, the three-dimensional inverse Fourier transform is applied to the Fourier spectrum $D(k_z, k_y, k_x)$ in adjoint mode:

$$d(x_1, \ldots, x_N) = F^{-1}(D) = \frac{1}{\sqrt{N_F}} \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} D(k_1, \ldots, k_N) e^{-j2\pi k_1 x_1} \cdots e^{-j2\pi k_N x_N} \, \mathrm{d}k_1 \cdots \mathrm{d}k_N$$

where N_F is the number of samples in the Fourier domain given by the product of the element of `nffts`. Both operators are effectively discretized and solved by a fast iterative algorithm known as Fast Fourier Transform. Note that the FFTND operator (using `norm="ortho"`) is a special operator in that the adjoint is also the inverse of the forward mode. For other norms, this does not hold (see `norm` help). However, for any norm, the N-dimensional Fourier transform is Hermitian for real input signals.

Attributes

dimsd
[`tuple`] Shape of the array after the forward, but before linearization.

For example, `y_reshaped = (Op * x.ravel()).reshape(Op.dimsd)`.

fs
[`tuple`] Each element of the tuple corresponds to the Discrete Fourier Transform sample frequencies along the respective direction given by `axes`.

real
[`bool`] When `True`, uses `rfftn/irfftn`

rdtype
[`bool`] Expected input type to the forward

cdtype
[`bool`] Output type of the forward. Complex equivalent to `rdtype`.

shape
[`tuple`] Operator shape

clinear
[`bool`] New in version 1.17.0.

Operator is complex-linear. Is false when either `real=True` or when `dtype` is not a complex type.

explicit
[`bool`] Operator contains a matrix that can be solved explicitly (`True`) or not (`False`)

Examples using `pylops.signalprocessing.FFTND`

- *Fourier Transform*
- *Patching*

pylops.signalprocessing.Shift

`pylops.signalprocessing.Shift`(*dims*, *shift*, *axis=-1*, *nfft=None*, *sampling=1.0*, *real=False*, *engine='numpy'*, *dtype='complex128'*, *name='S'*, ***kwargs_fftw*)

Shift operator

Apply fractional shift in the frequency domain along an `axis` of a multi-dimensional array of size `dims`.

Parameters

dims
[`tuple`] Number of samples for each dimension

shift
[`float` or `numpy.ndarray`] Fractional shift to apply in the same unit as `sampling`. For multi-dimensional inputs, this can be a scalar to apply to every trace along the chosen axis or an array of shifts to be applied to each trace.

axis
[`int`, optional] New in version 2.0.0.

Axis along which shift is applied

nfft
[`int`, optional] Number of samples in Fourier Transform (same as input if `nfft=None`)

sampling
[`float`, optional] Sampling step Δt.

real
[`bool`, optional] Model to which fft is applied has real numbers (`True`) or not (`False`). Used to enforce that the output of adjoint of a real model is real.

engine
[`str`, optional] Engine used for fft computation (`numpy`, `scipy`, or `fftw`). Choose `numpy` when working with CuPy arrays.

dtype
[`str`, optional] Type of elements in input array.

name
[`str`, optional] New in version 2.0.0.

Name of operator (to be used by *`pylops.utils.describe.describe`*)

****kwargs_fftw**
Arbitrary keyword arguments for `pyfftw.FTTW`

Raises

ValueError
If `dims` is provided and `axis` is bigger than `len(dims)`

NotImplementedError
If `engine` is neither `numpy`, `scipy`, nor `fftw`

Notes

The Shift operator applies the forward Fourier transform, an element-wise complex scaling, and inverse fourier transform

$$\mathbf{y} = \mathbf{F}^{-1}\mathbf{S}\mathbf{F}\mathbf{x}$$

Here $\mathbf{S}$ is a diagonal operator that scales the Fourier transformed input by $e^{-j2\pi f t_S}$, where t_S is the chosen `shift`.

Attributes

shape
[`tuple`] Operator shape

explicit
[`bool`] Operator contains a matrix that can be solved explicitly (`True`) or not (`False`)

Examples using `pylops.signalprocessing.Shift`

- *Shift*
- *18. Deblending*

pylops.signalprocessing.DWT

class pylops.signalprocessing.**DWT**(**args*, ***kwargs*)

One dimensional Wavelet operator.

Apply 1D-Wavelet Transform along an `axis` of a multi-dimensional array of size `dims`.

Note that the Wavelet operator is an overload of the `pywt` implementation of the wavelet transform. Refer to https://pywavelets.readthedocs.io for a detailed description of the input parameters.

Parameters

dims
[`int` or `tuple`] Number of samples for each dimension

axis
[`int`, optional] New in version 2.0.0.

Axis along which DWT is applied

wavelet
[`str`, optional] Name of wavelet type. Use `pywt.wavelist(kind='discrete')` for a list of available wavelets.

level
[`int`, optional] Number of scaling levels (must be >=0).

dtype
[str, optional] Type of elements in input array.

name
[str, optional] New in version 2.0.0.

Name of operator (to be used by *pylops.utils.describe.describe*)

Raises

ModuleNotFoundError
If `pywt` is not installed

ValueError
If `wavelet` does not belong to `pywt.families`

Notes

The Wavelet operator applies the multilevel Discrete Wavelet Transform (DWT) in forward mode and the multilevel Inverse Discrete Wavelet Transform (IDWT) in adjoint mode.

Wavelet transforms can be used to compress signals and present a key advantage over Fourier transforms in that they captures both frequency and location information in time. Consider using this operator as sparsifying transform when using L1 solvers.

Attributes

shape
[tuple] Operator shape

explicit
[bool] Operator contains a matrix that can be solved explicitly (`True`) or not (`False`)

Methods

`__init__`(dims[, axis, wavelet, level, ...])	Initialize this LinearOperator.
`adjoint`()	Hermitian adjoint.
`apply_columns`(cols)	Apply subset of columns of operator
`cond`([uselobpcg])	Condition number of linear operator.
`conj`()	Complex conjugate operator
`div`(y[, niter, densesolver])	Solve the linear problem $\mathbf{y} = \mathbf{A}\mathbf{x}$.
`dot`(x)	Matrix-matrix or matrix-vector multiplication.
`eigs`([neigs, symmetric, niter, uselobpcg])	Most significant eigenvalues of linear operator.
`matmat`(X)	Matrix-matrix multiplication.
`matvec`(x)	Matrix-vector multiplication.
`reset_count`()	Reset counters
`rmatmat`(X)	Matrix-matrix multiplication.
`rmatvec`(x)	Adjoint matrix-vector multiplication.
`todense`([backend])	Return dense matrix.
`toimag`([forw, adj])	Imag operator
`toreal`([forw, adj])	Real operator
`tosparse`()	Return sparse matrix.
`trace`([neval, method, backend])	Trace of linear operator.
`transpose`()	Transpose this linear operator.

Examples using `pylops.signalprocessing.DWT`

- *Wavelet transform*

pylops.signalprocessing.DWT2D

class pylops.signalprocessing.**DWT2D**(**args*, ***kwargs*)

Two dimensional Wavelet operator.

Apply 2D-Wavelet Transform along two `axes` of a multi-dimensional array of size `dims`.

Note that the Wavelet operator is an overload of the `pywt` implementation of the wavelet transform. Refer to https://pywavelets.readthedocs.io for a detailed description of the input parameters.

Parameters

dims
[`tuple`] Number of samples for each dimension

axes
[`int`, optional] New in version 2.0.0.

Axis along which DWT2D is applied

wavelet
[`str`, optional] Name of wavelet type. Use `pywt.wavelist(kind='discrete')` for a list of available wavelets.

level
[`int`, optional] Number of scaling levels (must be >=0).

dtype
[`str`, optional] Type of elements in input array.

name
[`str`, optional] New in version 2.0.0.

Name of operator (to be used by *`pylops.utils.describe.describe`*)

Raises

ModuleNotFoundError
If `pywt` is not installed

ValueError
If `wavelet` does not belong to `pywt.families`

Notes

The Wavelet operator applies the 2-dimensional multilevel Discrete Wavelet Transform (DWT2) in forward mode and the 2-dimensional multilevel Inverse Discrete Wavelet Transform (IDWT2) in adjoint mode.

Attributes

shape
[`tuple`] Operator shape

explicit
[`bool`] Operator contains a matrix that can be solved explicitly (`True`) or not (`False`)

Methods

`__init__`(dims[, axes, wavelet, level, ...])	Initialize this LinearOperator.
`adjoint`()	Hermitian adjoint.
`apply_columns`(cols)	Apply subset of columns of operator
`cond`([uselobpcg])	Condition number of linear operator.
`conj`()	Complex conjugate operator
`div`(y[, niter, densesolver])	Solve the linear problem $\mathbf{y} = \mathbf{A}\mathbf{x}$.
`dot`(x)	Matrix-matrix or matrix-vector multiplication.
`eigs`([neigs, symmetric, niter, uselobpcg])	Most significant eigenvalues of linear operator.
`matmat`(X)	Matrix-matrix multiplication.
`matvec`(x)	Matrix-vector multiplication.
`reset_count`()	Reset counters
`rmatmat`(X)	Matrix-matrix multiplication.
`rmatvec`(x)	Adjoint matrix-vector multiplication.
`todense`([backend])	Return dense matrix.
`toimag`([forw, adj])	Imag operator
`toreal`([forw, adj])	Real operator
`tosparse`()	Return sparse matrix.
`trace`([neval, method, backend])	Trace of linear operator.
`transpose`()	Transpose this linear operator.

Examples using `pylops.signalprocessing.DWT2D`

- *Wavelet transform*
- *05. Image deblurring*

pylops.signalprocessing.Seislet

class pylops.signalprocessing.**Seislet**(**args*, ***kwargs*)

Two dimensional Seislet operator.

Apply 2D-Seislet Transform to an input array given an estimate of its local `slopes`. In forward mode, the input array is reshaped into a two-dimensional array of size $n_x \times n_t$ and the transform is performed along the first (spatial) axis (see Notes for more details).

Parameters

slopes
[`numpy.ndarray`] Slope field of size $n_x \times n_t$

sampling
[`tuple`, optional] Sampling steps in x- and t-axis.

level
[`int`, optional] Number of scaling levels (must be >=0).

kind
[`str`, optional] Basis function used for predict and update steps: `haar` or `linear`.

inv
[`int`, optional] Apply inverse transform when invoking the adjoint (`True`) or not (`False`). Note that in some scenario it may be more appropriate to use the exact inverse as adjoint of

the Seislet operator even if this is not an orthogonal operator and the dot-test would not be satisfied (see Notes for details). Otherwise, the user can access the inverse directly as method of this class.

dtype
[str, optional] Type of elements in input array.

name
[str, optional] New in version 2.0.0.

Name of operator (to be used by *pylops.utils.describe.describe*)

Raises

NotImplementedError
If `kind` is different from haar or linear

ValueError
If `sampling` has more or less than two elements.

Notes

The Seislet transform [1] is implemented using the lifting scheme.

In its simplest form (i.e., corresponding to the Haar basis function for the Wavelet transform) the input dataset is separated into even ($\mathbf{e}$) and odd ($\mathbf{o}$) traces. Even traces are used to forward predict the odd traces using local slopes and the new odd traces (also referred to as residual) is defined as:

$$\mathbf{o}^{i+1} = \mathbf{r}^i = \mathbf{o}^i - P(\mathbf{e}^i)$$

where $P = P^+$ is the slope-based forward prediction operator (which is here implemented as a sinc-based resampling). The residual is then updated and summed to the even traces to obtain the new even traces (also referred to as coarse representation):

$$\mathbf{e}^{i+1} = \mathbf{c}^i = \mathbf{e}^i + U(\mathbf{o}^{i+1})$$

where $U = P^-/2$ is the update operator which performs a slope-based backward prediction. At this point $\mathbf{e}^{i+1}$ becomes the new data and the procedure is repeated *level* times (at maximum until $\mathbf{e}^{i+1}$ is a single trace. The Seislet transform is effectively composed of all residuals and the coarsest data representation.

In the inverse transform the two operations are reverted. Starting from the coarsest scale data representation $\mathbf{c}$ and residual $\mathbf{r}$, the even and odd parts of the previous scale are reconstructed as:

$$\mathbf{e}^i = \mathbf{c}^i - U(\mathbf{r}^i) = \mathbf{e}^{i+1} - U(\mathbf{o}^{i+1})$$

and:

$$\mathbf{o}^i = \mathbf{r}^i + P(\mathbf{e}^i) = \mathbf{o}^{i+1} + P(\mathbf{e}^i)$$

A new data is formed by interleaving $\mathbf{e}^i$ and $\mathbf{o}^i$ and the procedure repeated until the new data as the same number of traces as the original one.

Finally the adjoint operator can be easily derived by writing the lifting scheme in a matricial form:

$$\begin{bmatrix} \mathbf{r}_1 \\ \mathbf{r}_2 \\ \vdots \\ \mathbf{r}_N \\ \mathbf{c}_1 \\ \mathbf{c}_2 \\ \vdots \\ \mathbf{c}_N \end{bmatrix} = \begin{bmatrix} \mathbf{I} & \mathbf{0} & \dots & \mathbf{0} & -\mathbf{P} & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & \mathbf{I} & \dots & \mathbf{0} & \mathbf{0} & -\mathbf{P} & \dots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \dots & \mathbf{I} & \mathbf{0} & \mathbf{0} & \dots & -\mathbf{P} \\ \mathbf{U} & \mathbf{0} & \dots & \mathbf{0} & \mathbf{I}-\mathbf{UP} & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & \mathbf{U} & \dots & \mathbf{0} & \mathbf{0} & \mathbf{I}-\mathbf{UP} & \dots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \dots & \mathbf{U} & \mathbf{0} & \mathbf{0} & \dots & \mathbf{I}-\mathbf{UP} \end{bmatrix} \begin{bmatrix} \mathbf{o}_1 \\ \mathbf{o}_2 \\ \vdots \\ \mathbf{o}_N \\ \mathbf{e}_1 \\ \mathbf{e}_2 \\ \vdots \\ \mathbf{e}_N \end{bmatrix}$$

Transposing the operator leads to:

$$\begin{bmatrix} \mathbf{o}_1 \\ \mathbf{o}_2 \\ \vdots \\ \mathbf{o}_N \\ \mathbf{e}_1 \\ \mathbf{e}_2 \\ \vdots \\ \mathbf{e}_N \end{bmatrix} = \begin{bmatrix} \mathbf{I} & \mathbf{0} & \dots & \mathbf{0} & -\mathbf{U}^\mathbf{T} & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & \mathbf{I} & \dots & \mathbf{0} & \mathbf{0} & -\mathbf{U}^\mathbf{T} & \dots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \dots & \mathbf{I} & \mathbf{0} & \mathbf{0} & \dots & -\mathbf{U}^\mathbf{T} \\ \mathbf{P}^\mathbf{T} & \mathbf{0} & \dots & \mathbf{0} & \mathbf{I}-\mathbf{P}^\mathbf{T}\mathbf{U}^\mathbf{T} & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & \mathbf{P}^\mathbf{T} & \dots & \mathbf{0} & \mathbf{0} & \mathbf{I}-\mathbf{P}^\mathbf{T}\mathbf{U}^\mathbf{T} & \dots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \dots & \mathbf{P}^\mathbf{T} & \mathbf{0} & \mathbf{0} & \dots & \mathbf{I}-\mathbf{P}^\mathbf{T}\mathbf{U}^\mathbf{T} \end{bmatrix} \begin{bmatrix} \mathbf{r}_1 \\ \mathbf{r}_2 \\ \vdots \\ \mathbf{r}_N \\ \mathbf{c}_1 \\ \mathbf{c}_2 \\ \vdots \\ \mathbf{c}_N \end{bmatrix}$$

which can be written more easily in the following two steps:

$$\mathbf{o} = \mathbf{r} + \mathbf{U}^H\mathbf{c}$$

and:

$$\mathbf{e} = \mathbf{c} - \mathbf{P}^H(\mathbf{r} + \mathbf{U}^H(\mathbf{c})) = \mathbf{c} - \mathbf{P}^H\mathbf{o}$$

Similar derivations follow for more complex wavelet bases.

Attributes

shape
[tuple] Operator shape

explicit
[bool] Operator contains a matrix that can be solved explicitly (True) or not (False)

Methods

`__init__`(slopes[, sampling, level, kind, ...])	Initialize this LinearOperator.
`adjoint`()	Hermitian adjoint.
`apply_columns`(cols)	Apply subset of columns of operator
`cond`([uselobpcg])	Condition number of linear operator.
`conj`()	Complex conjugate operator
`div`(y[, niter, densesolver])	Solve the linear problem $\mathbf{y} = \mathbf{A}\mathbf{x}$.
`dot`(x)	Matrix-matrix or matrix-vector multiplication.
`eigs`([neigs, symmetric, niter, uselobpcg])	Most significant eigenvalues of linear operator.
`inverse`(x)	
`matmat`(X)	Matrix-matrix multiplication.
`matvec`(x)	Matrix-vector multiplication.
`reset_count`()	Reset counters
`rmatmat`(X)	Matrix-matrix multiplication.
`rmatvec`(x)	Adjoint matrix-vector multiplication.
`todense`([backend])	Return dense matrix.
`toimag`([forw, adj])	Imag operator
`toreal`([forw, adj])	Real operator
`tosparse`()	Return sparse matrix.
`trace`([neval, method, backend])	Trace of linear operator.
`transpose`()	Transpose this linear operator.

Examples using `pylops.signalprocessing.Seislet`

- *Seislet transform*
- *Slope estimation via Structure Tensor algorithm*

pylops.signalprocessing.Radon2D

`pylops.signalprocessing.`**`Radon2D`**(*taxis*, *haxis*, *pxaxis*, *kind='linear'*, *centeredh=True*, *interp=True*, *onthefly=False*, *engine='numpy'*, *dtype='float64'*, *name='R'*)

Two dimensional Radon transform.

Apply two dimensional Radon forward (and adjoint) transform to a 2-dimensional array of size $[n_{p_x} \times n_t]$ (and $[n_x \times n_t]$).

In forward mode this entails to spreading the model vector along parametric curves (lines, parabolas, or hyperbolas depending on the choice of `kind`), while stacking values in the data vector along the same parametric curves is performed in adjoint mode.

Parameters

taxis
[`np.ndarray`] Time axis

haxis
[`np.ndarray`] Spatial axis

pxaxis
[`np.ndarray`] Axis of scanning variable p_x of parametric curve

kind
[str, optional] Curve to be used for stacking/spreading (`linear`, `parabolic`, and `hyperbolic` are currently supported) or a function that takes (x, t_0, p_x) as input and returns t as output

centeredh
[bool, optional] Assume centered spatial axis (`True`) or not (`False`). If `True` the original `haxis` is ignored and a new axis is created.

interp
[bool, optional] Apply linear interpolation (`True`) or nearest interpolation (`False`) during stacking/spreading along parametric curve

onthefly
[bool, optional] Compute stacking parametric curves on-the-fly as part of forward and adjoint modelling (`True`) or at initialization and store them in look-up table (`False`). Using a look-up table is computationally more efficient but increases the memory burden

engine
[str, optional] Engine used for computation (`numpy` or `numba`)

dtype
[str, optional] Type of elements in input array.

name
[str, optional] New in version 2.0.0.

Name of operator (to be used by *pylops.utils.describe.describe*)

Returns

r2op
[*pylops.LinearOperator*] Radon operator

Raises

KeyError
If `engine` is neither `numpy` nor `numba`

NotImplementedError
If `kind` is not `linear`, `parabolic`, or `hyperbolic`

See also:

pylops.signalprocessing.Radon3D
Three dimensional Radon transform

pylops.Spread
Spread operator

Notes

The Radon2D operator applies the following linear transform in adjoint mode to the data after reshaping it into a 2-dimensional array of size $[n_x \times n_t]$ in adjoint mode:

$$m(p_x, t_0) = \int d(x, t = f(p_x, x, t))\, \mathrm{d}x$$

where $f(p_x, x, t) = t_0 + p_x x$ where $p_x = \sin(\theta)/v$ in linear mode, $f(p_x, x, t) = t_0 + p_x x^2$ in parabolic mode, and $f(p_x, x, t) = \sqrt{t_0^2 + x^2/p_x^2}$ in hyperbolic mode. Note that internally the p_x axis will be normalized by

the ratio of the spatial and time axes and used alongside unitless axes. Whilst this makes the linear mode fully unitless, users are required to apply additional scalings to the p_x axis for other relationships:

- p_x should be pre-multipled by d_x for the parabolic relationship;
- p_x should be pre-multipled by $(d_t/d_x)^2$ for the hyperbolic relationship.

As the adjoint operator can be interpreted as a repeated summation of sets of elements of the model vector along chosen parametric curves, the forward is implemented as spreading of values in the data vector along the same parametric curves. This operator is actually a thin wrapper around the `pylops.Spread` operator.

Examples using `pylops.signalprocessing.Radon2D`

- *1D, 2D and 3D Sliding*
- *Chirp Radon Transform*
- *Radon Transform*
- *Spread How-to*
- *11. Radon filtering*
- *12. Seismic regularization*
- *16. CT Scan Imaging*

pylops.signalprocessing.Radon3D

pylops.signalprocessing.**Radon3D**(*taxis*, *hyaxis*, *hxaxis*, *pyaxis*, *pxaxis*, *kind='linear'*, *centeredh=True*, *interp=True*, *onthefly=False*, *engine='numpy'*, *dtype='float64'*, *name='R'*)

Three dimensional Radon transform.

Apply three dimensional Radon forward (and adjoint) transform to a 3-dimensional array of size $[n_{p_y} \times n_{p_x} \times n_t]$ (and $[n_y \times n_x \times n_t]$).

In forward mode this entails to spreading the model vector along parametric curves (lines, parabolas, or hyperbolas depending on the choice of `kind`), while stacking values in the data vector along the same parametric curves is performed in adjoint mode.

Parameters

taxis
[`np.ndarray`] Time axis

hxaxis
[`np.ndarray`] Fast patial axis

hyaxis
[`np.ndarray`] Slow spatial axis

pyaxis
[`np.ndarray`] Axis of scanning variable p_y of parametric curve

pxaxis
[`np.ndarray`] Axis of scanning variable p_x of parametric curve

kind
[`str`, optional] Curve to be used for stacking/spreading (`linear`, `parabolic`, and `hyperbolic` are currently supported)

centeredh
[bool, optional] Assume centered spatial axis (`True`) or not (`False`). If `True` the original `haxis` is ignored and a new axis is created.

interp
[bool, optional] Apply linear interpolation (`True`) or nearest interpolation (`False`) during stacking/spreading along parametric curve

onthefly
[bool, optional] Compute stacking parametric curves on-the-fly as part of forward and adjoint modelling (`True`) or at initialization and store them in look-up table (`False`). Using a look-up table is computationally more efficient but increases the memory burden

engine
[str, optional] Engine used for computation (`numpy` or `numba`)

dtype
[str, optional] Type of elements in input array.

name
[str, optional] New in version 2.0.0.

Name of operator (to be used by *pylops.utils.describe.describe*)

Returns

r3op
[*pylops.LinearOperator*] Radon operator

Raises

KeyError
If `engine` is neither `numpy` nor `numba`

NotImplementedError
If `kind` is not `linear`, `parabolic`, or `hyperbolic`

See also:

pylops.signalprocessing.Radon2D
Two dimensional Radon transform

pylops.Spread
Spread operator

Notes

The Radon3D operator applies the following linear transform in adjoint mode to the data after reshaping it into a 3-dimensional array of size $[n_y \times n_x \times n_t]$ in adjoint mode:

$$m(p_y, p_x, t_0) = \int d(y, x, t = f(p_y, p_x, y, x, t))\, \mathrm{d}x\, \mathrm{d}y$$

where $f(p_y, p_x, y, x, t) = t_0 + p_y y + p_x x$ in linear mode, $f(p_y, p_x, y, x, t) = t_0 + p_y y^2 + p_x x^2$ in parabolic mode, and $f(p_y, p_x, y, x, t) = \sqrt{t_0^2 + y^2/p_y^2 + x^2/p_x^2}$ in hyperbolic mode. Note that internally the p_x and p_y axes will be normalized by the ratio of the spatial and time axes and used alongside unitless axes. Whilst this makes the linear mode fully unitless, users are required to apply additional scalings to the p_x axis for other relationships

- p_x should be pre-multipled by d_x/d_y for the parabolic relationship;
- p_x should be pre-multipled by $(d_t/d_x)^2/(d_t/d_y)^2$ for the hyperbolic relationship.

As the adjoint operator can be interpreted as a repeated summation of sets of elements of the model vector along chosen parametric curves, the forward is implemented as spreading of values in the data vector along the same parametric curves. This operator is actually a thin wrapper around the *pylops.Spread* operator.

Examples using `pylops.signalprocessing.Radon3D`

- *1D, 2D and 3D Sliding*
- *Chirp Radon Transform*
- *Radon Transform*
- *Spread How-to*

pylops.signalprocessing.ChirpRadon2D

class pylops.signalprocessing.**ChirpRadon2D**(**args*, ***kwargs*)

2D Chirp Radon transform

Apply Radon forward (and adjoint) transform using Fast Fourier Transform and Chirp functions to a 2-dimensional array of size $[n_x \times n_t]$ (both in forward and adjoint mode).

Note that forward and adjoint are swapped compared to the time-space implementation in *pylops.signalprocessing.Radon2D* and a direct *inverse* method is also available for this implementation.

Parameters

taxis
[np.ndarray] Time axis

haxis
[np.ndarray] Spatial axis

pmax
[np.ndarray] Maximum slope defined as $\tan$ of maximum stacking angle in x direction $p_{\text{max}} = \tan(\alpha_{x,\text{max}})$. If one operates in terms of minimum velocity c_0, set $p_{x,\text{max}} = c_0\,\mathrm{d}y/\mathrm{d}t$.

dtype
[str, optional] Type of elements in input array.

name
[str, optional] New in version 2.0.0.

Name of operator (to be used by *pylops.utils.describe.describe*)

Notes

Refer to [1] for the theoretical and implementation details.

Attributes

shape
[tuple] Operator shape

explicit
[bool] Operator contains a matrix that can be solved explicitly (`True`) or not (`False`)

Methods

`__init__`(taxis, haxis, pmax[, dtype, name])	Initialize this LinearOperator.
`adjoint`()	Hermitian adjoint.
`apply_columns`(cols)	Apply subset of columns of operator
`cond`([uselobpcg])	Condition number of linear operator.
`conj`()	Complex conjugate operator
`div`(y[, niter, densesolver])	Solve the linear problem $\mathbf{y} = \mathbf{A}\mathbf{x}$.
`dot`(x)	Matrix-matrix or matrix-vector multiplication.
`eigs`([neigs, symmetric, niter, uselobpcg])	Most significant eigenvalues of linear operator.
`inverse`(x)	
`matmat`(X)	Matrix-matrix multiplication.
`matvec`(x)	Matrix-vector multiplication.
`reset_count`()	Reset counters
`rmatmat`(X)	Matrix-matrix multiplication.
`rmatvec`(x)	Adjoint matrix-vector multiplication.
`todense`([backend])	Return dense matrix.
`toimag`([forw, adj])	Imag operator
`toreal`([forw, adj])	Real operator
`tosparse`()	Return sparse matrix.
`trace`([neval, method, backend])	Trace of linear operator.
`transpose`()	Transpose this linear operator.

Examples using `pylops.signalprocessing.ChirpRadon2D`

- *Chirp Radon Transform*

pylops.signalprocessing.ChirpRadon3D

class pylops.signalprocessing.**ChirpRadon3D**(**args*, ***kwargs*)

3D Chirp Radon transform

Apply Radon forward (and adjoint) transform using Fast Fourier Transform and Chirp functions to a 3-dimensional array of size $[n_y \times n_x \times n_t]$ (both in forward and adjoint mode).

Note that forward and adjoint are swapped compared to the time-space implementation in *pylops.signalprocessing.Radon3D* and a direct *inverse* method is also available for this implementation.

Parameters

taxis
[np.ndarray] Time axis

hxaxis
[np.ndarray] Fast patial axis

hyaxis
[np.ndarray] Slow spatial axis

pmax
[np.ndarray] Two element array $(p_{y,\text{max}}, p_{x,\text{max}})$ of tan of maximum stacking angles in y and x directions $(\tan(\alpha_{y,\text{max}}), \tan(\alpha_{x,\text{max}}))$. If one operates in terms of minimum velocity c_0, then $p_{y,\text{max}} = c_0\,\mathrm{d}y/\mathrm{d}t$ and $p_{x,\text{max}} = c_0\,\mathrm{d}x/\mathrm{d}t$

engine
[str, optional] Engine used for fft computation (`numpy` or `fftw`)

dtype
[str, optional] Type of elements in input array.

name
[str, optional] New in version 2.0.0.

Name of operator (to be used by *pylops.utils.describe.describe*)

****kwargs_fftw**
Arbitrary keyword arguments for `pyfftw.FTTW` (reccomended: `flags=('FFTW_ESTIMATE', ), threads=NTHREADS`)

Notes

Refer to [1] for the theoretical and implementation details.

Attributes

shape
[tuple] Operator shape

explicit
[bool] Operator contains a matrix that can be solved explicitly (`True`) or not (`False`)

Methods

`__init__`(taxis, hyaxis, hxaxis, pmax[, ...])	Initialize this LinearOperator.
`adjoint`()	Hermitian adjoint.
`apply_columns`(cols)	Apply subset of columns of operator
`cond`([uselobpcg])	Condition number of linear operator.
`conj`()	Complex conjugate operator
`div`(y[, niter, densesolver])	Solve the linear problem $\mathbf{y} = \mathbf{A}\mathbf{x}$.
`dot`(x)	Matrix-matrix or matrix-vector multiplication.
`eigs`([neigs, symmetric, niter, uselobpcg])	Most significant eigenvalues of linear operator.
`inverse`(x)	
`matmat`(X)	Matrix-matrix multiplication.
`matvec`(x)	Matrix-vector multiplication.
`reset_count`()	Reset counters
`rmatmat`(X)	Matrix-matrix multiplication.
`rmatvec`(x)	Adjoint matrix-vector multiplication.
`todense`([backend])	Return dense matrix.
`toimag`([forw, adj])	Imag operator
`toreal`([forw, adj])	Real operator
`tosparse`()	Return sparse matrix.
`trace`([neval, method, backend])	Trace of linear operator.
`transpose`()	Transpose this linear operator.

Examples using `pylops.signalprocessing.ChirpRadon3D`

- *Chirp Radon Transform*

pylops.signalprocessing.Sliding1D

`pylops.signalprocessing.`**`Sliding1D`**(*Op*, *dim*, *dimd*, *nwin*, *nover*, *tapertype='hanning'*, *name='S'*)

1D Sliding transform operator.

Apply a transform operator `Op` repeatedly to slices of the model vector in forward mode and slices of the data vector in adjoint mode. More specifically, in forward mode the model vector is divided into slices, each slice is transformed, and slices are then recombined in a sliding window fashion.

This operator can be used to perform local, overlapping transforms (e.g., *pylops.signalprocessing.FFT*) on 1-dimensional arrays.

Note: The shape of the model has to be consistent with the number of windows for this operator not to return an error. As the number of windows depends directly on the choice of `nwin` and `nover`, it is recommended to first run `sliding1d_design` to obtain the corresponding `dims` and number of windows.

Warning: Depending on the choice of *nwin* and *nover* as well as the size of the data, sliding windows may not cover the entire data. The start and end indices of each window will be displayed and returned with running `sliding1d_design`.

Parameters

Op
[*pylops.LinearOperator*] Transform operator

dim
[`tuple`] Shape of 1-dimensional model.

dimd
[`tuple`] Shape of 1-dimensional data

nwin
[`int`] Number of samples of window

nover
[`int`] Number of samples of overlapping part of window

tapertype
[`str`, optional] Type of taper (`hanning`, `cosine`, `cosinesquare` or `None`)

name
[`str`, optional] New in version 2.0.0.

Name of operator (to be used by *pylops.utils.describe.describe*)

Returns

Sop
[*pylops.LinearOperator*] Sliding operator

Raises

ValueError
Identified number of windows is not consistent with provided model shape (`dims`).

Examples using `pylops.signalprocessing.Sliding1D`

- *1D, 2D and 3D Sliding*

pylops.signalprocessing.Sliding2D

`pylops.signalprocessing.`**`Sliding2D`**(*Op*, *dims*, *dimsd*, *nwin*, *nover*, *tapertype='hanning'*, *name='S'*)

2D Sliding transform operator.

Apply a transform operator `Op` repeatedly to slices of the model vector in forward mode and slices of the data vector in adjoint mode. More specifically, in forward mode the model vector is divided into slices, each slice is transformed, and slices are then recombined in a sliding window fashion. Both model and data are internally reshaped and interpreted as 2-dimensional arrays: each slice contains a portion of the array in the first dimension (and the entire second dimension).

This operator can be used to perform local, overlapping transforms (e.g., *pylops.signalprocessing.FFT2D* or *pylops.signalprocessing.Radon2D*) on 2-dimensional arrays.

Note: The shape of the model has to be consistent with the number of windows for this operator not to return an error. As the number of windows depends directly on the choice of `nwin` and `nover`, it is recommended to first run `sliding2d_design` to obtain the corresponding `dims` and number of windows.

Warning: Depending on the choice of *nwin* and *nover* as well as the size of the data, sliding windows may not cover the entire data. The start and end indices of each window will be displayed and returned with running `sliding2d_design`.

Parameters

Op
[*pylops.LinearOperator*] Transform operator

dims
[`tuple`] Shape of 2-dimensional model. Note that `dims[0]` should be multiple of the model size of the transform in the first dimension

dimsd
[`tuple`] Shape of 2-dimensional data

nwin
[`int`] Number of samples of window

nover
[`int`] Number of samples of overlapping part of window

tapertype
[`str`, optional] Type of taper (`hanning`, `cosine`, `cosinesquare` or `None`)

name
[`str`, optional] New in version 2.0.0.

Name of operator (to be used by *pylops.utils.describe.describe*)

Returns

Sop
[pylops.LinearOperator] Sliding operator

Raises

ValueError
Identified number of windows is not consistent with provided model shape (dims).

Examples using pylops.signalprocessing.Sliding2D

- *1D, 2D and 3D Sliding*
- *Patching*
- *12. Seismic regularization*

pylops.signalprocessing.Sliding3D

pylops.signalprocessing.**Sliding3D**(*Op*, *dims*, *dimsd*, *nwin*, *nover*, *nop*, *tapertype='hanning'*, *nproc=1*, *name='P'*)

3D Sliding transform operator.w

Apply a transform operator Op repeatedly to patches of the model vector in forward mode and patches of the data vector in adjoint mode. More specifically, in forward mode the model vector is divided into patches each patch is transformed, and patches are then recombined in a sliding window fashion. Both model and data should be 3-dimensional arrays in nature as they are internally reshaped and interpreted as 3-dimensional arrays. Each patch contains in fact a portion of the array in the first and second dimensions (and the entire third dimension).

This operator can be used to perform local, overlapping transforms (e.g., *pylops.signalprocessing.FFTND* or *pylops.signalprocessing.Radon3D*) of 3-dimensional arrays.

Note: The shape of the model has to be consistent with the number of windows for this operator not to return an error. As the number of windows depends directly on the choice of nwin and nover, it is recommended to first run sliding3d_design to obtain the corresponding dims and number of windows.

Warning: Depending on the choice of *nwin* and *nover* as well as the size of the data, sliding windows may not cover the entire data. The start and end indices of each window will be displayed and returned with running sliding3d_design.

Parameters

Op
[pylops.LinearOperator] Transform operator

dims
[tuple] Shape of 3-dimensional model. Note that dims[0] and dims[1] should be multiple of the model sizes of the transform in the first and second dimensions

dimsd
[tuple] Shape of 3-dimensional data

nwin
[tuple] Number of samples of window

nover
[tuple] Number of samples of overlapping part of window

nop
[tuple] Number of samples in axes of transformed domain associated to spatial axes in the data

tapertype
[str, optional] Type of taper (hanning, cosine, cosinesquare or None)

nproc
[int, optional] Number of processes used to evaluate the N operators in parallel using multiprocessing. If nproc=1, work in serial mode.

name
[str, optional] New in version 2.0.0.

Name of operator (to be used by *pylops.utils.describe.describe*)

Returns

Sop
[*pylops.LinearOperator*] Sliding operator

Raises

ValueError
Identified number of windows is not consistent with provided model shape (dims).

Examples using pylops.signalprocessing.Sliding3D

- *1D, 2D and 3D Sliding*

pylops.signalprocessing.Patch2D

pylops.signalprocessing.**Patch2D**(*Op*, *dims*, *dimsd*, *nwin*, *nover*, *nop*, *tapertype='hanning'*, *scalings=None*, *name='P'*)

2D Patch transform operator.

Apply a transform operator Op repeatedly to patches of the model vector in forward mode and patches of the data vector in adjoint mode. More specifically, in forward mode the model vector is divided into patches, each patch is transformed, and patches are then recombined together. Both model and data are internally reshaped and interpreted as 2-dimensional arrays: each patch contains a portion of the array in both the first and second dimension.

This operator can be used to perform local, overlapping transforms (e.g., *pylops.signalprocessing.FFT2D* or *pylops.signalprocessing.Radon2D*) on 2-dimensional arrays.

Note: The shape of the model has to be consistent with the number of windows for this operator not to return an error. As the number of windows depends directly on the choice of nwin and nover, it is recommended to first run patch2d_design to obtain the corresponding dims and number of windows.

Warning: Depending on the choice of *nwin* and *nover* as well as the size of the data, sliding windows may not cover the entire data. The start and end indices of each window will be displayed and returned with running `patch2d_design`.

Parameters

Op
[*pylops.LinearOperator*] Transform operator

dims
[`tuple`] Shape of 2-dimensional model. Note that `dims[0]` and `dims[1]` should be multiple of the model size of the transform in their respective dimensions

dimsd
[`tuple`] Shape of 2-dimensional data

nwin
[`tuple`] Number of samples of window

nover
[`tuple`] Number of samples of overlapping part of window

nop
[`tuple`] Size of model in the transformed domain

tapertype
[`str`, optional] Type of taper (`hanning`, `cosine`, `cosinesquare` or `None`)

scalings
[`tuple` or `list`, optional] Set of scalings to apply to each patch. If `None`, no scale will be applied

name
[`str`, optional] New in version 2.0.0.

Name of operator (to be used by *pylops.utils.describe.describe*)

Returns

Sop
[*pylops.LinearOperator*] Sliding operator

Raises

ValueError
Identified number of windows is not consistent with provided model shape (`dims`).

See also:

Sliding1D
1D Sliding transform operator.

Sliding2D
2D Sliding transform operator.

Sliding3D
3D Sliding transform operator.

Patch3D
3D Patching transform operator.

Examples using `pylops.signalprocessing.Patch2D`

- *Patching*
- *18. Deblending*

pylops.signalprocessing.Patch3D

pylops.signalprocessing.**Patch3D**(*Op*, *dims*, *dimsd*, *nwin*, *nover*, *nop*, *tapertype='hanning'*, *scalings=None*, *name='P'*)

3D Patch transform operator.

Apply a transform operator `Op` repeatedly to patches of the model vector in forward mode and patches of the data vector in adjoint mode. More specifically, in forward mode the model vector is divided into patches, each patch is transformed, and patches are then recombined together. Both model and data are internally reshaped and interpreted as 3-dimensional arrays: each patch contains a portion of the array in every axis.

This operator can be used to perform local, overlapping transforms (e.g., *pylops.signalprocessing.FFTND* or *pylops.signalprocessing.Radon3D*) on 3-dimensional arrays.

Note: The shape of the model has to be consistent with the number of windows for this operator not to return an error. As the number of windows depends directly on the choice of `nwin` and `nover`, it is recommended to first run `patch3d_design` to obtain the corresponding `dims` and number of windows.

Warning: Depending on the choice of *nwin* and *nover* as well as the size of the data, sliding windows may not cover the entire data. The start and end indices of each window will be displayed and returned with running `patch3d_design`.

Parameters

Op
[*pylops.LinearOperator*] Transform operator

dims
[`tuple`] Shape of 3-dimensional model. Note that `dims[0]`, `dims[1]` and `dims[2]` should be multiple of the model size of the transform in their respective dimensions

dimsd
[`tuple`] Shape of 3-dimensional data

nwin
[`tuple`] Number of samples of window

nover
[`tuple`] Number of samples of overlapping part of window

nop
[`tuple`] Size of model in the transformed domain

tapertype
[`str`, optional] Type of taper (`hanning`, `cosine`, `cosinesquare` or `None`)

scalings
[`tuple` or `list`, optional] Set of scalings to apply to each patch. If `None`, no scale will be applied

name
[str, optional] Name of operator (to be used by *pylops.utils.describe.describe*)

Returns

Sop
[*pylops.LinearOperator*] Sliding operator

Raises

ValueError
Identified number of windows is not consistent with provided model shape (dims).

See also:

Sliding1D
1D Sliding transform operator.

Sliding2D
2D Sliding transform operator.

Sliding3D
3D Sliding transform operator.

Patch2D
2D Patching transform operator.

Examples using pylops.signalprocessing.Patch3D

- *Patching*

pylops.signalprocessing.Fredholm1

class pylops.signalprocessing.**Fredholm1**(**args*, ***kwargs*)

Fredholm integral of first kind.

Implement a multi-dimensional Fredholm integral of first kind. Note that if the integral is two dimensional, this can be directly implemented using pylops.basicoperators.MatrixMult. A multi-dimensional Fredholm integral can be performed as a pylops.basicoperators.BlockDiag operator of a series of pylops.basicoperators.MatrixMult. However, here we take advantage of the structure of the kernel and perform it in a more efficient manner.

Parameters

G
[numpy.ndarray] Multi-dimensional convolution kernel of size $[n_{\text{slice}} \times n_x \times n_y]$

nz
[int, optional] Additional dimension of model

saveGt
[bool, optional] Save G and G.H to speed up the computation of adjoint (True) or create G.H on-the-fly (False) Note that saveGt=True will double the amount of required memory

usematmul
[bool, optional] Use numpy.matmul (True) or for-loop with numpy.dot (False). As it is not possible to define which approach is more performant (this is highly dependent on the

size of G and input arrays as well as the hardware used in the computation), we advise users to time both methods for their specific problem prior to making a choice.

dtype
[str, optional] Type of elements in input array.

name
[str, optional] New in version 2.0.0.

Name of operator (to be used by *pylops.utils.describe.describe*)

Notes

A multi-dimensional Fredholm integral of first kind can be expressed as

$$d(k,x,z) = \int G(k,x,y) m(k,y,z)\,\mathrm{d}y \quad \forall k = 1,\ldots,n_{slice}$$

on the other hand its adjoin is expressed as

$$m(k,y,z) = \int G^*(k,y,x) d(k,x,z)\,\mathrm{d}x \quad \forall k = 1,\ldots,n_{\text{slice}}$$

In discrete form, this operator can be seen as a block-diagonal matrix multiplication:

$$\begin{bmatrix} \mathbf{G}_{k=1} & \mathbf{0} & \ldots & \mathbf{0} \\ \mathbf{0} & \mathbf{G}_{k=2} & \ldots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \ldots & \mathbf{G}_{k=N} \end{bmatrix} \begin{bmatrix} \mathbf{m}_{k=1} \\ \mathbf{m}_{k=2} \\ \vdots \\ \mathbf{m}_{k=N} \end{bmatrix}$$

Attributes

shape
[tuple] Operator shape

explicit
[bool] Operator contains a matrix that can be solved explicitly (True) or not (False)

Methods

`__init__`(G[, nz, saveGt, usematmul, dtype, name])	Initialize this LinearOperator.
`adjoint`()	Hermitian adjoint.
`apply_columns`(cols)	Apply subset of columns of operator
`cond`([uselobpcg])	Condition number of linear operator.
`conj`()	Complex conjugate operator
`div`(y[, niter, densesolver])	Solve the linear problem $\mathbf{y} = \mathbf{A}\mathbf{x}$.
`dot`(x)	Matrix-matrix or matrix-vector multiplication.
`eigs`([neigs, symmetric, niter, uselobpcg])	Most significant eigenvalues of linear operator.
`matmat`(X)	Matrix-matrix multiplication.
`matvec`(x)	Matrix-vector multiplication.
`reset_count`()	Reset counters
`rmatmat`(X)	Matrix-matrix multiplication.
`rmatvec`(x)	Adjoint matrix-vector multiplication.
`todense`([backend])	Return dense matrix.
`toimag`([forw, adj])	Imag operator
`toreal`([forw, adj])	Real operator
`tosparse`()	Return sparse matrix.
`trace`([neval, method, backend])	Trace of linear operator.
`transpose`()	Transpose this linear operator.

Wave-Equation processing

PressureToVelocity(nt, nr, dt, dr, rho, vel)	Pressure to Vertical velocity conversion.
UpDownComposition2D(nt, nr, dt, dr, rho, vel)	2D Up-down wavefield composition.
UpDownComposition3D(nt, nr, dt, dr, rho, vel)	3D Up-down wavefield composition.
MDC(G, nt, nv[, dt, dr, twosided, ...])	Multi-dimensional convolution.
PhaseShift(vel, dz, nt, freq, kx[, ky, ...])	Phase shift operator
Kirchhoff(*args, **kwargs)	Kirchhoff Demigration operator.
AcousticWave2D(*args, **kwargs)	Devito Acoustic propagator.

pylops.waveeqprocessing.PressureToVelocity

pylops.waveeqprocessing.**PressureToVelocity**(*nt*, *nr*, *dt*, *dr*, *rho*, *vel*, *nffts=(None, None, None)*, *critical=100.0*, *ntaper=10*, *topressure=False*, *backend='numpy'*, *dtype='complex128'*, *name='P'*)

Pressure to Vertical velocity conversion.

Apply conversion from pressure to vertical velocity seismic wavefield (or vertical velocity to pressure). The input model and data required by the operator should be created by flattening the a wavefield of size $([n_{r_y} \times n_{r_x} \times n_t]$.

Parameters

nt
[int] Number of samples along the time axis

nr
[int or tuple] Number of samples along the receiver axis (or axes)

dt
[float] Sampling along the time axis

dr
[float or tuple] Sampling(s) along the receiver array

rho
[float] Density ρ along the receiver array (must be constant)

vel
[float] Velocity c along the receiver array (must be constant)

nffts
[tuple, optional] Number of samples along the wavenumber and frequency axes

critical
[float, optional] Percentage of angles to retain in obliquity factor. For example, if `critical=100` only angles below the critical angle $\sqrt{k_y^2 + k_x^2} < \frac{\omega}{c}$ will be retained

ntaper
[float, optional] Number of samples of taper applied to obliquity factor around critical angle

topressure
[bool, optional] Perform conversion from particle velocity to pressure (`True`) or from pressure to particle velocity (`False`)

backend
[str, optional] Backend used for creation of obliquity factor operator (`numpy` or `cupy`)

dtype
[str, optional] Type of elements in input array.

name
[str, optional] New in version 2.0.0.

Name of operator (to be used by *pylops.utils.describe.describe*)

Returns

Cop
[*pylops.LinearOperator*] Pressure to particle velocity (or particle velocity to pressure) conversion operator

See also:

UpDownComposition2D
2D Wavefield composition

UpDownComposition3D
3D Wavefield composition

WavefieldDecomposition
Wavefield decomposition

Notes

A pressure wavefield $p(x,t)$ can be converted into an equivalent vertical particle velocity wavefield $v_z(x,t)$ by applying the following frequency-wavenumber dependant scaling [1]:

$$\hat{v}_z(k_x,\omega) = \frac{k_z}{\omega\rho}\hat{p}(k_x,\omega)$$

where the vertical wavenumber k_z is defined as $k_z = \sqrt{\frac{\omega^2}{c^2} - k_x^2}$.

Similarly a vertical particle velocity can be converted into an equivalent pressure wavefield by applying the following frequency-wavenumber dependant scaling [1]:

$$\hat{p}(k_x,\omega) = \frac{\omega\rho}{k_z}\hat{v}_z(k_x,\omega)$$

For 3-dimensional applications the only difference is represented by the vertical wavenumber k_z, which is defined as $k_z = \sqrt{\frac{\omega^2}{c^2} - k_x^2 - k_y^2}$.

In both cases, this operator is implemented as a concatanation of a 2 or 3-dimensional forward FFT (`pylops.signalprocessing.FFT2` or `pylops.signalprocessing.FFTN`), a weighting matrix implemented via `pylops.basicprocessing.Diagonal`, and 2 or 3-dimensional inverse FFT.

Examples using `pylops.waveeqprocessing.PressureToVelocity`

- *14. Seismic wavefield decomposition*

pylops.waveeqprocessing.UpDownComposition2D

pylops.waveeqprocessing.**UpDownComposition2D**(*nt*, *nr*, *dt*, *dr*, *rho*, *vel*, *nffts=(None, None)*, *critical=100.0*, *ntaper=10*, *scaling=1.0*, *backend='numpy'*, *dtype='complex128'*, *name='U'*)

2D Up-down wavefield composition.

Apply multi-component seismic wavefield composition from its up- and down-going constituents. The input model required by the operator should be created by flattening the separated wavefields of size $[n_r \times n_t]$ concatenated along the spatial axis.

Similarly, the data is also a flattened concatenation of pressure and vertical particle velocity wavefields.

Parameters

nt
[`int`] Number of samples along the time axis

nr
[`int`] Number of samples along the receiver axis

dt
[`float`] Sampling along the time axis

dr
[`float`] Sampling along the receiver array

rho
[`float`] Density ρ along the receiver array (must be constant)

vel
[float] Velocity c along the receiver array (must be constant)

nffts
[tuple, optional] Number of samples along the wavenumber and frequency axes

critical
[float, optional] Percentage of angles to retain in obliquity factor. For example, if `critical=100` only angles below the critical angle $|k_x| < \frac{f(k_x)}{c}$ will be retained will be retained

ntaper
[float, optional] Number of samples of taper applied to obliquity factor around critical angle

scaling
[float, optional] Scaling to apply to the operator (see Notes for more details)

backend
[str, optional] Backend used for creation of obliquity factor operator (`numpy` or `cupy`)

dtype
[str, optional] Type of elements in input array.

name
[str, optional] New in version 2.0.0.

Name of operator (to be used by *pylops.utils.describe.describe*)

Returns

UDop
[*pylops.LinearOperator*] Up-down wavefield composition operator

See also:

UpDownComposition3D
3D Wavefield composition

WavefieldDecomposition
Wavefield decomposition

Notes

Multi-component seismic data $p(x,t)$ and $v_z(x,t)$ can be synthesized in the frequency-wavenumber domain as the superposition of the up- and downgoing constituents of the pressure wavefield ($p^-(x,t)$ and $p^+(x,t)$) as follows [1]:

$$\begin{bmatrix} \hat{p} \\ \hat{v_z} \end{bmatrix} (k_x, \omega) = \begin{bmatrix} 1 & 1 \\ \frac{k_z}{\omega\rho} & -\frac{k_z}{\omega\rho} \end{bmatrix} \begin{bmatrix} \hat{p^+} \\ \hat{p^-} \end{bmatrix} (k_x, \omega)$$

where the vertical wavenumber k_z is defined as $k_z = \sqrt{\frac{\omega^2}{c^2} - k_x^2}$.

We can write the entire composition process in a compact matrix-vector notation as follows:

$$\begin{bmatrix} \mathbf{p} \\ s\mathbf{v_z} \end{bmatrix} = \begin{bmatrix} \mathbf{F} & 0 \\ 0 & s\mathbf{F} \end{bmatrix} \begin{bmatrix} \mathbf{I} & \mathbf{I} \\ \mathbf{W}^+ & \mathbf{W}^- \end{bmatrix} \begin{bmatrix} \mathbf{F}^H & 0 \\ 0 & \mathbf{F}^H \end{bmatrix} \mathbf{p}^{\pm}$$

where $\mathbf{F}$ is the 2-dimensional FFT (`pylops.signalprocessing.FFT2`), $\mathbf{W}^{\pm}$ are weighting matrices which contain the scalings $\pm\frac{k_z}{\omega\rho}$ implemented via `pylops.basicprocessing.Diagonal`, and s is a scaling factor that

is applied to both the particle velocity data and to the operator has shown above. Such a scaling is required to balance out the different dynamic range of pressure and particle velocity when solving the wavefield separation problem as an inverse problem.

As the operator is effectively obtained by chaining basic PyLops operators the adjoint is automatically implemented for this operator.

Examples using `pylops.waveeqprocessing.UpDownComposition2D`

- *14. Seismic wavefield decomposition*

pylops.waveeqprocessing.UpDownComposition3D

`pylops.waveeqprocessing.`**`UpDownComposition3D`**(*nt*, *nr*, *dt*, *dr*, *rho*, *vel*, *nffts=(None, None, None)*, *critical=100.0*, *ntaper=10*, *scaling=1.0*, *backend='numpy'*, *dtype='complex128'*, *name='U'*)

3D Up-down wavefield composition.

Apply multi-component seismic wavefield composition from its up- and down-going constituents. The input model required by the operator should be created by flattening the separated wavefields of size $[n_{r_y} \times n_{r_x} \times n_t]$ concatenated along the first spatial axis.

Similarly, the data is also a flattened concatenation of pressure and vertical particle velocity wavefields.

Parameters

nt
[`int`] Number of samples along the time axis

nr
[`tuple`] Number of samples along the receiver axes

dt
[`float`] Sampling along the time axis

dr
[`tuple`] Samplings along the receiver array

rho
[`float`] Density ρ along the receiver array (must be constant)

vel
[`float`] Velocity c along the receiver array (must be constant)

nffts
[`tuple`, optional] Number of samples along the wavenumbers and frequency axes (for the wavenumbers axes the same order as `nr` and `dr` must be followed)

critical
[`float`, optional] Percentage of angles to retain in obliquity factor. For example, if `critical=100` only angles below the critical angle $\sqrt{k_y^2 + k_x^2} < \frac{\omega}{c}$ will be retained

ntaper
[`float`, optional] Number of samples of taper applied to obliquity factor around critical angle

scaling
[`float`, optional] Scaling to apply to the operator (see Notes for more details)

backend
[str, optional] Backend used for creation of obliquity factor operator (numpy or cupy)

dtype
[str, optional] Type of elements in input array.

name
[str, optional] New in version 2.0.0.

Name of operator (to be used by *pylops.utils.describe.describe*)

Returns

UDop
[*pylops.LinearOperator*] Up-down wavefield composition operator

See also:

UpDownComposition2D
2D Wavefield composition

WavefieldDecomposition
Wavefield decomposition

Notes

Multi-component seismic data $p(y, x, t)$ and $v_z(y, x, t)$ can be synthesized in the frequency-wavenumber domain as the superposition of the up- and downgoing constituents of the pressure wavefield ($p^-(y, x, t)$ and $p^+(y, x, t)$) as described *pylops.waveeqprocessing.UpDownComposition2D*.

Here the vertical wavenumber k_z is defined as $k_z = \sqrt{\frac{\omega^2}{c^2} - k_y^2 - k_x^2}$.

pylops.waveeqprocessing.MDC

pylops.waveeqprocessing.**MDC**(*G*, *nt*, *nv*, *dt=1.0*, *dr=1.0*, *twosided=True*, *fftengine='numpy'*, *saveGt=True*, *conj=False*, *usematmul=False*, *prescaled=False*, *name='M'*)

Multi-dimensional convolution.

Apply multi-dimensional convolution between two datasets. Model and data should be provided after flattening 2- or 3-dimensional arrays of size $[n_t \times n_r \ (\times n_{vs})]$ and $[n_t \times n_s \ (\times n_{vs})]$ (or $2n_t - 1$ for twosided=True), respectively.

Parameters

G
[numpy.ndarray] Multi-dimensional convolution kernel in frequency domain of size $[n_{f_{\max}} \times n_s \times n_r]$

nt
[int] Number of samples along time axis for model and data (note that this must be equal to $2n_t - 1$ when working with twosided=True.

nv
[int] Number of samples along virtual source axis

dt
[float, optional] Sampling of time integration axis Δt

dr
[float, optional] Sampling of receiver integration axis Δr

twosided
[bool, optional] MDC operator has both negative and positive time (True) or only positive (False)

fftengine
[str, optional] Engine used for fft computation (numpy, scipy or fftw)

saveGt
[bool, optional] Save G and G.H to speed up the computation of adjoint of *pylops.signalprocessing.Fredholm1* (True) or create G.H on-the-fly (False) Note that saveGt=True will be faster but double the amount of required memory

conj
[str, optional] Perform Fredholm integral computation with complex conjugate of G

usematmul
[bool, optional] Use numpy.matmul (True) or for-loop with numpy.dot (False) in *pylops.signalprocessing.Fredholm1* operator. Refer to Fredholm1 documentation for details.

prescaled
[bool, optional] Apply scaling to kernel (False) or not (False) when performing spatial and temporal summations. In case prescaled=True, the kernel is assumed to have been pre-scaled when passed to the MDC routine.

name
[str, optional] New in version 2.0.0.

Name of operator (to be used by *pylops.utils.describe.describe*)

Raises

ValueError
If nt is even and twosided=True

See also:

MDD
Multi-dimensional deconvolution

Notes

The so-called multi-dimensional convolution (MDC) is a chained operator [1]. It is composed of a forward Fourier transform, a multi-dimensional integration, and an inverse Fourier transform:

$$y(t,s,v) = F^{-1}\Big(\int_S G(f,s,r)F(x(t,r,v))\,\mathrm{d}r\Big)$$

which is discretized as follows:

$$y(t,s,v) = \sqrt{n_t}\Delta t\Delta r F^{-1}\Big(\sum_{i_r=0}^{n_r} G(f,s,i_r)F(x(t,i_r,v))\Big)$$

where $\sqrt{n_t}\Delta t\Delta r$ is not applied if prescaled=True.

This operation can be discretized and performed by means of a linear operator

$$\mathbf{D} = \mathbf{F}^H\mathbf{G}\mathbf{F}$$

where $\mathbf{F}$ is the Fourier transform applied along the time axis and $\mathbf{G}$ is the multi-dimensional convolution kernel.

Examples using `pylops.waveeqprocessing.MDC`

- *Multi-Dimensional Convolution*
- *09. Multi-Dimensional Deconvolution*

pylops.waveeqprocessing.PhaseShift

pylops.waveeqprocessing.**PhaseShift**(*vel*, *dz*, *nt*, *freq*, *kx*, *ky=None*, *dtype='float64'*, *name='P'*)

Phase shift operator

Apply positive (forward) phase shift with constant velocity in forward mode, and negative (backward) phase shift with constant velocity in adjoint mode. Input model and data should be 2- or 3-dimensional arrays in time-space domain of size $[n_t \times n_x \ (\times n_y)]$.

Parameters

vel
[float, optional] Constant propagation velocity

dz
[float, optional] Depth step

nt
[int, optional] Number of time samples of model and data

freq
[numpy.ndarray] Positive frequency axis

kx
[int, optional] Horizontal wavenumber axis (centered around 0) of size $[n_x \times 1]$.

ky
[int, optional] Second horizontal wavenumber axis for 3d phase shift (centered around 0) of size $[n_y \times 1]$.

dtype
[str, optional] Type of elements in input array

name
[str, optional] New in version 2.0.0.

Name of operator (to be used by *pylops.utils.describe.describe*)

Returns

Pop
[*pylops.LinearOperator*] Phase shift operator

Notes

The phase shift operator implements a one-way wave equation forward propagation in frequency-wavenumber domain by applying the following transformation to the input model:

$$d(f, k_x, k_y) = m(f, k_x, k_y) e^{-j\Delta z \sqrt{\omega^2/v^2 - k_x^2 - k_y^2}}$$

where v is the constant propagation velocity and Δz is the propagation depth. In adjoint mode, the data is propagated backward using the following transformation:

$$m(f, k_x, k_y) = d(f, k_x, k_y) e^{j\Delta z \sqrt{\omega^2/v^2 - k_x^2 - k_y^2}}$$

Effectively, the input model and data are assumed to be in time-space domain and forward Fourier transform is applied to both dimensions, leading to the following operator:

$$\mathbf{d} = \mathbf{F}_t^H \mathbf{F}_x^H \mathbf{P} \mathbf{F}_x \mathbf{F}_t \mathbf{m}$$

where $\mathbf{P}$ perfoms the phase-shift as discussed above.

Examples using `pylops.waveeqprocessing.PhaseShift`

- *PhaseShift operator*

pylops.waveeqprocessing.Kirchhoff

class pylops.waveeqprocessing.**Kirchhoff**(**args*, ***kwargs*)

Kirchhoff Demigration operator.

Kirchhoff-based demigration/migration operator. Uses a high-frequency approximation of Green's function propagators based on `trav`.

Parameters

z
[numpy.ndarray] Depth axis

x
[numpy.ndarray] Spatial axis

t
[numpy.ndarray] Time axis for data

srcs
[numpy.ndarray] Sources in array of size $[2(3) \times n_s]$ The first axis should be ordered as (y,) x, z.

recs
[numpy.ndarray] Receivers in array of size $[2(3) \times n_r]$ The first axis should be ordered as (y,) x, z.

vel
[numpy.ndarray or float] Velocity model of size $[(n_y \times) n_x \times n_z]$ (or constant)

wav
[numpy.ndarray] Wavelet.

wavcenter
[int] Index of wavelet center

y
[numpy.ndarray] Additional spatial axis (for 3-dimensional problems)

mode
[str, optional] Computation mode (`analytic`, `eikonal` or `byot`, see Notes for more details)

wavfilter
[bool, optional] New in version 2.0.0.

Apply wavelet filter (`True`) or not (`False`)

dynamic
[bool, optional] New in version 2.0.0.

Include dynamic weights in computations (`True`) or not (`False`). Note that when `mode=byot`, the user is required to provide such weights in `amp`.

trav
[numpy.ndarray, optional] Traveltime table of size $[(n_y)n_x n_z \times n_s n_r]$ (to be provided if `mode='byot'`)

amp
[numpy.ndarray, optional] New in version 2.0.0.

Amplitude table of size $[(n_y)n_x n_z \times n_s n_r]$ (to be provided if `mode='byot'`)

aperture
[float or tuple, optional] New in version 2.0.0.

Maximum allowed aperture expressed as the ratio of offset over depth. If `None`, no aperture limitations are introduced. If scalar, a taper from 80% to 100% of aperture is applied. If tuple, apertures below the first value are accepted and those after the second value are rejected. A tapering is implemented for those between such values.

angleaperture
[float or tuple, optional] New in version 2.0.0.

Maximum allowed angle (either source or receiver side) in degrees. If `None`, angle aperture limitations are introduced. See `aperture` for implementation details regarding scalar and tuple cases.

anglerefl
[np.ndarray, optional] New in version 2.0.0.

Angle between the normal of the reflectors and the vertical axis in degrees

snell
[float or tuple, optional] New in version 2.0.0.

Threshold on Snell's law evaluation. If larger, the source-receiver-image point is discarded. If `None`, no check on the validity of the Snell's law is performed. See `aperture` for implementation details regarding scalar and tuple cases.

engine
[str, optional] Engine used for computations (`numpy` or `numba`).

dtype
[str, optional] Type of elements in input array.

name
[str, optional] New in version 2.0.0.

Name of operator (to be used by *pylops.utils.describe.describe*)

Raises

NotImplementedError
If `mode` is neither `analytic`, `eikonal`, or `byot`

Notes

The Kirchhoff demigration operator synthesizes seismic data given a propagation velocity model v and a reflectivity model m. In forward mode [1], [2]:

$$d(\mathbf{x_r}, \mathbf{x_s}, t) = \widetilde{w}(t) * \int_V G(\mathbf{x_r}, \mathbf{x}, t) m(\mathbf{x}) G(\mathbf{x}, \mathbf{x_s}, t) \, \mathrm{d}\mathbf{x}$$

where $m(\mathbf{x})$ represents the reflectivity at every location in the subsurface, $G(\mathbf{x}, \mathbf{x_s}, t)$ and $G(\mathbf{x_r}, \mathbf{x}, t)$ are the Green's functions from source-to-subsurface-to-receiver and finally $\widetilde{w}(t)$ is a filtered version of the wavelet $w(t)$ [3] (or the wavelet itself when `wavfilter=False`). In our implementation, the following high-frequency approximation of the Green's functions is adopted:

$$G(\mathbf{x_r}, \mathbf{x}, \omega) = a(\mathbf{x_r}, \mathbf{x}) e^{j\omega t(\mathbf{x_r}, \mathbf{x})}$$

where $a(\mathbf{x_r}, \mathbf{x})$ is the amplitude and $t(\mathbf{x_r}, \mathbf{x})$ is the traveltime. When `dynamic=False` the amplitude is disregarded leading to a kinematic-only Kirchhoff operator.

$$d(\mathbf{x_r}, \mathbf{x_s}, t) = \tilde{w}(t) * \int_V e^{j\omega(t(\mathbf{x_r}, \mathbf{x}) + t(\mathbf{x}, \mathbf{x_s}))} m(\mathbf{x}) \, \mathrm{d}\mathbf{x}$$

On the other hand, when `dynamic=True`, the amplitude scaling is defined as $a(\mathbf{x}, \mathbf{y}) = \frac{1}{\|\mathbf{x}-\mathbf{y}\|}$, that is, the reciprocal of the distance between the two points, approximating the geometrical spreading of the wavefront. Moreover an angle scaling is included in the modelling operator added as follows:

$$d(\mathbf{x_r}, \mathbf{x_s}, t) = \tilde{w}(t) * \int_V a(\mathbf{x}, \mathbf{x_s}) a(\mathbf{x}, \mathbf{x_r}) \frac{|cos\theta_s + cos\theta_r|}{v(\mathbf{x})} e^{j\omega(t(\mathbf{x_r}, \mathbf{x}) + t(\mathbf{x}, \mathbf{x_s}))} m(\mathbf{x}) \, \mathrm{d}\mathbf{x}$$

where θ_s and θ_r are the angles between the source-side and receiver-side rays and the normal to the reflector at the image point (or the vertical axis at the image point when `reflslope=None`), respectively.

Depending on the choice of `mode` the traveltime and amplitude of the Green's function will be also computed differently:

- `mode=analytic` or `mode=eikonal`: traveltimes, geometrical spreading, and angles are computed for every source-image point-receiver triplets and the Green's functions are implemented from traveltime look-up tables, placing scaled reflectivity values at corresponding source-to-receiver time in the data.
- `byot`: bring your own tables. Traveltime table are provided directly by user using `trav` input parameter. Similarly, in this case one can provide their own amplitude scaling `amp` (which should include the angle scaling too).

Three aperture limitations have been also implemented as defined by:

- `aperture`: the maximum allowed aperture is expressed as the ratio of offset over depth. This aperture limitation avoid including grazing angles whose contributions can introduce aliasing effects. A taper is added at the edges of the aperture;
- `angleaperture`: the maximum allowed angle aperture is expressed as the difference between the incident or emerging angle at every image point and the vertical axis (or the normal to the reflector if `anglerefl` is provided. This aperture limitation also avoid including grazing angles whose contributions can introduce aliasing effects. Note that for a homogenous medium and slowly varying heterogenous medium the offset and angle aperture limits may work in the same way;

- `snell`: the maximum allowed snell's angle is expressed as the absolute value of the sum between incident and emerging angles defined as in the `angleaperture` case. This aperture limitation is introduced to turn a scattering-based Kirchhoff engine into a reflection-based Kirchhoff engine where each image point is not considered as scatter but as a local horizontal (or straight) reflector.

Finally, the adjoint of the demigration operator is a *migration* operator which projects data in the model domain creating an image of the subsurface reflectivity.

Attributes

shape
[`tuple`] Operator shape

explicit
[`bool`] Operator contains a matrix that can be solved explicitly (`True`) or not (`False`)

Methods

`__init__`(z, x, t, srcs, recs, vel, wav, ...)	Initialize this LinearOperator.
`adjoint`()	Hermitian adjoint.
`apply_columns`(cols)	Apply subset of columns of operator
`cond`([uselobpcg])	Condition number of linear operator.
`conj`()	Complex conjugate operator
`div`(y[, niter, densesolver])	Solve the linear problem $\mathbf{y} = \mathbf{A}\mathbf{x}$.
`dot`(x)	Matrix-matrix or matrix-vector multiplication.
`eigs`([neigs, symmetric, niter, uselobpcg])	Most significant eigenvalues of linear operator.
`matmat`(X)	Matrix-matrix multiplication.
`matvec`(x)	Matrix-vector multiplication.
`reset_count`()	Reset counters
`rmatmat`(X)	Matrix-matrix multiplication.
`rmatvec`(x)	Adjoint matrix-vector multiplication.
`todense`([backend])	Return dense matrix.
`toimag`([forw, adj])	Imag operator
`toreal`([forw, adj])	Real operator
`tosparse`()	Return sparse matrix.
`trace`([neval, method, backend])	Trace of linear operator.
`transpose`()	Transpose this linear operator.

Examples using `pylops.waveeqprocessing.Kirchhoff`

- *15. Least-squares migration*

pylops.waveeqprocessing.AcousticWave2D

class pylops.waveeqprocessing.**AcousticWave2D**(**args*, ***kwargs*)

Devito Acoustic propagator.

Parameters

shape
[`tuple` or `numpy.ndarray`] Model shape `(nx, nz)`

origin
[tuple or numpy.ndarray] Model origin (ox, oz)

spacing
[tuple or numpy.ndarray] Model spacing (dx, dz)

vp
[numpy.ndarray] Velocity model in m/s

src_x
[numpy.ndarray] Source x-coordinates in m

src_z
[numpy.ndarray or float] Source z-coordinates in m

rec_x
[numpy.ndarray] Receiver x-coordinates in m

rec_z
[numpy.ndarray or float] Receiver z-coordinates in m

t0
[float] Initial time

tn
[int] Number of time samples

src_type
[str] Source type

space_order
[int, optional] Spatial ordering of FD stencil

nbl
[int, optional] Number ordering of samples in absorbing boundaries

f0
[float, optional] Source peak frequency (Hz)

checkpointing
[bool, optional] Use checkpointing (True) or not (False). Note that using checkpointing is needed when dealing with large models but it will slow down computations

dtype
[str, optional] Type of elements in input array.

name
[str, optional] Name of operator (to be used by *pylops.utils.describe.describe*)

Attributes

shape
[tuple] Operator shape

explicit
[bool] Operator contains a matrix that can be solved explicitly (True) or not (False)

Methods

`__init__`(shape, origin, spacing, vp, src_x, ...)	Initialize this LinearOperator.
`adjoint`()	Hermitian adjoint.
`apply_columns`(cols)	Apply subset of columns of operator
`cond`([uselobpcg])	Condition number of linear operator.
`conj`()	Complex conjugate operator
`div`(y[, niter, densesolver])	Solve the linear problem $\mathbf{y} = \mathbf{A}\mathbf{x}$.
`dot`(x)	Matrix-matrix or matrix-vector multiplication.
`eigs`([neigs, symmetric, niter, uselobpcg])	Most significant eigenvalues of linear operator.
`matmat`(X)	Matrix-matrix multiplication.
`matvec`(x)	Matrix-vector multiplication.
`reset_count`()	Reset counters
`rmatmat`(X)	Matrix-matrix multiplication.
`rmatvec`(x)	Adjoint matrix-vector multiplication.
`srcillumination_allshots`([savewav])	Source wavefield and illumination for all shots
`todense`([backend])	Return dense matrix.
`toimag`([forw, adj])	Imag operator
`toreal`([forw, adj])	Real operator
`tosparse`()	Return sparse matrix.
`trace`([neval, method, backend])	Trace of linear operator.
`transpose`()	Transpose this linear operator.

Geophysical subsurface characterization

avo.AVOLinearModelling(*args, **kwargs)	AVO Linearized modelling.
poststack.PoststackLinearModelling(wav, nt0)	Post-stack linearized seismic modelling operator.
prestack.PrestackLinearModelling(wav, theta)	Pre-stack linearized seismic modelling operator.
prestack.PrestackWaveletModelling(m, theta, nwav)	Pre-stack linearized seismic modelling operator for wavelet.

pylops.avo.avo.AVOLinearModelling

class pylops.avo.avo.**AVOLinearModelling**(**args*, ***kwargs*)

AVO Linearized modelling.

Create operator to be applied to a combination of elastic parameters for generation of seismic pre-stack reflectivity.

Parameters

theta
[np.ndarray] Incident angles in degrees

vsvp
[np.ndarray or float] V_S/V_P ratio

nt0
[int, optional] Number of samples (if vsvp is a scalar)

spatdims
[int or tuple, optional] Number of samples along spatial axis (or axes) (None if only one dimension is available)

linearization
[*{"akirich", "fatti", "PS"}*, optional]

- "akirich": Aki-Richards. See *pylops.avo.avo.akirichards*.
- "fatti": Fatti. See *pylops.avo.avo.fatti*.
- "PS": PS. See *pylops.avo.avo.ps*.

dtype
[str, optional] Type of elements in input array.

name
[str, optional] New in version 2.0.0.

Name of operator (to be used by *pylops.utils.describe.describe*)

Raises

NotImplementedError
If `linearization` is not an implemented linearization

Notes

The AVO linearized operator performs a linear combination of three (or two) elastic parameters arranged in input vector $\mathbf{m}$ of size $n_{t_0} \times N$ to create the so-called seismic reflectivity:

$$r(t, \theta, x, y) = \sum_{i=1}^{N} G_i(t, \theta) m_i(t, x, y) \qquad \forall\, t, \theta$$

where $N = 2, 3$. Note that the reflectivity can be in 1d, 2d or 3d and `spatdims` contains the dimensions of the spatial axis (or axes) x and y.

Attributes

shape
[tuple] Operator shape

explicit
[bool] Operator contains a matrix that can be solved explicitly (True) or not (False)

Methods

`__init__`(theta[, vsvp, nt0, spatdims, ...])	Initialize this LinearOperator.
`adjoint`()	Hermitian adjoint.
`apply_columns`(cols)	Apply subset of columns of operator
`cond`([uselobpcg])	Condition number of linear operator.
`conj`()	Complex conjugate operator
`div`(y[, niter, densesolver])	Solve the linear problem $\mathbf{y} = \mathbf{A}\mathbf{x}$.
`dot`(x)	Matrix-matrix or matrix-vector multiplication.
`eigs`([neigs, symmetric, niter, uselobpcg])	Most significant eigenvalues of linear operator.
`matmat`(X)	Matrix-matrix multiplication.
`matvec`(x)	Matrix-vector multiplication.
`reset_count`()	Reset counters
`rmatmat`(X)	Matrix-matrix multiplication.
`rmatvec`(x)	Adjoint matrix-vector multiplication.
`todense`([backend])	Return dense matrix.
`toimag`([forw, adj])	Imag operator
`toreal`([forw, adj])	Real operator
`tosparse`()	Return sparse matrix.
`trace`([neval, method, backend])	Trace of linear operator.
`transpose`()	Transpose this linear operator.

Examples using `pylops.avo.avo.AVOLinearModelling`

- *AVO modelling*

pylops.avo.poststack.PoststackLinearModelling

`pylops.avo.poststack.`**`PoststackLinearModelling`**(*wav*, *nt0*, *spatdims=None*, *explicit=False*, *sparse=False*, *kind='centered'*, *name=None*)

Post-stack linearized seismic modelling operator.

Create operator to be applied to an elastic parameter trace (or stack of traces) for generation of band-limited seismic post-stack data. The input model and data have shape $[n_{t_0} (\times n_x \times n_y)]$.

Parameters

wav
[`np.ndarray`] Wavelet in time domain (must have odd number of elements and centered to zero). If 1d, assume stationary wavelet for the entire time axis. If 2d, use as non-stationary wavelet (user must provide one wavelet per time sample in an array of size $[n_{t_0} \times n_{\text{wav}}]$ where n_{wav} is the length of each wavelet). Note that the `dtype` of this variable will define that of the operator

nt0
[`int`] Number of samples along time axis

spatdims
[`int` or `tuple`, optional] Number of samples along spatial axis (or axes) (`None` if only one dimension is available)

explicit
[bool, optional] Create a chained linear operator (`False`, preferred for large data) or a `MatrixMult` linear operator with dense matrix (`True`, preferred for small data)

sparse
[bool, optional] Create a sparse matrix (`True`) or dense (`False`) when `explicit=True`

kind
[str, optional] Derivative kind (`forward` or `centered`).

name
[str, optional] New in version 2.0.0.

Name of operator (to be used by *pylops.utils.describe.describe*)

Returns

Pop
[LinearOperator] post-stack modelling operator.

Raises

ValueError
If `wav` is two dimensional but does not contain `nt0` wavelets

Notes

Post-stack seismic modelling is the process of constructing seismic post-stack data from a profile of an elastic parameter of choice in time (or depth) domain. This can be easily achieved using the following forward model:

$$d(t, \theta = 0) = w(t) * \frac{\mathrm{d} \ln m(t)}{\mathrm{d}t}$$

where $m(t)$ is the elastic parameter profile and $w(t)$ is the time domain seismic wavelet. In compact form:

$$\mathbf{d} = \mathbf{WDm}$$

In the special case of acoustic impedance ($m(t) = AI(t)$), the modelling operator can be used to create zero-offset data:

$$d(t, \theta = 0) = \frac{1}{2} w(t) * \frac{\mathrm{d} \ln m(t)}{\mathrm{d}t}$$

where the scaling factor $\frac{1}{2}$ can be easily included in the wavelet.

pylops.avo.prestack.PrestackLinearModelling

pylops.avo.prestack.**PrestackLinearModelling**(*wav*, *theta*, *vsvp=0.5*, *nt0=1*, *spatdims=None*, *linearization='akirich'*, *explicit=False*, *kind='centered'*, *name=None*)

Pre-stack linearized seismic modelling operator.

Create operator to be applied to elastic property profiles for generation of band-limited seismic angle gathers from a linearized version of the Zoeppritz equation. The input model must be arranged in a vector of size $n_m \times n_{t_0}$ ($\times n_x \times n_y$) for `explicit=True` and $n_{t_0} \times n_m$ ($\times n_x \times n_y$) for `explicit=False`. Similarly the output data is arranged in a vector of size $n_\theta \times n_{t_0}$ ($\times n_x \times n_y$) for `explicit=True` and $n_{t_0} \times n_\theta$ ($\times n_x \times n_y$) for `explicit=False`.

Parameters

wav
[np.ndarray] Wavelet in time domain (must had odd number of elements and centered to zero). Note that the dtype of this variable will define that of the operator

theta
[np.ndarray] Incident angles in degrees. Must have same dtype of wav (or it will be automatically casted to it)

vsvp
[float or np.ndarray] V_S/V_P ratio (constant or time/depth variant)

nt0
[int, optional] number of samples (if vsvp is a scalar)

spatdims
[int or tuple, optional] Number of samples along spatial axis (or axes) (None if only one dimension is available)

linearization
[*{"akirich", "fatti", "PS"}* or callable, optional]

- "akirich": Aki-Richards. See *pylops.avo.avo.akirichards*.
- "fatti": Fatti. See *pylops.avo.avo.fatti*.
- "PS": PS. See *pylops.avo.avo.ps*.
- Function with the same signature as *pylops.avo.avo.akirichards*

explicit
[bool, optional] Create a chained linear operator (False, preferred for large data) or a MatrixMult linear operator with dense matrix (True, preferred for small data)

kind
[str, optional] Derivative kind (forward or centered).

name
[str, optional] New in version 2.0.0.

Name of operator (to be used by *pylops.utils.describe.describe*)

Returns

Preop
[LinearOperator] pre-stack modelling operator.

Raises

NotImplementedError
If linearization is not an implemented linearization

NotImplementedError
If kind is not forward nor centered

Notes

Pre-stack seismic modelling is the process of constructing seismic pre-stack data from three (or two) profiles of elastic parameters in time (or depth) domain. This can be easily achieved using the following forward model:

$$d(t, \theta) = w(t) * \sum_{i=1}^{n_m} G_i(t, \theta) m_i(t)$$

where $w(t)$ is the time domain seismic wavelet. In compact form:

$$\mathbf{d} = \mathbf{G}\mathbf{m}$$

On the other hand, pre-stack inversion aims at recovering the different profiles of elastic properties from the band-limited seismic pre-stack data.

pylops.avo.prestack.PrestackWaveletModelling

pylops.avo.prestack.**PrestackWaveletModelling**(*m*, *theta*, *nwav*, *wavc=None*, *vsvp=0.5*, *linearization='akirich'*, *name=None*)

Pre-stack linearized seismic modelling operator for wavelet.

Create operator to be applied to a wavelet for generation of band-limited seismic angle gathers using a linearized version of the Zoeppritz equation.

Parameters

m
[np.ndarray] elastic parameter profles of size $[n_{t_0} \times N]$ where $N = 3, 2$. Note that the `dtype` of this variable will define that of the operator

theta
[int] Incident angles in degrees. Must have same `dtype` of `m` (or it will be automatically cast to it)

nwav
[np.ndarray] Number of samples of wavelet to be applied/estimated

wavc
[int, optional] Index of the center of the wavelet

vsvp
[np.ndarray or float, optional] V_S/V_P ratio

linearization
[*{"akirich", "fatti", "PS"}* or callable, optional]

- "akirich": Aki-Richards. See *pylops.avo.avo.akirichards*.
- "fatti": Fatti. See *pylops.avo.avo.fatti*.
- "PS": PS. See *pylops.avo.avo.ps*.
- Function with the same signature as *pylops.avo.avo.akirichards*

name
[str, optional] New in version 2.0.0.

Name of operator (to be used by *pylops.utils.describe.describe*)

Returns

Mconv
[LinearOperator] pre-stack modelling operator for wavelet estimation.

Raises

NotImplementedError
If `linearization` is not an implemented linearization

Notes

Pre-stack seismic modelling for wavelet estimate is the process of constructing seismic reflectivities using three (or two) profiles of elastic parameters in time (or depth) domain arranged in an input vector $\mathbf{m}$ of size $n_{t_0} \times N$:

$$d(t, \theta) = \sum_{i=1}^{N} G_i(t, \theta) m_i(t) * w(t)$$

where $w(t)$ is the time domain seismic wavelet. In compact form:

$$\mathbf{d} = \mathbf{G}\mathbf{w}$$

On the other hand, pre-stack wavelet estimation aims at recovering the wavelet given knowledge of the band-limited seismic pre-stack data and the elastic parameter profiles.

3.7.2 Solvers

Template

Solver(Op[, callbacks])	This is a template class which a user must subclass when implementing a new solver.

pylops.optimization.basesolver.Solver

class pylops.optimization.basesolver.**Solver**(*Op*, *callbacks=None*)

This is a template class which a user must subclass when implementing a new solver. This class comprises of the following mandatory methods:

- `__init__`: initialization method to which the operator *Op* must be passed
- `setup`: a method that is invoked to setup the solver, basically it will create anything required prior to applying a step of the solver
- `step`: a method applying a single step of the solver
- `run`: a method applying multiple steps of the solver
- `finalize`: a method that is invoked at the end of the optimization process. It can be used to do some final clean-up of the properties of the operator that we want to expose to the user
- `solve`: a method applying the entire optimization loop of the solver for a certain number of steps

and optional methods:

- `_print_solver`: a method print on screen details of the solver (already implemented)
- `_print_setup`: a method print on screen details of the setup process

- `_print_step`: a method print on screen details of each step
- `_print_finalize`: a method print on screen details of the finalize process
- `callback`: a method implementing a callback function, which is called after every step of the solver

Parameters

Op
[*pylops.LinearOperator*] Operator to invert of

callbacks
[*pylops.optimization.callback.Callbacks*] Callbacks object used to implement custom callbacks

Methods

`__init__`(Op[, callbacks])	
`callback`(x, *args, **kwargs)	Callback routine
`finalize`(*args[, show])	Finalize solver
`run`(x, *args[, show])	Run multiple steps of solver
`setup`(y, *args[, show])	Setup solver
`solve`(y, *args[, show])	Solve
`step`(x, *args[, show])	Run one step of solver

Examples using `pylops.optimization.basesolver.Solver`

- *Linear Regression*
- *03. Solvers (Advanced)*

Basic

CG(Op[, callbacks])	Conjugate gradient
CGLS(Op[, callbacks])	Conjugate gradient least squares
LSQR(Op)	Solve an overdetermined system of equations given an operator `Op` and data `y` using LSQR iterations.

pylops.optimization.cls_basic.CG

class pylops.optimization.cls_basic.**CG**(*Op*, *callbacks=None*)

Conjugate gradient

Solve a square system of equations given an operator `Op` and data `y` using conjugate gradient iterations.

Parameters

Op
[*pylops.LinearOperator*] Operator to invert of size $[N \times N]$

Notes

Solve the $\mathbf{y} = \mathbf{Op}\,\mathbf{x}$ problem using conjugate gradient iterations [1].

Methods

`__init__`(Op[, callbacks])	

`callback`(x, *args, **kwargs)	Callback routine
`finalize`([show])	Finalize solver
`run`(x[, niter, show, itershow])	Run solver
`setup`(y[, x0, niter, tol, show])	Setup solver
`solve`(y[, x0, niter, tol, show, itershow])	Run entire solver
`step`(x[, show])	Run one step of solver

pylops.optimization.cls_basic.CGLS

class pylops.optimization.cls_basic.**CGLS**(*Op*, *callbacks=None*)

Conjugate gradient least squares

Solve an overdetermined system of equations given an operator `Op` and data `y` using conjugate gradient iterations.

Parameters

Op
[*pylops.LinearOperator*] Operator to invert of size $[N \times M]$

Notes

Minimize the following functional using conjugate gradient iterations:

$$J = ||\mathbf{y} - \mathbf{Op}\,\mathbf{x}||_2^2 + \epsilon^2 ||\mathbf{x}||_2^2$$

where ϵ is the damping coefficient.

Methods

`__init__`(Op[, callbacks])	

`callback`(x, *args, **kwargs)	Callback routine
`finalize`([show])	Finalize solver
`run`(x[, niter, show, itershow])	Run solver
`setup`(y[, x0, niter, damp, tol, show])	Setup solver
`solve`(y[, x0, niter, damp, tol, show, itershow])	Run entire solver
`step`(x[, show])	Run one step of solver

pylops.optimization.cls_basic.LSQR

class pylops.optimization.cls_basic.**LSQR**(*Op*)

Solve an overdetermined system of equations given an operator `Op` and data `y` using LSQR iterations.

$$cond$$

Parameters

Op
[*pylops.LinearOperator*] Operator to invert of size $[N \times M]$

Notes

Minimize the following functional using LSQR iterations [1]:

$$J = ||\mathbf{y} - \mathbf{Op}\,\mathbf{x}||_2^2 + \epsilon^2||\mathbf{x}||_2^2$$

where ϵ is the damping coefficient.

Methods

`__init__`(Op)	

`callback`(x, *args, **kwargs)	Callback routine
`finalize`([show])	Finalize solver
`run`(x[, niter, show, itershow])	Run solver
`setup`(y[, x0, damp, atol, btol, conlim, ...])	Setup solver
`solve`(y[, x0, damp, atol, btol, conlim, ...])	Run entire solver
`step`(x[, show])	Run one step of solver

cg(Op, y[, x0, niter, tol, show, itershow, ...])	Conjugate gradient
cgls(Op, y[, x0, niter, damp, tol, show, ...])	Conjugate gradient least squares
lsqr(Op, y[, x0, damp, atol, btol, conlim, ...])	LSQR

pylops.optimization.basic.cg

pylops.optimization.basic.**cg**(*Op*, *y*, *x0=None*, *niter=10*, *tol=0.0001*, *show=False*, *itershow=[10, 10, 10]*, *callback=None*)

Conjugate gradient

Solve a square system of equations given an operator `Op` and data `y` using conjugate gradient iterations.

Parameters

Op
[*pylops.LinearOperator*] Operator to invert of size $[N \times N]$

y
[`np.ndarray`] Data of size $[N \times 1]$

x0
[np.ndarray, optional] Initial guess

niter
[int, optional] Number of iterations

tol
[float, optional] Tolerance on residual norm

show
[bool, optional] Display iterations log

itershow
[list, optional] Display set log for the first N1 steps, last N2 steps, and every N3 steps in between where N1, N2, N3 are the three element of the list.

callback
[callable, optional] Function with signature (`callback(x)`) to call after each iteration where `x` is the current model vector

Returns

x
[np.ndarray] Estimated model of size $[N \times 1]$

iit
[int] Number of executed iterations

cost
[numpy.ndarray, optional] History of the L2 norm of the residual

Notes

See *pylops.optimization.cls_basic.CG*

pylops.optimization.basic.cgls

pylops.optimization.basic.**cgls**(*Op*, *y*, *x0=None*, *niter=10*, *damp=0.0*, *tol=0.0001*, *show=False*, *itershow=[10, 10, 10]*, *callback=None*)

Conjugate gradient least squares

Solve an overdetermined system of equations given an operator `Op` and data `y` using conjugate gradient iterations.

Parameters

Op
[*pylops.LinearOperator*] Operator to invert of size $[N \times M]$

y
[np.ndarray] Data of size $[N \times 1]$

x0
[np.ndarray, optional] Initial guess

niter
[int, optional] Number of iterations

damp
[float, optional] Damping coefficient

tol
[float, optional] Tolerance on residual norm

show
[bool, optional] Display iterations log

itershow
[list, optional] Display set log for the first N1 steps, last N2 steps, and every N3 steps in between where N1, N2, N3 are the three element of the list.

callback
[callable, optional] Function with signature (`callback(x)`) to call after each iteration where `x` is the current model vector

Returns

x
[np.ndarray] Estimated model of size $[M \times 1]$

istop
[int] Gives the reason for termination

1 means $\mathbf{x}$ is an approximate solution to $\mathbf{y} = \mathbf{Op}\,\mathbf{x}$

2 means $\mathbf{x}$ approximately solves the least-squares problem

iit
[int] Iteration number upon termination

r1norm
[float] $||\mathbf{r}||_2$, where $\mathbf{r} = \mathbf{y} - \mathbf{Op}\,\mathbf{x}$

r2norm
[float] $\sqrt{\mathbf{r}^T\mathbf{r} + \epsilon^2\mathbf{x}^T\mathbf{x}}$. Equal to `r1norm` if $\epsilon = 0$

cost
[numpy.ndarray, optional] History of r1norm through iterations

Notes

See *pylops.optimization.cls_basic.CGLS*

Examples using `pylops.optimization.basic.cgls`

- *CGLS and LSQR Solvers*

pylops.optimization.basic.lsqr

pylops.optimization.basic.**lsqr**(*Op*, *y*, *x0=None*, *damp=0.0*, *atol=1e-08*, *btol=1e-08*, *conlim=100000000.0*, *niter=10*, *calc_var=True*, *show=False*, *itershow=[10, 10, 10]*, *callback=None*)

LSQR

Solve an overdetermined system of equations given an operator `Op` and data `y` using LSQR iterations.

cond

Parameters

Op
[*pylops.LinearOperator*] Operator to invert of size $[N \times M]$

y
[np.ndarray] Data of size $[N \times 1]$

x0
[np.ndarray, optional] Initial guess of size $[M \times 1]$

damp
[float, optional] Damping coefficient

atol, btol
[float, optional] Stopping tolerances. If both are 1.0e-9, the final residual norm should be accurate to about 9 digits. (The solution will usually have fewer correct digits, depending on $(\mathbf{Op})$ and the size of damp.)

conlim
[float, optional] Stopping tolerance on $(\mathbf{Op})$ exceeds conlim. For square, conlim could be as large as 1.0e+12. For least-squares problems, conlim should be less than 1.0e+8. Maximum precision can be obtained by setting atol = btol = conlim = 0, but the number of iterations may then be excessive.

niter
[int, optional] Number of iterations

calc_var
[bool, optional] Estimate diagonals of $(\mathbf{Op}^H \mathbf{Op} + \epsilon^2 \mathbf{I})^{-1}$.

show
[bool, optional] Display iterations log

itershow
[list, optional] Display set log for the first N1 steps, last N2 steps, and every N3 steps in between where N1, N2, N3 are the three element of the list.

callback
[callable, optional] Function with signature (callback(x)) to call after each iteration where x is the current model vector

Returns

x
[np.ndarray] Estimated model of size $[M \times 1]$

istop
[int] Gives the reason for termination

0 means the exact solution is $\mathbf{x} = 0$

1 means $\mathbf{x}$ is an approximate solution to $\mathbf{y} = \mathbf{Op}\,\mathbf{x}$

2 means $\mathbf{x}$ approximately solves the least-squares problem

3 means the estimate of $(\overline{\mathbf{Op}})$ has exceeded conlim

4 means $\mathbf{y} - \mathbf{Op}\,\mathbf{x}$ is small enough for this machine

5 means the least-squares solution is good enough for this machine

6 means $(\overline{\mathbf{Op}})$ seems to be too large for this machine

7 means the iteration limit has been reached

r1norm
[float] $||\mathbf{r}||_2^2$, where $\mathbf{r} = \mathbf{y} - \mathbf{Op}\,\mathbf{x}$

r2norm
[float] $\sqrt{\mathbf{r}^T\mathbf{r} + \epsilon^2\mathbf{x}^T\mathbf{x}}$. Equal to `r1norm` if $\epsilon = 0$

anorm
[float] Estimate of Frobenius norm of $\overline{\mathbf{Op}} = [\mathbf{Op}\ \epsilon\mathbf{I}]$

acond
[float] Estimate of $(\overline{\mathbf{Op}})$

arnorm
[float] Estimate of norm of $(\mathbf{Op}^H\mathbf{r} - \epsilon^2\mathbf{x})$

var
[float] Diagonals of $(\mathbf{Op}^H\mathbf{Op})^{-1}$ (if `damp=0`) or more generally $(\mathbf{Op}^H\mathbf{Op} + \epsilon^2\mathbf{I})^{-1}$.

cost
[numpy.ndarray, optional] History of r1norm through iterations

Notes

See *pylops.optimization.cls_basic.LSQR*

Examples using `pylops.optimization.basic.lsqr`

- *CGLS and LSQR Solvers*

Least-squares

NormalEquationsInversion(Op[, callbacks])	Inversion of normal equations.
RegularizedInversion(Op[, callbacks])	Regularized inversion.
PreconditionedInversion(Op[, callbacks])	Preconditioned inversion.

pylops.optimization.cls_leastsquares.NormalEquationsInversion

class pylops.optimization.cls_leastsquares.**NormalEquationsInversion**(*Op*, *callbacks=None*)

Inversion of normal equations.

Solve the regularized normal equations for a system of equations given the operator `Op`, a data weighting operator `Weight` and optionally a list of regularization terms `Regs` and/or `NRegs`.

Parameters

Op
[*pylops.LinearOperator*] Operator to invert of size $[N \times M]$.

See also:

RegularizedInversion
Regularized inversion

PreconditionedInversion
Preconditioned inversion

Notes

Solve the following normal equations for a system of regularized equations given the operator $\mathbf{Op}$, a data weighting operator $\mathbf{W}$, a list of regularization terms ($\mathbf{R}_i$ and/or $\mathbf{N}_i$), the data $\mathbf{y}$ and regularization data $\mathbf{y}_{\mathbf{R}_i}$, and the damping factors ϵ_I, $\epsilon_{\mathbf{R}_i}$ and $\epsilon_{\mathbf{N}_i}$:

$$(\mathbf{Op}^T\mathbf{WOp} + \sum_i \epsilon_{\mathbf{R}_i}^2 \mathbf{R}_i^T\mathbf{R}_i + \sum_i \epsilon_{\mathbf{N}_i}^2 \mathbf{N}_i + \epsilon_I^2\mathbf{I})\mathbf{x} = \mathbf{Op}^T\mathbf{Wy} + \sum_i \epsilon_{\mathbf{R}_i}^2 \mathbf{R}_i^T\mathbf{y}_{\mathbf{R}_i}$$

Note that the data term of the regularizations $\mathbf{N}_i$ is implicitly assumed to be zero.

Methods

`__init__`(Op[, callbacks])	
`callback`(x, *args, **kwargs)	Callback routine
`finalize`(*args[, show])	Finalize solver
`run`(x[, engine, show])	Run solver
`setup`(y, Regs[, Weight, dataregs, epsI, ...])	Setup solver
`solve`(y, Regs[, x0, Weight, dataregs, epsI, ...])	Run entire solver
`step`()	Run one step of solver

pylops.optimization.cls_leastsquares.RegularizedInversion

class pylops.optimization.cls_leastsquares.**RegularizedInversion**(*Op*, *callbacks=None*)

Regularized inversion.

Solve a system of regularized equations given the operator `Op`, a data weighting operator `Weight`, and a list of regularization terms `Regs`.

Parameters

Op
[*pylops.LinearOperator*] Operator to invert of size $[N \times M]$.

See also:

RegularizedOperator
Regularized operator

NormalEquationsInversion
Normal equations inversion

PreconditionedInversion
Preconditioned inversion

Notes

Solve the following system of regularized equations given the operator $\mathbf{Op}$, a data weighting operator $\mathbf{W}^{1/2}$, a list of regularization terms $\mathbf{R}_i$, the data $\mathbf{y}$ and regularization data $\mathbf{y}_{\mathbf{R}_i}$, and the damping factors $\epsilon_\mathbf{I}$: and $\epsilon_{\mathbf{R}_i}$:

$$\begin{bmatrix} \mathbf{W}^{1/2}\mathbf{Op} \\ \epsilon_{\mathbf{R}_1}\mathbf{R}_1 \\ \vdots \\ \epsilon_{\mathbf{R}_N}\mathbf{R}_N \end{bmatrix} \mathbf{x} = \begin{bmatrix} \mathbf{W}^{1/2}\mathbf{y} \\ \epsilon_{\mathbf{R}_1}\mathbf{y}_{\mathbf{R}_1} \\ \vdots \\ \epsilon_{\mathbf{R}_N}\mathbf{y}_{\mathbf{R}_N} \end{bmatrix}$$

where the `Weight` provided here is equivalent to the square-root of the weight in `pylops.optimization.leastsquares.NormalEquationsInversion`. Note that this system is solved using the `scipy.sparse.linalg.lsqr` and an initial guess `x0` can be provided to this solver, despite the original solver does not allow so.

Methods

`__init__`(Op[, callbacks])	

`callback`(x, *args, **kwargs)	Callback routine
`finalize`(*args[, show])	Finalize solver
`run`(x[, engine, show])	Run solver
`setup`(y, Regs[, Weight, dataregs, epsRs, show])	Setup solver
`solve`(y, Regs[, x0, Weight, dataregs, ...])	Run entire solver
`step`()	Run one step of solver

pylops.optimization.cls_leastsquares.PreconditionedInversion

class pylops.optimization.cls_leastsquares.**PreconditionedInversion**(*Op*, *callbacks=None*)

Preconditioned inversion.

Solve a system of preconditioned equations given the operator `Op` and a preconditioner `P`.

Parameters

Op
[*pylops.LinearOperator*] Operator to invert of size $[N \times M]$.

See also:

RegularizedInversion
Regularized inversion

NormalEquationsInversion
Normal equations inversion

Notes

Solve the following system of preconditioned equations given the operator $\mathbf{Op}$, a preconditioner $\mathbf{P}$, the data $\mathbf{y}$

$$\mathbf{y} = \mathbf{Op}\,\mathbf{Pp}$$

where $\mathbf{p}$ is the solution in the preconditioned space and $\mathbf{x} = \mathbf{Pp}$ is the solution in the original space.

Methods

`__init__`(Op[, callbacks])	

`callback`(x, *args, **kwargs)	Callback routine
`finalize`(*args[, show])	Finalize solver
`run`(x[, engine, show])	Run solver
`setup`(y, P[, show])	Setup solver
`solve`(y, P[, x0, engine, show])	Run entire solver
`step`()	Run one step of solver

normal_equations_inversion(Op, y, Regs[, ...])	Inversion of normal equations.
regularized_inversion(Op, y, Regs[, x0, ...])	Regularized inversion.
preconditioned_inversion(Op, y, P[, x0, ...])	Preconditioned inversion.

pylops.optimization.leastsquares.normal_equations_inversion

pylops.optimization.leastsquares.**normal_equations_inversion**(*Op*, *y*, *Regs*, *x0=None*, *Weight=None*, *dataregs=None*, *epsI=0.0*, *epsRs=None*, *NRegs=None*, *epsNRs=None*, *engine='scipy'*, *show=False*, ***kwargs_solver*)

Inversion of normal equations.

Solve the regularized normal equations for a system of equations given the operator `Op`, a data weighting operator `Weight` and optionally a list of regularization terms `Regs` and/or `NRegs`.

Parameters

Op
[*pylops.LinearOperator*] Operator to invert of size $[N \times M]$

y
[`numpy.ndarray`] Data of size $[N \times 1]$

Regs
[`list`] Regularization operators (`None` to avoid adding regularization)

x0
[`numpy.ndarray`, optional] Initial guess of size $[M \times 1]$

Weight
[*pylops.LinearOperator*, optional] Weight operator

dataregs
[`list`, optional] Regularization data (must have the same number of elements as `Regs`)

epsI
[float, optional] Tikhonov damping

epsRs
[list, optional] Regularization dampings (must have the same number of elements as Regs)

NRegs
[list] Normal regularization operators (None to avoid adding regularization). Such operators must apply the chain of the forward and the adjoint in one go. This can be convenient in cases where a faster implementation is available compared to applying the forward followed by the adjoint.

epsNRs
[list, optional] Regularization dampings for normal operators (must have the same number of elements as NRegs)

engine
[str, optional] Solver to use (scipy or pylops)

show
[bool, optional] Display normal equations solver log

****kwargs_solver**
Arbitrary keyword arguments for chosen solver (scipy.sparse.linalg.cg and pylops.optimization.solver.cg are used for engine scipy and pylops, respectively)

Note: When user does not supply atol, it is set to "legacy".

Returns

xinv
[numpy.ndarray] Inverted model.

istop
[int] Convergence information (only when using scipy.sparse.linalg.cg):

0: successful exit

>0: convergence to tolerance not achieved, number of iterations

<0: illegal input or breakdown

See also:

RegularizedInversion
Regularized inversion

PreconditionedInversion
Preconditioned inversion

Notes

See *pylops.optimization.cls_leastsquares.NormalEquationsInversion*

Examples using pylops.optimization.leastsquares.normal_equations_inversion

- *Bilinear Interpolation*
- *03. Solvers*
- *05. Image deblurring*
- *06. 2D Interpolation*

pylops.optimization.leastsquares.regularized_inversion

pylops.optimization.leastsquares.**regularized_inversion**(*Op*, *y*, *Regs*, *x0=None*, *Weight=None*, *dataregs=None*, *epsRs=None*, *engine='scipy'*, *show=False*, ***kwargs_solver*)

Regularized inversion.

Solve a system of regularized equations given the operator `Op`, a data weighting operator `Weight`, and a list of regularization terms `Regs`.

Parameters

Op
[*pylops.LinearOperator*] Operator to invert of size $[N \times M]$

y
[numpy.ndarray] Data of size $[N \times 1]$

Regs
[list] Regularization operators (`None` to avoid adding regularization)

x0
[numpy.ndarray, optional] Initial guess of size $[M \times 1]$

Weight
[*pylops.LinearOperator*, optional] Weight operator

dataregs
[list, optional] Regularization data (if `None` a zero data will be used for every regularization operator in `Regs`)

epsRs
[list, optional] Regularization dampings

engine
[str, optional] Solver to use (`scipy` or `pylops`)

show
[bool, optional] Display normal equations solver log

****kwargs_solver**
Arbitrary keyword arguments for chosen solver (scipy.sparse.linalg.lsqr and `pylops.optimization.solver.cgls` are used for engine `scipy` and `pylops`, respectively)

Returns

xinv
[numpy.ndarray] Inverted model.

istop
[int] Gives the reason for termination

1 means $\mathbf{x}$ is an approximate solution to $\mathbf{y} = \mathbf{Op}\,\mathbf{x}$

2 means $\mathbf{x}$ approximately solves the least-squares problem

itn
[int] Iteration number upon termination

r1norm
[float] $||\mathbf{r}||_2^2$, where $\mathbf{r} = \mathbf{y} - \mathbf{Op}\,\mathbf{x}$

r2norm
[float] $\sqrt{\mathbf{r}^T\mathbf{r} + \epsilon^2\mathbf{x}^T\mathbf{x}}$. Equal to `r1norm` if $\epsilon = 0$

See also:

`RegularizedOperator`
Regularized operator

`NormalEquationsInversion`
Normal equations inversion

`PreconditionedInversion`
Preconditioned inversion

Notes

See *pylops.optimization.cls_leastsquares.RegularizedInversion*

Examples using `pylops.optimization.leastsquares.regularized_inversion`

- *Causal Integration*
- *MP, OMP, ISTA and FISTA*
- *Total Variation (TV) Regularization*
- *Wavelet estimation*
- *03. Solvers*
- *06. 2D Interpolation*
- *16. CT Scan Imaging*

pylops.optimization.leastsquares.preconditioned_inversion

pylops.optimization.leastsquares.**preconditioned_inversion**(*Op*, *y*, *P*, *x0=None*, *engine='scipy'*, *show=False*, ***kwargs_solver*)

Preconditioned inversion.

Solve a system of preconditioned equations given the operator `Op` and a preconditioner `P`.

Parameters

Op
[*pylops.LinearOperator*] Operator to invert of size $[N \times M]$

y
[numpy.ndarray] Data of size $[N \times 1]$

P
[*pylops.LinearOperator*] Preconditioner

x0
[numpy.ndarray] Initial guess of size $[M \times 1]$

engine
[str, optional] Solver to use (`scipy` or `pylops`)

show
[bool, optional] Display normal equations solver log

****kwargs_solver**
Arbitrary keyword arguments for chosen solver (scipy.sparse.linalg.lsqr and `pylops.optimization.solver.cgls` are used as default for numpy and cupy *data*, respectively)

Returns

xinv
[numpy.ndarray] Inverted model.

istop
[int] Gives the reason for termination

1 means $\mathbf{x}$ is an approximate solution to $\mathbf{y} = \mathbf{Op}\,\mathbf{x}$

2 means $\mathbf{x}$ approximately solves the least-squares problem

itn
[int] Iteration number upon termination

r1norm
[float] $||\mathbf{r}||_2^2$, where $\mathbf{r} = \mathbf{y} - \mathbf{Op}\,\mathbf{x}$

r2norm
[float] $\sqrt{\mathbf{r}^T\mathbf{r} + \epsilon^2\mathbf{x}^T\mathbf{x}}$. Equal to `r1norm` if $\epsilon = 0$

See also:

`RegularizedInversion`
Regularized inversion

`NormalEquationsInversion`
Normal equations inversion

Notes

See *pylops.optimization.cls_leastsquares.PreconditionedInversion*

Examples using pylops.optimization.leastsquares.preconditioned_inversion

- *Causal Integration*
- *Wavelet estimation*
- *03. Solvers*

Sparsity

IRLS(Op[, callbacks])	Iteratively reweighted least squares.
OMP(Op[, callbacks])	Orthogonal Matching Pursuit (OMP).
ISTA(Op[, callbacks])	Iterative Shrinkage-Thresholding Algorithm (ISTA).
FISTA(Op[, callbacks])	Fast Iterative Shrinkage-Thresholding Algorithm (FISTA).
SPGL1(Op[, callbacks])	Spectral Projected-Gradient for L1 norm.
SplitBregman(Op[, callbacks])	Split Bregman for mixed L2-L1 norms.

pylops.optimization.cls_sparsity.IRLS

class pylops.optimization.cls_sparsity.**IRLS**(*Op*, *callbacks=None*)

Iteratively reweighted least squares.

Solve an optimization problem with L_1 cost function (data IRLS) or L_1 regularization term (model IRLS) given the operator `Op` and data `y`.

In the *data IRLS*, the cost function is minimized by iteratively solving a weighted least squares problem with the weight at iteration i being based on the data residual at iteration $i-1$. This IRLS solver is robust to *outliers* since the L1 norm given less weight to large residuals than L2 norm does.

Similarly in the *model IRLS*, the weight at at iteration i is based on the model at iteration $i-1$. This IRLS solver inverts for a sparse model vector.

Parameters

Op
[*pylops.LinearOperator*] Operator to invert

Raises

NotImplementedError
If `kind` is different from model or data

Notes

Data IRLS solves the following optimization problem for the operator $\mathbf{Op}$ and the data $\mathbf{y}$:

$$J = \|\mathbf{y} - \mathbf{Op}\,\mathbf{x}\|_1$$

by a set of outer iterations which require to repeatedly solve a weighted least squares problem of the form:

$$*arg\,min \mathbf{x}^{(i+1)} =_{\mathbf{x}} \|\mathbf{y} - \mathbf{Op}\,\mathbf{x}\|^2_{2,\mathbf{R}^{(i)}} + \epsilon^2_{\mathbf{I}} \|\mathbf{x}\|^2_2$$

where $\mathbf{R}^{(i)}$ is a diagonal weight matrix whose diagonal elements at iteration i are equal to the absolute inverses of the residual vector $\mathbf{r}^{(i)} = \mathbf{y} - \mathbf{Op}\,\mathbf{x}^{(i)}$ at iteration i. More specifically the j-th element of the diagonal of $\mathbf{R}^{(i)}$ is

$$R^{(i)}_{j,j} = \frac{1}{\left|r^{(i)}_j\right| + \epsilon_{\mathbf{R}}}$$

or

$$R^{(i)}_{j,j} = \frac{1}{\max\{\left|r^{(i)}_j\right|, \epsilon_{\mathbf{R}}\}}$$

depending on the choice `threshR`. In either case, $\epsilon_{\mathbf{R}}$ is the user-defined stabilization/thresholding factor [1].

Similarly *model IRLS* solves the following optimization problem for the operator $\mathbf{Op}$ and the data $\mathbf{y}$:

$$J = \|\mathbf{x}\|_1 \quad \text{subject to} \quad \mathbf{y} = \mathbf{Op}\,\mathbf{x}$$

by a set of outer iterations which require to repeatedly solve a weighted least squares problem of the form [2]:

$$\mathbf{x}^{(i+1)} = \arg\min_{\mathbf{x}} \|\mathbf{x}\|^2_{2,\mathbf{R}^{(i)}} \quad \text{subject to} \quad \mathbf{y} = \mathbf{Op}\,\mathbf{x}$$

where $\mathbf{R}^{(i)}$ is a diagonal weight matrix whose diagonal elements at iteration i are equal to the absolutes of the model vector $\mathbf{x}^{(i)}$ at iteration i. More specifically the j-th element of the diagonal of $\mathbf{R}^{(i)}$ is

$$R^{(i)}_{j,j} = \left|x^{(i)}_j\right|.$$

Methods

`__init__`(Op[, callbacks])	

`callback`(x, *args, **kwargs)	Callback routine
`finalize`([show])	Finalize solver
`run`(x[, nouter, show, itershow])	Run solver
`setup`(y[, nouter, threshR, epsR, epsI, ...])	Setup solver
`solve`(y[, x0, nouter, threshR, epsR, epsI, ...])	Run entire solver
`step`(x[, show])	Run one step of solver

Examples using `pylops.optimization.cls_sparsity.IRLS`

- *Linear Regression*

pylops.optimization.cls_sparsity.OMP

class pylops.optimization.cls_sparsity.**OMP**(*Op*, *callbacks=None*)

Orthogonal Matching Pursuit (OMP).

Solve an optimization problem with L_0 regularization function given the operator `Op` and data `y`. The operator can be real or complex, and should ideally be either square $N = M$ or underdetermined $N < M$.

Parameters

Op
[*pylops.LinearOperator*] Operator to invert

See also:

ISTA
Iterative Shrinkage-Thresholding Algorithm (ISTA).

FISTA
Fast Iterative Shrinkage-Thresholding Algorithm (FISTA).

SPGL1
Spectral Projected-Gradient for L1 norm (SPGL1).

SplitBregman
Split Bregman for mixed L2-L1 norms.

Notes

Solves the following optimization problem for the operator $\mathbf{Op}$ and the data $\mathbf{y}$:

$$\|\mathbf{x}\|_0 \quad \text{subject to} \quad \|\mathbf{Op}\,\mathbf{x} - \mathbf{y}\|_2^2 \leq \sigma^2,$$

using Orthogonal Matching Pursuit (OMP). This is a very simple iterative algorithm which applies the following step:

$$*arg\,min * arg\,max \Lambda_k = \Lambda_{k-1} \cup \left\{ _j \left| \mathbf{Op}_j^H \, \mathbf{r}_k \right| \right\}$$
$$\mathbf{x}_k =_{\mathbf{x}} \left\| \mathbf{Op}_{\Lambda_k} \, \mathbf{x} - \mathbf{y} \right\|_2^2$$

Note that by choosing `niter_inner=0` the basic Matching Pursuit (MP) algorithm is implemented instead. In other words, instead of solving an optimization at each iteration to find the best $\mathbf{x}$ for the currently selected basis functions, the vector $\mathbf{x}$ is just updated at the new basis function by taking directly the value from the inner product $\mathbf{Op}_j^H \, \mathbf{r}_k$.

In this case it is highly recommended to provide a normalized basis function. If different basis have different norms, the solver is likely to diverge. Similar observations apply to OMP, even though mild unbalancing between the basis is generally properly handled.

Methods

`__init__`(Op[, callbacks])	

`callback`(x, *args, **kwargs)	Callback routine
`finalize`(x, cols[, show])	Finalize solver
`run`(x, cols[, show, itershow])	Run solver
`setup`(y[, niter_outer, niter_inner, sigma, ...])	Setup solver
`solve`(y[, niter_outer, niter_inner, sigma, ...])	Run entire solver
`step`(x, cols[, show])	Run one step of solver

pylops.optimization.cls_sparsity.ISTA

class pylops.optimization.cls_sparsity.**ISTA**(*Op*, *callbacks=None*)

Iterative Shrinkage-Thresholding Algorithm (ISTA).

Solve an optimization problem with L_p, $p = 0, 0.5, 1$ regularization, given the operator `Op` and data `y`. The operator can be real or complex, and should ideally be either square $N = M$ or underdetermined $N < M$.

Parameters

Op
[*pylops.LinearOperator*] Operator to invert

Raises

NotImplementedError
If `threshkind` is different from hard, soft, half, soft-percentile, or half-percentile

ValueError
If `perc=None` when `threshkind` is soft-percentile or half-percentile

ValueError
If `monitorres=True` and residual increases

See also:

OMP
Orthogonal Matching Pursuit (OMP).

FISTA
Fast Iterative Shrinkage-Thresholding Algorithm (FISTA).

SPGL1
Spectral Projected-Gradient for L1 norm (SPGL1).

SplitBregman
Split Bregman for mixed L2-L1 norms.

Notes

Solves the following synthesis problem for the operator $\mathbf{Op}$ and the data $\mathbf{y}$:

$$J = \|\mathbf{y} - \mathbf{Op}\,\mathbf{x}\|_2^2 + \epsilon\|\mathbf{x}\|_p$$

or the analysis problem:

$$J = \|\mathbf{y} - \mathbf{Op}\,\mathbf{x}\|_2^2 + \epsilon\|\mathbf{SOp}^H\,\mathbf{x}\|_p$$

if `SOp` is provided. Note that in the first case, `SOp` should be assimilated in the modelling operator (i.e., `Op=GOp * SOp`).

The Iterative Shrinkage-Thresholding Algorithms (ISTA) [1] is used, where $p = 0, 0.5, 1$. This is a very simple iterative algorithm which applies the following step:

$$\mathbf{x}^{(i+1)} = T_{(\epsilon\alpha/2,p)}\left(\mathbf{x}^{(i)} + \alpha\,\mathbf{Op}^H\left(\mathbf{y} - \mathbf{Op}\,\mathbf{x}^{(i)}\right)\right)$$

or

$$\mathbf{x}^{(i+1)} = \mathbf{SOp}\left\{T_{(\epsilon\alpha/2,p)}\mathbf{SOp}^H\left(\mathbf{x}^{(i)} + \alpha\,\mathbf{Op}^H\left(\mathbf{y} - \mathbf{Op}\,\mathbf{x}^{(i)}\right)\right)\right\}$$

where $\epsilon\alpha/2$ is the threshold and $T_{(\tau,p)}$ is the thresholding rule. The most common variant of ISTA uses the so-called soft-thresholding rule $T(\tau, p = 1)$. Alternatively an hard-thresholding rule is used in the case of $p = 0$ or a half-thresholding rule is used in the case of $p = 1/2$. Finally, percentile bases thresholds are also implemented: the damping factor is not used anymore an the threshold changes at every iteration based on the computed percentile.

Methods

`__init__`(Op[, callbacks])	

`callback`(x, *args, **kwargs)	Callback routine
`finalize`([show])	Finalize solver
`run`(x[, niter, show, itershow])	Run solver
`setup`(y[, x0, niter, SOp, eps, alpha, ...])	Setup solver
`solve`(y[, x0, niter, SOp, eps, alpha, ...])	Run entire solver
`step`(x[, show])	Run one step of solver

Examples using `pylops.optimization.cls_sparsity.ISTA`

- *03. Solvers (Advanced)*

pylops.optimization.cls_sparsity.FISTA

class pylops.optimization.cls_sparsity.**FISTA**(*Op*, *callbacks=None*)

Fast Iterative Shrinkage-Thresholding Algorithm (FISTA).

Solve an optimization problem with L_p, $p = 0, 0.5, 1$ regularization, given the operator `Op` and data `y`. The operator can be real or complex, and should ideally be either square $N = M$ or underdetermined $N < M$.

Parameters

Op
[*pylops.LinearOperator*] Operator to invert

Raises

NotImplementedError
If `threshkind` is different from hard, soft, half, soft-percentile, or half-percentile

ValueError
If `perc=None` when `threshkind` is soft-percentile or half-percentile

See also:

OMP
Orthogonal Matching Pursuit (OMP).

ISTA
Iterative Shrinkage-Thresholding Algorithm (ISTA).

SPGL1
Spectral Projected-Gradient for L1 norm (SPGL1).

SplitBregman
Split Bregman for mixed L2-L1 norms.

Notes

Solves the following synthesis problem for the operator $\mathbf{Op}$ and the data $\mathbf{y}$:

$$J = \|\mathbf{y} - \mathbf{Op}\,\mathbf{x}\|_2^2 + \epsilon\|\mathbf{x}\|_p$$

or the analysis problem:

$$J = \|\mathbf{y} - \mathbf{Op}\,\mathbf{x}\|_2^2 + \epsilon\|\mathbf{SOp}^H\,\mathbf{x}\|_p$$

if `SOp` is provided.

The Fast Iterative Shrinkage-Thresholding Algorithm (FISTA) [1] is used, where $p = 0, 0.5, 1$. This is a modified version of ISTA solver with improved convergence properties and limited additional computational cost. Similarly to the ISTA solver, the choice of the thresholding algorithm to apply at every iteration is based on the choice of p.

Methods

`__init__`(Op[, callbacks])	

`callback`(x, *args, **kwargs)	Callback routine
`finalize`([show])	Finalize solver
`run`(x[, niter, show, itershow])	Run solver
`setup`(y[, x0, niter, SOp, eps, alpha, ...])	Setup solver
`solve`(y[, x0, niter, SOp, eps, alpha, ...])	Run entire solver
`step`(x, z[, show])	Run one step of solver

Examples using `pylops.optimization.cls_sparsity.FISTA`

- *03. Solvers (Advanced)*

pylops.optimization.cls_sparsity.SPGL1

class pylops.optimization.cls_sparsity.**SPGL1**(*Op*, *callbacks=None*)

Spectral Projected-Gradient for L1 norm.

Solve a constrained system of equations given the operator `Op` and a sparsyfing transform `SOp` aiming to retrive a model that is sparse in the sparsifying domain.

This is a simple wrapper to `spgl1.spgl1` which is a porting of the well-known SPGL1 MATLAB solver into Python. In order to be able to use this solver you need to have installed the `spgl1` library.

Parameters

Op
[*pylops.LinearOperator*] Operator to invert of size $[N \times M]$.

Raises

ModuleNotFoundError
If the `spgl1` library is not installed

Notes

Solve different variations of sparsity-promoting inverse problem by imposing sparsity in the retrieved model [1].

The first problem is called *basis pursuit denoise (BPDN)* and its cost function is

$$\|\mathbf{x}\|_1 \quad \text{subject to} \quad \left\|\mathbf{Op}\,\mathbf{S}^H\mathbf{x} - \mathbf{y}\right\|_2^2 \leq \sigma,$$

while the second problem is the *$_1$-regularized least-squares or LASSO* problem and its cost function is

$$\left\|\mathbf{Op}\,\mathbf{S}^H\mathbf{x} - \mathbf{y}\right\|_2^2 \quad \text{subject to} \quad \|\mathbf{x}\|_1 \leq \tau$$

Methods

`__init__`(Op[, callbacks])	

`callback`(x, *args, **kwargs)	Callback routine
`finalize`(*args[, show])	Finalize solver
`run`(x[, show])	Run solver
`setup`(y[, SOp, tau, sigma, show])	Setup solver
`solve`(y[, x0, SOp, tau, sigma, show])	Run entire solver
`step`()	Run one step of solver

pylops.optimization.cls_sparsity.SplitBregman

class pylops.optimization.cls_sparsity.**SplitBregman**(*Op*, *callbacks=None*)

Split Bregman for mixed L2-L1 norms.

Solve an unconstrained system of equations with mixed L_2 and L_1 regularization terms given the operator `Op`, a list of L_1 regularization terms `RegsL1`, and an optional list of L_2 regularization terms `RegsL2`.

Parameters

Op
[*pylops.LinearOperator*] Operator to invert

Notes

Solve the following system of unconstrained, regularized equations given the operator $\mathbf{Op}$ and a set of mixed norm (L^2 and L_1) regularization terms $\mathbf{R}_{2,i}$ and $\mathbf{R}_{1,i}$, respectively:

$$J = \frac{\mu}{2}\|\mathbf{y} - \mathbf{Op}\,\mathbf{x}\|_2^2 + \frac{1}{2}\sum_i \epsilon_{\mathbf{R}_{2,i}}\|\mathbf{y}_{\mathbf{R}_{2,i}} - \mathbf{R}_{2,i}\mathbf{x}\|_2^2 + \sum_i \epsilon_{\mathbf{R}_{1,i}}\|\mathbf{R}_{1,i}\mathbf{x}\|_1$$

where μ is the reconstruction damping, $\epsilon_{\mathbf{R}_{2,i}}$ are the damping factors used to weight the different L^2 regularization terms of the cost function and $\epsilon_{\mathbf{R}_{1,i}}$ are the damping factors used to weight the different L_1 regularization terms of the cost function.

The generalized Split-Bergman algorithm [1] is used to solve such cost function: the algorithm is composed of a sequence of unconstrained inverse problems and Bregman updates.

The original system of equations is initially converted into a constrained problem:

$$J = \frac{\mu}{2}\|\mathbf{y} - \mathbf{Op}\,\mathbf{x}\|_2^2 + \frac{1}{2}\sum_i \epsilon_{\mathbf{R}_{2,i}}\|\mathbf{y}_{\mathbf{R}_{2,i}} - \mathbf{R}_{2,i}\mathbf{x}\|_2^2 + \sum_i \|\mathbf{y}_i\|_1 \quad \text{subject to} \quad \mathbf{y}_i = \mathbf{R}_{1,i}\mathbf{x} \quad \forall i$$

and solved as follows:

$$\begin{aligned}(\mathbf{x}^{k+1}, \mathbf{y}_i^{k+1}) = *arg\,min_{\mathbf{x},\mathbf{y}_i} \; & \|\mathbf{y} - \mathbf{Op}\,\mathbf{x}\|_2^2 + \frac{1}{2}\sum_i \epsilon_{\mathbf{R}_{2,i}}\|\mathbf{y}_{\mathbf{R}_{2,i}} - \mathbf{R}_{2,i}\mathbf{x}\|_2^2 \\ & + \frac{1}{2}\sum_i \epsilon_{\mathbf{R}_{1,i}}\|\mathbf{y}_i - \mathbf{R}_{1,i}\mathbf{x} - \mathbf{b}_i^k\|_2^2 \\ & + \sum_i \|(\mathbf{y}_i, \mathbf{b}_i^k)\|_1 \end{aligned} \tag{3.1}$$

$$\mathbf{b}_i^{k+1} = \mathbf{b}_i^k + (\mathbf{R}_{1,i}\mathbf{x}^{k+1} - \mathbf{y}^{k+1})$$

The `scipy.sparse.linalg.lsqr` solver and a fast shrinkage algorithm are used within a inner loop to solve the first step. The entire procedure is repeated `niter_outer` times until convergence.

Methods

`__init__`(Op[, callbacks])	

`callback`(x, *args, **kwargs)	Callback routine
`finalize`([show])	Finalize solver
`run`(x[, show, itershow, show_inner])	Run solver
`setup`(y, RegsL1[, x0, niter_outer, ...])	Setup solver
`solve`(y, RegsL1[, x0, niter_outer, ...])	Run entire solver
`step`(x[, show, show_inner])	Run one step of solver

irls(Op, y[, x0, nouter, threshR, epsR, ...])	Iteratively reweighted least squares.
omp(Op, y[, niter_outer, niter_inner, ...])	Orthogonal Matching Pursuit (OMP).
ista(Op, y[, x0, niter, SOp, eps, alpha, ...])	Iterative Shrinkage-Thresholding Algorithm (ISTA).
fista(Op, y[, x0, niter, SOp, eps, alpha, ...])	Fast Iterative Shrinkage-Thresholding Algorithm (FISTA).
spgl1(Op, y[, x0, SOp, tau, sigma, show])	Spectral Projected-Gradient for L1 norm.
splitbregman(Op, y, RegsL1[, x0, ...])	Split Bregman for mixed L2-L1 norms.

pylops.optimization.sparsity.irls

pylops.optimization.sparsity.**irls**(*Op*, *y*, *x0=None*, *nouter=10*, *threshR=False*, *epsR=1e-10*, *epsI=1e-10*, *tolIRLS=1e-10*, *kind='data'*, *show=False*, *itershow=[10, 10, 10]*, *callback=None*, ***kwargs_solver*)

Iteratively reweighted least squares.

Solve an optimization problem with L_1 cost function (data IRLS) or L_1 regularization term (model IRLS) given the operator `Op` and data `y`.

In the *data IRLS*, the cost function is minimized by iteratively solving a weighted least squares problem with the weight at iteration i being based on the data residual at iteration $i-1$. This IRLS solver is robust to *outliers* since the L1 norm given less weight to large residuals than L2 norm does.

Similarly in the *model IRLS*, the weight at at iteration i is based on the model at iteration $i-1$. This IRLS solver inverts for a sparse model vector.

Parameters

Op
[*pylops.LinearOperator*] Operator to invert

y
[numpy.ndarray] Data

x0
[numpy.ndarray, optional] Initial guess

nouter
[int, optional] Number of outer iterations

threshR
[bool, optional] Apply thresholding in creation of weight (`True`) or damping (`False`)

epsR
[float, optional] Damping to be applied to residuals for weighting term

epsI
[float, optional] Tikhonov damping

tolIRLS
[float, optional] Tolerance. Stop outer iterations if difference between inverted model at subsequent iterations is smaller than `tolIRLS`

kind
[str, optional] Kind of solver (`data` or `model`)

show
[bool, optional] Display logs

itershow
[list, optional] Display set log for the first N1 steps, last N2 steps, and every N3 steps in between where N1, N2, N3 are the three element of the list.

callback
[callable, optional] Function with signature (`callback(x)`) to call after each iteration where `x` is the current model vector

****kwargs_solver**
Arbitrary keyword arguments for `scipy.sparse.linalg.cg` solver for data IRLS and `scipy.sparse.linalg.lsqr` solver for model IRLS when using numpy data(or `pylops.optimization.solver.cg` and `pylops.optimization.solver.cgls` when using cupy data)

Returns

xinv
[numpy.ndarray] Inverted model

nouter
[int] Number of effective outer iterations

Notes

See *pylops.optimization.cls_sparsity.IRLS*

Examples using `pylops.optimization.sparsity.irls`

- *MP, OMP, ISTA and FISTA*
- *Polynomial Regression*

pylops.optimization.sparsity.omp

pylops.optimization.sparsity.**omp**(*Op*, *y*, *niter_outer=10*, *niter_inner=40*, *sigma=0.0001*, *normalizecols=False*, *show=False*, *itershow=[10, 10, 10]*, *callback=None*)

Orthogonal Matching Pursuit (OMP).

Solve an optimization problem with L^0 regularization function given the operator `Op` and data `y`. The operator can be real or complex, and should ideally be either square $N = M$ or underdetermined $N < M$.

Parameters

Op
[*pylops.LinearOperator*] Operator to invert

y
[numpy.ndarray] Data

niter_outer
[int, optional] Number of iterations of outer loop

niter_inner
[int, optional] Number of iterations of inner loop. By choosing `niter_inner=0`, the Matching Pursuit (MP) algorithm is implemented.

sigma
[list] Maximum L_2 norm of residual. When smaller stop iterations.

normalizecols
[list, optional] Normalize columns (True) or not (False). Note that this can be expensive as it requires applying the forward operator n_{cols} times to unit vectors (i.e., containing 1 at position j and zero otherwise); use only when the columns of the operator are expected to have highly varying norms.

show
[bool, optional] Display iterations log

itershow
[list, optional] Display set log for the first N1 steps, last N2 steps, and every N3 steps in between where N1, N2, N3 are the three element of the list.

callback
[callable, optional] Function with signature (callback(x)) to call after each iteration where x is the current model vector

Returns

xinv
[numpy.ndarray] Inverted model

niter_outer
[int] Number of effective outer iterations

cost
[numpy.ndarray] History of cost function

See also:

ISTA
Iterative Shrinkage-Thresholding Algorithm (ISTA).

FISTA
Fast Iterative Shrinkage-Thresholding Algorithm (FISTA).

SPGL1
Spectral Projected-Gradient for L1 norm (SPGL1).

SplitBregman
Split Bregman for mixed L2-L1 norms.

Notes

See *pylops.optimization.cls_sparsity.OMP*

Examples using `pylops.optimization.sparsity.omp`

- *MP, OMP, ISTA and FISTA*

pylops.optimization.sparsity.ista

pylops.optimization.sparsity.**ista**(*Op*, *y*, *x0=None*, *niter=10*, *SOp=None*, *eps=0.1*, *alpha=None*, *eigsdict=None*, *tol=1e-10*, *threshkind='soft'*, *perc=None*, *decay=None*, *monitorres=False*, *show=False*, *itershow=[10, 10, 10]*, *callback=None*)

Iterative Shrinkage-Thresholding Algorithm (ISTA).

Solve an optimization problem with L^p, $p = 0, 0.5, 1$ regularization, given the operator `Op` and data `y`. The operator can be real or complex, and should ideally be either square $N = M$ or underdetermined $N < M$.

Parameters

Op
[*pylops.LinearOperator*] Operator to invert

y
[numpy.ndarray] Data of size $[N \times 1]$

x0: :obj:`numpy.ndarray`, optional
Initial guess

niter
[int] Number of iterations

SOp
[*pylops.LinearOperator*, optional] Regularization operator (use when solving the analysis problem)

eps
[float, optional] Sparsity damping

alpha
[float, optional] Step size. To guarantee convergence, ensure $\alpha \leq 1/\lambda_{\text{max}}$, where λ_{max} is the largest eigenvalue of $\mathbf{Op}^H\mathbf{Op}$. If `None`, the maximum eigenvalue is estimated and the optimal step size is chosen as $1/\lambda_{\text{max}}$. If provided, the convergence criterion will not be checked internally.

eigsdict
[dict, optional] Dictionary of parameters to be passed to `pylops.LinearOperator.eigs` method when computing the maximum eigenvalue

tol
[float, optional] Tolerance. Stop iterations if difference between inverted model at subsequent iterations is smaller than `tol`

threshkind
[str, optional] Kind of thresholding ('hard', 'soft', 'half', 'hard-percentile', 'soft-percentile', or 'half-percentile' - 'soft' used as default)

perc
[float, optional] Percentile, as percentage of values to be kept by thresholding (to be provided when thresholding is soft-percentile or half-percentile)

decay
[numpy.ndarray, optional] Decay factor to be applied to thresholding during iterations

monitorres
[bool, optional] Monitor that residual is decreasing

show
[bool, optional] Display logs

itershow
[list, optional] Display set log for the first N1 steps, last N2 steps, and every N3 steps in between where N1, N2, N3 are the three element of the list.

callback
[callable, optional] Function with signature (`callback(x)`) to call after each iteration where `x` is the current model vector

Returns

xinv
[numpy.ndarray] Inverted model

niter
[int] Number of effective iterations

cost
[numpy.ndarray] History of cost function

Raises

NotImplementedError
If `threshkind` is different from hard, soft, half, soft-percentile, or half-percentile

ValueError
If `perc=None` when `threshkind` is soft-percentile or half-percentile

ValueError
If `monitorres=True` and residual increases

See also:

`OMP`
Orthogonal Matching Pursuit (OMP).

`FISTA`
Fast Iterative Shrinkage-Thresholding Algorithm (FISTA).

`SPGL1`
Spectral Projected-Gradient for L1 norm (SPGL1).

`SplitBregman`
Split Bregman for mixed L2-L1 norms.

Notes

See *pylops.optimization.cls_sparsity.ISTA*

Examples using `pylops.optimization.sparsity.ista`

- *MP, OMP, ISTA and FISTA*
- *03. Solvers*

pylops.optimization.sparsity.fista

`pylops.optimization.sparsity.`**`fista`**(*Op*, *y*, *x0=None*, *niter=10*, *SOp=None*, *eps=0.1*, *alpha=None*, *eigsdict=None*, *tol=1e-10*, *threshkind='soft'*, *perc=None*, *decay=None*, *monitorres=False*, *show=False*, *itershow=[10, 10, 10]*, *callback=None*)

Fast Iterative Shrinkage-Thresholding Algorithm (FISTA).

Solve an optimization problem with L^p, $p = 0, 0.5, 1$ regularization, given the operator `Op` and data `y`. The operator can be real or complex, and should ideally be either square $N = M$ or underdetermined $N < M$.

Parameters

Op
[*pylops.LinearOperator*] Operator to invert

y
[numpy.ndarray] Data

x0: :obj:`numpy.ndarray`, optional
Initial guess

niter
[int, optional] Number of iterations

SOp
[*pylops.LinearOperator*, optional] Regularization operator (use when solving the analysis problem)

eps
[float, optional] Sparsity damping

alpha
[float, optional] Step size. To guarantee convergence, ensure $\alpha \leq 1/\lambda_{\text{max}}$, where λ_{max} is the largest eigenvalue of $\mathbf{Op}^H\mathbf{Op}$. If `None`, the maximum eigenvalue is estimated and the optimal step size is chosen as $1/\lambda_{\text{max}}$. If provided, the convergence criterion will not be checked internally.

eigsdict
[dict, optional] Dictionary of parameters to be passed to `pylops.LinearOperator.eigs` method when computing the maximum eigenvalue

tol
[float, optional] Tolerance. Stop iterations if difference between inverted model at subsequent iterations is smaller than `tol`

threshkind
[str, optional] Kind of thresholding ('hard', 'soft', 'half', 'soft-percentile', or 'half-percentile' - 'soft' used as default)

perc
[float, optional] Percentile, as percentage of values to be kept by thresholding (to be provided when thresholding is soft-percentile or half-percentile)

decay
[numpy.ndarray, optional] Decay factor to be applied to thresholding during iterations

monitorres
[bool, optional] Monitor that residual is decreasing

show
[bool, optional] Display iterations log

itershow
[list, optional] Display set log for the first N1 steps, last N2 steps, and every N3 steps in between where N1, N2, N3 are the three element of the list.

callback
[callable, optional] Function with signature (`callback(x)`) to call after each iteration where `x` is the current model vector

Returns

xinv
[numpy.ndarray] Inverted model

niter
[int] Number of effective iterations

cost
[numpy.ndarray, optional] History of cost function

Raises

NotImplementedError
If `threshkind` is different from hard, soft, half, soft-percentile, or half-percentile

ValueError
If `perc=None` when `threshkind` is soft-percentile or half-percentile

See also:

`OMP`
Orthogonal Matching Pursuit (OMP).

`ISTA`
Iterative Shrinkage-Thresholding Algorithm (ISTA).

`SPGL1`
Spectral Projected-Gradient for L1 norm (SPGL1).

`SplitBregman`
Split Bregman for mixed L2-L1 norms.

Notes

See *pylops.optimization.cls_sparsity.FISTA*

Examples using `pylops.optimization.sparsity.fista`

- *MP, OMP, ISTA and FISTA*
- *03. Solvers*
- *05. Image deblurring*
- *11. Radon filtering*
- *15. Least-squares migration*
- *18. Deblending*

pylops.optimization.sparsity.spgl1

pylops.optimization.sparsity.**spgl1**(*Op*, *y*, *x0=None*, *SOp=None*, *tau=0.0*, *sigma=0.0*, *show=False*, ***kwargs_spgl1*)

Spectral Projected-Gradient for L1 norm.

Solve a constrained system of equations given the operator `Op` and a sparsyfing transform `SOp` aiming to retrive a model that is sparse in the sparsyfing domain.

This is a simple wrapper to `spgl1.spgl1` which is a porting of the well-known SPGL1 MATLAB solver into Python. In order to be able to use this solver you need to have installed the `spgl1` library.

Parameters

Op
[*pylops.LinearOperator*] Operator to invert

y
[`numpy.ndarray`] Data

x0
[`numpy.ndarray`, optional] Initial guess

SOp
[*pylops.LinearOperator*, optional] Sparsifying transform

tau
[`float`, optional] Non-negative LASSO scalar. If different from `0`, SPGL1 will solve LASSO problem

sigma
[`list`, optional] BPDN scalar. If different from `0`, SPGL1 will solve BPDN problem

show
[`bool`, optional] Display iterations log

****kwargs_spgl1**
Arbitrary keyword arguments for `spgl1.spgl1` solver

Returns

xinv
[`numpy.ndarray`] Inverted model in original domain.

pinv
[`numpy.ndarray`] Inverted model in sparse domain.

info
[dict] Dictionary with the following information:

- tau, final value of tau (see sigma above)
- rnorm, two-norm of the optimal residual
- rgap, relative duality gap (an optimality measure)
- gnorm, Lagrange multiplier of (LASSO)
- **stat, final status of solver**
  - 1: found a BPDN solution,
  - 2: found a BP solution; exit based on small gradient,
  - 3: found a BP solution; exit based on small residual,
  - 4: found a LASSO solution,
  - 5: error, too many iterations,
  - 6: error, linesearch failed,
  - 7: error, found suboptimal BP solution,
  - 8: error, too many matrix-vector products.
- niters, number of iterations
- nProdA, number of multiplications with A
- nProdAt, number of multiplications with A'
- n_newton, number of Newton steps
- time_project, projection time (seconds)
- time_matprod, matrix-vector multiplications time (seconds)
- time_total, total solution time (seconds)
- niters_lsqr, number of lsqr iterations (if subspace_min=True)
- xnorm1, L1-norm model solution history through iterations
- rnorm2, L2-norm residual history through iterations
- lambdaa, Lagrange multiplier history through iterations

Raises

ModuleNotFoundError
If the spgl1 library is not installed

Notes

See *pylops.optimization.cls_sparsity.SPGL1*

Examples using `pylops.optimization.sparsity.spgl1`

- *03. Solvers*

pylops.optimization.sparsity.splitbregman

pylops.optimization.sparsity.**splitbregman**(*Op*, *y*, *RegsL1*, *x0=None*, *niter_outer=3*, *niter_inner=5*, *RegsL2=None*, *dataregsL2=None*, *mu=1.0*, *epsRL1s=None*, *epsRL2s=None*, *tol=1e-10*, *tau=1.0*, *restart=False*, *show=False*, *itershow=[10, 10, 10]*, *show_inner=False*, *callback=None*, ***kwargs_lsqr*)

Split Bregman for mixed L2-L1 norms.

Solve an unconstrained system of equations with mixed L_2 and L_1 regularization terms given the operator `Op`, a list of L_1 regularization terms `RegsL1`, and an optional list of L_2 regularization terms `RegsL2`.

Parameters

Op
[*pylops.LinearOperator*] Operator to invert

y
[numpy.ndarray] Data

RegsL1
[list] L_1 regularization operators

x0
[numpy.ndarray, optional] Initial guess

niter_outer
[int] Number of iterations of outer loop

niter_inner
[int] Number of iterations of inner loop of first step of the Split Bregman algorithm. A small number of iterations is generally sufficient and for many applications optimal efficiency is obtained when only one iteration is performed.

RegsL2
[list] Additional L_2 regularization operators (if `None`, L_2 regularization is not added to the problem)

dataregsL2
[list, optional] L_2 Regularization data (must have the same number of elements of `RegsL2` or equal to `None` to use a zero data for every regularization operator in `RegsL2`)

mu
[float, optional] Data term damping

epsRL1s
[list] L_1 Regularization dampings (must have the same number of elements as `RegsL1`)

epsRL2s
[list] L_2 Regularization dampings (must have the same number of elements as `RegsL2`)

tol
[float, optional] Tolerance. Stop outer iterations if difference between inverted model at subsequent iterations is smaller than `tol`

tau
[float, optional] Scaling factor in the Bregman update (must be close to 1)

restart
[bool, optional] The unconstrained inverse problem in inner loop is initialized with the initial guess (True) or with the last estimate (False)

show
[bool, optional] Display iterations log

itershow
[list, optional] Display set log for the first N1 steps, last N2 steps, and every N3 steps in between where N1, N2, N3 are the three element of the list.

show_inner
[bool, optional] Display inner iteration logs of lsqr

callback
[callable, optional] Function with signature (`callback(x)`) to call after each iteration where `x` is the current model vector

****kwargs_lsqr**
Arbitrary keyword arguments for `scipy.sparse.linalg.lsqr` solver used to solve the first subproblem in the first step of the Split Bregman algorithm.

Returns

xinv
[numpy.ndarray] Inverted model

itn_out
[int] Iteration number of outer loop upon termination

cost
[numpy.ndarray, optional] History of cost function through iterations

Notes

See *pylops.optimization.cls_sparsity.SplitBregman*

Examples using `pylops.optimization.sparsity.splitbregman`

- *Total Variation (TV) Regularization*
- *05. Image deblurring*
- *16. CT Scan Imaging*

Callbacks

Callbacks()	This is a template class which a user must subclass when implementing callbacks for a solver.
MetricsCallback(xtrue[, Op, which])	Metrics callback

pylops.optimization.callback.Callbacks

class pylops.optimization.callback.**Callbacks**

This is a template class which a user must subclass when implementing callbacks for a solver. This class comprises of the following methods:

- `on_setup_begin`: a method that is invoked at the start of the setup method of the solver
- `on_setup_end`: a method that is invoked at the end of the setup method of the solver
- `on_step_begin`: a method that is invoked at the start of the step method of the solver
- `on_step_end`: a method that is invoked at the end of the setup step of the solver
- `on_run_begin`: a method that is invoked at the start of the run method of the solver
- `on_run_end`: a method that is invoked at the end of the run method of the solver

All methods take two input parameters: the solver itself, and the vector `x`.

Examples

```
>>> import numpy as np
>>> from pylops.basicoperators import MatrixMult
>>> from pylops.optimization.solver import CG
>>> from pylops.optimization.callback import Callbacks
>>> class StoreIterCallback(Callbacks):
...     def __init__(self):
...         self.stored = []
...     def on_step_end(self, solver, x):
...         self.stored.append(solver.iiter)
>>> cb_sto = StoreIterCallback()
>>> Aop = MatrixMult(np.random.normal(0., 1., 36).reshape(6, 6))
>>> Aop = Aop.H @ Aop
>>> y = Aop @ np.ones(6)
>>> cgsolve = CG(Aop, callbacks=[cb_sto, ])
>>> xest = cgsolve.solve(y=y, x0=np.zeros(6), tol=0, niter=6, show=False)[0]
>>> xest
array([1., 1., 1., 1., 1., 1.])
```

Methods

`__init__`()	

`on_run_begin`(solver, x)	Callback before entire solver run
`on_run_end`(solver, x)	Callback after entire solver run
`on_setup_begin`(solver, x0)	Callback before setup
`on_setup_end`(solver, x)	Callback after setup
`on_step_begin`(solver, x)	Callback before step of solver
`on_step_end`(solver, x)	Callback after step of solver

Examples using `pylops.optimization.callback.Callbacks`

- *Linear Regression*
- *03. Solvers (Advanced)*

pylops.optimization.callback.MetricsCallback

class pylops.optimization.callback.**MetricsCallback**(*xtrue*, *Op=None*, *which=('mae', 'mse', 'snr', 'psnr')*)

Metrics callback

This callback can be used to store different metrics from the `pylops.utils.metrics` module during iterations.

Parameters

xtrue
[`np.ndarray`] True model vector

Op
[*`pylops.LinearOperator`*, optional] Operator to apply to the solution prior to comparing it with *xtrue*

which
[`tuple`, optional] List of metrics to compute (currently available: "mae", "mse", "snr", and "psnr")

Methods

`__init__`(xtrue[, Op, which])	

`on_run_begin`(solver, x)	Callback before entire solver run
`on_run_end`(solver, x)	Callback after entire solver run
`on_setup_begin`(solver, x0)	Callback before setup
`on_setup_end`(solver, x)	Callback after setup
`on_step_begin`(solver, x)	Callback before step of solver
`on_step_end`(solver, x)	Callback after step of solver

Examples using `pylops.optimization.callback.MetricsCallback`

- *03. Solvers (Advanced)*

3.7.3 Applications

Wave-Equation processing

SeismicInterpolation(data, nrec, iava[, ...])	Seismic interpolation (or regularization).
Deghosting(p, nt, nr, dt, dr, vel, zrec[, ...])	Wavefield deghosting.
WavefieldDecomposition(p, vz, nt, nr, dt, ...)	Up-down wavefield decomposition.
MDD(G, d[, dt, dr, nfmax, wav, twosided, ...])	Multi-dimensional deconvolution.
Marchenko(R[, dt, nt, dr, nfmax, wav, toff, ...])	Marchenko redatuming
LSM(z, x, t, srcs, recs, vel, wav, wavcenter)	Least-squares Migration (LSM).

pylops.waveeqprocessing.SeismicInterpolation

pylops.waveeqprocessing.**SeismicInterpolation**(*data*, *nrec*, *iava*, *iava1=None*, *kind='fk'*, *nffts=None*, *sampling=None*, *spataxis=None*, *spat1axis=None*, *taxis=None*, *paxis=None*, *p1axis=None*, *centeredh=True*, *nwins=None*, *nwin=None*, *nover=None*, *engine='numba'*, *dottest=False*, ***kwargs_solver*)

Seismic interpolation (or regularization).

Interpolate seismic data from irregular to regular spatial grid. Depending on the size of the input `data`, interpolation is either 2- or 3-dimensional. In case of 3-dimensional interpolation, data can be irregularly sampled in either one or both spatial directions.

Parameters

data
[`np.ndarray`] Irregularly sampled seismic data of size $[n_{r_y} (\times n_{r_x} \times n_t)]$

nrec
[`int` or `tuple`] Number of elements in the regularly sampled (reconstructed) spatial array, n_{R_y} for 2-dimensional data and (n_{R_y}, n_{R_x}) for 3-dimensional data

iava
[`list` or `numpy.ndarray`] Integer (or floating) indices of locations of available samples in first dimension of regularly sampled spatial grid of interpolated signal. The `pylops.basicoperators.Restriction` operator is used in case of integer indices, while the `pylops.signalprocessing.Interp` operator is used in case of floating indices.

iava1
[`list` or `numpy.ndarray`, optional] Integer (or floating) indices of locations of available samples in second dimension of regularly sampled spatial grid of interpolated signal. Can be used only in case of 3-dimensional data.

kind
[`str`, optional] Type of inversion: `fk` (default), `spatial`, `radon-linear`, `chirpradon-linear`, `radon-parabolic` , `radon-hyperbolic`, `sliding`, or `chirp-sliding`

nffts
[int or tuple, optional] nffts : tuple, optional Number of samples in Fourier Transform for each direction. Required if `kind='fk'`

sampling
[tuple, optional] Sampling steps `dy` (, `dx`) and `dt`. Required if `kind='fk'` or `kind='radon-linear'`

spataxis
[np.ndarray, optional] First spatial axis. Required for `kind='radon-linear'`, `kind='chirpradon-linear'`, `kind='radon-parabolic'`, `kind='radon-hyperbolic'`, can also be provided instead of `sampling` for `kind='fk'`

spat1axis
[np.ndarray, optional] Second spatial axis. Required for `kind='radon-linear'`, `kind='chirpradon-linear'`, `kind='radon-parabolic'`, `kind='radon-hyperbolic'`, can also be provided instead of `sampling` for `kind='fk'`

taxis
[np.ndarray, optional] Time axis. Required for `kind='radon-linear'`, `kind='chirpradon-linear'`, `kind='radon-parabolic'`, `kind='radon-hyperbolic'`, can also be provided instead of `sampling` for `kind='fk'`

paxis
[np.ndarray, optional] First Radon axis. Required for `kind='radon-linear'`, `kind='chirpradon-linear'`, `kind='radon-parabolic'`, `kind='radon-hyperbolic'`, `kind='sliding'`, and `kind='chirp-sliding'`

p1axis
[np.ndarray, optional] Second Radon axis. Required for `kind='radon-linear'`, `kind='chirpradon-linear'`, `kind='radon-parabolic'`, `kind='radon-hyperbolic'`, `kind='sliding'`, and `kind='chirp-sliding'`

centeredh
[bool, optional] Assume centered spatial axis (`True`) or not (`False`). Required for `kind='radon-linear'`, `kind='radon-parabolic'` and `kind='radon-hyperbolic'`

nwins
[int or tuple, optional] Number of windows. Required for `kind='sliding'` and `kind='chirp-sliding'`

nwin
[int or tuple, optional] Number of samples of window. Required for `kind='sliding'` and `kind='chirp-sliding'`

nover
[int or tuple, optional] Number of samples of overlapping part of window. Required for `kind='sliding'` and `kind='chirp-sliding'`

engine
[str, optional] Engine used for Radon computations (`numpy/numba` for `Radon2D` and `Radon3D` or `numpy/fftw` for `ChirpRadon2D` and `ChirpRadon3D`)

dottest
[bool, optional] Apply dot-test

****kwargs_solver**
Arbitrary keyword arguments for *pylops.optimization.leastsquares.*

regularized_inversion solver if `kind='spatial'` or `pylops.optimization.sparsity.FISTA` solver otherwise

Returns

recdata
[`np.ndarray`] Reconstructed data of size $[n_{R_y} (\times n_{R_x} \times n_t)]$

recprec
[`np.ndarray`] Reconstructed data in the sparse or preconditioned domain in case of `kind='fk'`, `kind='radon-linear'`, `kind='radon-parabolic'`, `kind='radon-hyperbolic'` and `kind='sliding'`

cost
[`np.ndarray`] Cost function norm

Raises

KeyError
If `kind` is neither `spatial`, `fl`, `radon-linear`, `radon-parabolic`, `radon-hyperbolic` nor `sliding`

Notes

The problem of seismic data interpolation (or regularization) can be formally written as

$$\mathbf{y} = \mathbf{R}\mathbf{x}$$

where a restriction or interpolation operator is applied along the spatial direction(s). Here $\mathbf{y} = [\mathbf{y}_{R1}^T, \mathbf{y}_{R2}^T, \ldots, \mathbf{y}_{RN^T}]^T$ where each vector $\mathbf{y}_{Ri}$ contains all time samples recorded in the seismic data at the specific receiver R_i. Similarly, $\mathbf{x} = [\mathbf{x}_{r1}^T, \mathbf{x}_{r2}^T, \ldots, \mathbf{x}_{rM}^T]$, contains all traces at the regularly and finely sampled receiver locations r_i.

Several alternative approaches can be taken to solve such a problem. They mostly differ in the choice of the regularization (or preconditining) used to mitigate the ill-posedness of the problem:

- `spatial`: least-squares inversion in the original time-space domain with an additional spatial smoothing regularization term, corresponding to the cost function $J = ||\mathbf{y} - \mathbf{R}\mathbf{x}||_2 + \epsilon_\nabla \nabla ||\mathbf{x}||_2$ where ∇ is a second order space derivative implemented via `pylops.basicoperators.SecondDerivative` in 2-dimensional case and `pylops.basicoperators.Laplacian` in 3-dimensional case
- `fk`: L1 inversion in frequency-wavenumber preconditioned domain corresponding to the cost function $J = ||\mathbf{y} - \mathbf{R}\mathbf{F}\mathbf{x}||_2$ where $\mathbf{F}$ is frequency-wavenumber transform implemented via *pylops.signalprocessing.FFT2D* in 2-dimensional case and *pylops.signalprocessing.FFTND* in 3-dimensional case
- `radon-linear`: L1 inversion in linear Radon preconditioned domain using the same cost function as `fk` but with $\mathbf{F}$ being a Radon transform implemented via *pylops.signalprocessing.Radon2D* in 2-dimensional case and *pylops.signalprocessing.Radon3D* in 3-dimensional case
- `radon-parabolic`: L1 inversion in parabolic Radon preconditioned domain
- `radon-hyperbolic`: L1 inversion in hyperbolic Radon preconditioned domain
- `sliding`: L1 inversion in sliding-linear Radon preconditioned domain using the same cost function as `fk` but with $\mathbf{F}$ being a sliding Radon transform implemented via *pylops.signalprocessing.Sliding2D* in 2-dimensional case and *pylops.signalprocessing.Sliding3D* in 3-dimensional case

Examples using `pylops.waveeqprocessing.SeismicInterpolation`

- *12. Seismic regularization*

pylops.waveeqprocessing.Deghosting

pylops.waveeqprocessing.**Deghosting**(*p*, *nt*, *nr*, *dt*, *dr*, *vel*, *zrec*, *pd=None*, *win=None*, *npad=(11, 11)*, *ntaper=(11, 11)*, *restriction=None*, *sptransf=None*, *solver=<function lsqr>*, *dottest=False*, *dtype='complex128'*, ***kwargs_solver*)

Wavefield deghosting.

Apply seismic wavefield decomposition from single-component (pressure) data. This process is also generally referred to as model-based deghosting.

Parameters

p
[np.ndarray] Pressure data of of size $[n_{r_x} (\times n_{r_y}) \times n_t]$ (or $[n_{r_{x,\text{sub}}} (\times n_{r_{y,\text{sub}}}) \times n_t]$ in case a `restriction` operator is provided. Note that $n_{r_{x,\text{sub}}}$ (and $n_{r_{y,\text{sub}}}$) must agree with the size of the output of this operator)

nt
[int] Number of samples along the time axis

nr
[int or tuple] Number of samples along the receiver axis (or axes)

dt
[float] Sampling along the time axis

dr
[float or tuple] Sampling along the receiver array of the separated pressure consituents

vel
[float] Velocity along the receiver array (must be constant)

zrec
[float] Depth of receiver array

pd
[np.ndarray, optional] Direct arrival to be subtracted from p

win
[np.ndarray, optional] Time window to be applied to p to remove the direct arrival (if pd=None)

ntaper
[float or tuple, optional] Number of samples of taper applied to propagator to avoid edge effects

npad
[float or tuple, optional] Number of samples of padding applied to propagator to avoid edge effects angle

restriction
[*pylops.LinearOperator*, optional] Restriction operator

sptransf
[*pylops.LinearOperator*, optional] Sparsifying operator

solver
[float, optional] Function handle of solver to be used if `kind='inverse'`

dottest
[bool, optional] Apply dot-test

dtype
[str, optional] Type of elements in input array. If `None`, directly inferred from `p`

****kwargs_solver**
Arbitrary keyword arguments for chosen `solver`

Returns

pup
[np.ndarray] Up-going wavefield

pdown
[np.ndarray] Down-going wavefield

Notes

Up- and down-going components of seismic data $p^-(x,t)$ and $p^+(x,t)$ can be estimated from single-component data $p(x,t)$ using a ghost model.

The basic idea [1] is that of using a one-way propagator in the f-k domain (also referred to as ghost model) to predict the down-going field from the up-going one (excluded the direct arrival and its source ghost referred here to as $p_d(x,t)$):

$$p^+ - p_d = e^{-jk_z 2z_{\text{rec}}} p^-$$

where k_z is the vertical wavenumber and z_{rec} is the depth of the array of receivers

In a matrix form we can thus write the total wavefield as:

$$\mathbf{p} - \mathbf{p_d} = (\mathbf{I} + \Phi)\mathbf{p}^-$$

where Φ is one-way propagator implemented via the *pylops.waveeqprocessing.PhaseShift* operator.

Examples using `pylops.waveeqprocessing.Deghosting`

- *13. Deghosting*

pylops.waveeqprocessing.WavefieldDecomposition

pylops.waveeqprocessing.**WavefieldDecomposition**(*p*, *vz*, *nt*, *nr*, *dt*, *dr*, *rho*, *vel*, *nffts=(None, None, None)*, *critical=100.0*, *ntaper=10*, *scaling=1.0*, *kind='inverse'*, *restriction=None*, *sptransf=None*, *solver=<function lsqr>*, *dottest=False*, *dtype='complex128'*, ***kwargs_solver*)

Up-down wavefield decomposition.

Apply seismic wavefield decomposition from multi-component (pressure and vertical particle velocity) data. This process is also generally referred to as data-based deghosting.

Parameters

p
[np.ndarray] Pressure data of size $[n_{r_x} (\times n_{r_y}) \times n_t]$ (or $[n_{r_{x,\text{sub}}} (\times n_{r_{y,\text{sub}}}) \times n_t]$ in case a `restriction` operator is provided. Note that $n_{r_{x,\text{sub}}}$ (and $n_{r_{y,\text{sub}}}$) must agree with the size of the output of this operator.)

vz
[np.ndarray] Vertical particle velocity data of same size as pressure data

nt
[int] Number of samples along the time axis

nr
[int or tuple] Number of samples along the receiver axis (or axes)

dt
[float] Sampling along the time axis

dr
[float or tuple] Sampling along the receiver array (or axes)

rho
[float] Density ρ along the receiver array (must be constant)

vel
[float] Velocity c along the receiver array (must be constant)

nffts
[tuple, optional] Number of samples along the wavenumber and frequency axes

critical
[float, optional] Percentage of angles to retain in obliquity factor. For example, if `critical=100` only angles below the critical angle $\frac{f(k_x)}{c}$ will be retained

ntaper
[float, optional] Number of samples of taper applied to obliquity factor around critical angle

kind
[str, optional] Type of separation: `inverse` (default) or `analytical`

scaling
[float, optional] Scaling to apply to the operator (see Notes of `pylops.waveeqprocessing.wavedecomposition.UpDownComposition2D` for more details)

restriction
[*pylops.LinearOperator*, optional] Restriction operator

sptransf
[*pylops.LinearOperator*, optional] Sparsifying operator

solver
[float, optional] Function handle of solver to be used if `kind='inverse'`

dottest
[bool, optional] Apply dot-test

dtype
[str, optional] Type of elements in input array.

****kwargs_solver**
Arbitrary keyword arguments for chosen `solver`

Returns

pup
[np.ndarray] Up-going wavefield

pdown
[np.ndarray] Down-going wavefield

Raises

KeyError
If `kind` is neither `analytical` nor `inverse`

Notes

Up- and down-going components of seismic data $p^-(x,t)$ and $p^+(x,t)$ can be estimated from multi-component data $p(x,t)$ and $v_z(x,t)$ by computing the following expression [1]:

$$\begin{bmatrix} \hat{p}^+ \\ \hat{p}^- \end{bmatrix} (k_x, \omega) = \frac{1}{2} \begin{bmatrix} 1 & \frac{\omega \rho}{k_z} \\ 1 & -\frac{\omega \rho}{k_z} \end{bmatrix} \begin{bmatrix} \hat{p} \\ \hat{v}_z \end{bmatrix} (k_x, \omega)$$

if `kind='analytical'` or alternatively by solving the equation in `ptcpy.waveeqprocessing.UpDownComposition2D` as an inverse problem, if `kind='inverse'`.

The latter approach has several advantages as data regularization can be included as part of the separation process allowing the input data to be aliased. This is obtained by solving the following problem:

$$\begin{bmatrix} \mathbf{p} \\ s\mathbf{v_z} \end{bmatrix} = \begin{bmatrix} \mathbf{RF} & 0 \\ 0 & s\mathbf{RF} \end{bmatrix} \mathbf{W} \begin{bmatrix} \mathbf{F}^H\mathbf{S} & 0 \\ 0 & \mathbf{F}^H\mathbf{S} \end{bmatrix} \mathbf{p}^{\pm}$$

where $\mathbf{R}$ is a `ptcpy.basicoperators.Restriction` operator and $\mathbf{S}$ is sparsyfing transform operator (e.g., `ptcpy.signalprocessing.Radon2D`).

Examples using `pylops.waveeqprocessing.WavefieldDecomposition`

- *14. Seismic wavefield decomposition*

pylops.waveeqprocessing.MDD

`pylops.waveeqprocessing.MDD`(*G*, *d*, *dt=0.004*, *dr=1.0*, *nfmax=None*, *wav=None*, *twosided=True*, *causality_precond=False*, *adjoint=False*, *psf=False*, *dottest=False*, *saveGt=True*, *add_negative=True*, *smooth_precond=0*, *fftengine='numpy'*, ***kwargs_solver*)

Multi-dimensional deconvolution.

Solve multi-dimensional deconvolution problem using `scipy.sparse.linalg.lsqr` iterative solver.

Parameters

G
[numpy.ndarray] Multi-dimensional convolution kernel in time domain of size $[n_s \times n_r \times n_t]$ for `twosided=False` or `twosided=True` and `add_negative=True` (with only positive times) or size $[n_s \times n_r \times 2n_t - 1]$ for `twosided=True` and `add_negative=False` (with both positive and negative times)

d
[numpy.ndarray] Data in time domain $[n_s\,(\times n_{vs}) \times n_t]$ if `twosided=False` or `twosided=True` and `add_negative=True` (with only positive times) or size $[n_s\,(\times n_{vs}) \times 2n_t - 1]$ if `twosided=True`

dt
[float, optional] Sampling of time integration axis

dr
[float, optional] Sampling of receiver integration axis

nfmax
[int, optional] Index of max frequency to include in deconvolution process

wav
[numpy.ndarray, optional] Wavelet to convolve to the inverted model and psf (must be centered around its index in the middle of the array). If `None`, the outputs of the inversion are returned directly.

twosided
[bool, optional] MDC operator and data both negative and positive time (`True`) or only positive (`False`)

add_negative
[bool, optional] Add negative side to MDC operator and data (`True`) or not (`False`)- operator and data are already provided with both positive and negative sides. To be used only with `twosided=True`.

causality_precond
[bool, optional] Apply causality mask (`True`) or not (`False`)

smooth_precond
[int, optional] Lenght of smoothing to apply to causality preconditioner

adjoint
[bool, optional] Compute and return adjoint(s)

psf
[bool, optional] Compute and return Point Spread Function (PSF) and its inverse

dottest
[bool, optional] Apply dot-test

saveGt
[bool, optional] Save `G` and `G.H` to speed up the computation of adjoint of *pylops.signalprocessing.Fredholm1* (`True`) or create `G.H` on-the-fly (`False`) Note that `saveGt=True` will be faster but double the amount of required memory

fftengine
[str, optional] Engine used for fft computation (`numpy`, `scipy` or `fftw`)

****kwargs_solver**
Arbitrary keyword arguments for chosen solver (scipy.sparse.linalg.cg and `pylops.optimization.solver.cg` are used as default for numpy and cupy *data*, respectively)

Returns

minv
[numpy.ndarray] Inverted model of size $[n_r\,(\times n_{vs})\,\times\,n_t]$ for `twosided=False` or $[n_r\,(\times n_v s)\,\times\,2n_t\,-\,1]$ for `twosided=True`

madj
[numpy.ndarray] Adjoint model of size $[n_r\,(\times n_{vs})\,\times\,n_t]$ for `twosided=False` or $[n_r\,(\times n_r)\,\times\,2n_t\,-\,1]$ for `twosided=True`

psfinv
[numpy.ndarray] Inverted psf of size $[n_r \times n_r \times n_t]$ for `twosided=False` or $[n_r \times n_r \times 2n_t - 1]$ for `twosided=True`

psfadj
[numpy.ndarray] Adjoint psf of size $[n_r \times n_r \times n_t]$ for `twosided=False` or $[n_r \times n_r \times 2n_t - 1]$ for `twosided=True`

See also:

MDC
Multi-dimensional convolution

Notes

Multi-dimensional deconvolution (MDD) is a mathematical ill-solved problem, well-known in the image processing and geophysical community [1].

MDD aims at removing the effects of a Multi-dimensional Convolution (MDC) kernel or the so-called blurring operator or point-spread function (PSF) from a given data. It can be written as

$$\mathbf{d} = \mathbf{D}\mathbf{m}$$

or, equivalently, by means of its normal equation

$$\mathbf{m} = (\mathbf{D}^H\mathbf{D})^{-1}\mathbf{D}^H\mathbf{d}$$

where $\mathbf{D}^H\mathbf{D}$ is the PSF.

Examples using `pylops.waveeqprocessing.MDD`

- *09. Multi-Dimensional Deconvolution*

pylops.waveeqprocessing.Marchenko

class pylops.waveeqprocessing.**Marchenko**(*R*, *dt=0.004*, *nt=None*, *dr=1.0*, *nfmax=None*, *wav=None*, *toff=0.0*, *nsmooth=10*, *saveRt=True*, *prescaled=False*, *fftengine='numpy'*, *dtype='float64'*)

Marchenko redatuming

Solve multi-dimensional Marchenko redatuming problem using `scipy.sparse.linalg.lsqr` iterative solver.

Parameters

R
[numpy.ndarray] Multi-dimensional reflection response in time or frequency domain of size $[n_s \times n_r \times n_t(n_{f_{\max}})]$. If provided in time, `R` should not be of complex type. Note that the reflection response should have already been multiplied by 2.

dt
[float, optional] Sampling of time integration axis

nt
[float, optional] Number of samples in time (not required if `R` is in time)

dr
[float, optional] Sampling of receiver integration axis

nfmax
[int, optional] Index of max frequency to include in deconvolution process

wav
[numpy.ndarray, optional] Wavelet to apply to direct arrival when created using `trav`

toff
[float, optional] Time-offset to apply to traveltime

nsmooth
[int, optional] Number of samples of smoothing operator to apply to window

saveRt
[bool, optional] Save `R` and `R.H` to speed up the computation of adjoint of *pylops.signalprocessing.Fredholm1* (`True`) or create `R.H` on-the-fly (`False`) Note that `saveRt=True` will be faster but double the amount of required memory

prescaled
[bool, optional] Apply scaling to `R` (`False`) or not (`False`) when performing spatial and temporal summations within the *pylops.waveeqprocessing.MDC* operator. In case `prescaled=True`, the `R` is assumed to have been pre-scaled by the user.

fftengine
[str, optional] New in version 1.17.0.

Engine used for fft computation (`numpy`, `scipy` or `fftw`)

dtype
[bool, optional] Type of elements in input array.

Raises

TypeError
If `t` is not numpy.ndarray.

See also:

MDC
Multi-dimensional convolution

MDD
Multi-dimensional deconvolution

Notes

Marchenko redatuming is a method that allows to produce correct subsurface-to-surface responses given the availability of a reflection data and a macro-velocity model [1].

The Marchenko equations can be written in a compact matrix form [2] and solved by means of iterative solvers such as LSQR:

$$\begin{bmatrix} \Theta \mathbf{R} \mathbf{f}_\mathbf{d}^+ \\ \mathbf{0} \end{bmatrix} = \mathbf{I} - \begin{bmatrix} \mathbf{0} & \Theta \mathbf{R} \\ \Theta \mathbf{R}^* & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{f}^- \\ \mathbf{f}_\mathbf{m}^+ \end{bmatrix}$$

Finally the subsurface Green's functions can be obtained applying the following operator to the retrieved focusing functions

$$\begin{bmatrix} -\mathbf{g}^- \\ \mathbf{g}^{+*} \end{bmatrix} = \mathbf{I} - \begin{bmatrix} \mathbf{0} & \mathbf{R} \\ \mathbf{R}^* & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{f}^- \\ \mathbf{f}^+ \end{bmatrix}$$

Here $\mathbf{R}$ is the monopole-to-particle velocity seismic response (already multiplied by 2).

Attributes

ns
[int] Number of samples along source axis

nr
[int] Number of samples along receiver axis

shape
[tuple] Operator shape

explicit
[bool] Operator contains a matrix that can be solved explicitly (`True`) or not (`False`)

Methods

`__init__`(R[, dt, nt, dr, nfmax, wav, toff, ...])	

`apply_multiplepoints`(trav[, G0, nfft, rtm, ...])	Marchenko redatuming for multiple points
`apply_onepoint`(trav[, G0, nfft, rtm, ...])	Marchenko redatuming for one point

Examples using `pylops.waveeqprocessing.Marchenko`

- *10. Marchenko redatuming by inversion*

pylops.waveeqprocessing.LSM

class pylops.waveeqprocessing.**LSM**(*z*, *x*, *t*, *srcs*, *recs*, *vel*, *wav*, *wavcenter*, *y=None*, *kind='kirchhoff'*, *dottest=False*, ***kwargs_mod*)

Least-squares Migration (LSM).

Solve seismic migration as inverse problem given smooth velocity model `vel` and an acquisition setup identified by sources (`src`) and receivers (`recs`).

Parameters

z
[numpy.ndarray] Depth axis

x
[numpy.ndarray] Spatial axis

t
[numpy.ndarray] Time axis for data

srcs
[numpy.ndarray] Sources in array of size $[2(3) \times n_s]$

recs
[numpy.ndarray] Receivers in array of size $[2(3) \times n_r]$

vel
[numpy.ndarray or float] Velocity model of size $[(n_y \times) \, n_x \times n_z]$ (or constant)

wav
[numpy.ndarray] Wavelet

wavcenter
[int] Index of wavelet center

y
[numpy.ndarray] Additional spatial axis (for 3-dimensional problems)

kind
[:str`, optional] Kind of modelling operator (`kirchhoff`, `twoway`)

dottest
[bool, optional] Apply dot-test

****kwargs_mod**
[int, optional] Additional arguments to pass to modelling operators

See also:

pylops.waveeqprocessing.Kirchhoff
Kirchhoff operator

pylops.waveeqprocessing.AcousticWave2D
AcousticWave2D operator

Notes

Inverting a demigration operator is generally referred in the literature as least-squares migration (LSM) as historically a least-squares cost function has been used for this purpose. In practice any other cost function could be used, for examples if `solver='pylops.optimization.sparsity.FISTA'` a sparse representation of reflectivity is produced as result of the inversion.

This routines provides users with a easy-to-use, out-of-the-box least-squares migration application that currently implements:

- Kirchhoff LSM: this problem is parametrized in terms of reflectivity (i.e., vertical derivative of the acoustic impedance - or velocity in case of constant density). Currently, a ray-based modelling engine is used for this case (see *pylops.waveeqprocessing.Kirchhoff*).
- Born LSM: this problem is parametrized in terms of squared slowness perturbation (in the constant density case) and it is solved using an acoustic two-way eave equation modelling engine (see *pylops.waveeqprocessing.AcousticWave2D*).

The following table shows the current status of the LSM application:

	Kirchhoff integral	WKBJ	Wave eq
Reflectivity	V	X	X
Slowness-squared	X	X	V

Finally, it is worth noting that for both cases the first iteration of an iterative scheme aimed at inverting the demigration operator is a simple a projection of the recorded data into the model domain. An approximate (band-limited) image of the subsurface is therefore created. This process is referred to in the literature as *migration*.

Attributes

Demop
[*pylops.LinearOperator*] Demigration operator operator

Methods

`__init__`(z, x, t, srcs, recs, vel, wav, ...)	
`solve`(d[, solver])	Solve least-squares migration equations with chosen `solver`

Examples using `pylops.waveeqprocessing.LSM`

- *15. Least-squares migration*

Geophysical subsurface characterization

poststack.PoststackInversion(data, wav[, ...])	Post-stack linearized seismic inversion.
prestack.PrestackInversion(data, theta, wav)	Pre-stack linearized seismic inversion.

pylops.avo.poststack.PoststackInversion

pylops.avo.poststack.**PoststackInversion**(*data*, *wav*, *m0=None*, *explicit=False*, *simultaneous=False*, *epsI=None*, *epsR=None*, *dottest=False*, *epsRL1=None*, ***kwargs_solver*)

Post-stack linearized seismic inversion.

Invert post-stack seismic operator to retrieve an elastic parameter of choice from band-limited seismic post-stack data. Depending on the choice of input parameters, inversion can be trace-by-trace with explicit operator or global with either explicit or linear operator.

Parameters

data
[`np.ndarray`] Band-limited seismic post-stack data of size $[n_{t_0}\ (\times n_x \times n_y)]$

wav
[`np.ndarray`] Wavelet in time domain (must have odd number of elements and centered to zero). If 1d, assume stationary wavelet for the entire time axis. If 2d of size $[n_{t_0} \times n_h]$ use as non-stationary wavelet

m0
[`np.ndarray`, optional] Background model of size $[n_{t_0}\ (\times n_x \times n_y)]$

explicit
[`bool`, optional] Create a chained linear operator (`False`, preferred for large data) or a `MatrixMult` linear operator with dense matrix (`True`, preferred for small data)

simultaneous
[`bool`, optional] Simultaneously invert entire data (`True`) or invert trace-by-trace (`False`) when using `explicit` operator (note that the entire data is always inverted when working with linear operator)

epsI
[`float`, optional] Damping factor for Tikhonov regularization term

epsR
[float, optional] Damping factor for additional Laplacian regularization term

dottest
[bool, optional] Apply dot-test

epsRL1
[float, optional] Damping factor for additional blockiness regularization term

****kwargs_solver**
Arbitrary keyword arguments for `scipy.linalg.lstsq` solver (if `explicit=True` and `epsR=None`) or `scipy.sparse.linalg.lsqr` solver (if `explicit=False` and/or `epsR` is not `None`)

Returns

minv
[np.ndarray] Inverted model of size $[n_{t_0}\ (\times n_x \times n_y)]$

datar
[np.ndarray] Residual data (i.e., data - background data) of size $[n_{t_0}\ (\times n_x \times n_y)]$

Notes

The cost function and solver used in the seismic post-stack inversion module depends on the choice of `explicit`, `simultaneous`, `epsI`, and `epsR` parameters:

- `explicit=True`, `epsI=None` and `epsR=None`: the explicit solver `scipy.linalg.lstsq` is used if `simultaneous=False` (or the iterative solver `scipy.sparse.linalg.lsqr` is used if `simultaneous=True`)
- `explicit=True` with `epsI` and `epsR=None`: the regularized normal equations $\mathbf{W}^T\mathbf{d} = (\mathbf{W}^T\mathbf{W} + \epsilon_\mathbf{I}^2\mathbf{I})\mathbf{AI}$ are instead fed into the `scipy.linalg.lstsq` solver if `simultaneous=False` (or the iterative solver `scipy.sparse.linalg.lsqr` if `simultaneous=True`)
- `explicit=False` and `epsR=None`: the iterative solver `scipy.sparse.linalg.lsqr` is used
- `explicit=False` with `epsR` and `epsRL1=None`: the iterative solver *pylops.optimization.leastsquares.regularized_inversion* is used to solve the spatially regularized problem.
- `explicit=False` with `epsR` and `epsRL1`: the iterative solver `pylops.optimization.sparsity.SplitBregman` is used to solve the blockiness-promoting (in vertical direction) and spatially regularized (in additional horizontal directions) problem.

Note that the convergence of iterative solvers such as `scipy.sparse.linalg.lsqr` can be very slow for this type of operator. It is suggested to take a two steps approach with first a trace-by-trace inversion using the explicit operator, followed by a regularized global inversion using the outcome of the previous inversion as initial guess.

pylops.avo.prestack.PrestackInversion

pylops.avo.prestack.**PrestackInversion**(*data*, *theta*, *wav*, *m0=None*, *linearization='akirich'*, *explicit=False*, *simultaneous=False*, *epsI=None*, *epsR=None*, *dottest=False*, *returnres=False*, *epsRL1=None*, *kind='centered'*, *vsvp=0.5*, ***kwargs_solver*)

Pre-stack linearized seismic inversion.

Invert pre-stack seismic operator to retrieve a set of elastic property profiles from band-limited seismic pre-stack data (i.e., angle gathers). Depending on the choice of input parameters, inversion can be trace-by-trace with explicit operator or global with either explicit or linear operator.

Parameters

data
[np.ndarray] Band-limited seismic post-stack data of size $[(n_{\text{lins}}\times)\, n_{t_0}\times n_\theta\, (\times n_x\times n_y)]$

theta
[np.ndarray] Incident angles in degrees

wav
[np.ndarray] Wavelet in time domain (must had odd number of elements and centered to zero)

m0
[np.ndarray, optional] Background model of size $[n_{t_0}\times n_m\, (\times n_x\times n_y)]$

linearization
[*{"akirich", "fatti", "PS"}* or list, optional]

- "akirich": Aki-Richards. See *pylops.avo.avo.akirichards*.
- "fatti": Fatti. See *pylops.avo.avo.fatti*.
- "PS": PS. See *pylops.avo.avo.ps*.
- List which is a combination of previous options (required only when `m0 is None`).

explicit
[bool, optional] Create a chained linear operator (`False`, preferred for large data) or a `MatrixMult` linear operator with dense matrix (`True`, preferred for small data)

simultaneous
[bool, optional] Simultaneously invert entire data (`True`) or invert trace-by-trace (`False`) when using `explicit` operator (note that the entire data is always inverted when working with linear operator)

epsI
[float or list, optional] Damping factor(s) for Tikhonov regularization term. If a list of n_m elements is provided, the regularization term will have different strenght for each elastic property

epsR
[float, optional] Damping factor for additional Laplacian regularization term

dottest
[bool, optional] Apply dot-test

returnres
[bool, optional] Return residuals

epsRL1
[float, optional] Damping factor for additional blockiness regularization term

kind
[str, optional] Derivative kind (`forward` or `centered`).

vsvp
[float or np.ndarray] V_S/V_P ratio (constant or time/depth variant)

****kwargs_solver**
Arbitrary keyword arguments for scipy.linalg.lstsq solver (if `explicit=True` and `epsR=None`) or scipy.sparse.linalg.lsqr solver (if `explicit=False` and/or `epsR` is not `None`))

Returns

minv
[np.ndarray] Inverted model of size $[n_{t_0} \times n_m \ (\times n_x \times n_y)]$

datar
[np.ndarray] Residual data (i.e., data - background data) of size $[n_{t_0} \times n_\theta \ (\times n_x \times n_y)]$

Notes

The different choices of cost functions and solvers used in the seismic pre-stack inversion module follow the same convention of the seismic post-stack inversion module.

Refer to *pylops.avo.poststack.PoststackInversion* for more details.

3.8 PyLops Utilities

Alongside with its *Linear Operators* and *Solvers*, PyLops contains also a number of auxiliary routines performing universal tasks that are used by several operators or simply within one or more *Tutorials* for the preparation of input data and subsequent visualization of results.

3.8.1 Dot-test

dottest(Op[, nr, nc, rtol, atol, ...])	Dot test.

pylops.utils.dottest

pylops.utils.**dottest**(*Op*, *nr=None*, *nc=None*, *rtol=1e-06*, *atol=1e-21*, *complexflag=0*, *raiseerror=True*, *verb=False*, *backend='numpy'*)

Dot test.

Generate random vectors $\mathbf{u}$ and $\mathbf{v}$ and perform dot-test to verify the validity of forward and adjoint operators. This test can help to detect errors in the operator implementation.

Parameters

Op
[*pylops.LinearOperator*] Linear operator to test.

nr
[int] Number of rows of operator (i.e., elements in data)

nc
[int] Number of columns of operator (i.e., elements in model)

rtol
[float, optional] Relative dottest tolerance

atol
[float, optional] Absolute dottest tolerance .. versionadded:: 2.0.0

complexflag
[bool, optional] Generate random vectors with

- 0: Real entries for model and data
- 1: Complex entries for model and real entries for data

- 2: Real entries for model and complex entries for data
- 3: Complex entries for model and data

raiseerror
[bool, optional] Raise error or simply return False when dottest fails

verb
[bool, optional] Verbosity

backend
[str, optional] Backend used for dot test computations (numpy or cupy). This parameter will be used to choose how to create the random vectors.

Returns

passed
[bool] Passed flag.

Raises

AssertionError
If dot-test is not verified within chosen tolerances.

Notes

A dot-test is mathematical tool used in the development of numerical linear operators.

More specifically, a correct implementation of forward and adjoint for a linear operator should verify the following *equality* within a numerical tolerance:

$$(\mathbf{Op}\,\mathbf{u})^H\mathbf{v} = \mathbf{u}^H(\mathbf{Op}^H\mathbf{v})$$

Examples using `pylops.utils.dottest`

- *Normal Moveout (NMO) Correction*
- *Seislet transform*
- *02. The Dot-Test*

3.8.2 Decorators

add_ndarray_support_to_solver(func)	Decorator which converts a solver-type function that only supports a 1d-array into one that supports one (dimsd-shaped) ndarray.
disable_ndarray_multiplication(func)	Decorator which disables ndarray multiplication.
reshaped([func, forward, swapaxis])	Decorator for the common reshape/flatten pattern used in many operators.

pylops.utils.decorators.add_ndarray_support_to_solver

pylops.utils.decorators.**add_ndarray_support_to_solver**(*func*)

Decorator which converts a solver-type function that only supports a 1d-array into one that supports one (dimsd-shaped) ndarray.

Parameters

func
[callable] Solver type function. Its signature must be `func(A, b, *args, **kwargs)`. Its output must be a result-type tuple: `(xinv, ...)`.

Returns

wrapper
[callable] Decorated function

pylops.utils.decorators.disable_ndarray_multiplication

pylops.utils.decorators.**disable_ndarray_multiplication**(*func*)

Decorator which disables ndarray multiplication.

Parameters

func
[callable] Generic function

Returns

wrapper
[callable] Decorated function

pylops.utils.decorators.reshaped

pylops.utils.decorators.**reshaped**(*func=None*, *forward=None*, *swapaxis=False*)

Decorator for the common reshape/flatten pattern used in many operators.

Parameters

func
[callable, optional] Function to be decorated when no arguments are provided

forward
[bool, optional] Reshape to `dims` if True, otherwise to `dimsd`. If not provided, the decorated function's name will be inspected to infer the mode. Any operator having a name with 'rmat' as substring or whose name is 'div' or '__truediv__' will reshape to `dimsd`.

swapaxis
[bool, optional] If True, swaps the last axis of the input array of the decorated function with `self.axis`. Only use if the decorated LinearOperator has `axis` attribute.

Notes

A `_matvec` (forward) function can be simplified from

```
def _matvec(self, x):
    x = x.reshape(self.dims)
    x = x.swapaxes(self.axis, -1)
    y = do_things_to_reshaped_swapped(y)
    y = y.swapaxes(self.axis, -1)
    y = y.ravel()
    return y
```

to

```
@reshaped(swapaxis=True)
def _matvec(self, x):
    y = do_things_to_reshaped_swapped(y)
    return y
```

When the decorator has no arguments, it can be called without parenthesis, e.g.:

```
@reshaped
def _matvec(self, x):
    y = do_things_to_reshaped(y)
    return y
```

Examples using `pylops.utils.decorators.reshaped`

- *Normal Moveout (NMO) Correction*

3.8.3 Describe

describe(Op)	Describe a PyLops operator

pylops.utils.describe.describe

pylops.utils.describe.**describe**(*Op*)

Describe a PyLops operator

New in version 1.17.0.

Convert a PyLops operator into a `sympy` mathematical formula. This routine is useful both for debugging and educational purposes.

Note that users can add a name to each operator prior to running the describe method, i.e. `Op.name='Opname'`. Alternatively, each of the PyLops operator that composes the operator `Op` is automatically assigned a name. Moreover, note that the symbols T and † are used in the mathematical expressions to indicate transposed and adjoint operators, respectively.

Parameters

Op

[*pylops.LinearOperator*] Linear Operator to describe

Examples using `pylops.utils.describe.describe`

- *Describe*

3.8.4 Estimators

trace_hutchinson(Op[, neval, batch_size, ...])	Trace of linear operator using the Hutchinson method.
trace_hutchpp(Op[, neval, sampler, backend])	Trace of linear operator using the Hutch++ method.
trace_nahutchpp(Op[, neval, sampler, c1, ...])	Trace of linear operator using the NA-Hutch++ method.

pylops.utils.estimators.trace_hutchinson

pylops.utils.estimators.**trace_hutchinson**(*Op*, *neval=None*, *batch_size=None*, *sampler='rademacher'*, *backend='numpy'*)

Trace of linear operator using the Hutchinson method.

Returns an estimate of the trace of a linear operator using the Hutchinson method [1].

Parameters

Op
[*pylops.LinearOperator*] Linear operator to compute trace on.

neval
[int, optional] Maximum number of matrix-vector products compute. Defaults to 10% of `shape[1]`.

batch_size
[int, optional] Vectorize computations by sampling sketching matrices instead of vectors. Set this value to as high as permitted by memory, but there is no guarantee of speedup. Coerced to never exceed `neval`. When using "unitvector" as sampler, is coerced to not exceed `shape[1]`. Defaults to 100 or `neval`.

sampler
[str, optional] Sample sketching matrices from the following distributions:

- "gaussian": Mean zero, unit variance Gaussian.
- "rayleigh": Sample from mean zero, unit variance Gaussian and normalize the columns.
- "rademacher": Random sign.
- "unitvector": Samples from the unit vectors e_i without replacement.

backend
[str, optional] Backend used to densify matrix (`numpy` or `cupy`). Note that this must be consistent with how the operator has been created.

Returns

trace
[float] Operator trace.

Raises

ValueError
If `neval` is smaller than 3.

NotImplementedError
If the `sampler` is not one of the available samplers.

Notes

Let m = `shape[1]` and k = `neval`. This algorithm estimates the trace via

$$\frac{1}{k}\sum_{i=1}^{k} \mathbf{z}_i^T \, \mathbf{Op} \, \mathbf{z}_i$$

where vectors $\mathbf{z}_i$ are sampled according to the sampling function. See [2] for a description of the variance and ϵ-approximation of different samplers.

Prefer the Rademacher sampler if the goal is to minimize variance, but the Gaussian for a better probability of approximating the correct value. Use the Unit Vector approach if you are sampling a large number of `neval` (compared to `shape[1]`), especially if the operator is highly-structured.

pylops.utils.estimators.trace_hutchpp

`pylops.utils.estimators.`**`trace_hutchpp`**(*Op*, *neval=None*, *sampler='rademacher'*, *backend='numpy'*)

Trace of linear operator using the Hutch++ method.

Returns an estimate of the trace of a linear operator using the Hutch++ method [1].

Parameters

Op
[*pylops.LinearOperator*] Linear operator to compute trace on.

neval
[`int`, optional] Maximum number of matrix-vector products compute. Defaults to 10% of `shape[1]`.

sampler
[`str`, optional] Sample sketching matrices from the following distributions:

- "gaussian": Mean zero, unit variance Gaussian.
- "rayleigh": Sample from mean zero, unit variance Gaussian and normalize the columns.
- "rademacher": Random sign.

backend
[`str`, optional] Backend used to densify matrix (`numpy` or `cupy`). Note that this must be consistent with how the operator has been created.

Returns

trace
[`float`] Operator trace.

Raises

ValueError
If `neval` is smaller than 3.

NotImplementedError
If the `sampler` is not one of the available samplers.

Notes

This function follows Algorithm 1 of [1]. Let m = `shape[1]` and k = `neval`.

1. Sample sketching matrices $\mathbf{S} \in \mathbb{R}^{m \times \lfloor k/3 \rfloor}$, and $\mathbf{G} \in \mathbb{R}^{m \times \lfloor k/3 \rfloor}$, from sub-Gaussian distributions.
2. Compute reduced QR decomposition of $\mathbf{Op}\,\mathbf{S}$, retaining only $\mathbf{Q}$.
3. Return $\mathrm{tr}(\mathbf{Q}^T\,\mathbf{Op}\,\mathbf{Q}) + \frac{1}{\lfloor k/3 \rfloor}\,\mathrm{tr}\left(\mathbf{G}^T(\mathbf{I} - \mathbf{Q}\mathbf{Q}^T)\,\mathbf{Op}\,(\mathbf{I} - \mathbf{Q}\mathbf{Q}^T)\mathbf{G}\right)$

Use the Rademacher sampler unless you know what you are doing.

pylops.utils.estimators.trace_nahutchpp

pylops.utils.estimators.**trace_nahutchpp**(*Op*, *neval=None*, *sampler='rademacher'*, *c1=0.16666666666666666*, *c2=0.3333333333333333*, *backend='numpy'*)

Trace of linear operator using the NA-Hutch++ method.

Returns an estimate of the trace of a linear operator using the Non-Adaptive variant of Hutch++ method [1].

Parameters

Op
[*pylops.LinearOperator*] Linear operator to compute trace on.

neval
[int, optional] Maximum number of matrix-vector products compute. Defaults to 10% of `shape[1]`.

sampler
[str, optional] Sample sketching matrices from the following distributions:

- "gaussian": Mean zero, unit variance Gaussian.
- "rayleigh": Sample from mean zero, unit variance Gaussian and normalize the columns.
- "rademacher": Random sign.

c1
[float, optional] Fraction of `neval` for sketching matrix $\mathbf{S}$.

c2
[float, optional] Fraction of `neval` for sketching matrix $\mathbf{R}$. Must be larger than `c2`, ideally by a factor of at least 2.

backend
[str, optional] Backend used to densify matrix (`numpy` or `cupy`). Note that this must be consistent with how the operator has been created.

Returns

trace
[float] Operator trace.

Raises

ValueError
If `neval` not large enough to accomodate `c1` and `c2`.

NotImplementedError
If the `sampler` is not one of the available samplers.

Notes

This function follows Algorithm 2 of [1]. Let m = `shape[1]` and k = `neval`.

1. Fix constants c_1, c_2, c_3 such that $c_1 < c_2$ and $c_1 + c_2 + c_3 = 1$.
2. Sample sketching matrices $\mathbf{S} \in \mathbb{R}^{m \times c_1 k}$, $\mathbf{R} \in \mathbb{R}^{m \times c_2 k}$, and $\mathbf{G} \in \mathbb{R}^{m \times c_3 k}$ from sub-Gaussian distributions.
3. Compute $\mathbf{Z} = \mathbf{Op}\,\mathbf{R}$, $\mathbf{W} = \mathbf{Op}\,\mathbf{S}$, and $\mathbf{Y} = (\mathbf{S}^T\mathbf{Z})^+$, where $+$ denotes the Moore–Penrose inverse.
4. Return $\mathrm{tr}(\mathbf{Y}\mathbf{W}^T\mathbf{Z}) + \frac{1}{c_3 k}\left[\mathrm{tr}(\mathbf{G}^T\,\mathbf{Op}\,\mathbf{G}) - \mathrm{tr}(\mathbf{G}^T\mathbf{Z}\mathbf{Y}\mathbf{W}^T\mathbf{G})\right]$

The default values for c_1 and c_2 are set to $1/6$ and $1/3$, respectively, but [1] suggests $1/4$ and $1/2$.

Use the Rademacher sampler unless you know what you are doing.

3.8.5 Metrics

mae(xref, xcmp)	Mean Absolute Error (MAE)
mse(xref, xcmp)	Mean Square Error (MSE)
snr(xref, xcmp)	Signal to Noise Ratio (SNR)
psnr(xref, xcmp[, xmax])	Peak Signal to Noise Ratio (PSNR)

pylops.utils.metrics.mae

pylops.utils.metrics.**mae**(*xref*, *xcmp*)

Mean Absolute Error (MAE)

Compute Mean Absolute Error between two vectors

Parameters

xref
[numpy.ndarray] Reference vector

xcmp
[numpy.ndarray] Comparison vector

Returns

mae
[float] Mean Absolute Error

pylops.utils.metrics.mse

pylops.utils.metrics.**mse**(*xref*, *xcmp*)

Mean Square Error (MSE)

Compute Mean Square Error between two vectors

Parameters

xref
[numpy.ndarray] Reference vector

xcmp
[numpy.ndarray] Comparison vector

Returns

mse
[float] Mean Square Error

pylops.utils.metrics.snr

pylops.utils.metrics.**snr**(*xref*, *xcmp*)

Signal to Noise Ratio (SNR)

Compute Signal to Noise Ratio between two vectors

Parameters

xref
[numpy.ndarray] Reference vector

xcmp
[numpy.ndarray] Comparison vector

Returns

snr
[float] Signal to Noise Ratio of `xcmp` with respect to `xref`

pylops.utils.metrics.psnr

pylops.utils.metrics.**psnr**(*xref*, *xcmp*, *xmax=None*)

Peak Signal to Noise Ratio (PSNR)

Compute Peak Signal to Noise Ratio between two vectors.

Parameters

xref
[numpy.ndarray] Reference vector

xcmp
[numpy.ndarray] Comparison vector

xmax
[float, optional] Maximum value to use. If `None`, the actual maximum of the reference vector is used

Returns

psnr
[float] Peak Signal to Noise Ratio of `xcmp` with respect to `xref`

3.8.6 Geophysical Reservoir characterization

avo.zoeppritz_scattering(vp1, vs1, rho1, ...)	Zoeppritz solution.
avo.zoeppritz_element(vp1, vs1, rho1, vp0, ...)	Single element of Zoeppritz solution.
avo.zoeppritz_pp(vp1, vs1, rho1, vp0, vs0, ...)	PP reflection coefficient from the Zoeppritz scattering matrix.
avo.approx_zoeppritz_pp(vp1, vs1, rho1, vp0, ...)	PP reflection coefficient from the approximate Zoeppritz equation.
avo.akirichards(theta, vsvp[, n])	Three terms Aki-Richards approximation.
avo.fatti(theta, vsvp[, n])	Three terms Fatti approximation.
avo.ps(theta, vsvp[, n])	PS reflection coefficient

pylops.avo.avo.zoeppritz_scattering

pylops.avo.avo.**zoeppritz_scattering**(*vp1*, *vs1*, *rho1*, *vp0*, *vs0*, *rho0*, *theta1*)

Zoeppritz solution.

Calculates the angle dependent p-wave reflectivity of an interface between two media for a set of incident angles.

Parameters

vp1
[float] P-wave velocity of the upper medium

vs1
[float] S-wave velocity of the upper medium

rho1
[float] Density of the upper medium

vp0
[float] P-wave velocity of the lower medium

vs0
[float] S-wave velocity of the lower medium

rho0
[float] Density of the lower medium

theta1
[np.ndarray or float] Incident angles in degrees

Returns

zoep
[np.ndarray] 4×4 matrix representing the scattering matrix for the incident angle `theta1`

See also:

zoeppritz_element
Single reflectivity element of Zoeppritz solution

zoeppritz_pp
PP reflectivity element of Zoeppritz solution

pylops.avo.avo.zoeppritz_element

pylops.avo.avo.**zoeppritz_element**(*vp1*, *vs1*, *rho1*, *vp0*, *vs0*, *rho0*, *theta1*, *element='PdPu'*)

Single element of Zoeppritz solution.

Simple wrapper to `pylops.avo.avo.scattering_matrix`, returning any mode reflection coefficient from the Zoeppritz scattering matrix for specific combination of incident and reflected wave and a set of incident angles

Parameters

vp1
[float] P-wave velocity of the upper medium

vs1
[float] S-wave velocity of the upper medium

rho1
[float] Density of the upper medium

vp0
[float] P-wave velocity of the lower medium

vs0
[float] S-wave velocity of the lower medium

rho0
[float] Density of the lower medium

theta1
[np.ndarray or float] Incident angles in degrees

element
[str, optional] Specific choice of incident and reflected wave combining any two of the following strings: Pd P-wave downgoing, Sd S-wave downgoing, Pu P-wave upgoing, Su S-wave upgoing (e.g., PdPu)

Returns

refl
[np.ndarray] reflectivity values for all input angles for specific combination of incident and reflected wave.

See also:

zoeppritz_scattering
Zoeppritz solution

zoeppritz_pp
PP reflectivity element of Zoeppritz solution

pylops.avo.avo.zoeppritz_pp

pylops.avo.avo.**zoeppritz_pp**(*vp1*, *vs1*, *rho1*, *vp0*, *vs0*, *rho0*, *theta1*)

PP reflection coefficient from the Zoeppritz scattering matrix.

Simple wrapper to `pylops.avo.avo.scattering_matrix`, returning the PP reflection coefficient from the Zoeppritz scattering matrix for a set of incident angles

Parameters

vp1
[float] P-wave velocity of the upper medium

vs1
[float] S-wave velocity of the upper medium

rho1
[float] Density of the upper medium

vp0
[float] P-wave velocity of the lower medium

vs0
[float] S-wave velocity of the lower medium

rho0
[float] Density of the lower medium

theta1
[np.ndarray or float] Incident angles in degrees

Returns

PPrefl
[np.ndarray] PP reflectivity values for all input angles.

See also:

zoeppritz_scattering
Zoeppritz solution

zoeppritz_element
Single reflectivity element of Zoeppritz solution

pylops.avo.avo.approx_zoeppritz_pp

pylops.avo.avo.**approx_zoeppritz_pp**(*vp1*, *vs1*, *rho1*, *vp0*, *vs0*, *rho0*, *theta1*)

PP reflection coefficient from the approximate Zoeppritz equation.

Approximate calculation of PP reflection from the Zoeppritz scattering matrix for a set of incident angles [1].

Parameters

vp1
[np.ndarray or list or tuple] P-wave velocity of the upper medium

vs1
[np.ndarray or list or tuple] S-wave velocity of the upper medium

rho1
[np.ndarray or list or tuple] Density of the upper medium

vp0
[np.ndarray or list or tuple] P-wave velocity of the lower medium

vs0
[np.ndarray or list or tuple] S-wave velocity of the lower medium

rho0
[np.ndarray or list or tuple] Density of the lower medium

theta1
[np.ndarray or float] Incident angles in degrees

Returns

PPrefl
[np.ndarray] PP reflectivity values for all input angles.

See also:

zoeppritz_scattering
Zoeppritz solution

zoeppritz_element
Single reflectivity element of Zoeppritz solution

zoeppritz_pp
PP reflectivity element of Zoeppritz solution

pylops.avo.avo.akirichards

pylops.avo.avo.**akirichards**(*theta*, *vsvp*, *n=1*)

Three terms Aki-Richards approximation.

Computes the coefficients of the of three terms Aki-Richards approximation for a set of angles and a constant or variable VS/VP ratio.

Parameters

theta
[np.ndarray] Incident angles in degrees

vsvp
[np.ndarray or float] V_S/V_P ratio

n
[int, optional] Number of samples (if vsvp is a scalar)

Returns

G1
[np.ndarray] First coefficient of three terms Aki-Richards approximation $[n_\theta \times n_{\text{vsvp}}]$

G2
[np.ndarray] Second coefficient of three terms Aki-Richards approximation $[n_\theta \times n_{\text{vsvp}}]$

G3
[np.ndarray] Third coefficient of three terms Aki-Richards approximation $[n_\theta \times n_{\text{vsvp}}]$

Notes

The three terms Aki-Richards approximation [1], [2], is used to compute the reflection coefficient as linear combination of contrasts in V_P, V_S, and ρ. More specifically:

$$R(\theta) = G_1(\theta)\frac{\Delta V_P}{\overline{V}_P} + G_2(\theta)\frac{\Delta V_S}{\overline{V}_S} + G_3(\theta)\frac{\Delta \rho}{\overline{\rho}}$$

where

$$
\begin{aligned}
G_1(\theta) &= \frac{1}{2\cos^2\theta}, \\
G_2(\theta) &= -4(V_S/V_P)^2 \sin^2\theta, \\
G_3(\theta) &= 0.5 - 2(V_S/V_P)^2 \sin^2\theta, \\
\frac{\Delta V_P}{\overline{V}_P} &= 2\frac{V_{P,2} - V_{P,1}}{V_{P,2} + V_{P,1}}, \\
\frac{\Delta V_S}{\overline{V}_S} &= 2\frac{V_{S,2} - V_{S,1}}{V_{S,2} + V_{S,1}}, \\
\frac{\Delta \rho}{\overline{\rho}} &= 2\frac{\rho_2 - \rho_1}{\rho_2 + \rho_1}
\end{aligned} \tag{3.4}
$$

pylops.avo.avo.fatti

`pylops.avo.avo.fatti`(*theta*, *vsvp*, *n=1*)

Three terms Fatti approximation.

Computes the coefficients of the three terms Fatti approximation for a set of angles and a constant or variable VS/VP ratio.

Parameters

theta
[`np.ndarray`] Incident angles in degrees

vsvp
[`np.ndarray` or `float`] V_S/V_P ratio

n
[`int`, optional] Number of samples (if `vsvp` is a scalar)

Returns

G1
[`np.ndarray`] First coefficient of three terms Smith-Gidlow approximation $[n_\theta \times n_{\text{vsvp}}]$

G2
[`np.ndarray`] Second coefficient of three terms Smith-Gidlow approximation $[n_\theta \times n_{\text{vsvp}}]$

G3
[`np.ndarray`] Third coefficient of three terms Smith-Gidlow approximation $[n_\theta \times n_{\text{vsvp}}]$

Notes

The three terms Fatti approximation [1], [2], is used to compute the reflection coefficient as linear combination of contrasts in AI, SI, and ρ. More specifically:

$$
R(\theta) = G_1(\theta)\frac{\Delta \text{AI}}{\bar{\text{AI}}} + G_2(\theta)\frac{\Delta \text{SI}}{\overline{\text{SI}}} + G_3(\theta)\frac{\Delta \rho}{\overline{\rho}}
$$

where

$$G_1(\theta) = 0.5(1 + \tan^2\theta), \tag{3.10}$$
$$G_2(\theta) = -4(V_S/V_P)^2 \sin^2\theta, \tag{3.11}$$
$$G_3(\theta) = 0.5\left(4(V_S/V_P)^2 \sin^2\theta - \tan^2\theta\right), \tag{3.12}$$
$$\frac{\Delta \text{AI}}{\overline{\text{AI}}} = 2\frac{\text{AI}_2 - \text{AI}_1}{\text{AI}_2 + \text{AI}_1}, \tag{3.13}$$
$$\frac{\Delta \text{SI}}{\overline{\text{SI}}} = 2\frac{\text{SI}_2 - \text{SI}_1}{\text{SI}_2 + \text{SI}_1}, \tag{3.14}$$
$$\frac{\Delta \rho}{\overline{\rho}} = 2\frac{\rho_2 - \rho_1}{\rho_2 + \rho_1} \tag{3.15}$$

pylops.avo.avo.ps

pylops.avo.avo.**ps**(*theta*, *vsvp*, *n=1*)

PS reflection coefficient

Computes the coefficients for the PS approximation for a set of angles and a constant or variable VS/VP ratio.

Parameters

theta
[np.ndarray] Incident angles in degrees

vsvp
[np.ndarray or float] V_S/V_P ratio

n
[int, optional] Number of samples (if vsvp is a scalar)

Returns

G1
[np.ndarray] First coefficient for VP $[n_\theta \times n_{\text{vsvp}}]$. Since the PS reflection at zero angle is zero, this value is not used and is only available to ensure function signature compatibility with other linearization routines.

G2
[np.ndarray] Second coefficient for VS $[n_\theta \times n_{\text{vsvp}}]$

G3
[np.ndarray] Third coefficient for density $[n_\theta \times n_{\text{vsvp}}]$

Notes

The approximation in [1] is used to compute the PS reflection coefficient as linear combination of contrasts in V_P, V_S, and ρ. More specifically:

$$R(\theta) = G_2(\theta)\frac{\Delta V_S}{\bar{V}_S} + G_3(\theta)\frac{\Delta \rho}{\bar{\rho}}$$

where

$$G_2(\theta) = \tan\frac{\theta}{2}\left\{4(V_S/V_P)^2\sin^2\theta - 4(V_S/V_P)\cos\theta\cos\phi\right\}, \tag{3.16}$$

$$G_3(\theta) = -\tan\frac{\theta}{2}\left\{1 - 2(V_S/V_P)^2\sin^2\theta + 2(V_S/V_P)\cos\theta\cos\phi\right\}, \tag{3.17}$$

$$\frac{\Delta V_S}{\overline{V_S}} = 2\frac{V_{S,2} - V_{S,1}}{V_{S,2} + V_{S,1}} \tag{3.18}$$

$$\frac{\Delta \rho}{\overline{\rho}} = 2\frac{\rho_2 - \rho_1}{\rho_2 + \rho_1} \tag{3.19}$$

Note that θ is the P-incidence angle whilst ϕ is the S-reflected angle which is computed using Snell's law and the average V_S/V_P ratio.

3.8.7 Scalability test

scalability_test(Op, x[, workers, forward])	Scalability test.

pylops.utils.scalability_test

pylops.utils.**scalability_test**(*Op*, *x*, *workers=[1, 2, 4]*, *forward=True*)

Scalability test.

Small auxiliary routine to test the performance of operators using `multiprocessing`. This helps identifying the maximum number of workers beyond which no performance gain is observed.

Parameters

Op
[*pylops.LinearOperator*] Operator to test. It must allow for multiprocessing.

x
[numpy.ndarray, optional] Input vector.

workers
[list, optional] Number of workers to test out.

forward
[bool, optional] Apply forward (`True`) or adjoint (`False`)

Returns

compute_times
[list] Compute times as function of workers

speedup
[list] Speedup as function of workers

Examples using `pylops.utils.scalability_test`

- *Operators with Multiprocessing*

3.8.8 Sliding and Patching

sliding1d.sliding1d_design(dimd, nwin, ...)	Design Sliding1D operator
sliding2d.sliding2d_design(dimsd, nwin, ...)	Design Sliding2D operator
sliding3d.sliding3d_design(dimsd, nwin, ...)	Design Sliding3D operator
patch2d.patch2d_design(dimsd, nwin, nover, nop)	Design Patch2D operator
patch3d.patch3d_design(dimsd, nwin, nover, nop)	Design Patch3D operator

pylops.signalprocessing.sliding1d.sliding1d_design

pylops.signalprocessing.sliding1d.**sliding1d_design**(*dimd*, *nwin*, *nover*, *nop*)

Design Sliding1D operator

This routine can be used prior to creating the *pylops.signalprocessing.Sliding1D* operator to identify the correct number of windows to be used based on the dimension of the data (`dimsd`), dimension of the window (`nwin`), overlap (`nover`),a and dimension of the operator acting in the model space.

Parameters

dimsd
[tuple] Shape of 2-dimensional data.

nwin
[tuple] Number of samples of window.

nover
[tuple] Number of samples of overlapping part of window.

nop
[tuple] Size of model in the transformed domain.

Returns

nwins
[int] Number of windows.

dim
[int] Shape of 2-dimensional model.

mwins_inends
[tuple] Start and end indices for model patches.

dwins_inends
[tuple] Start and end indices for data patches.

pylops.signalprocessing.sliding2d.sliding2d_design

pylops.signalprocessing.sliding2d.**sliding2d_design**(*dimsd*, *nwin*, *nover*, *nop*)

Design Sliding2D operator

This routine can be used prior to creating the *pylops.signalprocessing.Sliding2D* operator to identify the correct number of windows to be used based on the dimension of the data (`dimsd`), dimension of the window (`nwin`), overlap (`nover`),a and dimension of the operator acting in the model space.

Parameters

dimsd
[tuple] Shape of 2-dimensional data.

nwin
[int] Number of samples of window.

nover
[int] Number of samples of overlapping part of window.

nop
[tuple] Size of model in the transformed domain.

Returns

nwins
[int] Number of windows.

dims
[tuple] Size of 2-dimensional model.

mwins_inends
[tuple] Start and end indices for model patches (stored as tuple of tuples).

dwins_inends
[tuple] Start and end indices for data patches (stored as tuple of tuples).

pylops.signalprocessing.sliding3d.sliding3d_design

pylops.signalprocessing.sliding3d.**sliding3d_design**(*dimsd*, *nwin*, *nover*, *nop*)

Design Sliding3D operator

This routine can be used prior to creating the *pylops.signalprocessing.Sliding3D* operator to identify the correct number of windows to be used based on the dimension of the data (`dimsd`), dimension of the window (`nwin`), overlap (`nover`),a and dimension of the operator acting in the model space.

Parameters

dimsd
[tuple] Shape of 2-dimensional data.

nwin
[tuple] Number of samples of window.

nover
[tuple] Number of samples of overlapping part of window.

nop
[tuple] Size of model in the transformed domain.

Returns

nwins
[tuple] Number of windows.

dims
[tuple] Shape of 2-dimensional model.

mwins_inends
[tuple] Start and end indices for model patches (stored as tuple of tuples).

dwins_inends
[tuple] Start and end indices for data patches (stored as tuple of tuples).

pylops.signalprocessing.patch2d.patch2d_design

pylops.signalprocessing.patch2d.**patch2d_design**(*dimsd*, *nwin*, *nover*, *nop*)

Design Patch2D operator

This routine can be used prior to creating the *pylops.signalprocessing.Patch2D* operator to identify the correct number of windows to be used based on the dimension of the data (`dimsd`), dimension of the window (`nwin`), overlap (`nover`),a and dimension of the operator acting in the model space.

Parameters

dimsd
[tuple] Shape of 2-dimensional data.

nwin
[tuple] Number of samples of window.

nover
[tuple] Number of samples of overlapping part of window.

nop
[tuple] Size of model in the transformed domain.

Returns

nwins
[tuple] Number of windows.

dims
[tuple] Shape of 2-dimensional model.

mwins_inends
[tuple] Start and end indices for model patches (stored as tuple of tuples).

dwins_inends
[tuple] Start and end indices for data patches (stored as tuple of tuples).

pylops.signalprocessing.patch3d.patch3d_design

pylops.signalprocessing.patch3d.**patch3d_design**(*dimsd*, *nwin*, *nover*, *nop*)

Design Patch3D operator

This routine can be used prior to creating the *pylops.signalprocessing.Patch3D* operator to identify the correct number of windows to be used based on the dimension of the data (`dimsd`), dimension of the window (`nwin`), overlap (`nover`),a and dimension of the operator acting in the model space.

Parameters

dimsd
[tuple] Shape of 3-dimensional data.

nwin
[tuple] Number of samples of window.

nover
[tuple] Number of samples of overlapping part of window.

nop
[tuple] Size of model in the transformed domain.

Returns

nwins
[tuple] Number of windows.

dims
[tuple] Shape of 3-dimensional model.

mwins_inends
[tuple] Start and end indices for model patches (stored as tuple of tuples).

dwins_inends
[tuple] Start and end indices for data patches (stored as tuple of tuples).

3.8.9 Synthetics

seismicevents.makeaxis(par)	Create axes t, x, and y axes
seismicevents.linear2d(x, t, v, t0, theta, ...)	Linear 2D events
seismicevents.parabolic2d(x, t, t0, px, pxx, ...)	Parabolic 2D events
seismicevents.hyperbolic2d(x, t, t0, vrms, ...)	Hyperbolic 2D events
seismicevents.linear3d(x, y, t, v, t0, ...)	Linear 3D events
seismicevents.hyperbolic3d(x, y, t, t0, ...)	Hyperbolic 3D events

pylops.utils.seismicevents.makeaxis

pylops.utils.seismicevents.**makeaxis**(*par*)

Create axes t, x, and y axes

Create space and time axes from dictionary containing initial values `ot`, `ox`, `oy`, sampling steps `dt`, dx``, `dy` and number of elements `nt`, nx``, `ny` for each axis.

Parameters

par
[dict] Dictionary containing initial values, sampling steps, and number of elements

Returns

t
[numpy.ndarray] Time axis

t2
[numpy.ndarray] Symmetric time axis

x
[numpy.ndarray] x axis

y
[numpy.ndarray] y axis (None, if oy, dy or ny are not provided)

Examples

```
>>> par = {'ox':0, 'dx':2, 'nx':60,
>>>        'oy':0, 'dy':2, 'ny':100,
>>>        'ot':0, 'dt':4, 'nt':400}
>>> # Create axis
>>> t, t2, x, y = makeaxis(par)
```

Examples using `pylops.utils.seismicevents.makeaxis`

- *1D, 2D and 3D Sliding*
- *Chirp Radon Transform*
- *Multi-Dimensional Convolution*
- *Normal Moveout (NMO) Correction*
- *Patching*
- *PhaseShift operator*
- *Spread How-to*
- *Synthetic seismic*
- *09. Multi-Dimensional Deconvolution*
- *11. Radon filtering*
- *12. Seismic regularization*
- *14. Seismic wavefield decomposition*

pylops.utils.seismicevents.linear2d

pylops.utils.seismicevents.**linear2d**(*x*, *t*, *v*, *t0*, *theta*, *amp*, *wav*)

Linear 2D events

Create 2d linear events given propagation velocity, intercept time, angle, and amplitude of each event

Parameters

x
[numpy.ndarray] space axis

t
[numpy.ndarray] time axis

v
[float] propagation velocity

t0
[tuple or float] intercept time at $x = 0$ of each linear event

theta
[tuple or float] angle (in degrees) of each linear event

amp
[tuple or float] amplitude of each linear event

wav
[numpy.ndarray] wavelet to be applied to data

Returns

d
[numpy.ndarray] data without wavelet of size $[n_x \times n_t]$

dwav
[numpy.ndarray] data with wavelet of size $[n_x \times n_t]$

Notes

Each event is created using the following relation:

$$t_i(x) = t_{0,i} + p_{x,i}x$$

where $p_{x,i} = \sin(\theta_i)/v$

Examples using `pylops.utils.seismicevents.linear2d`

- *Chirp Radon Transform*
- *Spread How-to*
- *Synthetic seismic*
- *11. Radon filtering*
- *12. Seismic regularization*

pylops.utils.seismicevents.parabolic2d

pylops.utils.seismicevents.**parabolic2d**(*x*, *t*, *t0*, *px*, *pxx*, *amp*, *wav*)

Parabolic 2D events

Create 2d parabolic events given intercept time, slowness, curvature, and amplitude of each event

Parameters

x
[numpy.ndarray] space axis

t
[numpy.ndarray] time axis

t0
[tuple or float] intercept time at $x = 0$ of each parabolic event

px
[tuple or float] slowness of each parabolic event

pxx
[tuple or float] curvature of each parabolic event

amp
[tuple or float] amplitude of each parabolic event

wav
[numpy.ndarray] wavelet to be applied to data

Returns

d
[numpy.ndarray] data without wavelet of size $[n_x \times n_t]$

dwav
[numpy.ndarray] data with wavelet of size $[n_x \times n_t]$

Notes

Each event is created using the following relation:

$$t_i(x) = t_{0,i} + p_{x,i}x + p_{xx,i}x^2$$

Examples using `pylops.utils.seismicevents.parabolic2d`

- *1D, 2D and 3D Sliding*
- *Patching*
- *Synthetic seismic*
- *11. Radon filtering*

pylops.utils.seismicevents.hyperbolic2d

pylops.utils.seismicevents.**hyperbolic2d**(*x*, *t*, *t0*, *vrms*, *amp*, *wav*)

Hyperbolic 2D events

Create 2d hyperbolic events given intercept time, root-mean-square velocity, and amplitude of each event

Parameters

x
[numpy.ndarray] space axis

t
[numpy.ndarray] time axis

t0
[tuple or float] intercept time at $x = 0$ of each of hyperbolic event

vrms
[tuple or float] root-mean-square velocity of each hyperbolic event

amp
[tuple or float] amplitude of each hyperbolic event

wav
[numpy.ndarray] wavelet to be applied to data

Returns

d
[numpy.ndarray] data without wavelet of size $[n_x \times n_t]$

dwav
[numpy.ndarray] data with wavelet of size $[n_x \times n_t]$

Notes

Each event is created using the following relation:

$$t_i(x) = \sqrt{t_{0,i}^2 + \frac{x^2}{v_{\text{rms},i}^2}}$$

Examples using `pylops.utils.seismicevents.hyperbolic2d`

- *Multi-Dimensional Convolution*
- *Normal Moveout (NMO) Correction*
- *PhaseShift operator*
- *Synthetic seismic*
- *09. Multi-Dimensional Deconvolution*
- *14. Seismic wavefield decomposition*

pylops.utils.seismicevents.linear3d

pylops.utils.seismicevents.**linear3d**(*x*, *y*, *t*, *v*, *t0*, *theta*, *phi*, *amp*, *wav*)

Linear 3D events

Create 3d linear events given propagation velocity, intercept time, angles, and amplitude of each event.

Parameters

x
[numpy.ndarray] space axis in x direction

y
[numpy.ndarray] space axis in y direction

t
[numpy.ndarray] time axis

v
[float] propagation velocity

t0
[tuple or float] intercept time at $x = 0$ of each linear event

theta
[tuple or float] angle in x direction (in degrees) of each linear event

phi
[tuple or float] angle in y direction (in degrees) of each linear event

amp
[tuple or float] amplitude of each linear event

wav
[numpy.ndarray] wavelet to be applied to data

Returns

d
[numpy.ndarray] data without wavelet of size $[n_y \times n_x \times n_t]$

dwav
[numpy.ndarray] data with wavelet of size $[n_y \times n_x \times n_t]$

Notes

Each event is created using the following relation:

$$t_i(x, y) = t_{0,i} + p_{x,i}x + p_{y,i}y$$

where $p_{x,i} = \frac{1}{v}\sin(\theta_i)\cos(\phi_i)$ and $p_{x,i} = \frac{1}{v}\sin(\theta_i)\sin(\phi_i)$.

Examples using `pylops.utils.seismicevents.linear3d`

- *Chirp Radon Transform*
- *Synthetic seismic*

pylops.utils.seismicevents.hyperbolic3d

pylops.utils.seismicevents.**hyperbolic3d**(*x*, *y*, *t*, *t0*, *vrms_x*, *vrms_y*, *amp*, *wav*)

Hyperbolic 3D events

Create 3d hyperbolic events given intercept time, root-mean-square velocities, and amplitude of each event

Parameters

x
[numpy.ndarray] space axis in x direction

y
[numpy.ndarray] space axis in y direction

t
[numpy.ndarray] time axis

t0
[tuple or float] intercept time at $x = 0$ of each of hyperbolic event

vrms_x
[tuple or float] root-mean-square velocity in x direction for each hyperbolic event

vrms_y
[tuple or float] root-mean-square velocity in y direction for each hyperbolic event

amp
[tuple or float] amplitude of each hyperbolic event

wav
[numpy.ndarray] wavelet to be applied to data

Returns

d
[numpy.ndarray] data without wavelet of size $[n_y \times n_x \times n_t]$

dwav
[numpy.ndarray] data with wavelet of size $[n_y \times n_x \times n_t]$

Notes

Each event is created using the following relation:

$$t_i(x, y) = \sqrt{t_{0,i}^2 + \frac{x^2}{v_{\text{rms}_x,i}^2} + \frac{y^2}{v_{\text{rms}_y,i}^2}}$$

Note that velocities do not have a physical meaning here (compared to the corresponding *pylops.utils.seismicevents.hyperbolic2d*), they rather simply control the curvature of the hyperboloid along the spatial axes.

Examples using `pylops.utils.seismicevents.hyperbolic3d`

- *1D, 2D and 3D Sliding*
- *Patching*
- *PhaseShift operator*
- *Synthetic seismic*

marchenko.directwave(wav, trav, nt, dt[, ...])	Analytical direct wave in acoustic media

pylops.waveeqprocessing.marchenko.directwave

pylops.waveeqprocessing.marchenko.**directwave**(*wav*, *trav*, *nt*, *dt*, *nfft=None*, *dist=None*, *kind='2d'*, *derivative=True*)

Analytical direct wave in acoustic media

Compute the analytical acoustic 2d or 3d Green's function in frequency domain given a wavelet `wav`, traveltime curve `trav` and distance `dist` (for 3d case only).

Parameters

wav
[numpy.ndarray] Wavelet in time domain to apply to direct arrival when created using `trav`. Phase will be discarded resulting in a zero-phase wavelet with same amplitude spectrum as provided by `wav`

trav
[numpy.ndarray] Traveltime of first arrival from subsurface point to surface receivers of size $[n_r \times 1]$

nt
[float, optional] Number of samples in time

dt
[float, optional] Sampling in time

nfft
[int, optional] Number of samples in fft time (if `None`, `nfft=nt`)

dist: :obj:`numpy.ndarray`
Distance between subsurface point to surface receivers of size $[n_r \times 1]$

kind
[str, optional] 2-dimensional (2d) or 3-dimensional (3d)

derivative
[bool, optional] Apply time derivative (True) or not (False)

Returns

direct
[numpy.ndarray] Direct arrival in time domain of size $[n_t \times n_r]$

Notes

The analytical Green's function in 2D [1] is :

$$G^{2D}(\mathbf{r}) = -\frac{i}{4} H_0^{(1)}(k|\mathbf{r}|)$$

and in 3D [1] is:

$$G^{3D}(\mathbf{r}) = \frac{e^{-jk\mathbf{r}}}{4\pi\mathbf{r}}$$

Note that these Green's functions represent the acoustic response to a point source of volume injection. In case the response to a point source of volume injection rate is desired, a $j\omega$ scaling (which is equivalent to applying a first derivative in time domain) must be applied. Here this is accomplished by setting `derivative=True`.

3.8.10 Signal-processing

signalprocessing.convmtx(h, n)	Convolution matrix
signalprocessing.nonstationary_convmtx(H, n)	Convolution matrix from a bank of filters
signalprocessing.dip_estimate(d[, dz, dx, ...])	Local dip estimation
signalprocessing.slope_estimate(d[, dz, dx, ...])	Local slope estimation

pylops.utils.signalprocessing.convmtx

pylops.utils.signalprocessing.**convmtx**(h, n)

Convolution matrix

Equivalent of MATLAB's convmtx function . Makes a dense convolution matrix $\mathbf{C}$ such that the dot product `np.dot(C, x)` is the convolution of the filter h and the input signal x.

Parameters

h
[np.ndarray] Convolution filter (1D array)

n
[int] Number of columns (if len$(h) < n$) or rows (if len$(h) \geq n$) of convolution matrix

Returns

C
[np.ndarray] Convolution matrix of size len$(h) + n - 1 \times n$ (if len$(h) < n$) or $n \times$ len$(h) + n - 1$ (if len$(h) \geq n$)

pylops.utils.signalprocessing.nonstationary_convmtx

pylops.utils.signalprocessing.**nonstationary_convmtx**(*H*, *n*, *hc=0*, *pad=(0, 0)*)

Convolution matrix from a bank of filters

Makes a dense convolution matrix $\mathbf{C}$ such that the dot product `np.dot(C, x)` is the nonstationary convolution of the bank of filters $H = [h_1, h_2, h_n]$ and the input signal x.

Parameters

H
[np.ndarray] Convolution filters (2D array of shape $[n_{\text{filters}} \times n_h]$

n
[int] Number of columns of convolution matrix

hc
[np.ndarray, optional] Index of center of first filter

pad
[np.ndarray] Zero-padding to apply to the bank of filters before and after the provided values (use it to avoid wrap-around or pass filters with enough padding)

Returns

C
[np.ndarray] Convolution matrix

pylops.utils.signalprocessing.dip_estimate

pylops.utils.signalprocessing.**dip_estimate**(*d*, *dz=1.0*, *dx=1.0*, *smooth=5*, *eps=0.0*)

Local dip estimation

Local dips are estimated using the *Structure Tensor* algorithm [1]_.

Note: For stability purposes, it is important to ensure that the orders of magnitude of the samplings are similar.

Parameters

d
[np.ndarray] Input dataset of size $n_z \times n_x$

dz
[float, optional] Sampling in z-axis, Δz

dx
[float, optional] Sampling in x-axis, Δx

smooth
[float or np.ndarray, optional] Standard deviation for Gaussian kernel. The standard deviations of the Gaussian filter are given for each axis as a sequence, or as a single number, in which case it is equal for all axes.

eps
[float, optional] Regularization term. All anisotropies where $\lambda_{\text{max}} < \epsilon$ are also set to zero. See Notes. When using with small values of `smooth`, start from a very small number (e.g. 1e-10) and start increasing by a power of 10 until results are satisfactory.

Returns

dips
[np.ndarray] Estimated local dips. The unit is radians, in the range of $-\frac{\pi}{2}$ to $\frac{\pi}{2}$.

anisotropies
[np.ndarray] Estimated local anisotropies: $1 - \lambda_{\text{min}}/\lambda_{\text{max}}$

Notes

Thin wrapper around `pylops.utils.signalprocessing.dip_estimate` with `slopes==True`. See the Notes of `pylops.utils.signalprocessing.dip_estimate` for details.

Examples using `pylops.utils.signalprocessing.dip_estimate`

- *Slope estimation via Structure Tensor algorithm*

pylops.utils.signalprocessing.slope_estimate

pylops.utils.signalprocessing.**slope_estimate**(*d*, *dz=1.0*, *dx=1.0*, *smooth=5*, *eps=0.0*, *dips=False*)

Local slope estimation

Local slopes are estimated using the *Structure Tensor* algorithm [1]. Slopes are returned as $\tan\theta$, defined in a RHS coordinate system with z-axis pointing upward.

Note: For stability purposes, it is important to ensure that the orders of magnitude of the samplings are similar.

Parameters

d
[np.ndarray] Input dataset of size $n_z \times n_x$

dz
[float, optional] Sampling in z-axis, Δz

Warning: Since version 1.17.0, defaults to 1.0.

dx
[float, optional] Sampling in x-axis, Δx

Warning: Since version 1.17.0, defaults to 1.0.

smooth
[float or np.ndarray, optional] Standard deviation for Gaussian kernel. The standard deviations of the Gaussian filter are given for each axis as a sequence, or as a single number, in which case it is equal for all axes.

Warning: Default changed in version 1.17.0 to 5 from previous value of 20.

eps
[float, optional] New in version 1.17.0.

Regularization term. All slopes where $|g_{zx}| < \epsilon \max_{(x,z)}\{|g_{zx}|, |g_{zz}|, |g_{xx}|\}$ are set to zero. All anisotropies where $\lambda_{\text{max}} < \epsilon$ are also set to zero. See Notes. When using with small values of `smooth`, start from a very small number (e.g. 1e-10) and start increasing by a power of 10 until results are satisfactory.

dips
[bool, optional] New in version 2.0.0.

Return dips (`True`) instead of slopes (`False`).

Returns

slopes
[np.ndarray] Estimated local slopes. The unit is that of $\Delta z/\Delta x$.

> **Warning:** Prior to version 1.17.0, always returned dips.

anisotropies
[np.ndarray] Estimated local anisotropies: $1 - \lambda_{\text{min}}/\lambda_{\text{max}}$

> **Note:** Since 1.17.0, changed name from `linearity` to `anisotropies`. Definition remains the same.

Notes

For each pixel of the input dataset $\mathbf{d}$ the local gradients $g_z = \frac{\partial \mathbf{d}}{\partial z}$ and $g_x = \frac{\partial \mathbf{d}}{\partial x}$ are computed and used to define the following three quantities:

$$g_{zz} = \left(\frac{\partial \mathbf{d}}{\partial z}\right)^2 \tag{3.20}$$

$$g_{xx} = \left(\frac{\partial \mathbf{d}}{\partial x}\right)^2 \tag{3.21}$$

$$g_{zx} = \frac{\partial \mathbf{d}}{\partial z}\frac{\partial \mathbf{d}}{\partial x} \tag{3.22}$$

They are then spatially smoothed and at each pixel their smoothed versions are arranged in a 2×2 matrix called the *smoothed gradient-square tensor*:

$$\mathbf{G} = \begin{bmatrix} g_{zz} & g_{zx} \\ g_{zx} & g_{xx} \end{bmatrix}$$

Local slopes can be expressed as $p = \frac{\lambda_{\text{max}} - g_{zz}}{g_{zx}}$, where λ_{max} is the largest eigenvalue of $\mathbf{G}$.

Similarly, local dips can be expressed as $\tan(2\theta) = 2g_{zx}/(g_{zz} - g_{xx})$.

Moreover, we can obtain a measure of local anisotropy, defined as

$$a = 1 - \lambda_{\text{min}}/\lambda_{\text{max}}$$

where λ_{min} is the smallest eigenvalue of $\mathbf{G}$. A value of $a = 0$ indicates perfect isotropy whereas $a = 1$ indicates perfect anisotropy.

Examples using `pylops.utils.signalprocessing.slope_estimate`

- *Seislet transform*
- *Slope estimation via Structure Tensor algorithm*

3.8.11 Tapers

`tapers.taper2d`(nt, nmask, ntap[, tapertype])	2D taper
`tapers.taper3d`(nt, nmask, ntap[, tapertype])	3D taper
`tapers.tapernd`(nmask, ntap[, tapertype])	ND taper

pylops.utils.tapers.taper2d

pylops.utils.tapers.**taper2d**(*nt*, *nmask*, *ntap*, *tapertype='hanning'*)

2D taper

Create 2d mask of size $[n_{\text{mask}} \times n_t]$ with tapering of size `ntap` along the first (and possibly second) dimensions

Parameters

nt
[`int`] Number of samples along second dimension

nmask
[`int`] Number of samples along first dimension

ntap
[`int` or `list`] Number of samples of tapering at edges of first dimension (or both dimensions).

tapertype
[`str`, optional] Type of taper (`hanning`, `cosine`, `cosinesquare` or `None`)

Returns

taper
[`numpy.ndarray`] 2d mask with tapering along first dimension of size $[n_{\text{mask}} \times n_t]$

Examples using `pylops.utils.tapers.taper2d`

- *PhaseShift operator*
- *Tapers*

pylops.utils.tapers.taper3d

pylops.utils.tapers.**taper3d**(*nt*, *nmask*, *ntap*, *tapertype='hanning'*)

3D taper

Create 3d mask of size $[n_{\text{mask}}[0] \times n_{\text{mask}}[1] \times n_t]$ with tapering of size `ntap` along the first and second dimension

Parameters

nt
[int] Number of time samples of mask along third dimension

nmask
[tuple] Number of space samples of mask along first and second dimensions

ntap
[tuple] Number of samples of tapering at edges of first and second dimensions

tapertype
[int] Type of taper (`hanning`, `cosine`, `cosinesquare` or `None`)

Returns

taper
[numpy.ndarray] 3d mask with tapering along first dimension of size $[n_{\text{mask,0}} \times n_{\text{mask,1}} \times n_t]$

Examples using pylops.utils.tapers.taper3d

- *Multi-Dimensional Convolution*
- *PhaseShift operator*
- *Tapers*
- *09. Multi-Dimensional Deconvolution*

pylops.utils.tapers.tapernd

pylops.utils.tapers.**tapernd**(*nmask*, *ntap*, *tapertype='hanning'*)

ND taper

Create nd mask of size $[n_{\text{mask}}[0] \times n_{\text{mask}}[1] \times ... \times n_{\text{mask}}[N-1]]$ with tapering of size `ntap` along all dimensions

Parameters

nmask
[tuple] Number of space samples of mask along every dimension

ntap
[tuple] Number of samples of tapering at edges of every dimension

tapertype
[int] Type of taper (`hanning`, `cosine`, `cosinesquare` or `None`)

Returns

taper
[numpy.ndarray] Nd mask with tapering along first dimension of size $[n_{\text{mask,0}} \times n_{\text{mask,1}} \times ... \times n_{\text{mask,N-1}}]$

3.8.12 Wavelets

wavelets.gaussian(t[, std])	Gaussian wavelet
wavelets.klauder(t[, f, taper])	Klauder wavelet
wavelets.ormsby(t[, f, taper])	Ormsby wavelet
wavelets.ricker(t[, f0, taper])	Ricker wavelet

pylops.utils.wavelets.gaussian

pylops.utils.wavelets.**gaussian**(*t*, *std=1.0*)

Gaussian wavelet

Create a Gaussian wavelet given time axis `t` and standard deviation `std` using `scipy.signal.windows.gaussian`.

Parameters

t
[numpy.ndarray] Time axis (positive part including zero sample)

std
[float, optional] Standard deviation of gaussian

Returns

w
[numpy.ndarray] Wavelet

t
[numpy.ndarray] Symmetric time axis

wcenter
[int] Index of center of wavelet

Examples using `pylops.utils.wavelets.gaussian`

- *Wavelets*

pylops.utils.wavelets.klauder

pylops.utils.wavelets.**klauder**(*t*, *f=(5.0, 20.0)*, *taper=None*)

Klauder wavelet

Create a Klauder wavelet given time axis `t` and standard deviation `std`. This wavelet mimics the autocorrelation of a linear frequency modulated sweep.

Parameters

t
[numpy.ndarray] Time axis (positive part including zero sample)

f
[tuple, optional] Frequency sweep

taper
[func, optional] Taper to apply to wavelet (must be a function that takes the size of the window as input

Returns

w
[numpy.ndarray] Wavelet

t
[numpy.ndarray] Symmetric time axis

wcenter
[int] Index of center of wavelet

Examples using `pylops.utils.wavelets.klauder`

- *Wavelets*

pylops.utils.wavelets.ormsby

`pylops.utils.wavelets.ormsby`(*t*, *f=(5.0, 10.0, 45.0, 50.0)*, *taper=None*)

Ormsby wavelet

Create a Ormsby wavelet given time axis `t` and frequency range defined by four frequencies which parametrize a trapezoidal shape in the frequency spectrum.

Parameters

t
[numpy.ndarray] Time axis (positive part including zero sample)

f
[tuple, optional] Frequency range

taper
[func, optional] Taper to apply to wavelet (must be a function that takes the size of the window as input

Returns

w
[numpy.ndarray] Wavelet

t
[numpy.ndarray] Symmetric time axis

wcenter
[int] Index of center of wavelet

Examples using `pylops.utils.wavelets.ormsby`

- *Wavelets*

pylops.utils.wavelets.ricker

pylops.utils.wavelets.**ricker**(*t*, *f0=10*, *taper=None*)

Ricker wavelet

Create a Ricker wavelet given time axis `t` and central frequency `f_0`

Parameters

t
[numpy.ndarray] Time axis (positive part including zero sample)

f0
[float, optional] Central frequency

taper
[func, optional] Taper to apply to wavelet (must be a function that takes the size of the window as input

Returns

w
[numpy.ndarray] Wavelet

t
[numpy.ndarray] Symmetric time axis

wcenter
[int] Index of center of wavelet

Examples using `pylops.utils.wavelets.ricker`

- *1D, 2D and 3D Sliding*
- *AVO modelling*
- *Chirp Radon Transform*
- *Convolution*
- *MP, OMP, ISTA and FISTA*
- *Multi-Dimensional Convolution*
- *Normal Moveout (NMO) Correction*
- *Patching*
- *PhaseShift operator*
- *Pre-stack modelling*
- *Shift*
- *Slope estimation via Structure Tensor algorithm*
- *Spread How-to*
- *Synthetic seismic*
- *Wavelet estimation*
- *Wavelets*
- *07. Post-stack inversion*

- *08. Pre-stack (AVO) inversion*
- *09. Multi-Dimensional Deconvolution*
- *11. Radon filtering*
- *12. Seismic regularization*
- *14. Seismic wavefield decomposition*
- *15. Least-squares migration*

3.9 Implementing new operators

Users are welcome to create new operators and add them to the PyLops library.

In this tutorial, we will go through the key steps in the definition of an operator, using the *pylops.Diagonal* as an example. This is a very simple operator that applies a diagonal matrix to the model in forward mode and to the data in adjoint mode.

3.9.1 Creating the operator

The first thing we need to do is to create a new file with the name of the operator we would like to implement. Note that as the operator will be a class, we need to follow the UpperCaseCamelCase convention both for the class itself and for the filename.

Once we have created the file, we will start by importing the modules that will be needed by the operator. While this varies from operator to operator, you will always need to import the *pylops.LinearOperator* class, which will be used as *parent* class for any of our operators:

```
from pylops import LinearOperator
```

This class is a child of the `scipy.sparse.linalg.LinearOperator` class itself which implements the same methods of its parent class as well as an additional method for quick inversion: such method can be easily accessed by using `\` between the operator and the data (e.g., `A \ y`).

After that we define our new object:

```
class Diagonal(LinearOperator):
```

followed by a numpydoc docstring (starting with `r"""` and ending with `"""`) containing the documentation of the operator. Such docstring should contain at least a short description of the operator, a `Parameters` section with a detailed description of the input parameters and a `Notes` section providing a mathematical explanation of the operator. Take a look at some of the core operators of PyLops to get a feeling of the level of details of the mathematical explanation.

Initialization (`__init__`)

We then need to create the `__init__` where the input parameters are passed and saved as members of our class. While the input parameters change from operator to operator, it is always required to create three members, the first called `shape` with a tuple containing the dimensions of the operator in the data and model space, the second called `dtype` with the data type object (`np.dtype`) of the model and data, and the third called `explicit` with a boolean (`True` or `False`) identifying if the operator can be inverted by a direct solver or requires an iterative solver. This member is `True` if the operator has also a member `A` that contains the matrix to be inverted like for example in the *pylops.MatrixMult* operator, and it will be `False` otherwise. In this case we have another member called `d` which is equal to the input vector containing the diagonal elements of the matrix we want to multiply to the model and data.

```
def __init__(self, d, dtype=None):
    self.d = d.ravel()
    self.shape = (len(self.d), len(self.d))
    self.dtype = np.dtype(dtype)
    self.explicit = False
```

Alternatively, since version 2.0.0, the recommended way of initializing operators derived from the base *pylops.LinearOperator* class is to invoke `super` to assign the required attributes:

```
def __init__(self, d, dtype=None):
    self.d = d.ravel()
    super().__init__(dtype=np.dtype(dtype), shape=(len(self.d), len(self.d)))
```

In this case, there is no need to declare `explicit` as it already defaults to `False`. Since version 2.0.0, every *pylops.LinearOperator* class is imbued with `dims`, `dimsd`, `clinear` and `explicit`, in addition to the required `dtype` and `shape`. See the docs of *pylops.LinearOperator* for more information about what these attributes mean.

Forward mode (_matvec)

We can then move onto writing the *forward mode* in the method `_matvec`. In other words, we will need to write the piece of code that will implement the following operation $\mathbf{y} = \mathbf{A}\mathbf{x}$. Such method is always composed of two inputs (the object itself `self` and the input model `x`). In our case the code to be added to the forward is very simple, we will just need to apply element-wise multiplication between the model $\mathbf{x}$ and the elements along the diagonal contained in the array $\mathbf{d}$. We will finally need to `return` the result of this operation:

```
def _matvec(self, x):
    return self.d * x
```

Adjoint mode (_rmatvec)

Finally we need to implement the *adjoint mode* in the method `_rmatvec`. In other words, we will need to write the piece of code that will implement the following operation $\mathbf{x} = \mathbf{A}^H\mathbf{y}$. Such method is also composed of two inputs (the object itself `self` and the input data `y`). In our case the code to be added to the forward is the same as the one from the forward (but this will obviously be different from operator to operator):

```
def _rmatvec(self, x):
    return self.d * x
```

And that's it, we have implemented our first linear operator!

3.9.2 Testing the operator

Being able to write an operator is not yet a guarantee of the fact that the operator is correct, or in other words that the adjoint code is actually the *adjoint* of the forward code. Luckily for us, a simple test can be performed to check the validity of forward and adjoint operators, the so called *dot-test*.

We can generate random vectors $\mathbf{u}$ and $\mathbf{v}$ and verify the the following *equality* within a numerical tolerance:

$$(\mathbf{A} * \mathbf{u})^H * \mathbf{v} = \mathbf{u}^H * (\mathbf{A}^H * \mathbf{v})$$

The method *pylops.utils.dottest* implements such a test for you, all you need to do is create a new test within an existing `test_*.py` file in the `pytests` folder (or in a new file).

Generally a test file will start with a number of dictionaries containing different parameters we would like to use in the testing of one or more operators. The test itself starts with a *decorator* that contains a list of all (or some) of dictionaries that will would like to use for our specific operator, followed by the definition of the test

```
@pytest.mark.parametrize("par", [(par1),(par2)])
def test_Diagonal(par):
```

At this point we can first of all create the operator and run the *pylops.utils.dottest* preceded by the `assert` command. Moreover, the forward and adjoint methods should tested towards expected outputs or even better, when the operator allows it (i.e., operator is invertible), a small inversion should be run and the inverted model tested towards the input model.

```
"""Dot-test and inversion for diagonal operator
"""
d = np.arange(par['nx']) + 1.

Dop = Diagonal(d)
assert dottest(Dop, par['nx'], par['nx'],
               complexflag=0 if par['imag'] == 1 else 3)

x = np.ones(par['nx'])
xlsqr = lsqr(Dop, Dop * x, damp=1e-20, iter_lim=300, show=0)[0]
assert_array_almost_equal(x, xlsqr, decimal=4)
```

3.9.3 Documenting the operator

Once the operator has been created, we can add it to the documentation of PyLops. To do so, simply add the name of the operator within the `index.rst` file in `docs/source/api` directory.

Moreover, in order to facilitate the user of your operator by other users, a simple example should be provided as part of the Sphinx-gallery of the documentation of the PyLops library. The directory `examples` contains several scripts that can be used as template.

3.9.4 Final checklist

Before submitting your new operator for review, use the following **checklist** to ensure that your code adheres to the guidelines of PyLops:

- you have created a new file containing a single class (or a function when the new operator is a simple combination of existing operators - see *pylops.Laplacian* for an example of such operator) and added to a new or existing directory within the `pylops` package.
- the new class contains at least `__init__`, `_matvec` and `_matvec` methods.
- the new class (or function) has a numpydoc docstring documenting at least the input `Parameters` and with a `Notes` section providing a mathematical explanation of the operator
- a new test has been added to an existing `test_*.py` file within the `pytests` folder. The test should verify that the new operator passes the *pylops.utils.dottest*. Moreover it is advisable to create a small toy example where the operator is applied in forward mode and the resulting data is inverted using \ from *pylops.LinearOperator*.
- the new operator is used within at least one *example* (in `examples` directory) or one *tutorial* (in `tutorials` directory).

3.10 Implementing new solvers

Users are welcome to create new solvers and add them to the PyLops library.

In this tutorial, we will go through the key steps in the definition of a solver, using the `pylops.optimization.basic.CG` as an example.

Note: In case the solver that you are planning to create falls within the category of proximal solvers, we encourage to consider adding it to the PyProximal project.

3.10.1 Creating the solver

The first thing we need to do is to locate a file containing solvers in the same family of the solver we plan to include, or create a new file with the name of the solver we would like to implement (or preferably its family). Note that as the solver will be a class, we need to follow the UpperCaseCamelCase convention for the class itself but not for the filename.

At this point we can start by importing the modules that will be needed by the solver. This varies from solver to solver, however you will always need to import the *pylops.optimization.basesolver.Solver* which will be used as *parent* class for any of our solvers. Moreover, we always recommend to import `pylops.utils.backend.get_array_module` as solvers should be written in such a way that it can work both with `numpy` and `cupy` arrays. See later for details.

```
import time

import numpy as np

from pylops.optimization.basesolver import Solver
from pylops.utils.backend import get_array_module
```

After that we define our new object:

```
class CG(Solver):
```

followed by a numpydoc docstring (starting with `r"""` and ending with `"""`) containing the documentation of the solver. Such docstring should contain at least a short description of the solver, a `Parameters` section with a description of the input parameters of the associated `__init__` method and a `Notes` section providing a reference to the original solver and possibly a concise mathematical explanation of the solver. Take a look at some of the core solver of PyLops to get a feeling of the level of details of the mathematical explanation.

As for any Python class, our solver will need an `__init__` method. In this case, however, we will just rely on that of the base class. Two input parameters are passed to the `__init__` method and saved as members of our class, namely the operator $\mathbf{Op}$ associated with the system of equations we wish to solve, $\mathbf{y} = \mathbf{Op}\,\mathbf{x}$, and optionally a *pylops.optimization.callback.Callbacks* object. Moreover, an additional parameters is created that contains the current time (this is used later to report the execution time of the solver). Here is the `__init__` method of the base class:

```
def __init__(self, Op, callbacks=None):
    self.Op = Op
    self.callbacks = callbacks
    self._registercallbacks()
    self.tstart = time.time()
```

We can now move onto writing the *setup* of the solver in the method `setup`. We will need to write a piece of code that prepares the solver prior to being able to apply a step. In general, this requires defining the data vector `y` and the initial guess of the solver `x0` (if not provided, this will be automatically set to be a zero vector), alongside various hyperparameters of the solver — e.g., those involved in the stopping criterion. For example in this case we only have two parameters: `niter` refers to the maximum allowed number of iterations, and `tol` to tolerance on the residual norm (the solver will be stopped if this is smaller than the chosen tolerance). Moreover, we always have the possibility to decide whether we want to operate the solver (in this case its setup part) in verbose or silent mode. This is driven by the `show` parameter. We will soon discuss how to choose what to print on screen in case of verbose mode (`show=True`).

The setup method can be loosely seen as composed of three parts. First, the data vector and hyperparameters are stored as members of the class. Moreover the type of the `y` vector is checked to evaluate whether to use `numpy` or `cupy` for algebraic operations (this is done by `self.ncp = get_array_module(y)`). Second, the starting guess is initialized using either the provided vector `x0` or a zero vector. Finally, a number of variables are initialized to be used inside the `step` method to keep track of the optimization process. Moreover, note that the `setup` method returns the created starting guess `x` (does not store it as member of the class).

```
def setup(self, y, x0=None, niter=None, tol=1e-4, show=False):

    self.y = y
    self.tol = tol
    self.niter = niter
    self.ncp = get_array_module(y)

    # initialize solver
    if x0 is None:
        x = self.ncp.zeros(self.Op.shape[1], dtype=self.y.dtype)
        self.r = self.y.copy()
    else:
        x = x0.copy()
        self.r = self.y - self.Op.matvec(x)
    self.c = self.r.copy()
    self.kold = self.ncp.abs(self.r.dot(self.r.conj()))

    # create variables to track the residual norm and iterations
    self.cost = []
    self.cost.append(np.sqrt(self.kold))
    self.iiter = 0

    # print setup
    if show:
        self._print_setup(np.iscomplexobj(x))
    return x
```

At this point, we need to implement the core of the solver, the `step` method. Here, we take the input at the previous iterate, update it following the rule of the solver of choice, and return it. The other input parameter required by this method is `show` to choose whether we want to print a report of the step on screen or not. However, if appropriate, a user can add additional input parameters. For CG, the step is:

```
def step(self, x, show=False):
    Opc = self.Op.matvec(self.c)
    cOpc = self.ncp.abs(self.c.dot(Opc.conj()))
    a = self.kold / cOpc
    x += a * self.c
    self.r -= a * Opc
```

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```
    k = self.ncp.abs(self.r.dot(self.r.conj()))
    b = k / self.kold
    self.c = self.r + b * self.c
    self.kold = k
    self.iiter += 1
    self.cost.append(np.sqrt(self.kold))
    if show:
        self._print_step(x)
    return x
```

Similarly, we also implement a `run` method that is in charge of running a number of iterations by repeatedly calling the `step` method. It is also usually convenient to implement a finalize method; this method can do any required post-processing that should not be applied at the end of each step, rather at the end of the entire optimization process. For CG, this is as simple as converting the `cost` variable from a list to a `numpy` array. For more details, see our implementations for CG.

Last but not least, we can wrap it all up in the `solve` method. This method takes as input the data, the initial model and the same hyperparameters of the setup method and runs the entire optimization process. For CG:

```
def solve(self, y, x0=None, niter=10, tol=1e-4, show=False, itershow=[10, 10, 10]):
    x = self.setup(y=y, x0=x0, niter=niter, tol=tol, show=show)
    x = self.run(x, niter, show=show, itershow=itershow)
    self.finalize(show)
    return x, self.iiter, self.cost
```

And that's it, we have implemented our first solver operator!

Although the methods that we just described are enough to implement any solver of choice, we find important to provide users with feedback during the inversion process. Imagine that the modelling operator is very expensive and can take minutes (or even hours to run), we don't want to leave a user waiting for hours before they can tell if the solver has done something meaningful. To avoid such scenario, we can implement so called *_print_** methods where *=`solver`, `setup`, `step`, `finalize` that print on screen some useful information (e.g., first value of the current estimate, norm of residual, etc.). The `solver` and `finalize` print are already implemented in the base class, the other two must be implemented when creating a new solver. When these methods are implemented and a user passes `show=True` to the associated method, our solver will provide such information on screen throughout the inverse process. To better understand how to write such methods, we suggest to look into the source code of the CG method.

Finally, to be backward compatible with versions of PyLops <*v2.0.0*, we also want to create a function with the same name of the class-based solver (but in small letters) which simply instantiates the solver and runs it. This function is usually placed in the same file of the class-based solver and snake_case should be used for its name. This function generally takes all the mandatory and optional parameters of the solver as input and returns some of the most valuable properties of the class-based solver object. An example for *CG* is:

```
def cg(Op, y, x0, niter=10, tol=1e-4, show=False, itershow=[10, 10, 10], callback=None):
    cgsolve = CG(Op)
    if callback is not None:
        cgsolve.callback = callback
    x, iiter, cost = cgsolve.solve(
        y=y, x0=x0, tol=tol, niter=niter, show=show, itershow=itershow
    )
    return x, iiter, cost
```

3.10.2 Testing the solver

Being able to write a solver is not yet a guarantee of the fact that the solver is correct, or in other words that the solver can converge to a correct solution (at least in the case of full rank operator).

We encourage to create a new test within an existing `test_*.py` file in the `pytests` folder (or in a new file). We also encourage to test the function-bases solver, as this will implicitly test the underlying class-based solver.

Generally a test file will start with a number of dictionaries containing different parameters we would like to use in the testing of one or more solvers. The test itself starts with a *decorator* that contains a list of all (or some) of dictionaries that will would like to use for our specific operator, followed by the definition of the test:

```
@pytest.mark.parametrize("par", [(par1),(par2)])
def test_CG(par):
```

At this point we can first create a full-rank operator, an input vector and compute the associated data. We can then run the solver for a certain number of iterations, checking that the solution agrees with the true x within a certain tolerance:

```
"""CG with linear operator
"""
np.random.seed(10)

A = np.random.normal(0, 10, (par["ny"], par["nx"]))
A = np.conj(A).T @ A  # to ensure definite positive matrix
Aop = MatrixMult(A, dtype=par["dtype"])

x = np.ones(par["nx"])
x0 = np.random.normal(0, 10, par["nx"])

y = Aop * x
xinv = cg(Aop, y, x0=x0, niter=par["nx"], tol=1e-5, show=True)[0]
assert_array_almost_equal(x, xinv, decimal=4)
```

3.10.3 Documenting the solver

Once the solver has been created, we can add it to the documentation of PyLops. To do so, simply add the name of the operator within the `index.rst` file in `docs/source/api` directory.

Moreover, in order to facilitate the user of your operator by other users, a simple example should be provided as part of the Sphinx-gallery of the documentation of the PyLops library. The directory `examples` contains several scripts that can be used as template.

3.10.4 Final checklist

Before submitting your new solver for review, use the following **checklist** to ensure that your code adheres to the guidelines of PyLops:

- you have added the new solver to a new or existing file in the `optimization` directory within the `pylops` package.
- the new class contains at least `__init__`, `setup`, `step`, `run`, `finalize`, and `solve` methods.
- each of the above methods have a numpydoc docstring documenting at least the input `Parameters` and the `__init__` method contains also a `Notes` section providing a mathematical explanation of the solver.

- a new test has been added to an existing `test_*.py` file within the `pytests` folder. The test should verify that the new solver converges to the true solution for a well-designed inverse problem (i.e., full rank operator).
- the new solver is used within at least one *example* (in `examples` directory) or one *tutorial* (in `tutorials` directory).

3.11 Contributing

Contributions are welcome and greatly appreciated!

The best way to get in touch with the core developers and maintainers is to join the PyLops slack channel as well as open new *Issues* directly from the GitHub repo.

Moreover, take a look at the *Roadmap* page for a list of current ideas for improvements and additions to the PyLops library.

3.11.1 Welcomed contributions

Bug reports

Report bugs at https://github.com/PyLops/pylops/issues.

If you are playing with the PyLops library and find a bug, please report it including:

- Your operating system name and version.
- Any details about your Python environment.
- Detailed steps to reproduce the bug.

New operators and features

Open an issue at https://github.com/PyLops/pylops/issues with tag *enhancement*.

If you are proposing a new operator or a new feature:

- Explain in detail how it should work.
- Keep the scope as narrow as possible, to make it easier to implement.

Fix issues

There is always a backlog of issues that need to be dealt with. Look through the GitHub Issues for operator/feature requests or bugfixes.

Add examples or improve documentation

Writing new operators is not the only way to get involved and contribute. Create examples with existing operators as well as improving the documentation of existing operators is as important as making new operators and very much encouraged.

3.11.2 Step-by-step instructions for contributing

Ready to contribute?

1. Follow all instructions in *Step-by-step installation for developers*.
2. Create a branch for local development, usually starting from the dev branch:

```
>> git checkout -b name-of-your-branch dev
```

Now you can make your changes locally.

3. When you're done making changes, check that your code follows the guidelines for *Implementing new operators* and that the both old and new tests pass successfully:

```
>> make tests
```

4. Run flake8 to check the quality of your code:

```
>> make lint
```

Note that PyLops does not enforce full compliance with flake8, rather this is used as a guideline and will also be run as part of our CI. Make sure to limit to a minimum flake8 warnings before making a PR.

5. Update the docs

```
>> make docupdate
```

6. Commit your changes and push your branch to GitHub:

```
>> git add .
>> git commit -m "Your detailed description of your changes."
>> git push origin name-of-your-branch
```

Remember to add `-u` when pushing the branch for the first time. We recommend using Conventional Commits to format your commit messages, but this is not enforced.

7. Submit a pull request through the GitHub website.

3.11.3 Pull Request Guidelines

Before you submit a pull request, check that it meets these guidelines:

1. The pull request should include new tests for all the core routines that have been developed.
2. If the pull request adds functionality, the docs should be updated accordingly.
3. Ensure that the updated code passes all tests.

3.12 Project structure

This repository is organized as follows: * **pylops**: Python library containing various linear operators and auxiliary routines * **pytests**: set of pytests * **testdata**: sample datasets used in pytests and documentation * **docs**: Sphinx documentation * **examples**: set of python script examples for each linear operator to be embedded in documentation using sphinx-gallery * **tutorials**: set of python script tutorials to be embedded in documentation using sphinx-gallery

3.13 Changelog

3.13.1 Version 2.0.0

Released on: 12/08/2022

PyLops has undergone significant changes in this release, including new `LinearOperator` s, more features, new examples and bugfixes. To aid users in navigating the breaking changes, we provide the following document MIGRATION_V1_V2.md.

New Features

- Multiplication of linear operators by N-dimensional arrays is now supported via the new `dims`/`dimsd` properties. Users do not need to use `.ravel` and `.reshape` as often anymore. See the migration guide for more information.
- Typing annotations for several submodules (`avo`, `basicoperators`, `signalprocessing`, `utils`, `optimization`, `waveeqprocessing`)
- New *pylops.TorchOperator* wraps a Pylops operator into a PyTorch function
- New *pylops.signalprocessing.Patch3D* applies a linear operator repeatedly to patches of the model vector
- Each of *pylops.signalprocessing.Sliding1D*, *pylops.signalprocessing.Sliding2D*, *pylops.signalprocessing.Sliding3D*, *pylops.signalprocessing.Patch2D* and *pylops.signalprocessing.Patch3D* have an associated `slidingXd_design` or `patchXd_design` functions associated with them to aid the user in designing the windows
- *pylops.FirstDerivative* and *pylops.SecondDerivative*, and therefore other derivative operators which rely on the (e.g., *pylops.Gradient*) support higher order stencils
- *pylops.waveeqprocessing.Kirchhoff* substitutes `pylops.waveeqprocessing.Demigration` and incorporates a variety of new functionalities
- New *pylops.waveeqprocessing.AcousticWave2D* wraps the Devito acoutic wave propagator providing a wave-equation based Born modeling operator with a reverse-time migration adjoint
- Solvers can now be implemented via the *pylops.optimization.basesolver.Solver* class. They can now be used through a functional interface with lowercase name (e.g., *pylops.optimization.sparsity.splitbregman*) or via class interface with CamelCase name (e.g., *pylops.optimization.cls_sparsity.SplitBregman*. Moreover, solvers now accept callbacks defined by the *pylops.optimization.callback.Callbacks* interface (see e.g., *pylops.optimization.callback.MetricsCallback*).
- Metrics such as *pylops.utils.metrics.mae* and *pylops.utils.metrics.mse* and others
- New *pylops.utils.signalprocessing.dip_estimate* estimates local dips in an image (measured in radians) in a stabler way than the old *pylops.utils.signalprocessing.dip_estimate* did for slopes.
- New *pylops.utils.tapers.tapernd* for N-dimensional tapers
- New wavelets *pylops.utils.wavelets.klauder* and *pylops.utils.wavelets.ormsby*

Documentation

- Installation has been revamped
- Revamped guide on how to implement a new LinearOperator from scratch
- New guide on how to implement a new solver from scratch
- New tutorials:
  - Solvers (Advanced)
  - Deblending
  - Automatic Differentiation
- New gallery examples:
  - Patching
  - Wavelets

3.13.2 Version 1.18.3

Released on: 30/07/2022

- Refractored *pylops.utils.dottest*, and added two new optional input parameters (*atol* and *rtol*)
- Added optional parameter *densesolver* to `pylops.LinearOperator.div`
- Fixed *pylops.optimization.basic.lsqr*, `pylops.optimization.sparsity.ISTA`, and `pylops.optimization.sparsity.FISTA` to work with cupy arrays. This change was required by how recent cupy versions handle scalars, which are not converted directly into float types, rather kept as cupy arrays.
- Fix bug in *pylops.waveeqprocessing.Deghosting* introduced in commit 7e596d4.

3.13.3 Version 1.18.2

Released on: 29/04/2022

- Refractored *pylops.utils.dottest*, and added two new optional input parameters (*atol* and *rtol*)
- Added optional parameter *densesolver* to `pylops.LinearOperator.div`

3.13.4 Version 1.18.1

Released on: 29/04/2022

- !DELETED! due to a mistake in the release process

3.13.5 Version 1.18.0

Released on: 19/02/2022

- Added *NMO* example to gallery
- Extended *pylops.Laplacian* to N-dimensional arrays
- Added *forward* kind to *pylops.SecondDerivative* and *pylops.Laplacian*
- Added *chirp-sliding* kind to `pylops.waveeqprocessing.seismicinterpolation.SeismicInterpolation`

- Fixed bug due to the new internal structure of *LinearOperator* submodule introduced in *scipy1.8.0*

3.13.6 Version 1.17.0

Released on: 29/01/2022

- Added *pylops.utils.describe.describe* method
- Added `fftengine` to *pylops.waveeqprocessing.Marchenko*
- Added `ifftshift_before` and `fftshift_after` optional input parameters in *pylops.signalprocessing.FFT*
- Added `norm` optional input parameter to *pylops.signalprocessing.FFT2D* and *pylops.signalprocessing.FFTND*
- Added `scipy` backend to *pylops.signalprocessing.FFT* and *pylops.signalprocessing.FFT2D* and *pylops.signalprocessing.FFTND*
- Added `eps` optional input parameter in *pylops.utils.signalprocessing.slope_estimate*
- Added pre-commit hooks
- Improved pre-commit hooks
- Vectorized *pylops.utils.signalprocessing.slope_estimate*
- Handlexd `nfft<nt` case in *pylops.signalprocessing.FFT* and *pylops.signalprocessing.FFT2D* and *pylops.signalprocessing.FFTND*
- Introduced automatic casting of dtype in *pylops.MatrixMult*
- Improved documentation and definition of optinal parameters of *pylops.Spread*
- Major clean up of documentation and mathematical formulas
- Major refractoring of the inner structure of *pylops.signalprocessing.FFT* and *pylops.signalprocessing.FFT2D* and *pylops.signalprocessing.FFTND*
- Reduced warnings in test suite
- Reduced computational time of `test_wavedecomposition` in the test suite
- Fixed bug in *pylops.signalprocessing.Sliding1D*, *pylops.signalprocessing.Sliding2D* and *pylops.signalprocessing.Sliding3D* where the `dtype` of the Restriction operator is inffered from `Op`
- Fixed bug in *pylops.signalprocessing.Radon2D* and *pylops.signalprocessing.Radon3D* when using centered spatial axes
- Fixed scaling in *pylops.signalprocessing.FFT* with `real=True` to pass the dot-test

3.13.7 Version 1.16.0

Released on: 11/12/2021

- Added `pylops.utils.estimators` submodule for trace estimation
- Added *x0* in `pylops.optimization.sparsity.ISTA` and `pylops.optimization.sparsity.FISTA` to handle non-zero initial guess
- Modified `pylops.optimization.sparsity.ISTA` and `pylops.optimization.sparsity.FISTA` to handle multiple right hand sides

- Modified creation of *haxis* in *pylops.signalprocessing.Radon2D* and *pylops.signalprocessing.Radon3D* to allow for uncentered spatial axes
- Fixed *_rmatvec* for explicit in `pylops.LinearOperator._ColumnLinearOperator`

3.13.8 Version 1.15.0

Released on: 23/10/2021

- Added *pylops.signalprocessing.Shift* operator.
- Added option to choose derivative kind in *pylops.avo.poststack.PoststackInversion* and *pylops.avo.prestack.PrestackInversion*.
- Improved efficiency of adjoint of *pylops.signalprocessing.Fredholm1* by applying complex conjugation to the vectors.
- Added *vsvp* to *pylops.avo.prestack.PrestackInversion* allowing to use user defined VS/VP ratio.
- Added *kind* to `pylops.basicoperators.CausalIntegration` allowing `full`, `half`, or `trapezoidal` integration.
- Fixed *_hardthreshold_percentile* in `pylops.optimization.sparsity` - Issue #249.
- Fixed r2norm in `pylops.optimization.solver.cgls`.

3.13.9 Version 1.14.0

Released on: 09/07/2021

- Added `pylops.optimization.solver.lsqr` solver
- Added utility routine *pylops.utils.scalability_test* for scalability tests when using `multiprocessing`
- Added *pylops.avo.avo.ps* AVO modelling option and restructured *pylops.avo.prestack.PrestackLinearModelling* to allow passing any function handle that can perform AVO modelling apart from those directly available
- Added R-linear operators (when setting the property *clinear=False* of a linear operator). `pylops.basicoperators.Real`, `pylops.basicoperators.Imag`, and `pylops.basicoperators.Conj`
- Added possibility to run operators `pylops.basicoperators.HStack`, `pylops.basicoperators.VStack`, `pylops.basicoperators.Block` `pylops.basicoperators.BlockDiag`, and *pylops.signalprocessing.Sliding3D* using `multiprocessing`
- Added dtype to vector *X* when using `scipy.sparse.linalg.lobpcg` in *eigs* method of *pylops.LinearOperator*
- Use *kind=forward* fot FirstDerivative in *pylops.avo.poststack.PoststackInversion* inversion when dealing with L1 regularized inversion as it makes the inverse problem more stable (no ringing in solution)
- Changed *cost* in `pylops.optimization.solver.cg` and `pylops.optimization.solver.cgls` to be L2 norms of residuals
- Fixed `pylops.utils.dottest.dottest` for imaginary vectors and to ensure *u* and *v* vectors are of same dtype of the operator

3.13.10 Version 1.13.0

Released on: 26/03/2021

- Added *pylops.signalprocessing.Sliding1D* and *pylops.signalprocessing.Patch2D* operators
- Added `pylops.basicoperators.MemoizeOperator` operator
- Added decay and analysis option in `pylops.optimization.sparsity.ISTA` and `pylops.optimization.sparsity.FISTA` solvers
- Added *toreal* and *toimag* methods to *pylops.LinearOperator*
- Make *nr* and *nc* optional in `pylops.utils.dottest.dottest`
- Fixed complex check in `pylops.basicoperators.MatrixMult` when working with complex-valued cupy arrays
- Fixed bug in data reshaping in check in *pylops.avo.prestack.PrestackInversion*
- Fixed loading error when using old cupy and/or cusignal (see Issue #201)

3.13.11 Version 1.12.0

Released on: 22/11/2020

- Modified all operators and solvers to work with cupy arrays
- Added `eigs` and `solver` submodules to `pylops.optimization`
- Added `deps` and `backend` submodules to `pylops.utils`
- Fixed bug in *pylops.signalprocessing.Convolve2D*. and *pylops.signalprocessing.ConvolveND*. when dealing with filters that have less dimensions than the input vector.

3.13.12 Version 1.11.1

Released on: 24/10/2020

- Fixed import of `pyfttw` when not available in :py:class:``pylops.signalprocessing.ChirpRadon3D`

3.13.13 Version 1.11.0

Released on: 24/10/2020

- Added *pylops.signalprocessing.ChirpRadon2D* and *pylops.signalprocessing.ChirpRadon3D* operators.
- Fixed bug in the inferred dimensions for regularization data creation in `pylops.optimization.leastsquares.NormalEquationsInversion`, `pylops.optimization.leastsquares.RegularizedInversion`, and `pylops.optimization.sparsity.SplitBregman`.
- Changed dtype of *pylops.HStack* to allow automatic inference from dtypes of input operator.
- Modified dtype of *pylops.waveeqprocessing.Marchenko* operator to ensure that outputs of forward and adjoint are real arrays.
- Reverted to previous complex-friendly implementation of `pylops.optimization.sparsity._softthreshold` to avoid division by 0.

3.13.14 Version 1.10.0

Released on: 13/08/2020

- Added `tosparse` method to *pylops.LinearOperator*.
- Added `kind=linear` in *pylops.signalprocessing.Seislet* operator.
- Added `kind` to *pylops.FirstDerivative*. operator to perform forward and backward (as well as centered) derivatives.
- Added `kind` to `pylops.optimization.sparsity.IRLS` solver to choose between data or model sparsity.
- Added possibility to use `scipy.sparse.linalg.lobpcg` in `pylops.LinearOperator.eigs` and `pylops.LinearOperator.cond`
- Added possibility to use `scipy.signal.oaconvolve` in *pylops.signalprocessing.Convolve1D*.
- Added `NRegs` to `pylops.optimization.leastsquares.NormalEquationsInversion` to allow providing regularization terms directly in the form of `H^T H`.

3.13.15 Version 1.9.1

Released on: 25/05/2020

- Changed internal behaviour of `pylops.sparsity.OMP` when `niter_inner=0`. Automatically reverts to Matching Pursuit algorithm.
- Changed handling of `dtype` in *pylops.signalprocessing.FFT* and *pylops.signalprocessing.FFT2D* to ensure that the type of the input vector is retained when applying forward and adjoint.
- Added `dtype` parameter to the `FFT` calls in the definition of the *pylops.waveeqprocessing.MDD* operation. This ensure that the type of the real part of `G` input is enforced to the output vectors of the forward and adjoint operations.

3.13.16 Version 1.9.0

Released on: 13/04/2020

- Added *pylops.waveeqprocessing.Deghosting* and *pylops.signalprocessing.Seislet* operators
- Added hard and half thresholds in `pylops.optimization.sparsity.ISTA` and `pylops.optimization.sparsity.FISTA` solvers
- Added `prescaled` input parameter to *pylops.waveeqprocessing.MDC* and *pylops.waveeqprocessing.Marchenko*
- Added sinc interpolation to *pylops.signalprocessing.Interp* (`kind == 'sinc'`)
- Modified *pylops.waveeqprocessing.marchenko.directwave* to to model analytical responses from both sources of volume injection (`derivative=False`) and source of volume injection rate (`derivative=True`)
- Added `pylops.LinearOperator.asoperator` method to *pylops.LinearOperator*
- Added *pylops.utils.signalprocessing.slope_estimate* function
- Fix bug in *pylops.signalprocessing.Radon2D* and *pylops.signalprocessing.Radon3D* when `onthefly=True` returning the same result as when `onthefly=False`

3.13.17 Version 1.8.0

Released on: 12/01/2020

- Added `pylops.LinearOperator.todense` method to *pylops.LinearOperator*
- Added *pylops.signalprocessing.Bilinear*, *pylops.signalprocessing.DWT*, and `pylops.signalprocessing.DWT2` operators
- Added *pylops.waveeqprocessing.PressureToVelocity*, `pylops.waveeqprocessing.UpDownComposition3Doperator`, and *pylops.waveeqprocessing.PhaseShift* operators
- Fix bug in `pylops.basicoperators.Kronecker` (see Issue #125)

3.13.18 Version 1.7.0

Released on: 10/11/2019

- Added *pylops.Gradient*, *pylops.Sum*, *pylops.FirstDirectionalDerivative*, and *pylops.SecondDirectionalDerivative* operators
- Added `pylops.LinearOperator._ColumnLinearOperator` private operator
- Added possibility to directly mix Linear operators and numpy/scipy 2d arrays in *pylops.VStack* and *pylops.HStack* and *pylops.BlockDiag* operators
- Added `pylops.optimization.sparsity.OMP` solver

3.13.19 Version 1.6.0

Released on: 10/08/2019

- Added *pylops.signalprocessing.ConvolveND* operator
- Added *pylops.utils.signalprocessing.nonstationary_convmtx* to create matrix for non-stationary convolution
- Added possibility to perform seismic modelling (and inversion) with non-stationary wavelet in *pylops.avo.poststack.PoststackLinearModelling*
- Create private methods for *pylops.Block*, *pylops.avo.poststack.PoststackLinearModelling*, *pylops.waveeqprocessing.MDC* to allow calling different operators (e.g., from pylops-distributed or pylops-gpu) within the method

3.13.20 Version 1.5.0

Released on: 30/06/2019

- Added `conj` method to *pylops.LinearOperator*
- Added *pylops.Kronecker*, *pylops.Roll*, and *pylops.Transpose* operators
- Added *pylops.signalprocessing.Fredholm1* operator
- Added `pylops.optimization.sparsity.SPGL1` and `pylops.optimization.sparsity.SplitBregman` solvers
- Sped up *pylops.signalprocessing.Convolve1D* using `scipy.signal.fftconvolve` for multi-dimensional signals

- Changes in implementation of *pylops.waveeqprocessing.MDC* and *pylops.waveeqprocessing.Marchenko* to take advantage of primitives operators
- Added `epsRL1` option to *pylops.avo.poststack.PoststackInversion* and *pylops.avo.prestack.PrestackInversion* to include TV-regularization terms by means of `pylops.optimization.sparsity.SplitBregman` solver

3.13.21 Version 1.4.0

Released on: 01/05/2019

- Added `numba` engine to *pylops.Spread* and *pylops.signalprocessing.Radon2D* operators
- Added *pylops.signalprocessing.Radon3D* operator
- Added *pylops.signalprocessing.Sliding2D* and *pylops.signalprocessing.Sliding3D* operators
- Added *pylops.signalprocessing.FFTND* operator
- Added *pylops.signalprocessing.Radon3D* operator
- Added `niter` option to `pylops.LinearOperator.eigs` method
- Added `show` option to `pylops.optimization.sparsity.ISTA` and `pylops.optimization.sparsity.FISTA` solvers
- Added `pylops.waveeqprocessing.seismicinterpolation`, `pylops.waveeqprocessing.waveeqdecomposition` and `pylops.waveeqprocessing.lsm` submodules
- Added tests for `engine` in various operators
- Added documentation regarding usage of `pylops` Docker container

3.13.22 Version 1.3.0

Released on: 24/02/2019

- Added `fftw` engine to *pylops.signalprocessing.FFT* operator
- Added `pylops.optimization.sparsity.ISTA` and `pylops.optimization.sparsity.FISTA` sparse solvers
- Added possibility to broadcast (handle multi-dimensional arrays) to *pylops.Diagonal* and `pylops..Restriction` operators
- Added *pylops.signalprocessing.Interp* operator
- Added *pylops.Spread* operator
- Added *pylops.signalprocessing.Radon2D* operator

3.13.23 Version 1.2.0

Released on: 13/01/2019

- Added `pylops.LinearOperator.eigs` and `pylops.LinearOperator.cond` methods to estimate estimate eigenvalues and conditioning number using scipy wrapping of ARPACK
- Modified default `dtype` for all operators to be `float64` (or `complex128`) to be consistent with default dtypes used by numpy (and scipy) for real and complex floating point numbers.
- Added *pylops.Flip* operator
- Added *pylops.Symmetrize* operator
- Added *pylops.Block* operator
- Added *pylops.Regression* operator performing polynomial regression and modified *pylops.LinearRegression* to be a simple wrapper of *pylops.Regression* when `order=1`
- Modified *pylops.MatrixMult* operator to work with both numpy ndarrays and scipy sparse matrices
- Added *pylops.avo.prestack.PrestackInversion* routine
- Added possibility to have a data weight via `Weight` input parameter to `pylops.optimization.leastsquares.NormalEquationsInversion` and `pylops.optimization.leastsquares.RegularizedInversion` solvers
- Added `pylops.optimization.sparsity.IRLS` solver

3.13.24 Version 1.1.0

Released on: 13/12/2018

- Added *pylops.CausalIntegration* operator

3.13.25 Version 1.0.1

Released on: 09/12/2018

- Changed module from `lops` to `pylops` for consistency with library name (and pip install).
- Removed quickplots from utilities and `matplotlib` from requirements of *PyLops*.

3.13.26 Version 1.0.0

Released on: 04/12/2018

- First official release.

3.14 Roadmap

This roadmap is aimed at providing an high-level overview on the bug fixes, improvements and new functionality that are planned for the PyLops library.

Any of the fixes/additions mentioned in the roadmap are directly linked to a *Github Issue* that provides more details onto the reason and initial thoughts for the implementation of such a fix/addition.

Striked tasks have been completed and related github issue closed with more details regarding how this task has been carried out.

3.14.1 Library structure

- Create a child repository and python library called `geolops` (just a suggestion) where geoscience-related operators and examples are moved across, keeping the core `pylops` library very generic and multi-purpose - Issue #22.

3.14.2 Code cleaning

- Change all ``np.flatten()`` into ``np.ravel()`` - Issue #24.
- Fix all `if: return ... else: ...` statements to enforce a single return with the same number of outputs - Issue #26.
- Protected attributes and @property attributes in linear operator classes? - Issue #27.

3.14.3 Code optimization

- Investigate speed-up given by decorating `_matvec` and `_rmatvec` methods with numba `@jit` and `@stencil` decorators - Issue #23.
- Replace np.fft.* routines used in several submodules with double engine, numpy and pyFFTW - Issue #20.

3.14.4 Modules

avo

- Add possibility to choose different damping factors for each elastic parameter to invert for in *pylops.avo.prestack.PrestackInversion* - Issue #25.

basicoperators

- Create Kronecker operator - Issue #28.
- Deal with edges in FirstDerivative and SecondDerivative operators - Issue #34.

optimization

- Sparse solvers - Issue #44.

signalprocessing

- Compare performance in FTT operator of performing np.swap+np.fft.fft(..., axis=-1) versus np.fft.fft(..., axis=chosen) - Issue #33.
- Add Wavelet operator performing the wavelet transform - Issue #21.
- Fredholm1 operator applying Fredholm integrals of first kind - Issue #31.
- Fredholm2 operators applying Fredholm integrals of second kind - Issue #31.

utils

Nothing so far

waveeqprocessing

- numpy.matmul as a way to speed up integral computation (i.e., inner for loop) in MDC operator - Issue #32.
- `NMO` operator performing NMO modelling - Issue #29.
- WavefieldDecomposition operator performing acoustic wavefield separation by inversion - Issue #30.

3.15 Papers using PyLops

This section lists various conference abstracts and papers using the PyLops framework. If you publish a paper using PyLops, we'd love to hear about it!

3.16 How to cite

When using PyLops in scientific publications, please cite the following paper:

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